# Frustration and Entanglement

ICTP international workshop on

Current Trends in Frustrated Magnetism

SPS, JNU

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HRI, Allahabad

# Outline

1. Understanding entanglement

2. Entanglement in many-body physics

3. What is frustration?

4. Characterizing "classical" frustration in q systems

5. Frustration and Entanglement
   I. Area Law
   II. Genuine multipartite entanglement

6. End remarks

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# Understanding entanglement

## LOCC paradigm in quantum info

- If the state is shared between two or more parties, the parties would only be able to act locally.

  Allowed operations: LOCC.

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- What do we mean by LOCC?

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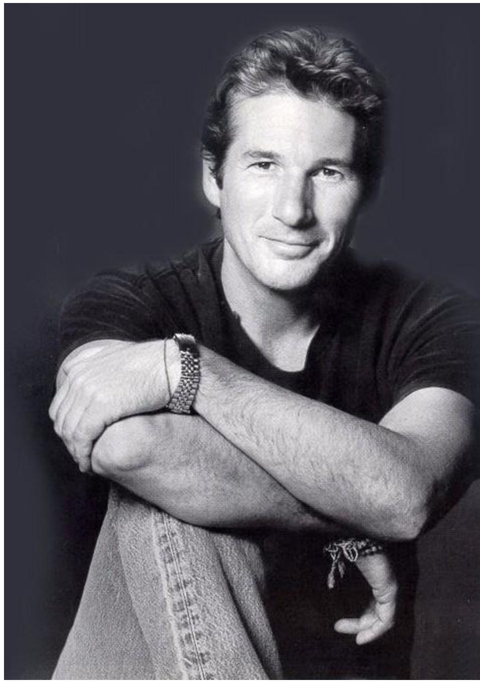
Not this!!

# Understanding entanglement

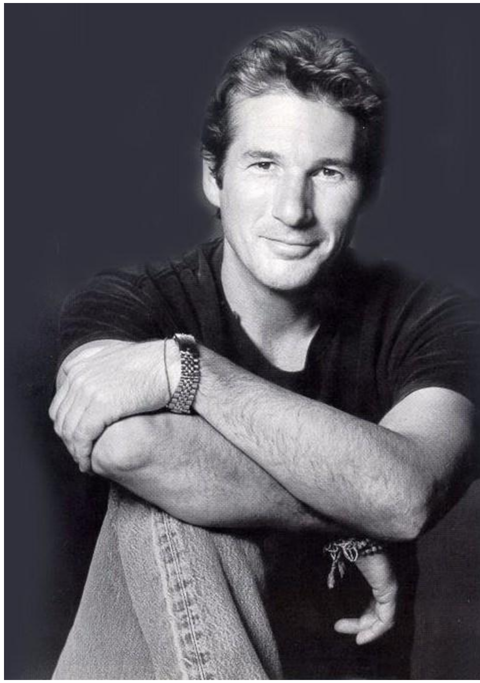
What do we mean by LOCC?



- Alice makes a measurement and communicates her result to Bob (say, by a phone call).

# Understanding entanglement

## What do we mean by LOCC?



- Alice makes a measurement and communicates her result to Bob (say, by a phone call).
- Then depending on her result, Bob will make his measurement and communicate his result to Alice.
- And so on.

# Understanding entanglement

## Separable and Entangled states

- Quantum states that can be prepared by LOCC → Separable states.

- Otherwise → Entangled states.

# Understanding entanglement

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- How do they look like?

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- Separable *pure* states: products over pure states of individual systems.

# Separable and Entangled states

- Quantum states that can be prepared by LOCC $\rightarrow$ Separable states.
- How do they look like? Mathematically?
- Separable states: mixtures of products over pure states of individual systems.

# Understanding entanglement

## Which "entanglements" can we ***compute***?

## Circa 2000

- Nielsen, Preskill, Wootters *et al.*

# Understanding entanglement

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Circa 2000

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Idea of using entanglement-like concepts
in quantum many-body phenomena was put forward.

# Understanding entanglement

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## Circa 2000

- Nielsen, Preskill, Wootters *et al.*
- Osborne and Nielsen, QIP'02, PRA'02
- Osterloh, Amico, Falci, Fazio, Nature'02

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To see the behavior of entanglement in real systems,
it is ***not*** sufficient
to understand an entanglement measure conceptually.
We must also be able to ***compute*** it
for the states of the real systems.

# Understanding entanglement

Which "entanglements" can we ***compute***?

- Bipartite states.

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- In higher dimensions, logarithmic negativity can be calculated. But it cannot detect bound entanglement.

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Logneg of a two-party state is $\log_2(2N + 1)$.

