

**ICTP School On Medical Physics For  
Radiation Therapy:  
Dosimetry And Treatment Planning For Basic And Advanced  
Applications**  
13 - 24 April 2015  
*Miramare, Trieste, Italy*

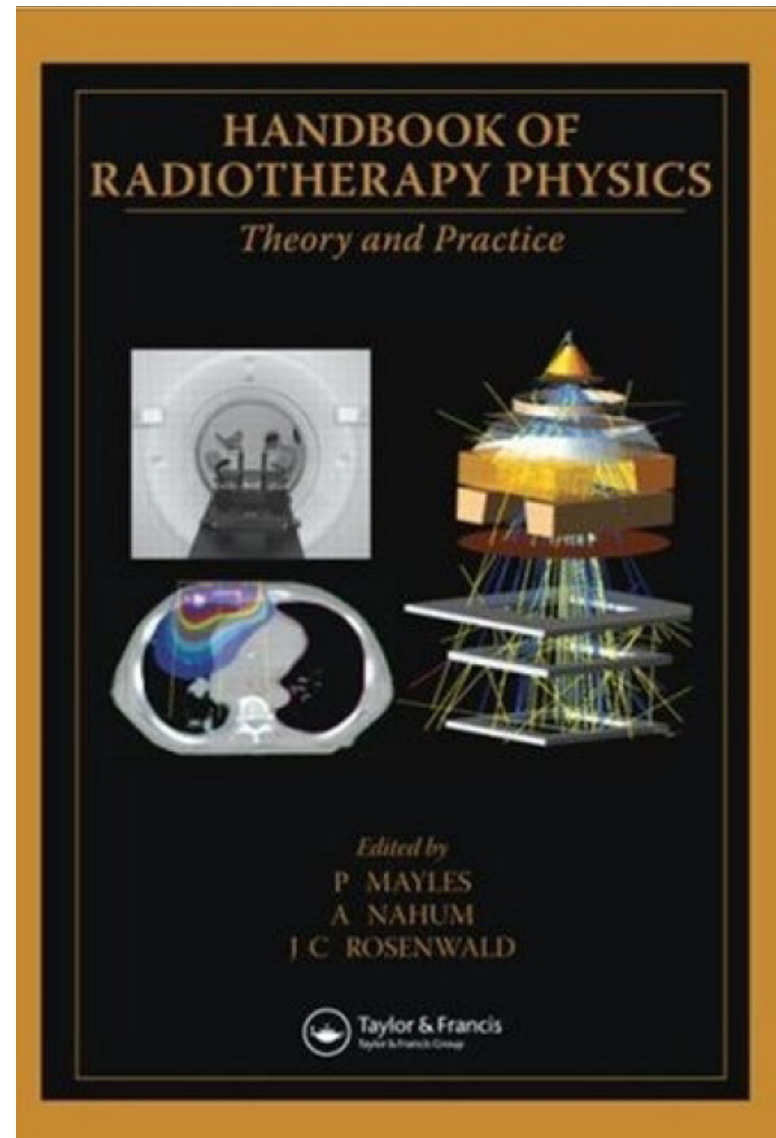
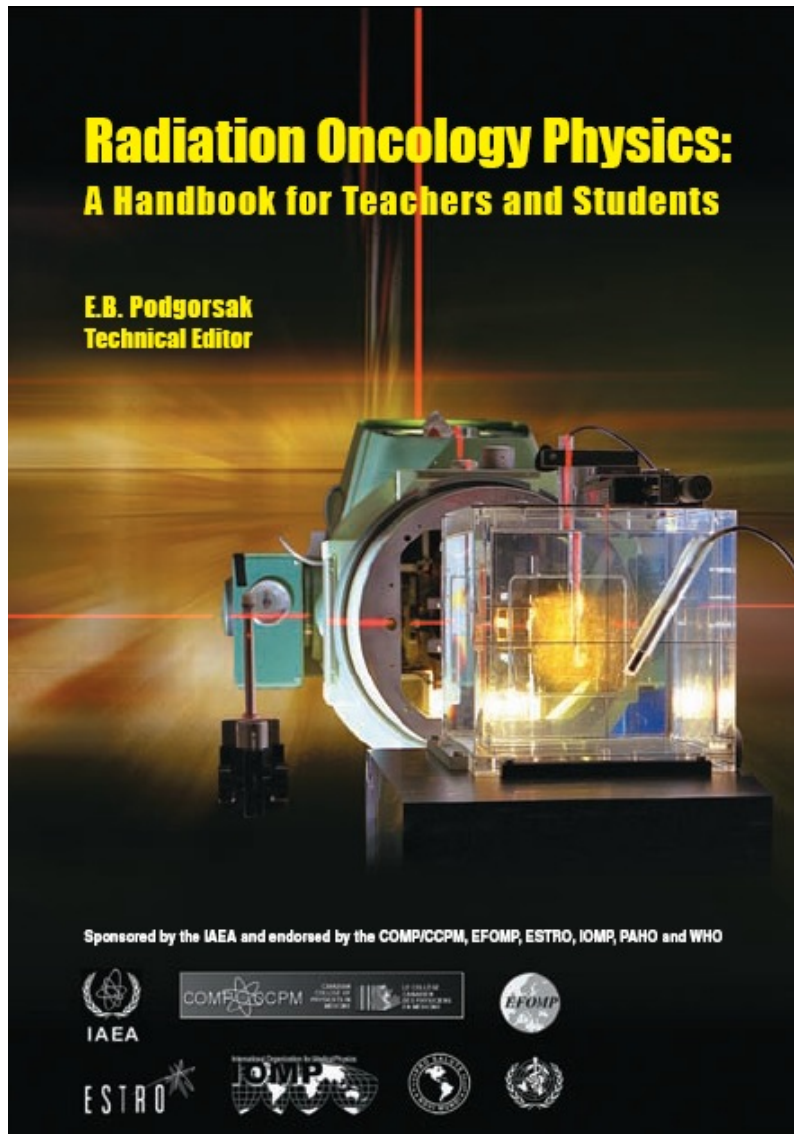
# **Dosimetry: Fundamentals**

G. Hartmann  
EFOMP & German Cancer Research Center (DKFZ)  
[g.hartmann@dkfz.de](mailto:g.hartmann@dkfz.de)

## Content:

- (1) Introduction: Definition of "radiation dose"
- (2) General methods of dose measurement
- (3) Principles of dosimetry with ionization chambers:
  - Dose in air
  - Stopping Power
  - Conversion into dose in water, Bragg Gray Conditions
  - Spencer-Attix Formulation

This lesson is partly based on:



# IAEA Website:



The image shows a screenshot of the IAEA website. At the top is a blue header with the IAEA logo and the text 'IAEA.org International Atomic Energy Agency'. To the right of the logo are links: 'Contact Us | Help | Site Index | Signup for News'. Below the header is a navigation bar with buttons: 'About IAEA', 'Our Work', 'News Centre', 'Publications', and 'Data Centre'. To the right of these buttons is a 'FONT SIZE' section with 'A + A -'.

A red arrow points to a yellow box containing a blue button labeled 'Division of Human Health'.

Below the yellow box are three columns of links:

- Links**
  - Programme Activities
  - Co-ordinated Research Projects
  - Meetings and Training Courses
  - Publications
  - Organizational Chart and Personnel
- Section Links**
  - Dosimetry Codes of Practice and Worksheets
  - Radiation Oncology Physics handbook
  - Radiation Oncology Physics Slides
  - Imaging in Radiotherapy
  - DIRAC
- Regular Programme Activities**
  - Quality Audits in Radiotherapy Dosimetry
  - Radiation Metrology Supporting the Network of SSDs
  - Dosimetry Codes of Practice and Guidelines for Radiation Measurements
  - Medical Physics Developments for Quality Assurance and Clinical Applications of Ionizing Radiation

## 1. Introduction

### Exact physical meaning of "dose of radiation"

"Dose" is a sloppy expression to describe a quantity of radiation and should be used only when the communication really knows its meaning.

A dose of radiation is correctly defined as a quantity of **absorbed dose**,  $D$ .

The most fundamental definition of dose is given in Report ICRU 85a

ICRU REPORT No. 85

#### FUNDAMENTAL QUANTITIES AND UNITS FOR IONIZING RADIATION (*Revised*)

THE INTERNATIONAL COMMISSION ON  
RADIATION UNITS AND  
MEASUREMENTS

OCTOBER 2011

Journal of the ICRU Volume 11 No 1 2011  
Published by Oxford University Press

## 1. Introduction

### Exact physical meaning of "dose of radiation"

- According to ICRU Report 85a, the absorbed dose  $D$  is defined by:

$$D = \frac{d\bar{\epsilon}}{dm}$$

where  $d\bar{\epsilon}$  is the mean energy imparted to matter of mass

$dm$  is a small element of mass

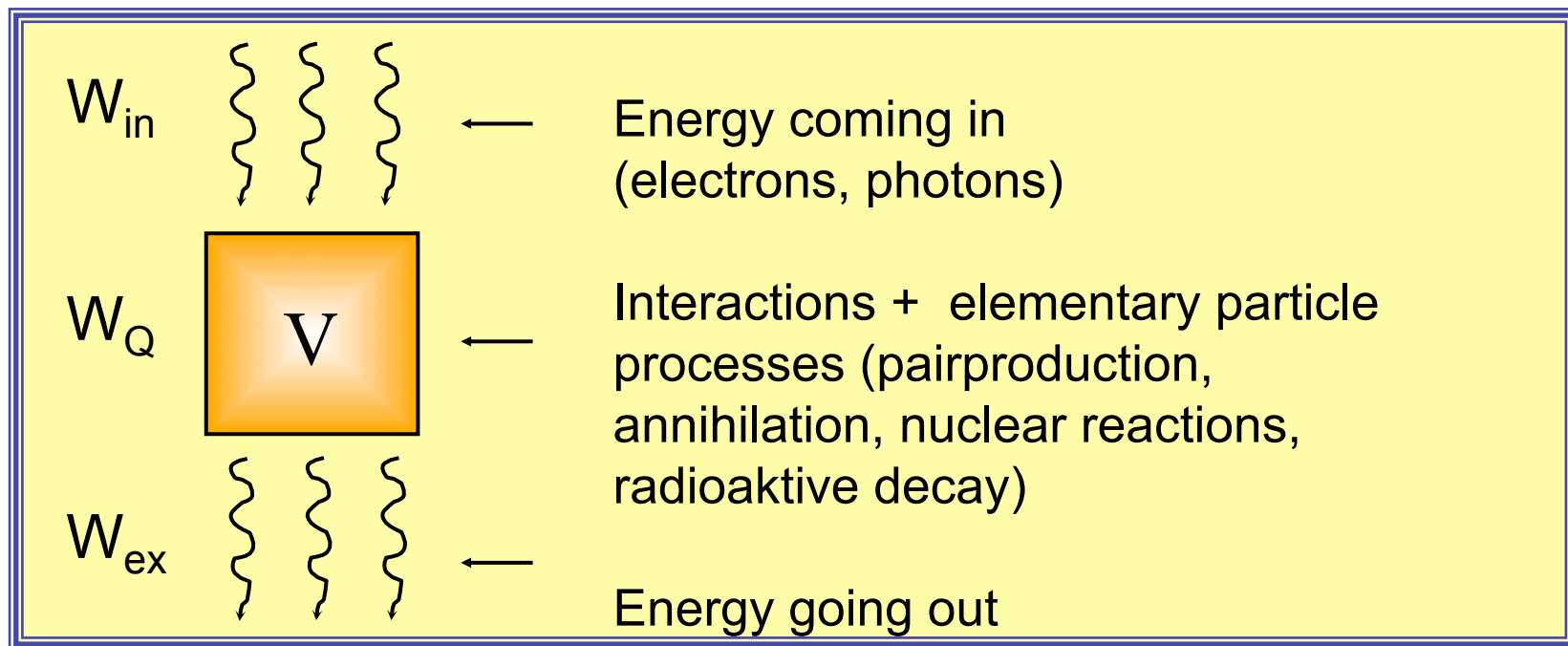
- The unit of absorbed dose is joule per kilogram (J/kg), the special name for this unit is gray (Gy).

