Solutions to Nuclear Medicine Practical Session Problems from

Joint IAEA/ICTP workshop on the Determination of Uncertainties in Radiation Dosimetry

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1. Decay correction uncertainties and errors

A. *Using the data provided, calculate the uncertainty in the activity arising from the uncertainty in the radionuclide half-life for decay correction times of 20 minutes and 30 minutes.*

This is solved using a sensitivity analysis method (a Type B evaluation method) that was shown in the lecture notes. Essentially, the idea is to find the difference in decay-corrected activity when the half-life used is replaced by the half-life plus (or minus) its uncertainty. For the 20-minute case, we want to find the ratio:

$$\frac{A'}{A} = \frac{e^{-\ln(2)*(20\ minutes)/(9.9670\ minutes)}}{e^{-\ln(2)*(20\ minutes)/(9.9670+0.0037\ minutes)}}$$

$$\frac{A'}{A} = \frac{e^{-1.390884}}{e^{-1.390368}}$$

$$\frac{A'}{A} = 0.99484$$

Subtracting this ratio from 1 gives us 0.000516 **= 0.052 %** as our answer. Repeating the procedure, using 30 minutes instead of 20 minutes as our counting time, gives us **0.077 %** as our standard uncertainty.

B. *Based on the decay data provided, calculate as a function of time the error in the activity arising from errors in the measured time of 1 minute, 5 minutes, 10 minutes, and 15 minutes. Assume the intended counting time is 20 minutes.*

This is solved the same way as the problem above, except that we now substitute the error in our counting time instead of in the half-life:

$$\frac{A'}{A} = \frac{e^{-\ln(2)*(20\ minutes)/(9.9670\ minutes)}}{e^{-\ln(2)*(20\ minutes+1\ minute)/(9.9670\ minutes)}}$$

$$\frac{A'}{A} = \frac{e^{-1.390884}}{e^{-1.460428}}$$

$$\frac{A'}{A} = 1.072020$$

Subtracting this value from 1 gives us -0.072020, which tells us that our error in the activity is **-7.2 %** (the sign is important!) because of the 1 minute error in the counting time. Following the same method, we obtain the following errors for the 5-minute, 10-minute, and 15-minute cases, respectively: -41.6 %, -100.5 %, and -183.8 %.

C. *Based on the provided decay data, calculate the time error required to give a 1 % error in the activity. Again, assume the intended counting time is 20 minutes.*

This is essentially an algebra problem using the techniques above, in which we are to find the value of *x* so that

$$\frac{A'}{A} = 1.01 = \frac{e^{-\ln(2)*(20\ minutes)/(9.9670\ minutes)}}{e^{-\ln(2)*(20\ minutes+x\ minutes)/(9.9670\ minutes)}}$$

First we simplify the right hand side:

$$\frac{e^{-\ln(2)*(20\ minutes)/(9.9670\ minutes)}}{e^{-\ln(2)*(20\ minutes+x\ minutes)/(9.9670\ minutes)}} = e^{-1.390884-(-\ln(2)*\frac{(20\ minutes+x\ minutes)}{(9,9670\ minutes)})}$$

Taking the natural logarithm of both sides, we get

$$0.00995 = -1.390884 + \ln(2) * (20\ minutes + x\ minutes)/(9.9670\ minutes)$$

Rearranging, we obtain

$$1.400834 = \ln(2) * \frac{20\ minutes + x\ minutes}{9.9670\ minutes}$$

$$20.143074\ minutes = 20\ minutes + x\ minutes$$

$$x = 0.143074$$

Therefore, an error of **0.14 minutes**, will give us an error of 1 % in the activity. Note that you could have set the first ratio to 0.01 and gotten the same answer (with a different sign).