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Logneg of a two-party state is
$\log_2(2N + 1)$.

N = sum of mod of negative eigenvalues
in partial transpose of state.

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Possible in arbitrary dimensions.

# Understanding entanglement

Which "entanglements" can we ***compute***?

- Bipartite states.

This sets the stage for the

QI – many-body interface.

Possible in arbitrary dimensions.

# Understanding entanglement

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Indeed, two of the main directions of study are

1. EoF of reduced densities of spin-1/2 ground states
2. Scaling of local entropy in ground state partitions

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"Area Law"

Possible in arbitrary dimensions.

# Understanding entanglement
# Multiparty entanglement

- Many notions available.

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a. Geometric measure

Wei, Goldbart, PRA'03
Balsone, DellAnno, DeSiene, Illuminatti, PRA'08
+ ……

# Understanding entanglement
# Multiparty entanglement

- Many ***notions*** available.
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a. Geometric measure
b. Global measure

# Understanding entanglement
# Multiparty entanglement

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Meyer, Wallach, JMP'02
\+ ……

# Understanding entanglement
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A. Sen(De), US, PRA'10

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# Entanglement in many-body physics

# Quantum Phase Transitions

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- Transitions at zero temperature.

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# Entanglement in many-body physics

# Quantum Phase Transitions

- Transitions at zero temperature.
- Implying, transition not temp. driven.
- Driven by system parameter, like a magnetic field.

# Entanglement in many-body physics

# Quantum Phase Transitions

**Typical situation:**

- H = H(int) + a H(field)

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- “a” can be changed.