## 1. Introduction

### Exact physical meaning of "dose of radiation"

#### □ 4 characteristics of absorbed dose:

(1) The term "**energy imparted**" can be considered to be the radiation energy absorbed in a volume:



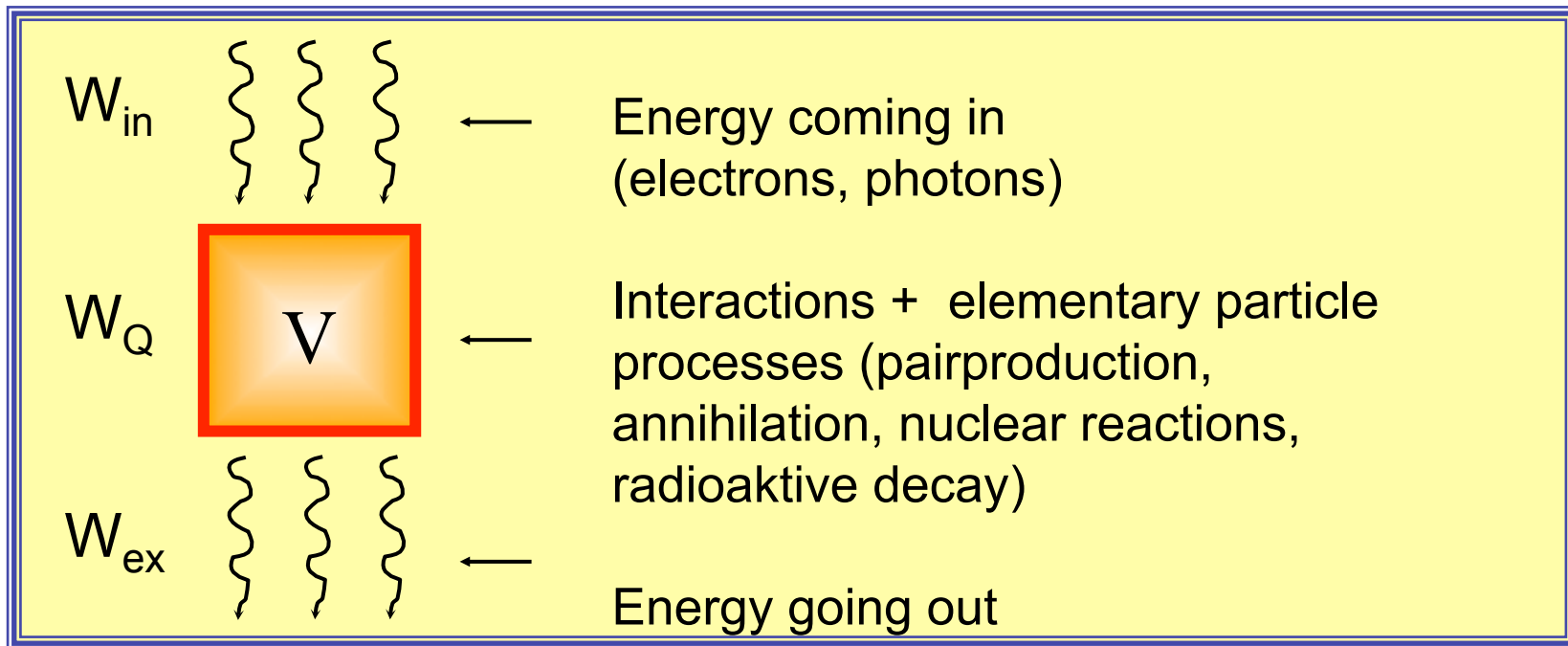
$$\text{Energy absorbed} = W_{in} - W_{ex} + W_Q$$

## 1. Introduction

### Exact physical meaning of "dose of radiation"

❑ Four characteristics of absorbed dose :

(2) The term "**absorbed dose**" refers to an exactly defined volume and only to the volume  $V$ :



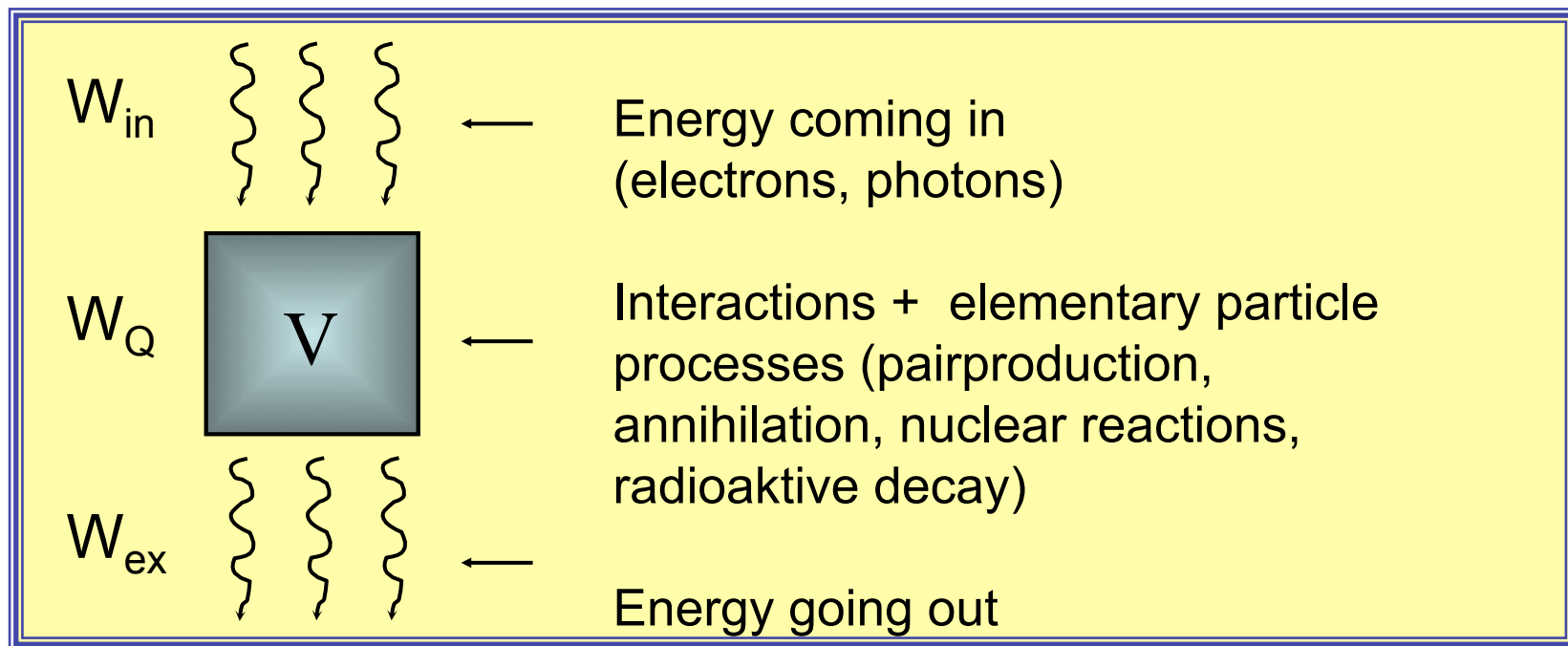


## 1. Introduction

### Exact physical meaning of "dose of radiation"

❑ Four characteristics of absorbed dose :

(3) The term "**absorbed dose**" refers to the material of the volume :



= air:  $D_{air}$



= water:  $D_{water}$

## 1. Introduction

### Exact physical meaning of "dose of radiation"

- Four characteristics of absorbed dose:
  - (4) "**absorbed dose**" is a macroscopic quantity that refers to a point  $\overset{1}{r}$  in space:

$$D = D\left(\overset{1}{r}\right)$$

This is associated with:

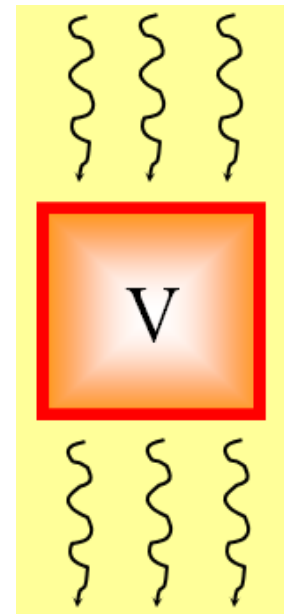
- (a)  $D$  is steadily in space and time
- (b)  $D$  can be differentiated in space and time

This last statement on absorbed dose:

"absorbed dose is a macroscopic quantity that refers to a mathematical **point in space,  $r$** "

seems to be a contradiction to:

"The term absorbed dose refers to an exactly defined **volume**"



**We need a closer look into:**

**What is happening in an irradiated volume?**

In particular, facing our initial definition:

$$D = \frac{d\bar{\varepsilon}}{dm}$$

**this question is synonym to the question, what  
energy imparted really means !!!**

## 1. Introduction

### "Absorbed dose" and "energy imparted"

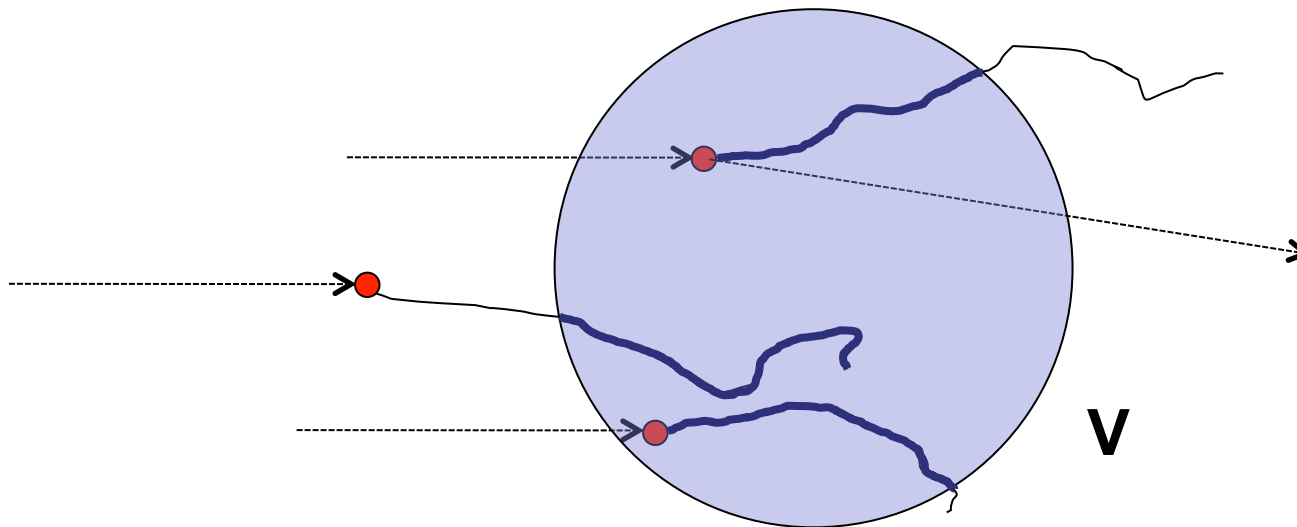
The absorbed dose  $D$  is defined by:

$$D = \frac{d\bar{\epsilon}}{dm}$$

← energy imparted

We need a definition of energy imparted  $\epsilon$  :

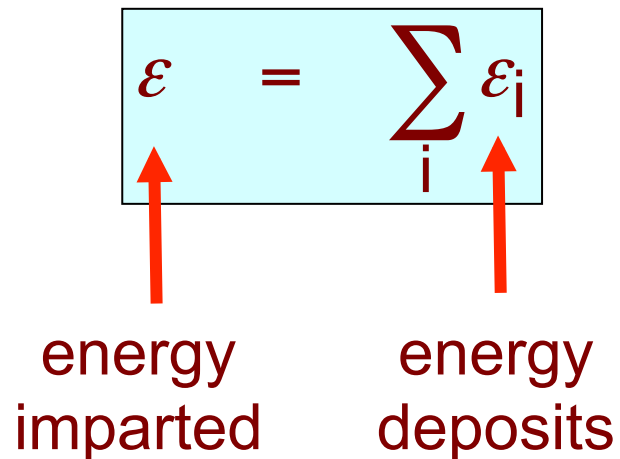
The energy imparted,  $\epsilon$ , to matter in a given volume is the sum of all **energy deposits** in that volume.



## 1. Introduction

### "Absorbed dose" and "energy imparted"

The energy imparted  $\varepsilon$  is the sum of all elemental **energy deposits** by those basic interaction processes which have occurred **in the volume** during a time interval considered:



The diagram shows the equation  $\varepsilon = \sum_i \varepsilon_i$  enclosed in a light blue rectangular box. Below the box, two red arrows point upwards. The first arrow points to the symbol  $\varepsilon$  on the left side of the equation, and the second arrow points to the symbol  $\varepsilon_i$  on the right side of the equation. Below the first arrow is the text "energy imparted", and below the second arrow is the text "energy deposits".

$$\varepsilon = \sum_i \varepsilon_i$$

energy imparted      energy deposits

## 1. Introduction

**"Absorbed dose" and "energy imparted"**

Now we need a definition of an **energy deposit** (symbol:  $\epsilon_i$ ).  
The **energy deposit** is the elemental absorption of radiation energy as

$$\epsilon_i = \epsilon_{in} - \epsilon_{out} + Q \quad \text{Unit: J}$$

**in a single interaction process.**

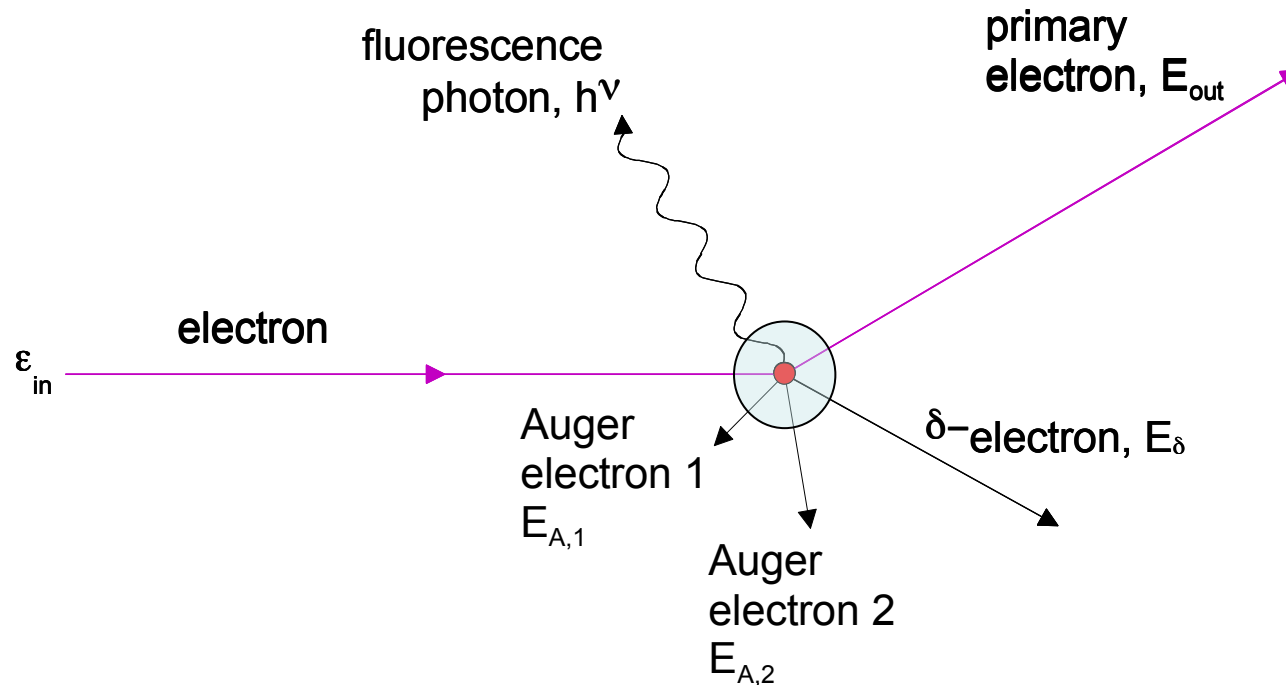
□ Three examples will be given for that:

- electron knock-on interaction
- pair production
- positron annihilation

# 1. Introduction

## "Absorbed dose" and "energy imparted"

Energy deposit  $\varepsilon_i$  by electron knock-on interaction:



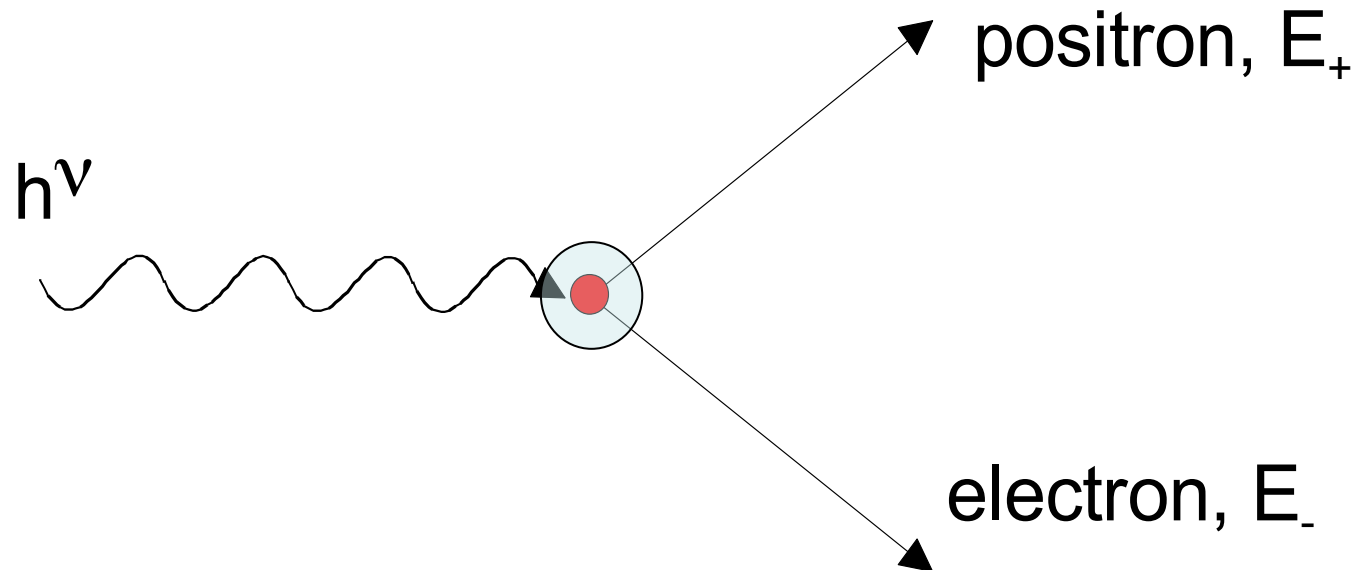
$$\varepsilon_i = \varepsilon_{in} - (E_{out} + E_{\delta} + h\nu + E_{A,1} + E_{A,2})$$



## 1. Introduction

### "Absorbed dose" and "energy imparted"

Energy deposit  $\varepsilon_i$  by pair production:



Note: **The rest energy of the positron and electron is also escaping!**

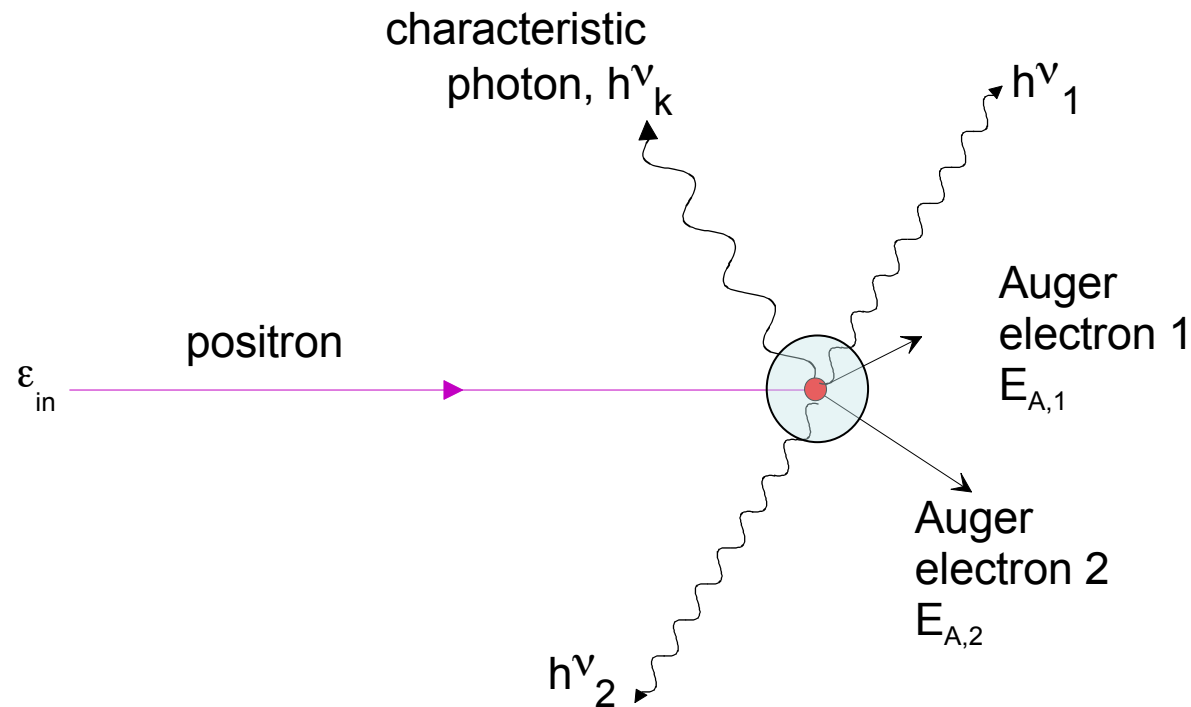
$$\varepsilon_i = h\nu - (E_+ + E_-) - 2m_0c^2$$

# 1. Introduction

## "Absorbed dose" and "energy imparted"

Energy deposit  $\varepsilon_i$  by positron annihilation:

Note: The rest energies have to be added !



$$\varepsilon_i = \varepsilon_{in} - (h\nu_1 + h\nu_2 + h\nu_k + E_{A,1} + E_{A,2}) + 2m_0c^2$$

## 1. Introduction

### Energy imparted and energy deposit

- The energy deposit  $\varepsilon_i$  is the energy deposited in a single interaction  $i$

$$\varepsilon_i = \varepsilon_{\text{in}} - \varepsilon_{\text{out}} + Q \quad \text{Unit: J}$$

where

$\varepsilon_{\text{in}}$  = the energy of the incident ionizing particle (excluding rest energy)

$\varepsilon_{\text{out}}$  = the sum of energies of all ionizing particles leaving the interaction (excluding rest energy),

$Q$  = is the change in the rest energies of the nucleus and of all particles involved in the interaction.

## 1. Introduction

### Energy imparted and energy deposit

Application to dosimetry:

A radiation detector responds to irradiation with a signal ***M*** which is basically related to the energy imparted  $\varepsilon$  in the detector volume.

$$M \sim \varepsilon = \sum_i \varepsilon_i$$

## 1. Introduction

### Stochastic of energy deposit events

By nature, a single energy deposit  $\varepsilon_i$  is a stochastic quantity.

It follows:

energy imparted is  
**also** a stochastic  
quantity:

$$\varepsilon = \sum_i \varepsilon_i$$

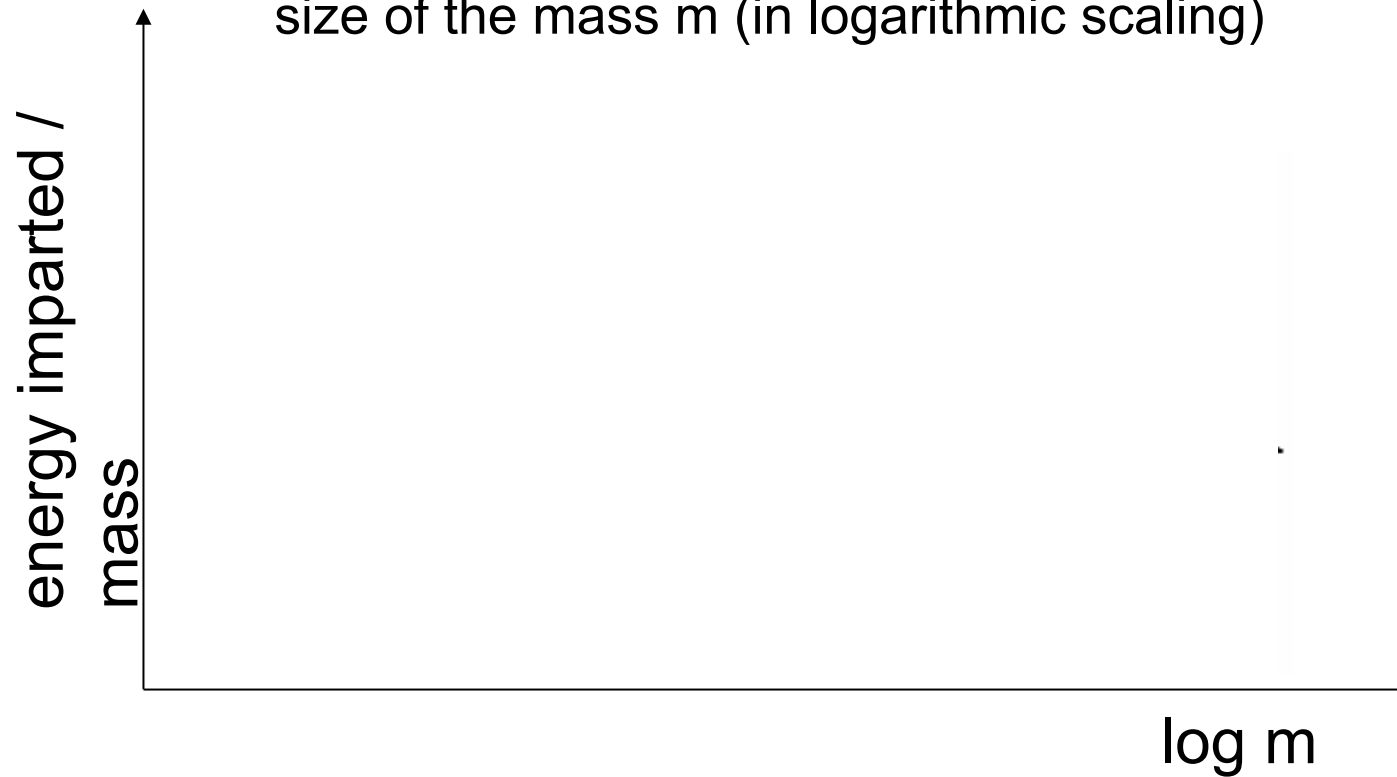
The diagram shows the equation  $\varepsilon = \sum_i \varepsilon_i$  enclosed in a light blue box. Below the box, the text "energy imparted" is positioned under the  $\varepsilon$  on the left, and "energy deposits" is positioned under the  $\varepsilon_i$  on the right. Two red arrows point upwards from these labels to the corresponding terms in the equation.

That means with respect to repeated measurements of energy imparted:

If the determination of  $\varepsilon$  is repeated, it will never will yield the same value.

As a consequence we can observe the following:

Shown below is the value of  $(\epsilon/m)$  as a function of the size of the mass  $m$  (in logarithmic scaling)

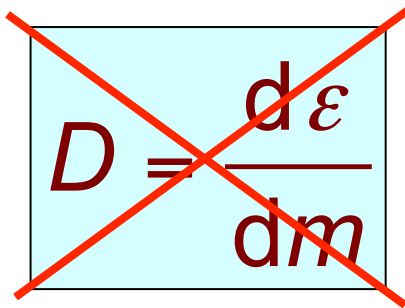


The distribution of  $(\epsilon/m)$  will be larger and larger with decreasing size of  $m$  !