2. ^{99m}**Tc Linearity Exercise**

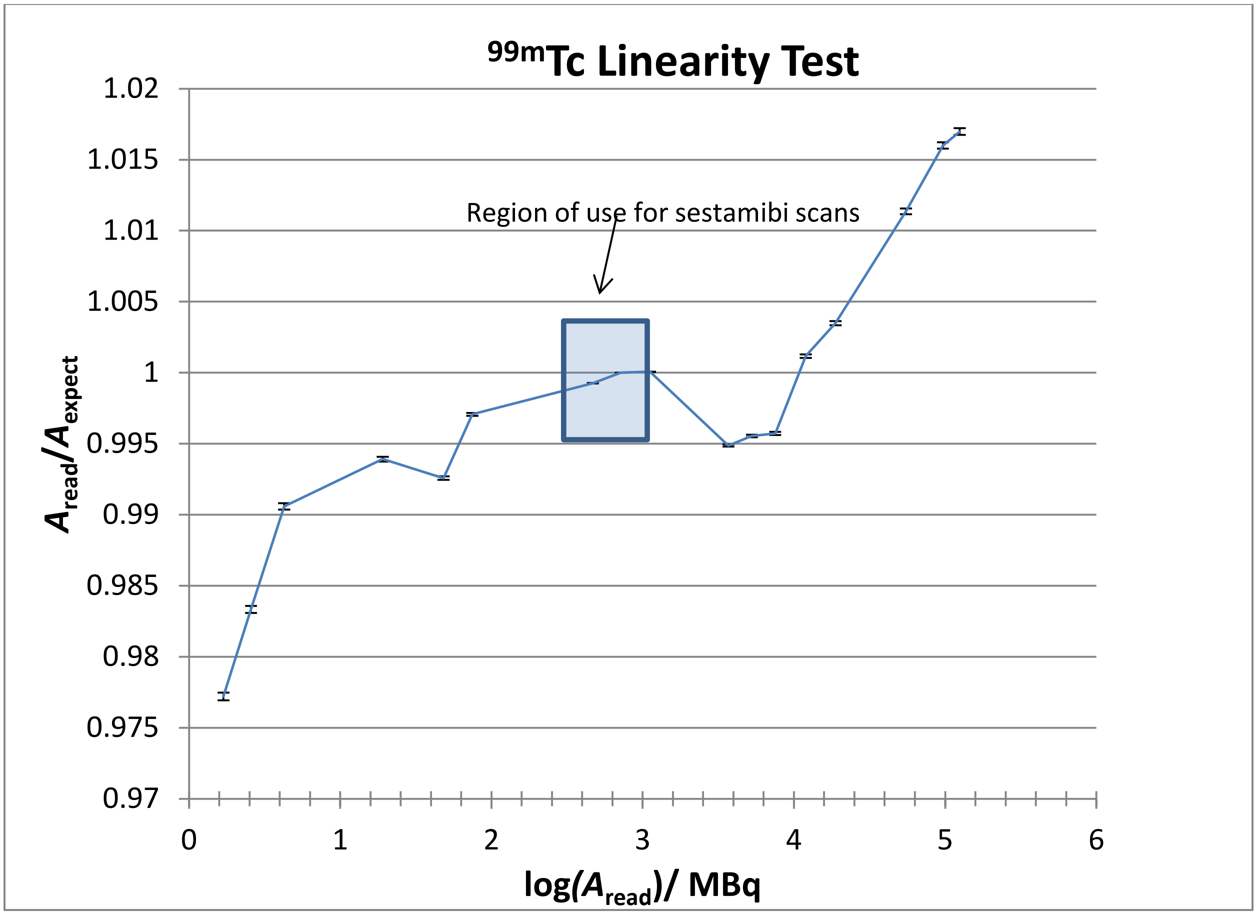
This exercise required you to take a set of ^{99m}Tc activity measurements that were made over time in order to measure the response linearity of one of the activity calibrators maintained at NIST. The student was asked to determine the linearity correction factors (and uncertainty) as a function of the activity reading. I have included a copy of my spreadsheet, along with notations, to demonstrate the calculations.