# Entanglement in many-body physics

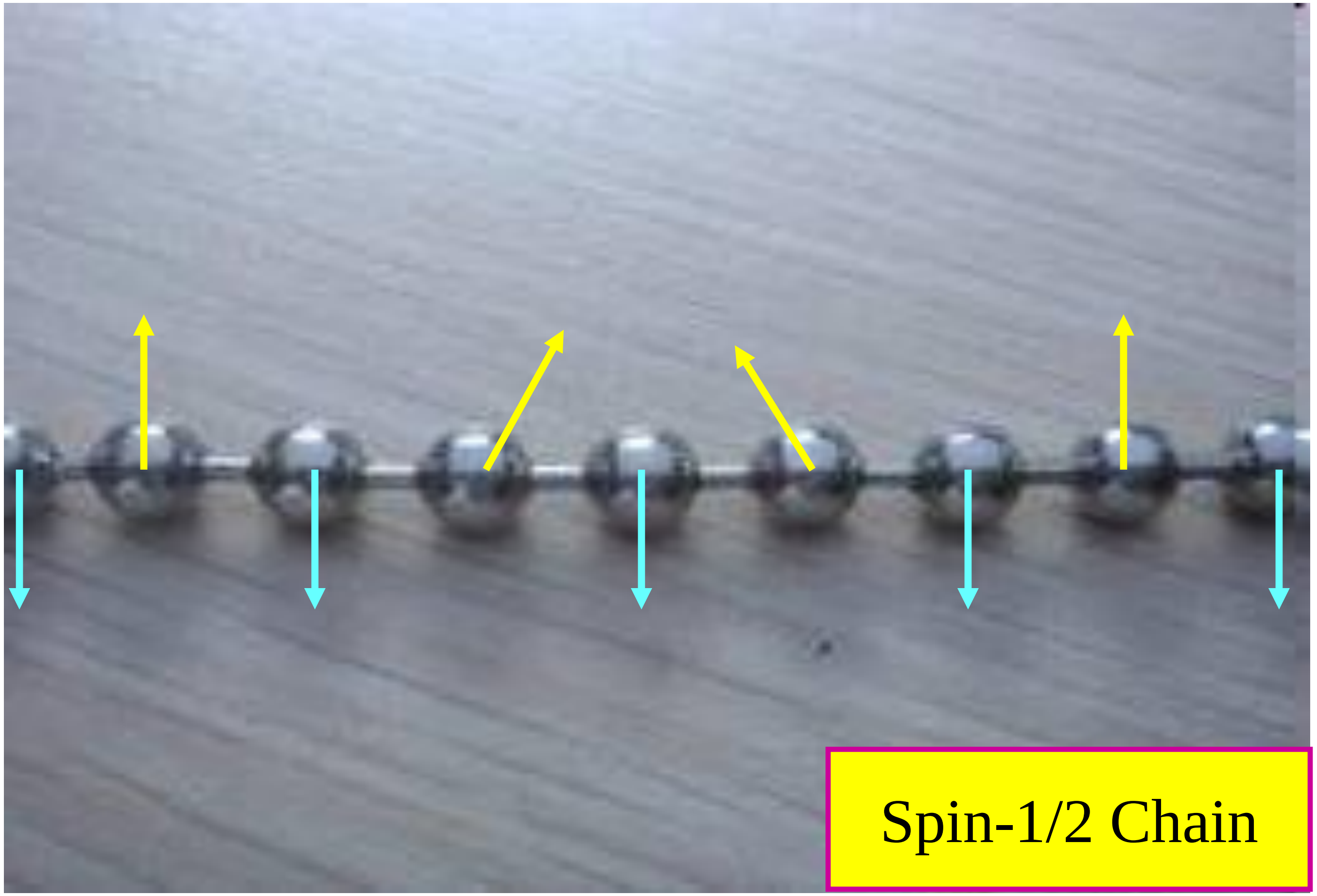
# Quantum Phase Transitions

**Typical situation:**

- H = H(int) + a H(field)
- Ground state of H ← guarantees T=0
- GS depends on “a”.
- “a” can be changed.
- Nonanalyticity appears in some physical quantity as “a” is changed.

# Entanglement in many-body physics

# Two-site densities



# Entanglement in many-body physics

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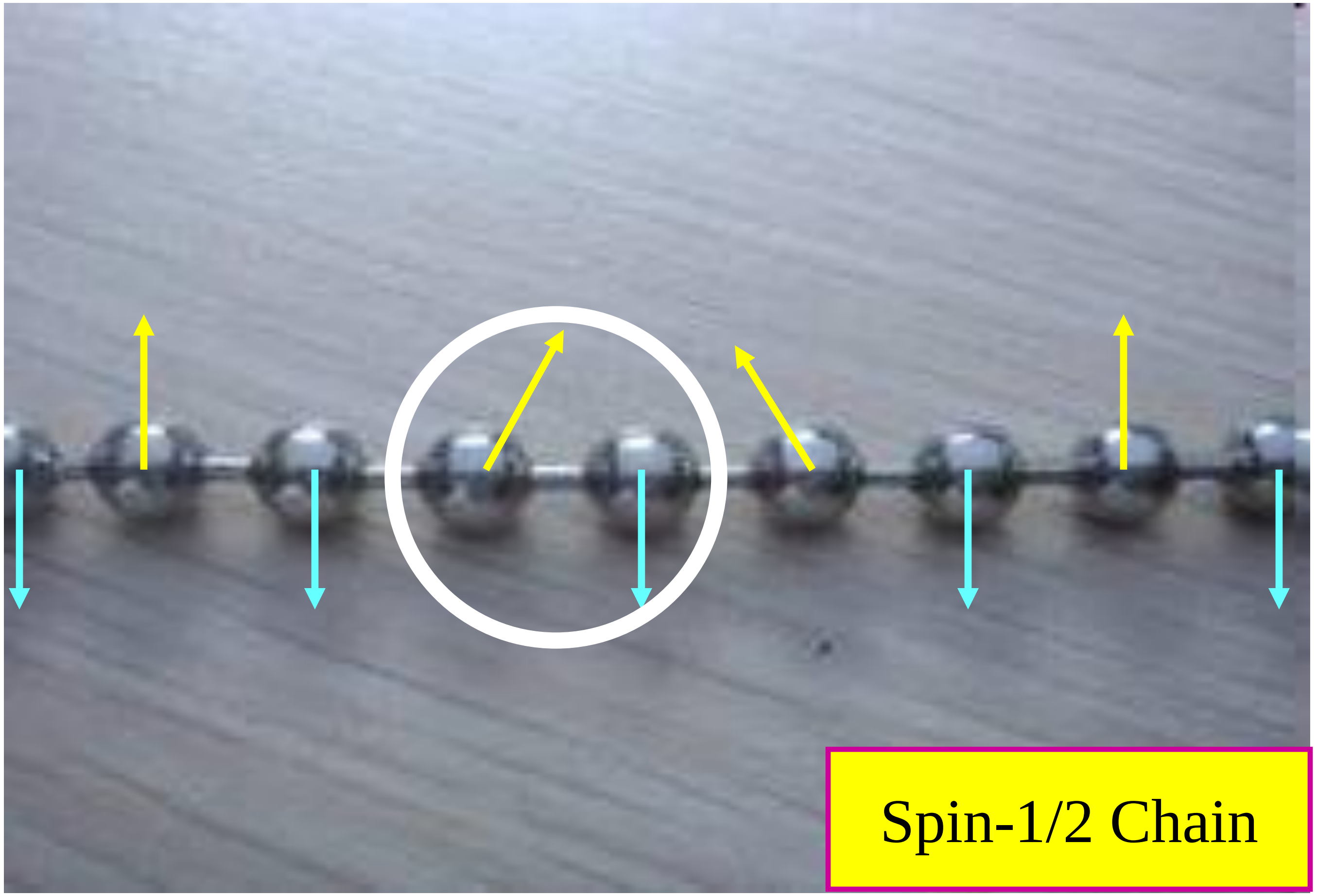