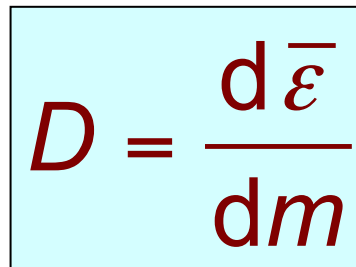
## 1. Introduction

### Exact physical meaning of "dose of radiation"

- That is the reason why the absorbed dose  $D$  is **not** defined by:


$$D = \frac{d\epsilon}{dm}$$

but by:

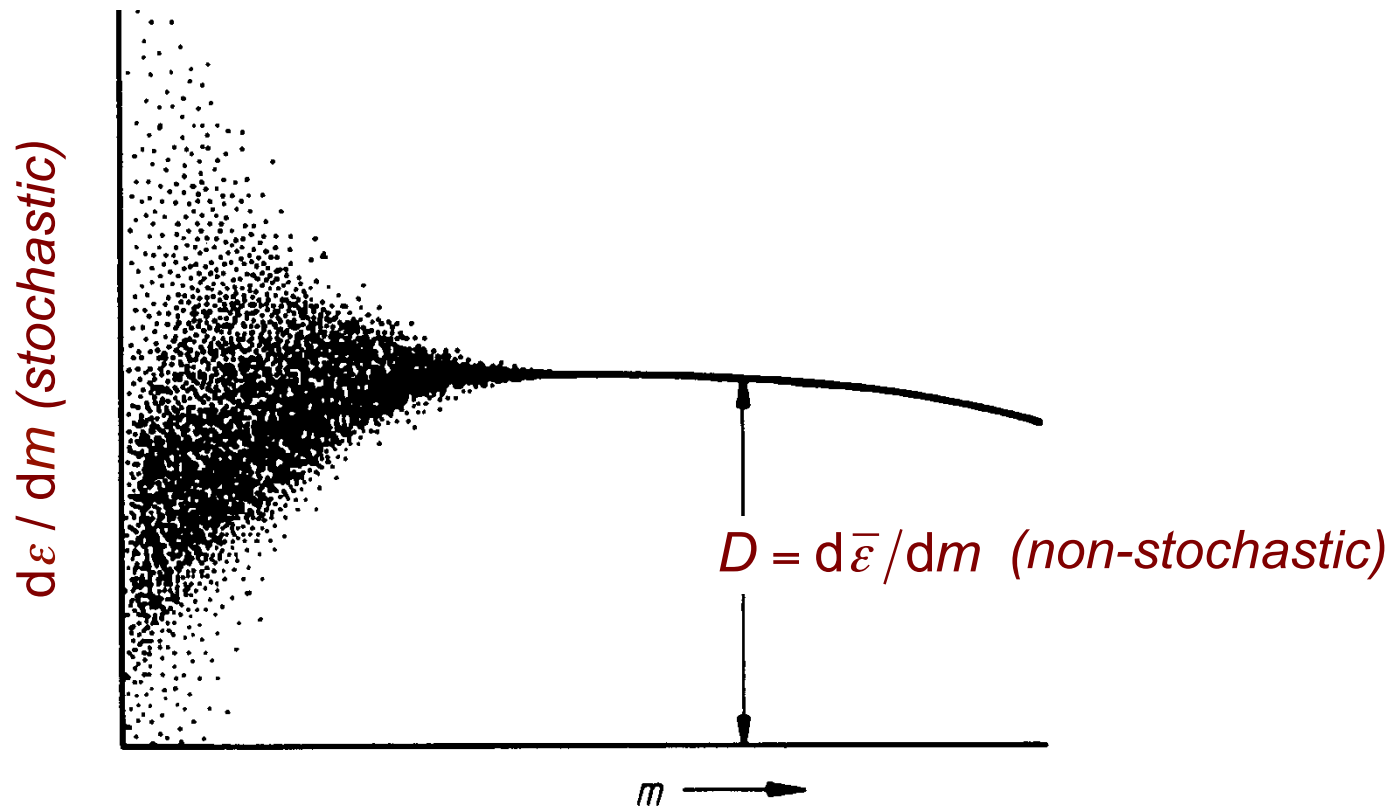

$$D = \frac{d\bar{\epsilon}}{dm}$$

where  $d\bar{\epsilon}$  is the **mean** energy imparted

$dm$  is a small element of mass

## The difference between energy imparted and absorbed dose

- ❑ The energy imparted  $\varepsilon$  is a **stochastic quantity**
- ❑ The absorbed dose  $D$  is a **non-stochastic quantity**





## 1. Introduction

### What is meant by "radiation dose"

- ❑ Often, the definition of absorbed dose is expressed in a simplified manner as:

$$D = \frac{dE}{dm}$$

- ❑ But remember:  
The correct definition of absorbed dose  $D$  as being a non-stochastic quantity is:

$$D = \frac{d\bar{\varepsilon}}{dm}$$

Now we should have a precise idea of what is meant with a dose of radiation.

However, there are also further dose quantities which are frequently used.

One important example is the **KERMA**.

The **kerma**,  $K$ , is the quotient of  $dE_{\text{tr}}$  by  $dm$ , where  $dE_{\text{tr}}$  is the sum of the initial kinetic energies of all the charged particles liberated by uncharged particles in a mass  $dm$  of material, thus

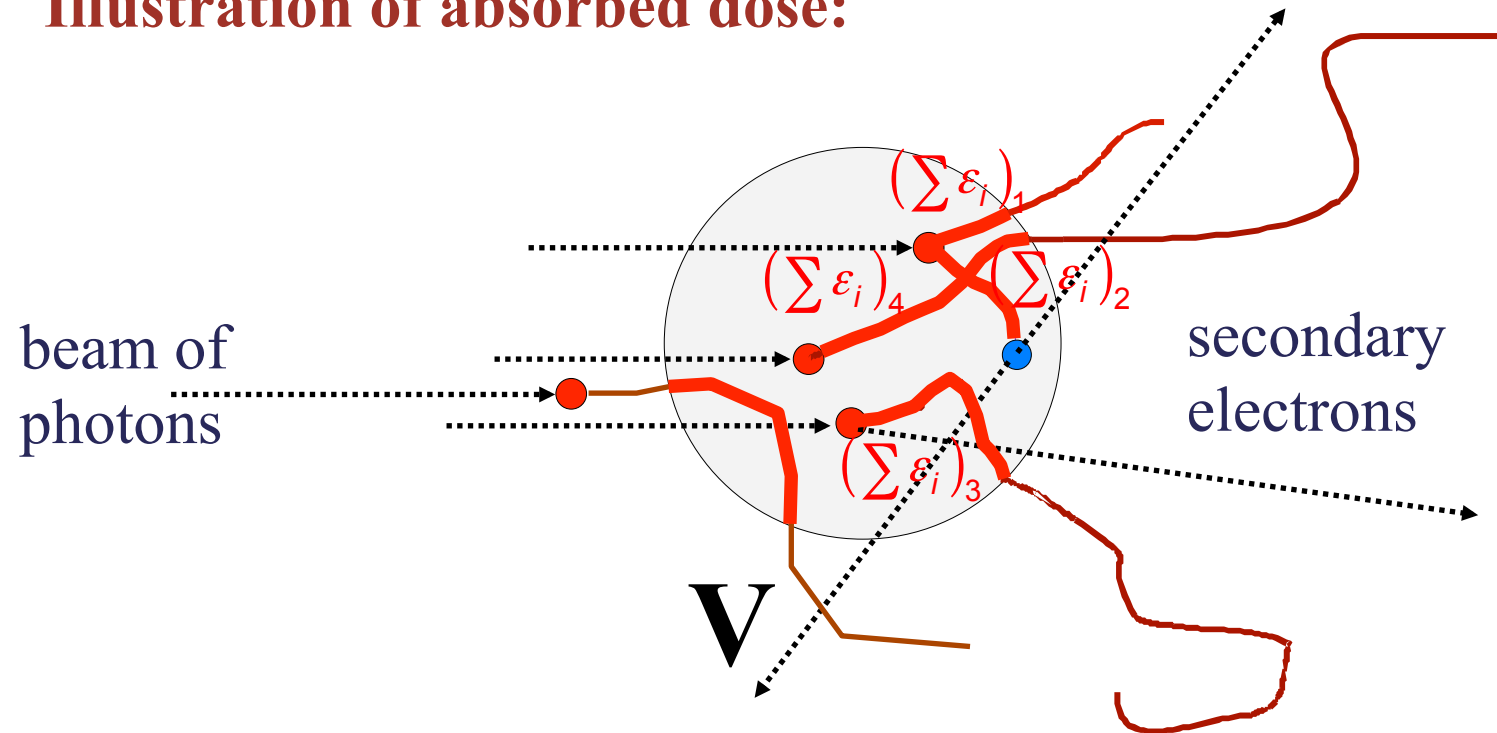
$$K = \frac{dE_{\text{tr}}}{dm} .$$

Unit:  $\text{J kg}^{-1}$

The special name for the unit of kerma is gray (Gy).

## Absorbed dose

### Illustration of absorbed dose:

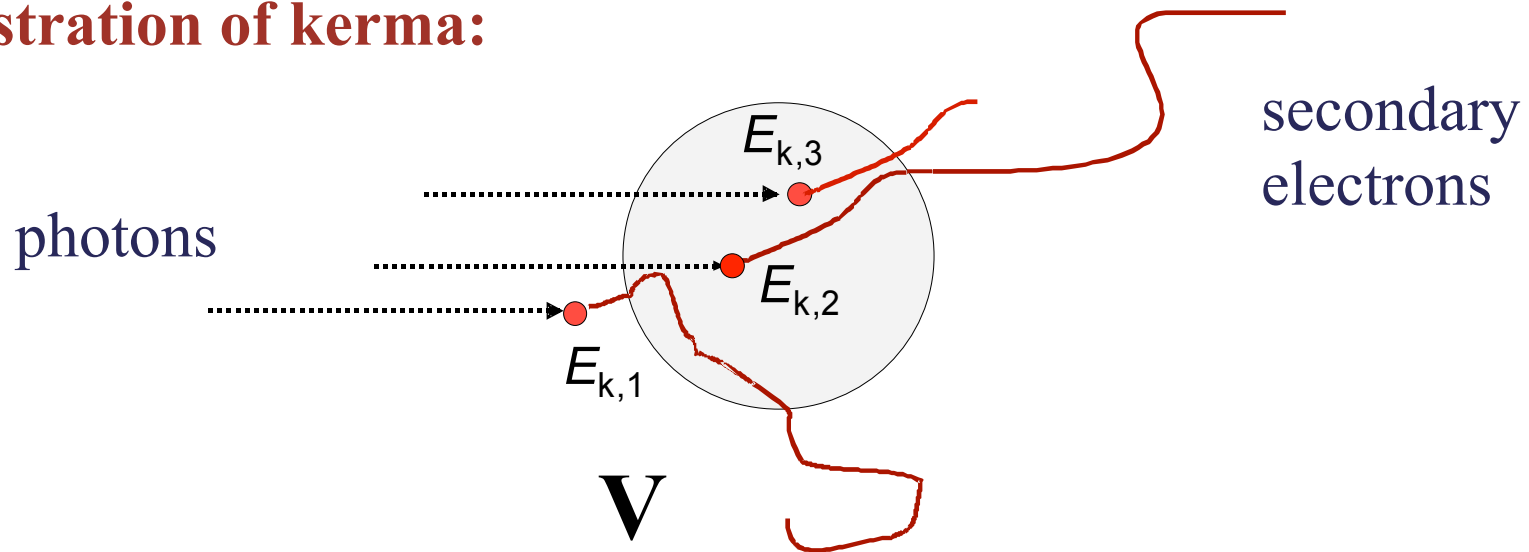


$(\sum \epsilon_i)$  is the sum of energy losts by collisions along the track of the secondary particles **within the volume V**.

$$\text{energy absorbed in the volume} = (\sum \epsilon_i)_1 + (\sum \epsilon_i)_2 + (\sum \epsilon_i)_3 + (\sum \epsilon_i)_4$$

# Kerma

## Illustration of kerma:



The collision energy transferred **within the volume** is:

$$E_{\text{tr}} = E_{k,2} + E_{k,3}$$

where  $E_k$  is the initial kinetic energy of the secondary electrons.

Note:  $E_{k,1}$  is transferred **outside the volume** and is therefore not taken into account in the definition of kerma!

Kerma, as well as the following dosimetical quantities can be calculated, if the energy fluence of photons is known:

Terma

$$\int \Phi_E \cdot \left( \frac{E\mu}{\rho} \right) \cdot dE \quad \left[ \frac{\text{J}}{\text{kg}} \right]$$

Kerma

$$\int \Phi_E \cdot \left( \frac{E\mu_{tr}}{\rho} \right) \cdot dE \quad \left[ \frac{\text{J}}{\text{kg}} \right]$$

Collision Kerma

$$\int \Phi_E \cdot \left( \frac{E\mu_{en}}{\rho} \right) \cdot dE \quad \left[ \frac{\text{J}}{\text{kg}} \right]$$

} for photons

## **A further difference between absorbed dose and KERMA**

The absorbed dose  $D$  is a quantity which is accessible mainly by a measurement

KERMA is a dosimetical quantity which cannot be measured but calculated only (based on the knowledge of photon fluence differential in energy)

## Absorbed dose from charged particle:

This requires the introduction of the concept of stopping power

### ICRU REPORT No. 85

#### 4.4 Mass Stopping Power

The *mass stopping power*,  $S/\rho$ , of a material, for charged particles of a given type and energy, is the quotient of  $dE$  by  $\rho dl$ , where  $dE$  is the mean energy lost by the charged particles in traversing a distance  $dl$  in the material of density  $\rho$ , thus

$$\frac{S}{\rho} = \frac{1}{\rho} \frac{dE}{dl}.$$

Unit:  $\text{J m}^2 \text{kg}^{-1}$

# Stopping Power and Mass Stopping Power

The mass stopping power can be expressed as a sum of independent components by

$$\frac{S}{\rho} = \frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{el}} + \frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{rad}} + \frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{nuc}}, \quad (4.4.1)$$

$\frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{el}} = \frac{1}{\rho} S_{\text{el}}$  is the *mass electronic (or collision)<sup>4</sup> stopping power* due to interactions with atomic electrons resulting in ionization or excitation.