There are actually a number of ways to approach this problem. I developed this particular method because it avoids some of the difficulties in calculating the uncertainty when using the more traditional method of linearizing the function by taking logarithms.

The key to successfully completing the task was to identify a measurement in which the activity calibrator response is expected to be linear. I chose this activity to be below 1 GBq. The next step is to use the time that this measurement was taken and use it as the reference time. The activity at this time is then decay-corrected to all the other measurement times in order to calculate an "expected activity" at those times. This "expected activity" is then compared to the reading to obtain a ratio that expresses the bias in the reading due to linearity effects.

The primary uncertainty component in this exercise is the ^{99m}Tc half-life. This is calculated in a separate column, since the magnitude of the uncertainty is different for each measurement (see exercises above).

The plot of the ratio (bias) versus the $\log_{10}$ of activity, along with the (standard) uncertainty bars is given in the plot below.



99mTc Linearity Test
Region of use for sestamibi scans
Aread/Aexpect
log(Aread)/ MBq
1.02
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
0.975
0.97
0
1
2
3
4
5
6

3. ^{18}**F calibration coefficient calculation**

In this exercise, you were asked to calculate the ^{18}F calibration coefficient for an activity calibrator that uses an external electrometer. All the necessary measurement data were provided in four files and some additional information that would be needed was provided in a Word file. An example Excel spreadsheet with my usual setup for approaching this problem is also included. All these files are on the USB stick that was given to you during the workshop.

The first step is to calculate the average current reading and its standard deviation for each of the four ampoules. This will need to be done in a separate spreadsheet tab for each ampoule. Since the measurements were taken over the course of 200 s, there is actually 2 % decay that takes places between the start of the count and the end. Therefore, you will need to first decay each current reading to a common reference time. I took the end of the count time as the reference for each count.

Once you have the average current measurements for the four ampoules, these need to be background- and decay corrected. The mass of solution for each ampoule can be found from the differences in filled and empty masses for each. The calibration coefficient if then calculated from the current divided by the total activity (activity concentration*solution mass) and is expressed (after the appropriate multiplication for units) as pA/MBq.

Here are the uncertainty components that I analyzed from the data provided:

Component	Comment	u_i, %
Measurement repeatability	Average standard deviation of the mean on 100 current measurements for each of 4 ampoules. See spreadsheet for calculation	9×10^{-3}
Measurement reproducibility	Standard deviation on the independent determination of the calibration coefficient for 4 separately prepared ampoules	0.23
Mass determinations	Standard uncertainty on mass determination for any ampoule. Provided in data set.	0.1
Half-life	Standard uncertainty on calibration coefficient for a 0.02 % standard uncertainty on the ^{18}F half-life propagated over decay intervals of between 4.8 h and 6.4 h.	0.04
	$U_c = \sqrt{\sum u_i^2}$	0.25

The measurements were taken in the linear range of the activity calibrator, thus no linearity correction was necessary.

One very important piece of information that was left out of the data set that was included on the USB stick was the standard uncertainty on the ^{18}F calibrated activity. For this particular case, the activity calibration has a relative combined standard uncertainty of 0.48 %. When this is added in quadrature to the combined standard uncertainty calculated in the table above, the relative combined standard uncertainty becomes 0.54 %.

Therefore, the calibration coefficient for this chamber for ^{18}F in the geometry defined in this experiment is **10.407(56) pA/MBq**, where the calibration factor is calculated as the average value obtained from independent measurements on 4 different ampoules. The uncertainty is a standard (k=1) uncertainty calculated from the quadratic addition of the components given in the table above and an additional component of 0.48 % due to the standard uncertainty on the ^{18}F calibrated activity.

Nuclide	99mTc	
Half Life	0.250279	days
Lambda	2.768908	d-1
Lambda(unc)	2.769035	d-1

The "expected activity" is calculated by decay-correcting the activity at the reference time to the time of each measurement.