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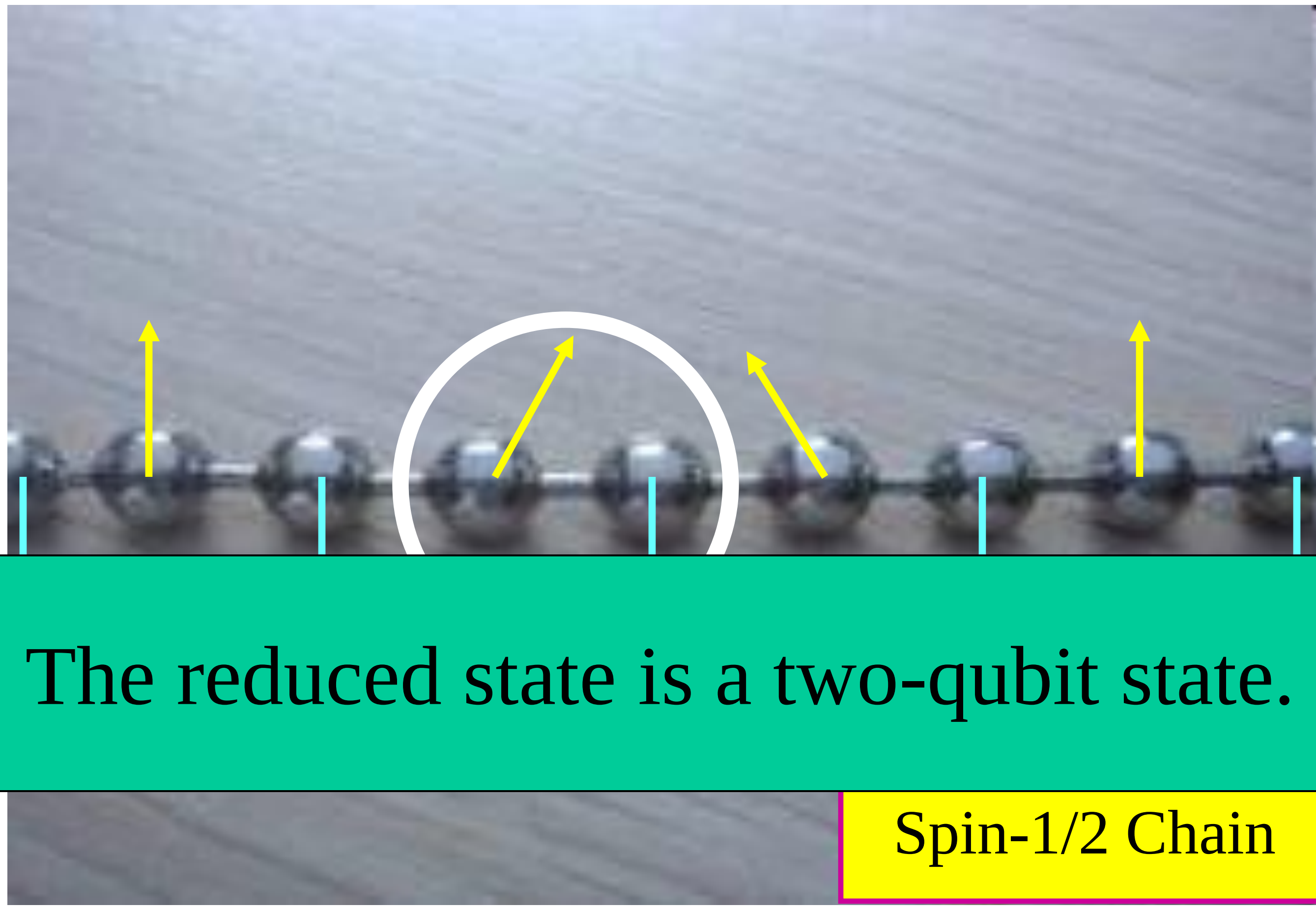


The reduced state is a two-qubit state.