$\frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{rad}} = \frac{1}{\rho} S_{\text{rad}}$  is the *mass radiative stopping power* due to emission of bremsstrahlung in the electric fields of atomic nuclei or atomic electrons, and

$\frac{1}{\rho} \left( \frac{dE}{dl} \right)_{\text{nuc}} = \frac{1}{\rho} S_{\text{nuc}}$  is the *mass nuclear stopping power<sup>5</sup>* due to elastic Coulomb interactions in which recoil energy is imparted to atoms.



## Stopping Power and Mass Stopping Power

Why **stopping power, i.e.** the energy lost of electrons is such an important concept in dosimetry?

Answer 1: The energy lost is at the same time the energy absorbed

Answer 2: There is a fundamental relationship between the **absorbed dose from charged particles** and the mass electronic stopping power

Absorbed dose of charged particles is approximately equal to CEMA.

Exact definition of CEMA:

(**CEMA** = **C** onverted **E** nergy per **Ma** ss)

The *cema*,  $C$ , for ionizing charged particles, is the quotient of  $dE_{\text{el}}$  by  $dm$ , where  $dE_{\text{el}}$  is the mean energy lost in electronic interactions in a mass  $dm$  of a material by the charged particles, except secondary electrons, incident on  $dm$ , thus

$$C = \frac{dE_{\text{el}}}{dm} = \int \Phi_{\text{B}}(E) \frac{S_{\text{el}}}{\rho} dE$$

Unit:  $\text{J kg}^{-1}$

The special name of the unit of cema is gray (Gy).

## Summary: Energy absorption and absorbed dose

☐ absorbed dose

$$D = \frac{d\bar{\varepsilon}}{dm}$$

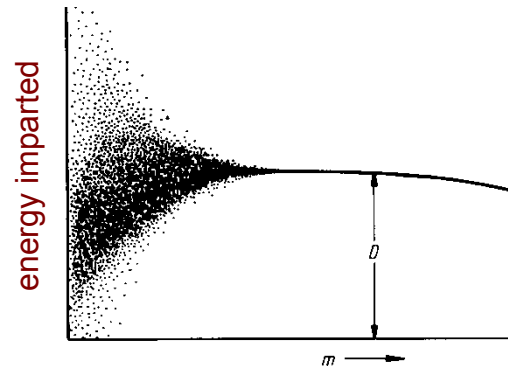
☐ energy imparted

$$\varepsilon = \sum_i \varepsilon_i$$

☐ energy deposit

$$\varepsilon_i = \varepsilon_{\text{in}} - \varepsilon_{\text{out}} + Q$$

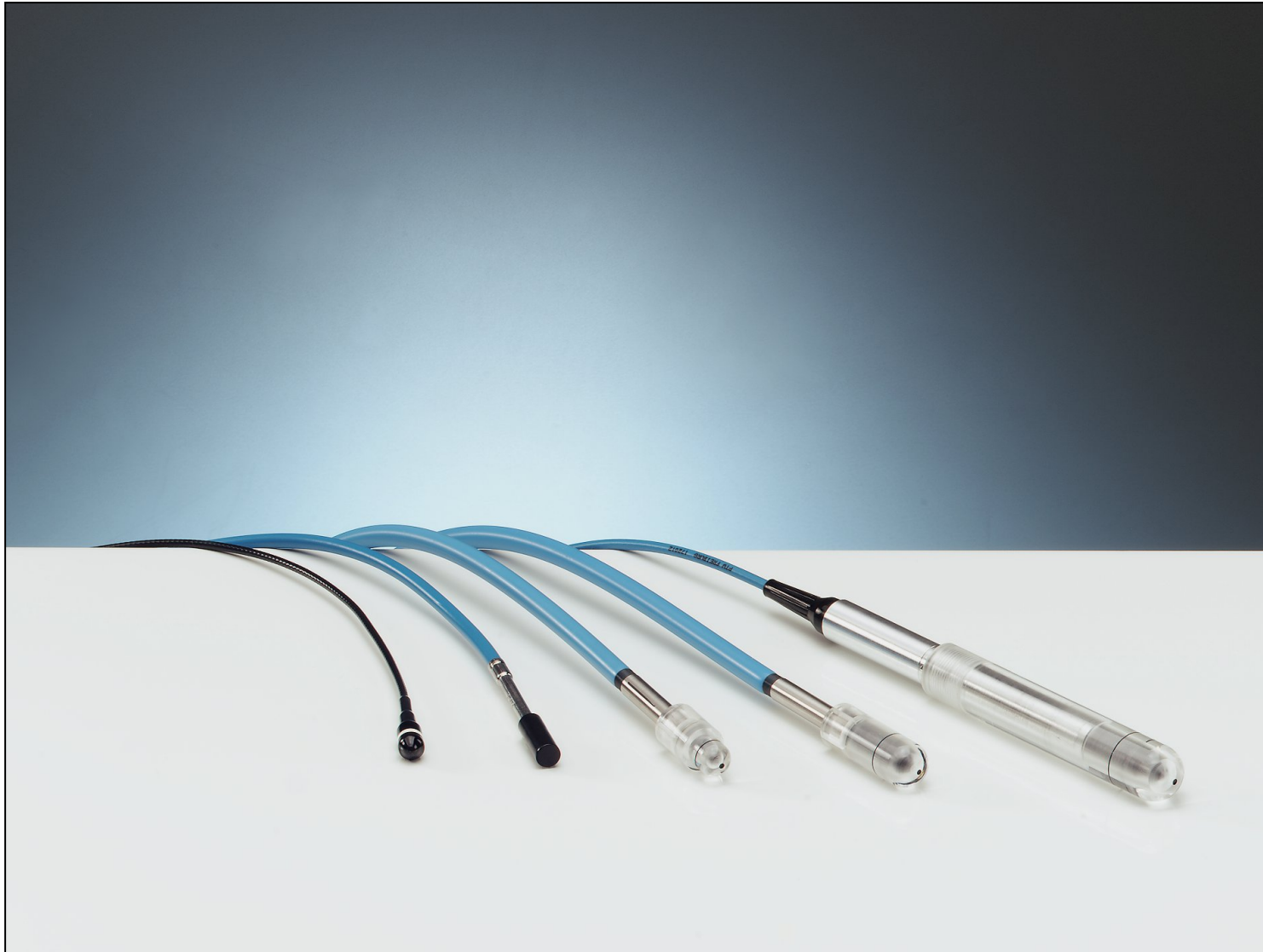
☐ stochastic character of energy absorption



## 2. General methods of dose measurement

- ❑ Absorbed dose is measured with a (radiation) **dosimeter**
- ❑ The four most commonly used radiation dosimeters are:
  - **Ionization chambers**
  - **Radiographic films**
  - **TLDs**
  - **Diodes**

## 2. General methods of dose measurement: **ionization chambers**



## 2. General methods of dose measurement: **ionization chambers**

Advantage	(small) Disadvantage
<input type="checkbox"/> Accurate and precise	<input type="checkbox"/> Connecting cables required
<input type="checkbox"/> Recommended for beam calibration	<input type="checkbox"/> High voltage supply required
<input type="checkbox"/> Necessary corrections well understood	<input type="checkbox"/> Many corrections required
<input type="checkbox"/> Instant readout	

## 2. General methods of dose measurement:

### Film

Advantage	Disadvantage
<ul style="list-style-type: none"><li><input type="checkbox"/> 2-D spatial resolution</li><li><input type="checkbox"/> Very thin: does not perturb the beam</li></ul>	<ul style="list-style-type: none"><li><input type="checkbox"/> Darkroom and processing facilities required</li><li><input type="checkbox"/> Processing difficult to control</li><li><input type="checkbox"/> Variation between films &amp; batches</li><li><input type="checkbox"/> Needs proper calibration against ionization chambers</li><li><input type="checkbox"/> Energy dependence problems</li><li><input type="checkbox"/> Cannot be used for beam calibration</li></ul>

## 2. General methods of dose measurement:

### Radiochromic film

Advantage	Disadvantage
<input type="checkbox"/> 2-D spatial resolution	<input type="checkbox"/> <del>Darkroom and processing facilities required</del>
<input type="checkbox"/> Very thin: does not perturb the beam	<input type="checkbox"/> <del>Processing difficult to control</del>
	<input type="checkbox"/> Variation between films & batches
	<input type="checkbox"/> Needs proper calibration against ionization chambers
	<input type="checkbox"/> Energy dependence problems
	<input type="checkbox"/> <del>Needs an appropriate scanner!</del>



## 2. General methods of dose measurement:

### Thermo-Luminescence-Dosimeter (TLD)

Advantage	Disadvantage
<ul style="list-style-type: none"><li><input type="checkbox"/> Small in size: point dose measurements possible</li><li><input type="checkbox"/> Many TLDs can be exposed in a single exposure</li><li><input type="checkbox"/> Available in various forms</li><li><input type="checkbox"/> Some are reasonably tissue equivalent</li><li><input type="checkbox"/> Not expensive</li></ul>	<ul style="list-style-type: none"><li><input type="checkbox"/> Signal erased during readout</li><li><input type="checkbox"/> Easy to lose reading</li><li><input type="checkbox"/> No instant readout</li><li><input type="checkbox"/> Accurate results require care</li><li><input type="checkbox"/> Readout and calibration time consuming</li><li><input type="checkbox"/> Not recommended for beam calibration</li></ul>

## 2. General methods of dose measurement:

### Diode

Advantage	Disadvantage
<ul style="list-style-type: none"><li>❑ Small size</li><li>❑ High sensitivity</li><li>❑ Instant readout</li><li>❑ No external bias voltage</li><li>❑ Simple instrumentation</li><li>❑ Good to measure relative distributions!</li></ul>	<ul style="list-style-type: none"><li>❑ Requires connecting cables</li><li>❑ Variability of calibration with temperature</li><li>❑ Change in sensitivity with accumulated dose</li><li>❑ Special care needed to ensure constancy of response</li><li>❑ Should not be used for beam calibration</li></ul>

## 2. General methods of dose measurement:

The remaining lesson is  
exclusively dedicated to the  
determination of "**absorbed dose  
to water**"  
by ionization chambers  
in terms of Gray!

### 3. Some principles of dosimetry with ionization chambers

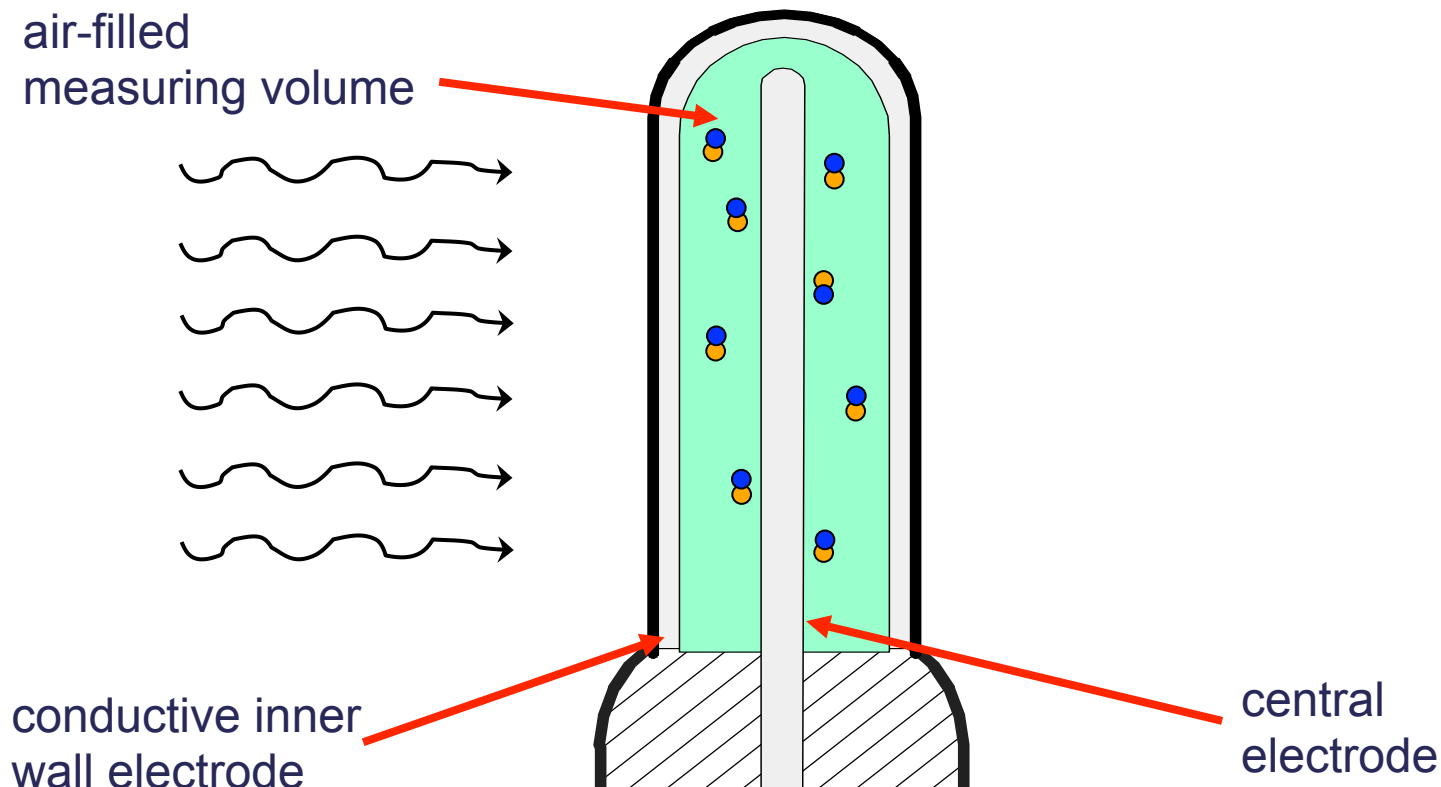
#### **Ionization**

- ❑ Measurement of absorbed dose requires the measurement of the mean energy imparted in small volume by various interaction processes.
- ❑ Such interaction processes normally result in the creation of ion pairs.