Time (EST)	Setting	Reading (MBq)	Log (Reading)	ΔT (days)	Expected (from reading cloest to tref)	R(read/expect)	$U_{half\text{-}life}$ (absolute)
6/14/2010 15:11	99mTc	124700	5.095866453	-1.857465278	122618.1988	1.016977913	0.000240016
6/14/2010 17:23	99mTc	96700	4.985426474	-1.765972222	95177.12172	1.016000466	0.000227975
6/14/2010 22:14	99mTc	55000	4.740362689	-1.563831019	54381.78092	1.011368129	0.000200962
6/15/2010 7:26	99mTc	18860	4.275541688	-1.180115741	18794.36077	1.003492496	0.000150475
6/15/2010 11:21	99mTc	11990	4.078819183	-1.017361111	11975.99538	1.001169391	0.000129423
6/15/2010 15:20	99mTc	7530	3.876794976	-0.851331019	7562.351605	0.995722018	0.000107714
6/15/2010 18:25	99mTc	5270	3.721810615	-0.722511574	5293.570673	0.995547302	9.13998E-05
6/15/2010 21:30	99mTc	3690	3.567026366	-0.594039352	3709.010884	0.994874406	7.50975E-05
6/16/2010 7:55	99mTc	1117	3.048053173	-0.160590278	1116.930839	1.000061921	2.0408E-05
6/16/2010 11:46	99mTc	716	2.854913022	0	716	1	0
6/16/2010 15:27	99mTc	468	2.670245853	0.153298611	468.3474374	0.999258163	1.94661E-05
6/17/2010 7:20	99mTc	74.7	1.873320602	0.815219907	74.91927501	0.997073183	0.000103296
6/17/2010 11:05	99mTc	48.2	1.683047038	0.97181713	48.56037606	0.992578804	0.000122584
6/17/2010 19:07	99mTc	19.11	1.281260687	1.306423611	19.2269571	0.993917025	0.000165017
6/18/2010 8:06	99mTc	4.26	0.629409599	1.847280093	4.300500637	0.990582344	0.000232559
6/18/2010 12:25	99mTc	2.57	0.409933123	2.027141204	2.613558138	0.983333779	0.000253337
6/18/2010 16:00	99mTc	1.69	0.227886705	2.176273148	1.72940933	0.97721226	0.000270284
6/21/2010 0:00	99mTc	NA					

6/16/2010 11:46

1. Assume response in this activity range is linear and use this time as the reference time.

Source	i (A)	s	Time (EST)	ΔT (h)	i Corrected (A)	s	Mass	K-value (pA/MBq)
Ampoule 1	-4.52076E-10	3.57749E-12	5/2/2012 11:49	6.1833	-4.34027E-11	3.43466E-14	4.9553	10.40984115
Ampoule 2	-4.88057E-10	4.06862E-12	5/2/2012 11:36	6.4000	-4.3163E-11	3.59823E-14	4.9387	10.38715389
Ampoule 3	-2.68490E-10	2.75883E-12	5/2/2012 13:11	4.8167	-4.32707E-11	4.44622E-14	4.9255	10.44096903
Ampoule 4	-3.41393E-10	3.69238E-12	5/2/2012 12:34	5.4333	-4.35525E-11	4.71048E-14	4.9817	10.39042802

Remember to convert to pA/MBq from A/Bq!

Calculated from mass differences using data provided.

T_ref	5/2/2012 18:00	
lambda_F	0.379017487	h^{-1}
Vinten_back	1.13780E-14	A
new_CA	8.41E+05	Bq/g

Average = 10.40709802 pA/MBq

Standard deviation = 0.024701059 pA/MBq

relative standard deviations of the mean:	
Ampoule 1	7.91346E-05
Ampoule 2	8.33636E-05
Ampoule 3	0.000102754
Ampoule 4	0.000108156
Average =	0.009%