# Entanglement in many-body physics

# Two-site densities

***The prescription:***

# Entanglement in many-body physics

# Two-site densities

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1. Find ground state of spin-1/2 system

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2. Remove all spins except two NNs

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# Two-site densities

***The prescription:***

1. Find ground state of spin-1/2 system
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4. Investigate it wrt the relevant system parameter

# Entanglement in many-body physics
## Quantum XY spin model

$$\Sigma J [(1 + \gamma) S_x^i S_x^{i+1} + (1 - \gamma) S_y^i S_y^{i+1}] - a S_z^i$$

# Entanglement in many-body physics
# Quantum XY spin model

$$\Sigma J\,[(1+\gamma)\,S_x^i S_x^{i+1} + (1-\gamma)\,S_y^i S_y^{i+1}] - a\,S_z^i$$

S are half of Pauli matrices.

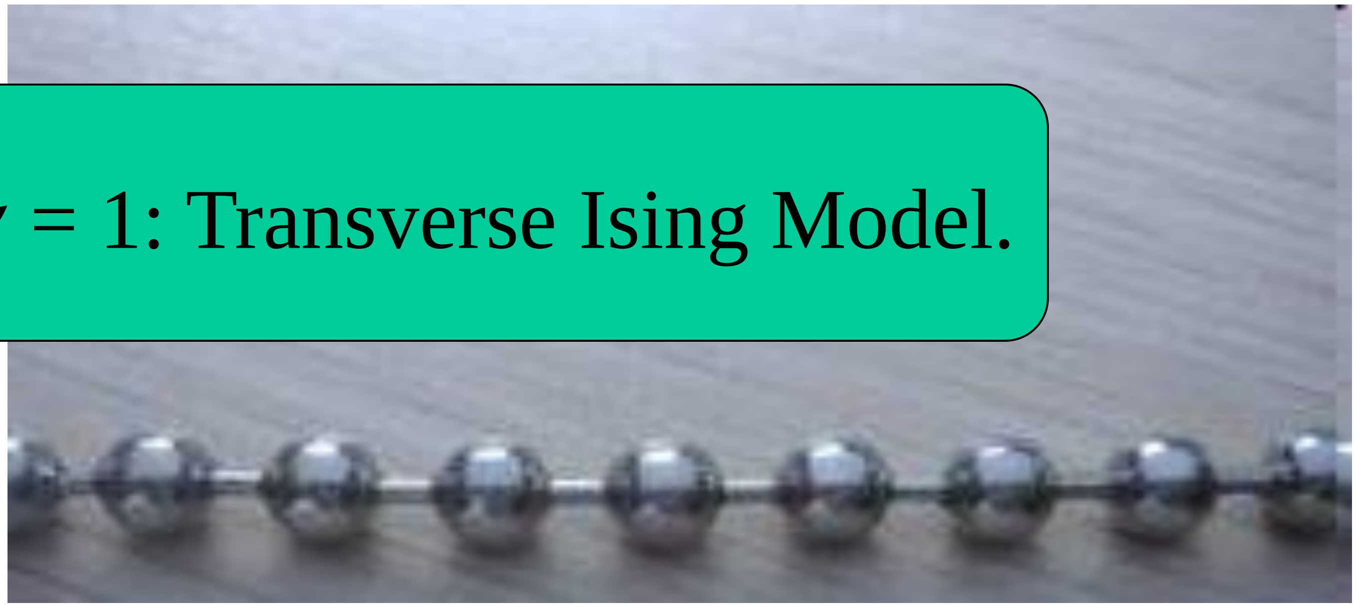
# Entanglement in many-body physics
# Quantum XY spin model

$$\Sigma J\,[(1+\gamma)\,S_x^i S_x^{i+1} + (1-\gamma)\,S_y^i S_y^{i+1}] - a\,S_z^i$$

*Quantum* phase transition at h=1.

# Entanglement in many-body physics
# Quantum XY spin model

For $\gamma = 1$: Transverse Ising Model.