### 3. Some principles of dosimetry with ionization chambers

#### **Ionization**

- ❑ Example: Creation of charge carriers in an ionization chamber



### 3. Some principles of dosimetry with ionization chambers

#### **Ionization**

- ❑ The creation and measurement of ionization in a gas is the basis for dosimetry with ionization chambers.



- ❑ **Because of the key role that ionization chambers play in radiotherapy dosimetry, it is vital that practicing physicists have a thorough knowledge of the characteristics of ionization chambers.**

### 3. Some principles of dosimetry with ionization chambers

#### **Ionization chambers**

- ❑ The **ionization chamber** is the most practical and most widely used type of dosimeter for accurate measurement of machine output in radiotherapy.
- ❑ It may be used as an absolute or relative dosimeter.
- ❑ Its sensitive volume is usually filled with ambient air and:
  - The dose related measured quantity is charge  $Q$ ,
  - The dose rate related measured quantity is current  $I$ ,  
produced by radiation in the chamber sensitive volume.

### 3. Some principles of dosimetry with ionization chambers

#### Absorbed dose in air

- ❑ Measured charge  $Q$  and sensitive air mass  $m_{\text{air}}$  are related to absorbed dose in air  $D_{\text{air}}$  by:

$$D_{\text{air}} = \frac{Q}{m_{\text{air}}} \left( \frac{\overline{W}_{\text{air}}}{e} \right)$$
$$D = \frac{d\overline{\varepsilon}}{dm}$$

$\overline{W}_{\text{air}}/e$  is the mean energy required to produce an ion pair in air per unit charge  $e$ .



### 3. Some principles of dosimetry with ionization chambers

#### Values of $(\overline{W}_{\text{air}}/e)$

- ❑ It is generally assumed that for  $\overline{W}_{\text{air}}/e$  a constant value can be used, valid for the complete photon and electron energy range used in radiotherapy dosimetry.

- ❑  $\overline{W}_{\text{air}}/e$  depends on relative humidity of air:

- For air at relative humidity of 50%:

$$(\overline{W}_{\text{air}}/e) = 33.77 \text{ J/C}$$



- For dry air:

$$(\overline{W}_{\text{air}}/e) = 33.97 \text{ J/C}$$

### 3. Some principles of dosimetry with ionization chambers

#### Absorbed dose in water

- ❑ Thus the absorbed dose in air can be easily obtained by:

$$D_{\text{air}} = \frac{Q}{m_{\text{air}}} \left( \frac{\overline{W}_{\text{air}}}{e} \right)$$

- ❑ Next the measured absorbed dose in air of the ionization chamber  $D_{\text{air}}$  must be converted into absorbed dose in water  $D_{\text{w}}$ .
- ❑ This conversion depends on several conditions such as:
  - type and energy of radiation
  - type and volume of the ionization chamber

### 3. Some principles of dosimetry with ionization chambers

#### Absorbed dose in water

- ❑ For this conversion and for most cases of dosimetry in clinically applied radiation fields such as:
  - high energy photons ( $E > 1 \text{ MeV}$ )
  - high energy electrons

the so-called **Bragg-Gray Cavity Theory** can be applied.

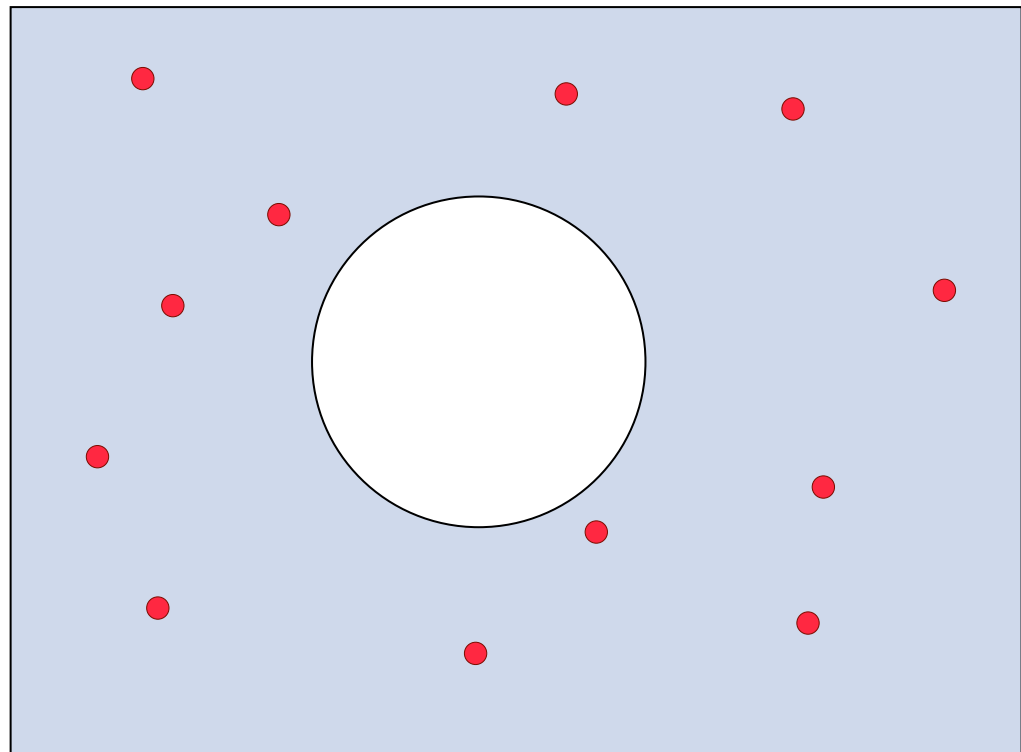
To enter the discussion of what is meant by:

## **Bragg-Gray Theory**

we start to analyze the dose absorbed in the detector and assume, that the detector is an air-filled ionization chamber in water:

The primary interactions within a radiation field of photons then are photon interactions.

- photon interaction

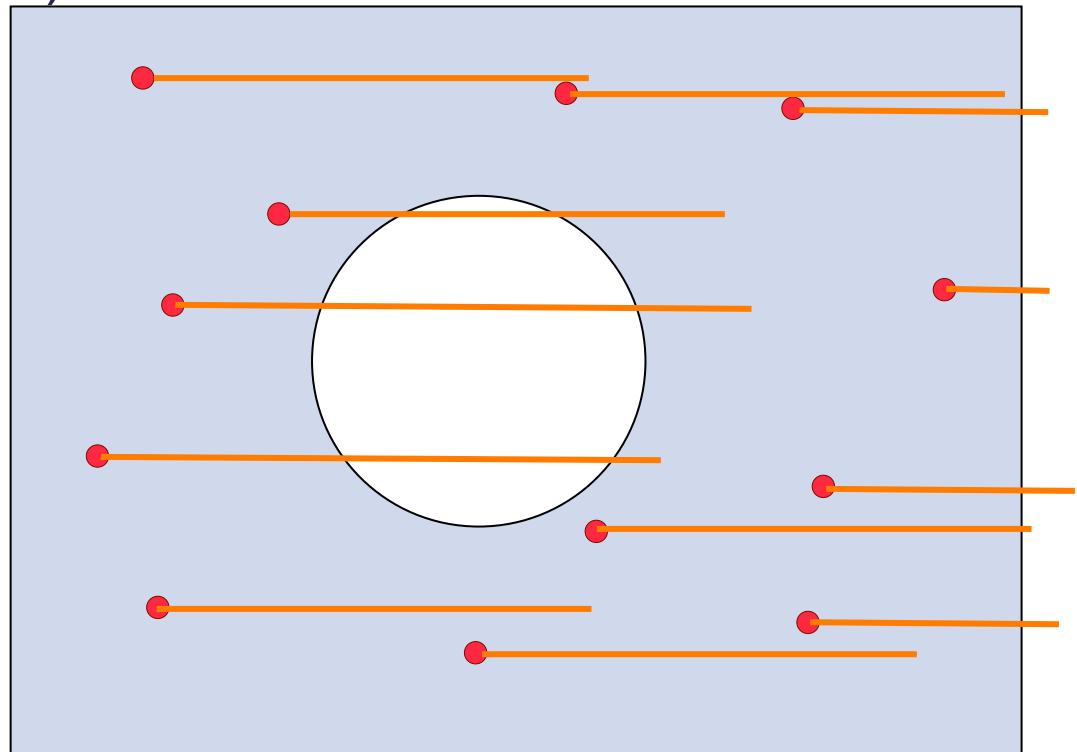


Note:

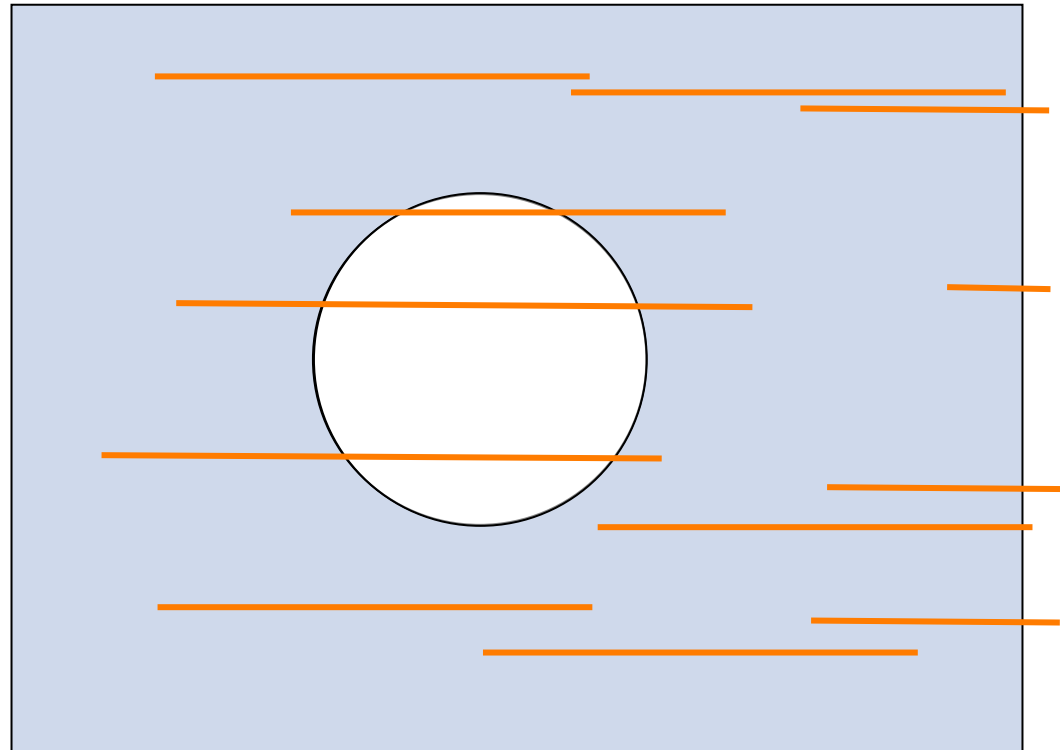
We assume that the number of interactions in the air cavity itself is negligible (due to the ratio of density between air and water)

The primary interactions of the photon radiation mainly consist of those producing secondary electrons

— electron track



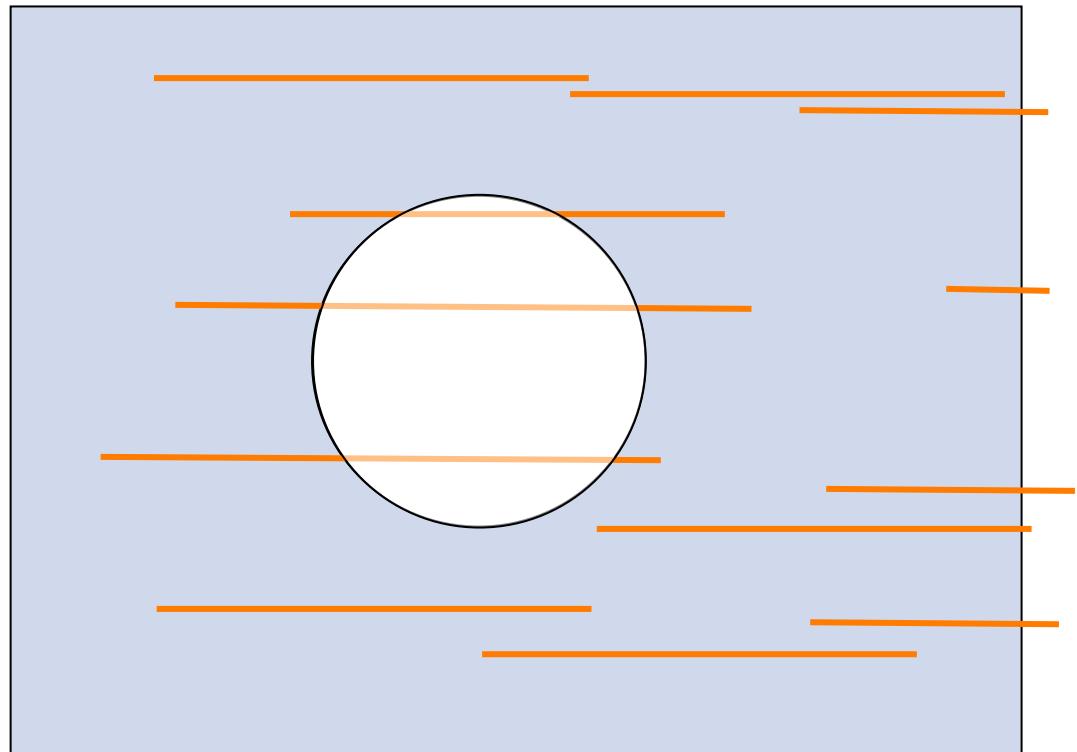
We know: Interactions of the secondary electrons in any medium are characterized by the **stopping power**.



Consequently, the types of interactions  
**within the air cavity**  
are exclusively those of electrons characterized by stopping  
power.

Absorbed dose D in the  
air can be  
calculated D as:

$$D_{air} = \int \Phi_E \cdot \left( \frac{S_{el}}{\rho} \right)_{air} \cdot dE$$



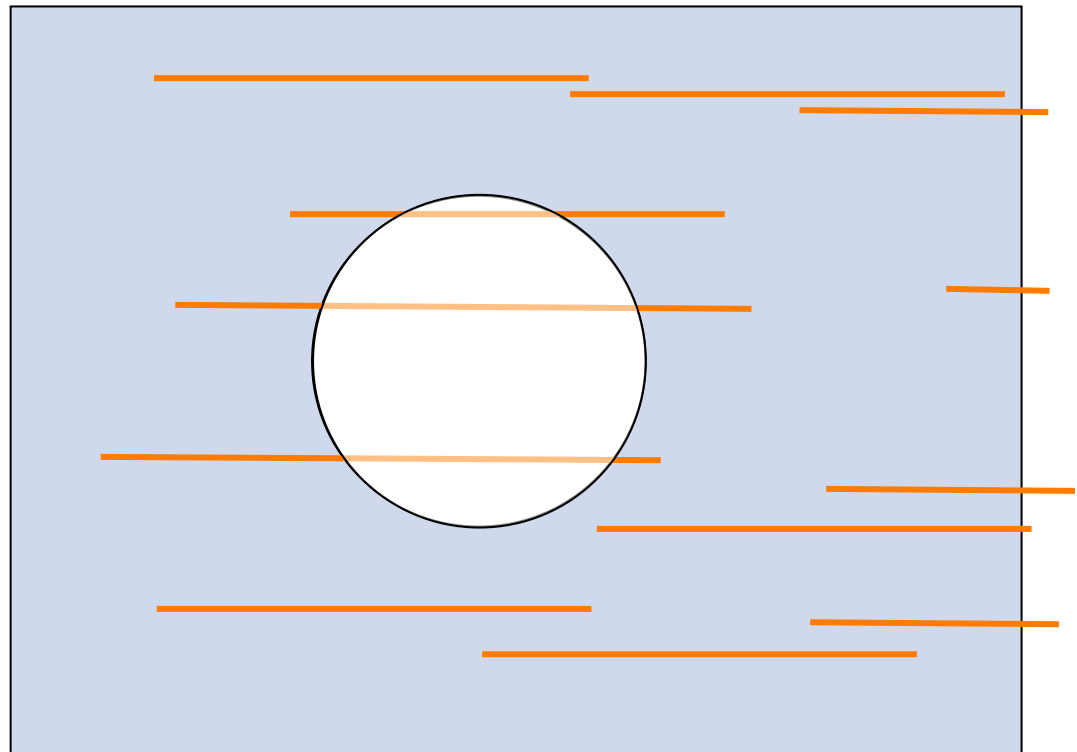
Let us further assume, that exactly the **same fluence** of the secondary electrons exists, independent from whether there is the **air cavity** or **water**.

We would have in air:

$$D_{air} = \int \Phi_E \cdot \left( \frac{S_{el}}{\rho} \right)_{air} \cdot dE$$

and we would have in water:

$$D_{water} = \int \Phi_E \cdot \left( \frac{S_{el}}{\rho} \right)_{water} \cdot dE$$





We further introduce the **mean mass stopping** power as:

$$\left( \frac{\bar{S}_{el}}{\rho} \right) = \frac{\int_0^{E_{max}} \Phi_E(E) \cdot \left( \frac{S_{el}}{\rho} \right) dE}{\int_0^{E_{max}} \Phi_E(E) dE}$$

Because of  $\int_0^{E_{max}} \Phi_E(E) dE = \Phi$  , we obtain:

absorbed dose in water

$$D_{water} = \left( \frac{\bar{S}_{el}}{\rho} \right)_{water} \times \Phi$$

absorbed dose in air

$$D_{air} = \left( \frac{\bar{S}_{el}}{\rho} \right)_{air} \times \Phi$$

□ ...and it follows from:

$$D_{water} = \left( \frac{\bar{S}_{el}}{\rho} \right)_{water} \Phi$$

$$D_{air} = \left( \frac{\bar{S}_{el}}{\rho} \right)_{air} \Phi$$

the relationship which is fundamental in dosimetry

$$D_{water} = D_{air} \cdot \frac{\left( \frac{\bar{S}_{el}}{\rho} \right)_{water}}{\left( \frac{\bar{S}_{el}}{\rho} \right)_{air}}$$

Summary of the derivation of the equation:

$$D_{water} = D_{air} \cdot \left( \frac{\bar{S}_{el}}{\rho} \right)_{water} / \left( \frac{\bar{S}_{el}}{\rho} \right)_{air}$$

This conversion formula is valid under the two conditions:

- 1) The cavity must be **small** when compared with the range of charged particles incident on it, so that its presence **does not perturb the fluence** of the electrons in the medium;
- 2) The absorbed dose in the cavity is deposited **solely by the electrons** crossing it (i.e. photon interactions in the cavity are assumed to be negligible and thus can be ignored).

## Conversion of absorbed dose

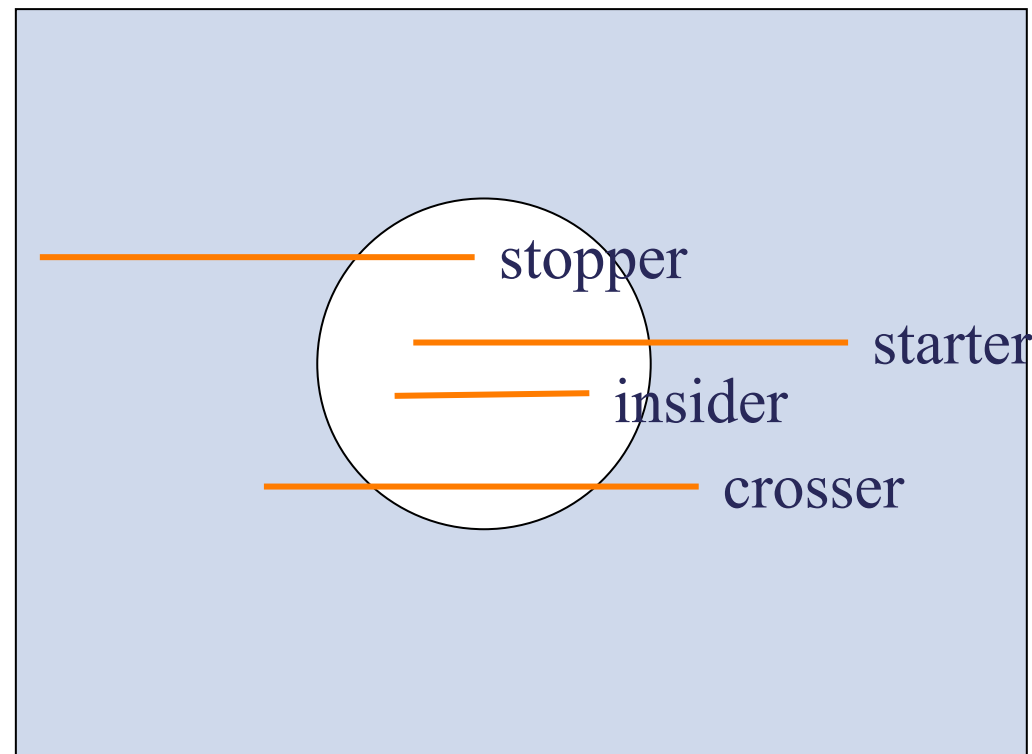
- ❑ These considerations are the essence of the Bragg-Gray theory, and the two conditions are hence called the

**two Bragg-Gray conditions.**

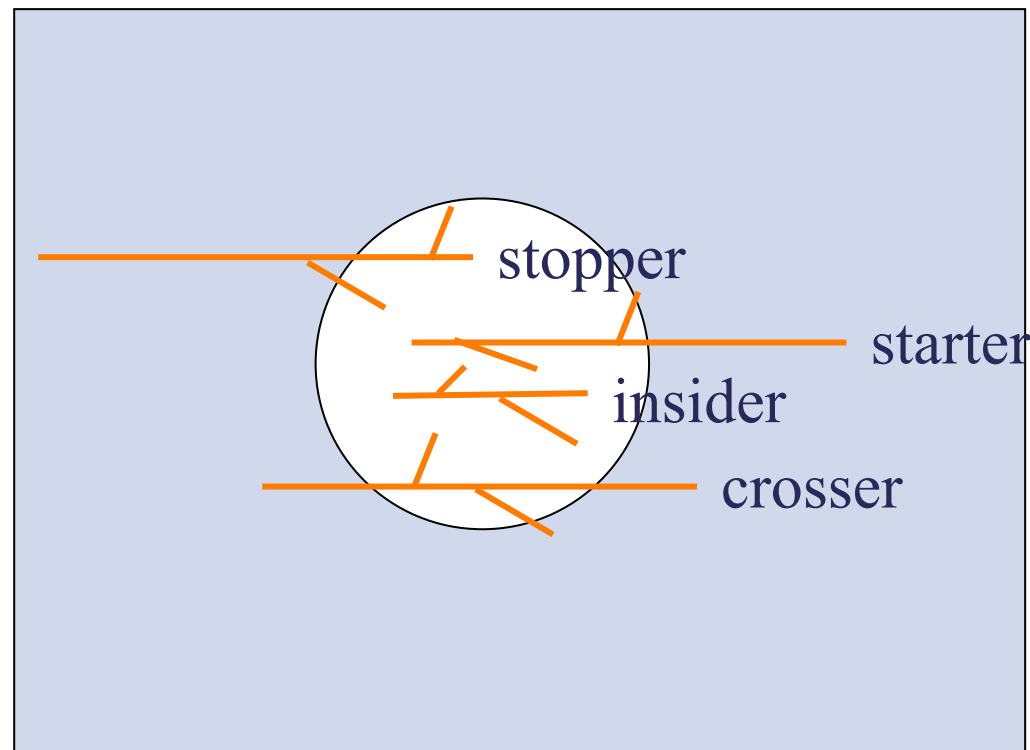
- ❑ Thus Bragg-Gray theory provides the most important mean to determine water absorbed dose from a detector measurement which is not made of water:
- ❑ If the two Bragg-Gray conditions are fulfilled, the absorbed dose in water can be obtained by the absorbed dose measured in the detector using

$$D_{\text{water}} = \frac{Q}{m_{\text{air}}} \cdot \left( \frac{\overline{W}_{\text{air}}}{e} \right) \cdot \frac{(S_{\text{el}}/\rho)_{\text{water}}}{(S_{\text{el}}/\rho)_{\text{air}}}$$

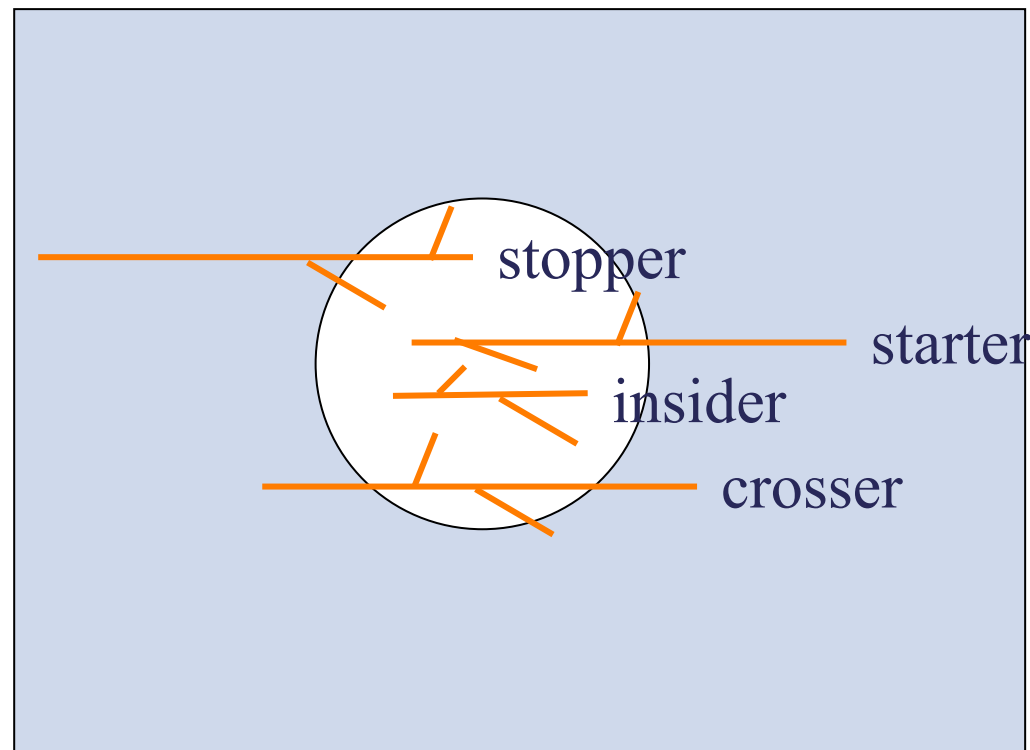
How well are the two Bragg-Gray conditions really fulfilled??  
To discuss this question, we need a closer look on the cavity and all **possible electron tracks** in the following:



In addition, the electron tracks must also include the production of so-called  $\delta$  electrons:

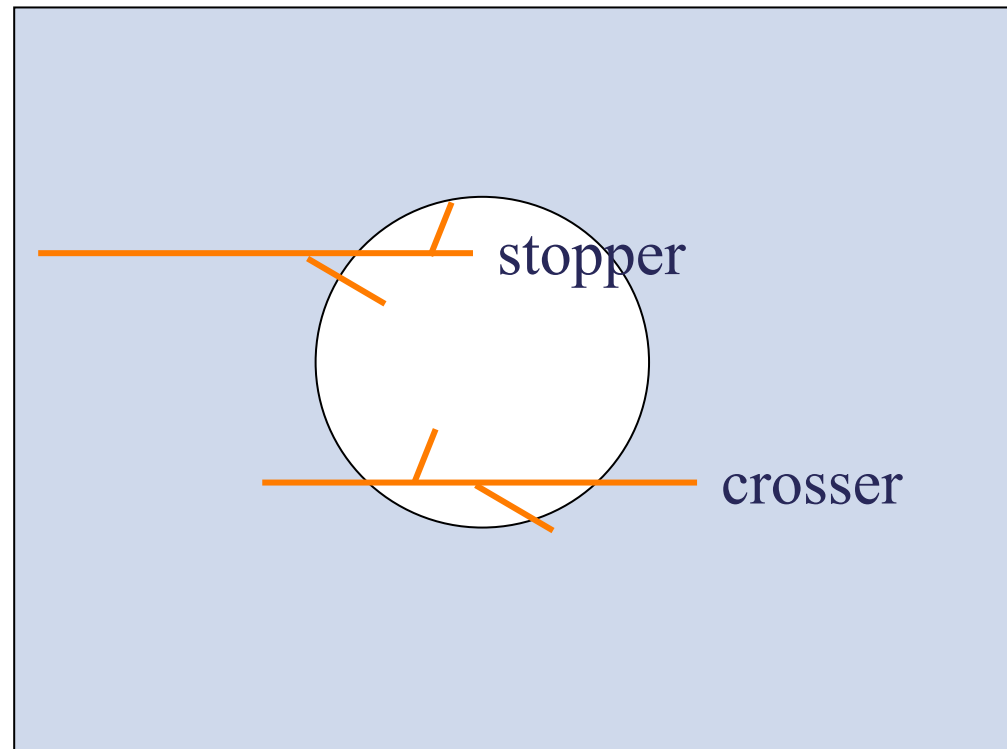


- ❑ In a very good approximation we can neglect photon interactions within the cavity.
- ❑ Thus we will neglect the starters and insiders!



In a very good approximation, also the fluence of the pure crossers and stoppers is not changed (a density change does not change the fluence!).

However, the fluence of the  $\delta$  electrons is slightly changed close to the border of the cavity (the number of  $\delta$  electrons entering and leaving the cavity is unbalanced).

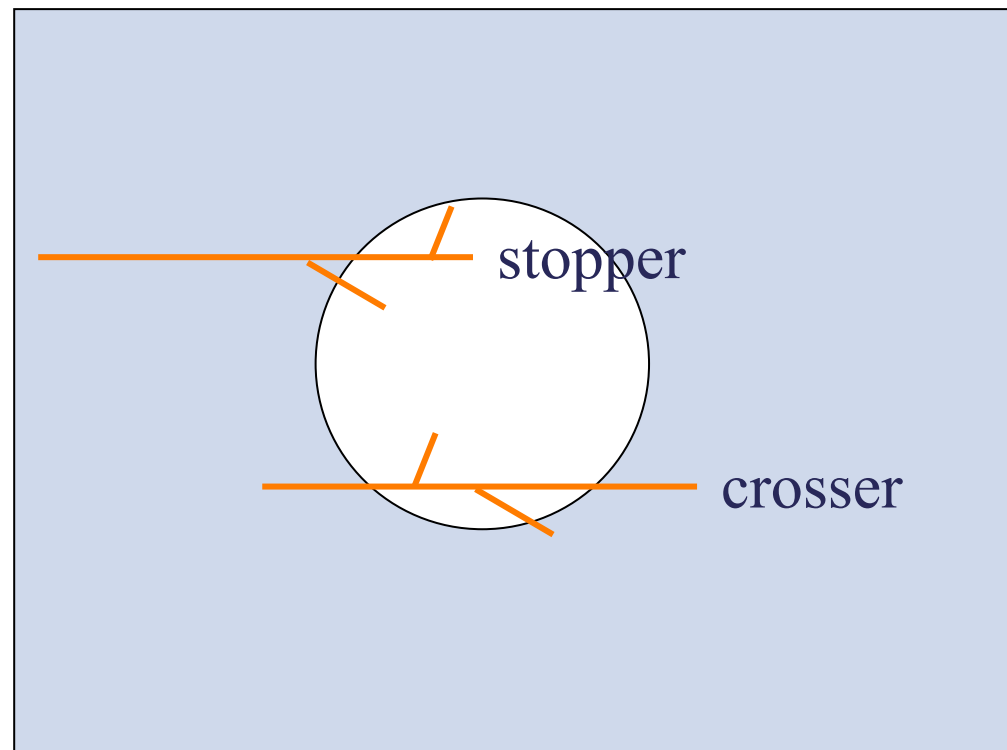




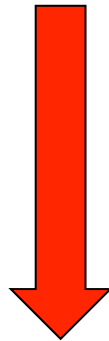
It follows:

Thus the Bragg-Gray condition, that the fluence of **all electrons** must not be disturbed, cannot be exactly fulfilled.

Hence this must be taken into account by a so-called perturbation factor when converting dose in air to that in water.

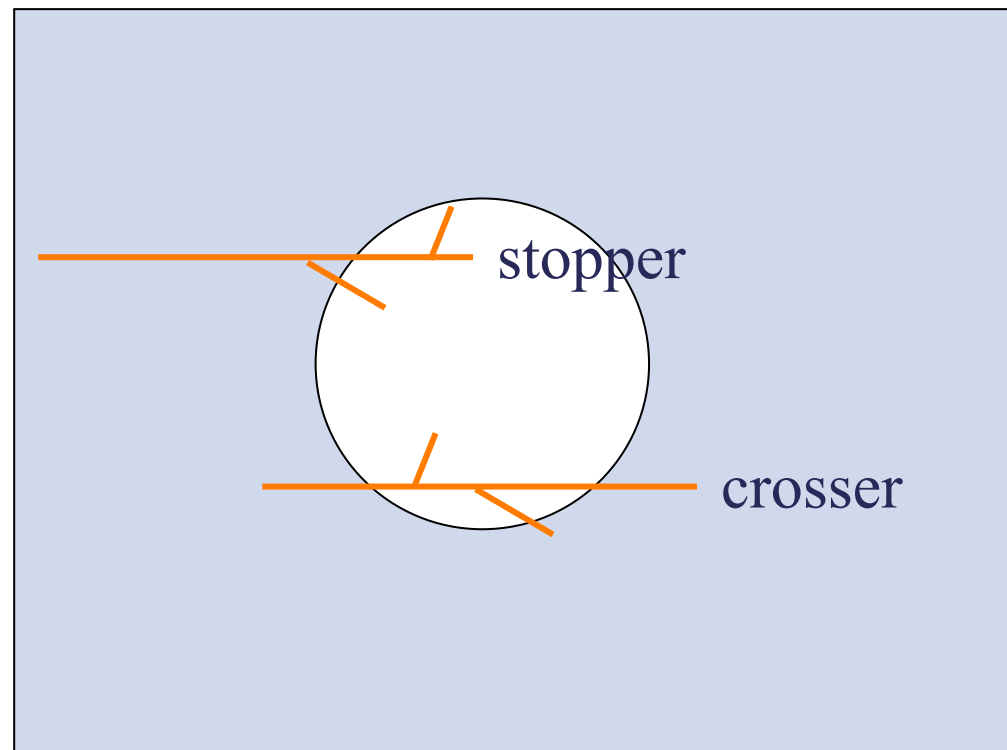


$$D_{\text{water}} = \frac{Q}{m_{\text{air}}} \cdot \left( \frac{\overline{W}_{\text{air}}}{e} \right) \cdot \frac{(s_{\text{el}}/\rho)_{\text{water}}}{(s_{\text{el}}/\rho)_{\text{air}}}$$



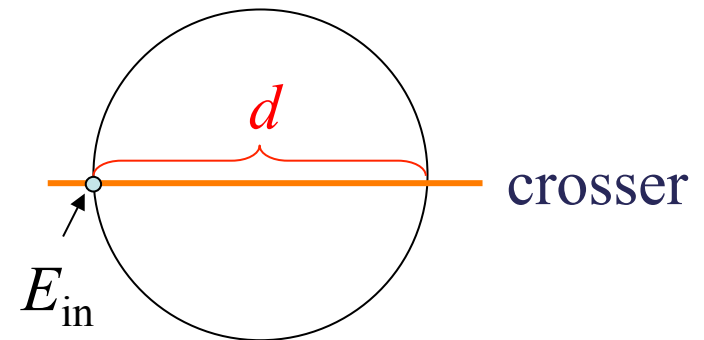
$$D_{\text{water}} = \frac{Q}{m_{\text{air}}} \cdot \left( \frac{\overline{W}_{\text{air}}}{e} \right) \cdot \frac{(s_{\text{el}}/\rho)_{\text{water}}}{(s_{\text{el}}/\rho)_{\text{air}}} \cdot p$$

- ❑ What about the stoppers ???? Do they create a problem???
- ❑ The answer is: Yes, they do!



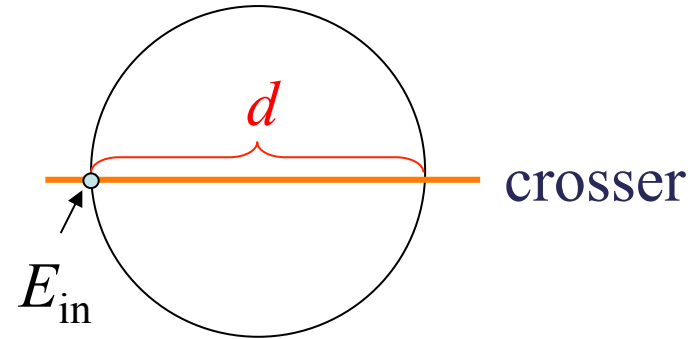
- ❑ Let us exactly analyze the process of energy absorption of a crosser:
- ❑ We assume that the energy  $E_{in}$  of the electron entering the cavity is almost not changed when moving along its track length  $d$  within the cavity.
- ❑ Then the energy imparted  $\varepsilon$  is:

$$\varepsilon = S_{el}(E_{in}) \times d$$



We compare this situation:

$$\varepsilon = S_{el}(E_{in}) \times d$$



With the energy absorption of a stopper:

$$\varepsilon = E_{in}$$



- Therefore, the calculation of absorbed dose using the stopping power according to the formula:

$$D_{air} = \int \Phi_E \cdot \left( \frac{S_{el}}{\rho} \right)_{air} \cdot dE$$

only works for crossers!

**As a consequence, the calculation of the ratio of the mean mass collision stopping power also works only for crossers**

$$S_{water,air} = \left( \frac{\bar{S}_{el}}{\rho} \right)_{water} / \left( \frac{\bar{S}_{el}}{\rho} \right)_{air}$$

**and hence needs some corrections for the stoppers!**

## Spencer-Attix stopping power ratio

- ❑ Spencer & Attix have developed a method in the calculation of the water to air stopping power ratio which explicitly takes into account the problem of the stoppers!

$$\left(\frac{\bar{S}}{\rho}\right)_{w,a}^{SA} = \frac{\int_{\Delta}^{E_{\max}} \Phi_E^{w,\delta}(E) \cdot \frac{L_{\Delta,w}(E)}{\rho} dE + \Phi_E^{w,\delta}(\Delta) \cdot \frac{S_w(\Delta)}{\rho} \cdot \Delta}{\int_{\Delta}^{E_{\max}} \Phi_E^{w,\delta}(E) \cdot \frac{L_{\Delta,\text{air}}(E)}{\rho} dE + \Phi_E^{w,\delta}(\Delta) \cdot \frac{S_{\text{air}}(\Delta)}{\rho} \cdot \Delta}$$

## Summary: Determination of Absorbed dose in water

The absorbed dose in water is obtained from the measured charge in an ionization chamber by:

$$D_{\text{water}} = \frac{Q}{m_{\text{air}}} \cdot \left( \frac{\overline{W}_{\text{air}}}{e} \right) \cdot s_{w,a}^{\text{SA}} \cdot p$$

where:

- $s_{w,air}^{\text{SA}}$  is now the water to air ratio of the mean mass **Spencer-Attix stopping power**
- $p$  is for all perturbation correction factors required to take into account deviations from BG-conditions.