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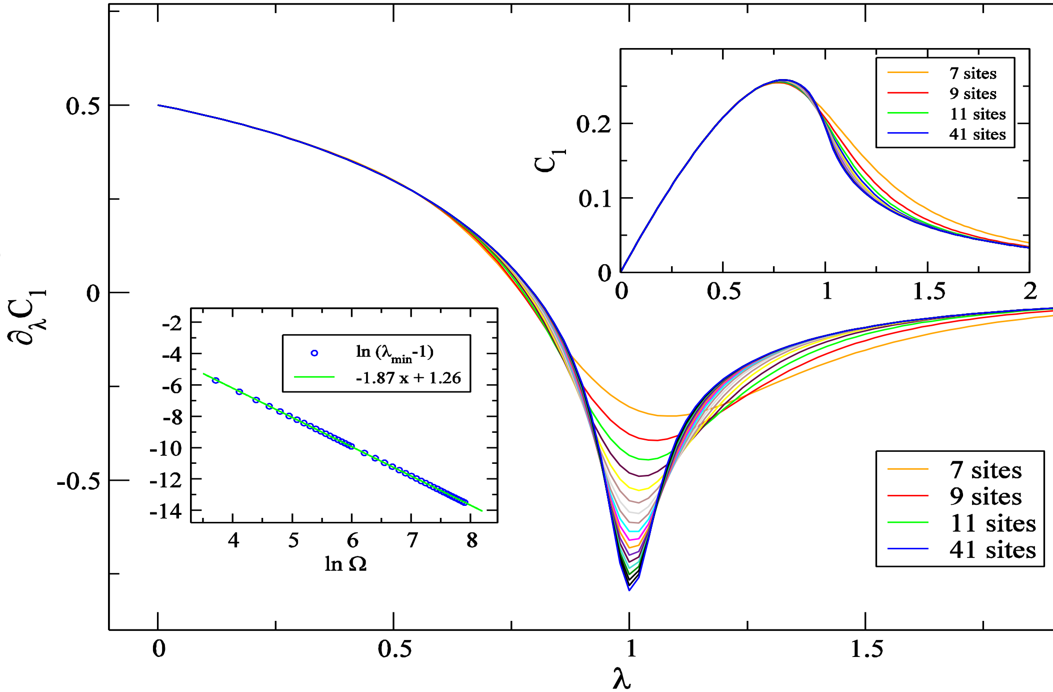
*Quantum* phase transition at h=1.

# Entanglement in many-body physics

## Linking QI with concepts in quantum statistical mechanics and quantum phase transitions.

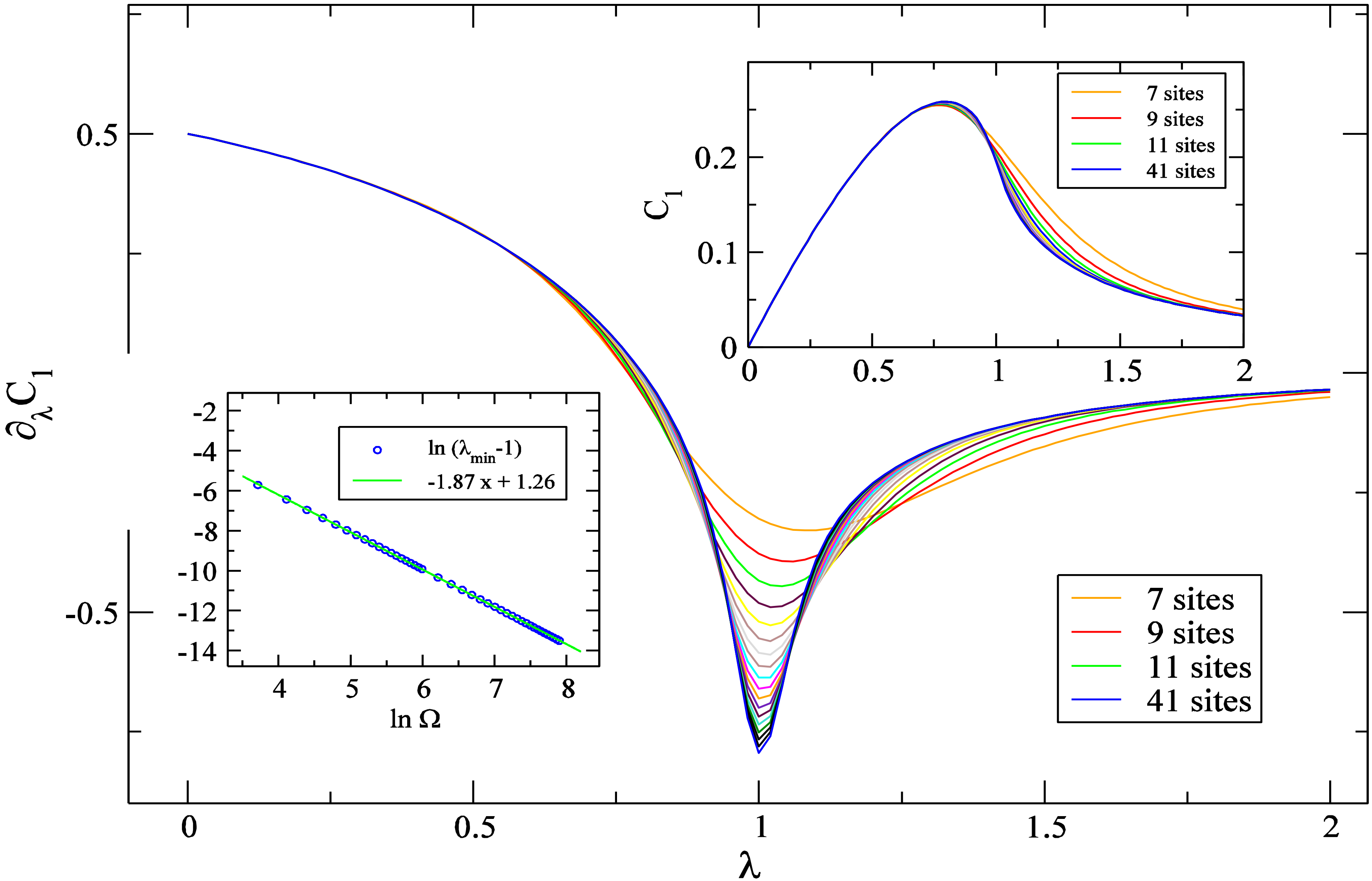
Near QPT in 1D transverse Ising model, 2-site entanglement remains short ranged, while 2-site correlation length diverges.

Entanglement, however, does show signs of criticality.




Osterloh, Amico, Falci, & Fazio, Nature 2002; Osborne & Nielsen, Phys. Rev. A 2002.

# Entanglement in many-body physics



# Entanglement in many-body physics

# Two-site densities

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# Entanglement in many-body physics

## Two-site densities

Why ***ground*** state?

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# Entanglement in many-body physics

# Two-site densities

Why ***ground*** state?

***The prescription:***

1. Find ground s
2. Remove all sp
3. Find EoF of r
4. Investigate it er

Guarantees that there are no thermal effects.

# Entanglement in many-body physics

# Two-site densities

Why ***ground*** state?

***The prescription:***

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Thermal states,
time-evolved states
also considered.

# Entanglement in many-body physics

## Two-site densities

Why ***NN***?

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# Entanglement in many-body physics

# Two-site densities

Why ***NN***?

***The prescription:***

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In many instances,
but NOT all,
NNN and so on
have little to no entanglement.

# Entanglement in many-body physics

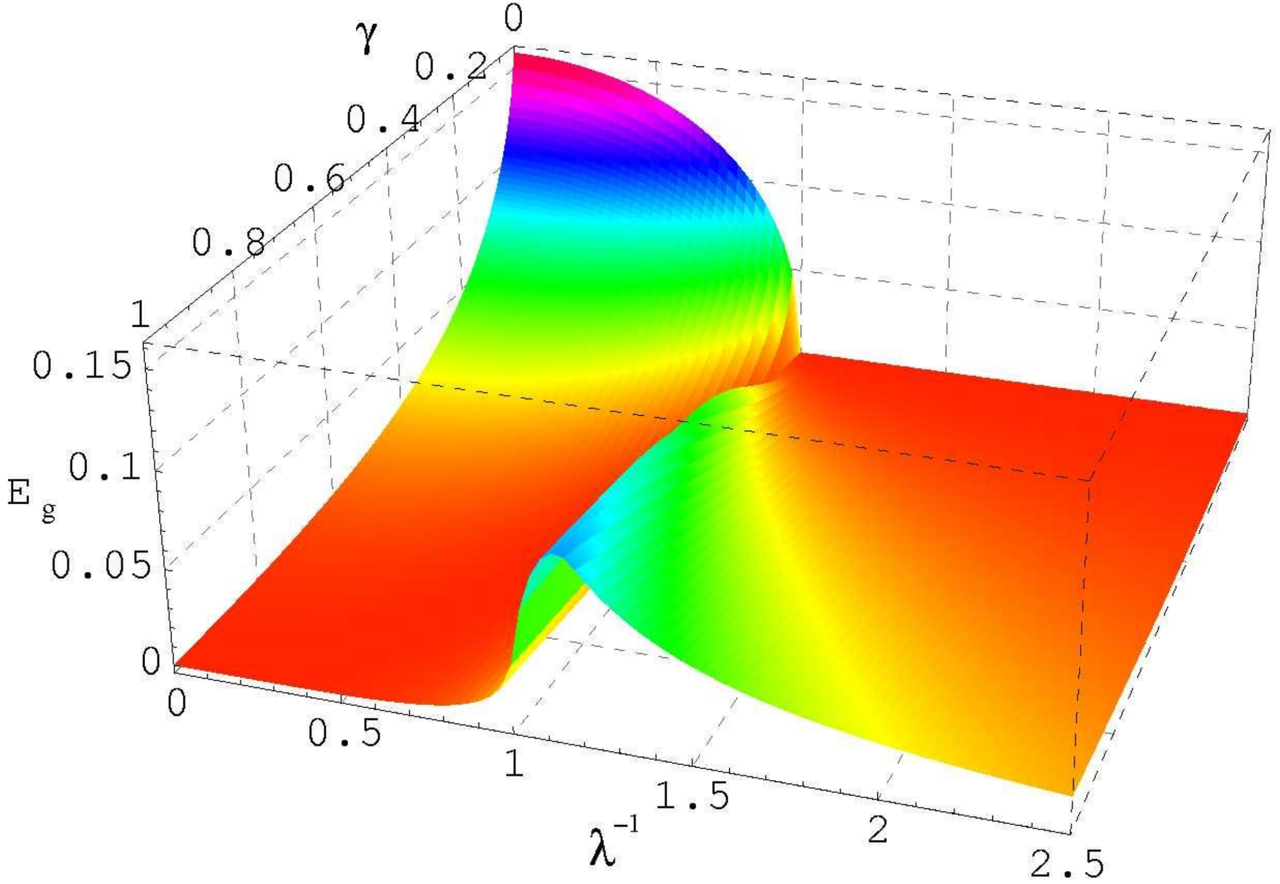
## Multiparty entanglement

Multiparty entanglement detects QPT

a. Geometric measure
b. Global measure
c. Generalized geometric measure (GGM)

# Entanglement in many-body physics

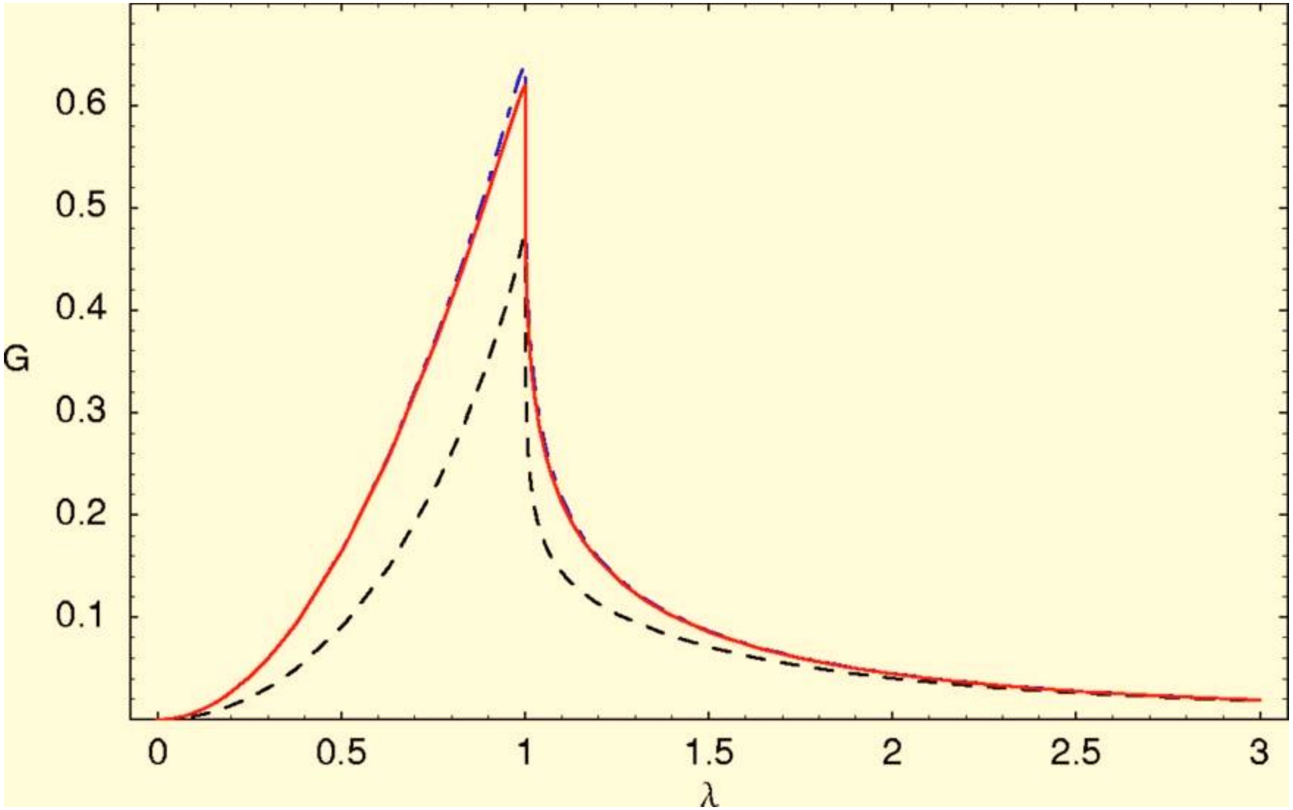
## Geometric measure detects QPT




Wei, Das, Mukhopadhyay, Vishveshwara, Goldbart, PRA'05

# Entanglement in many-body physics

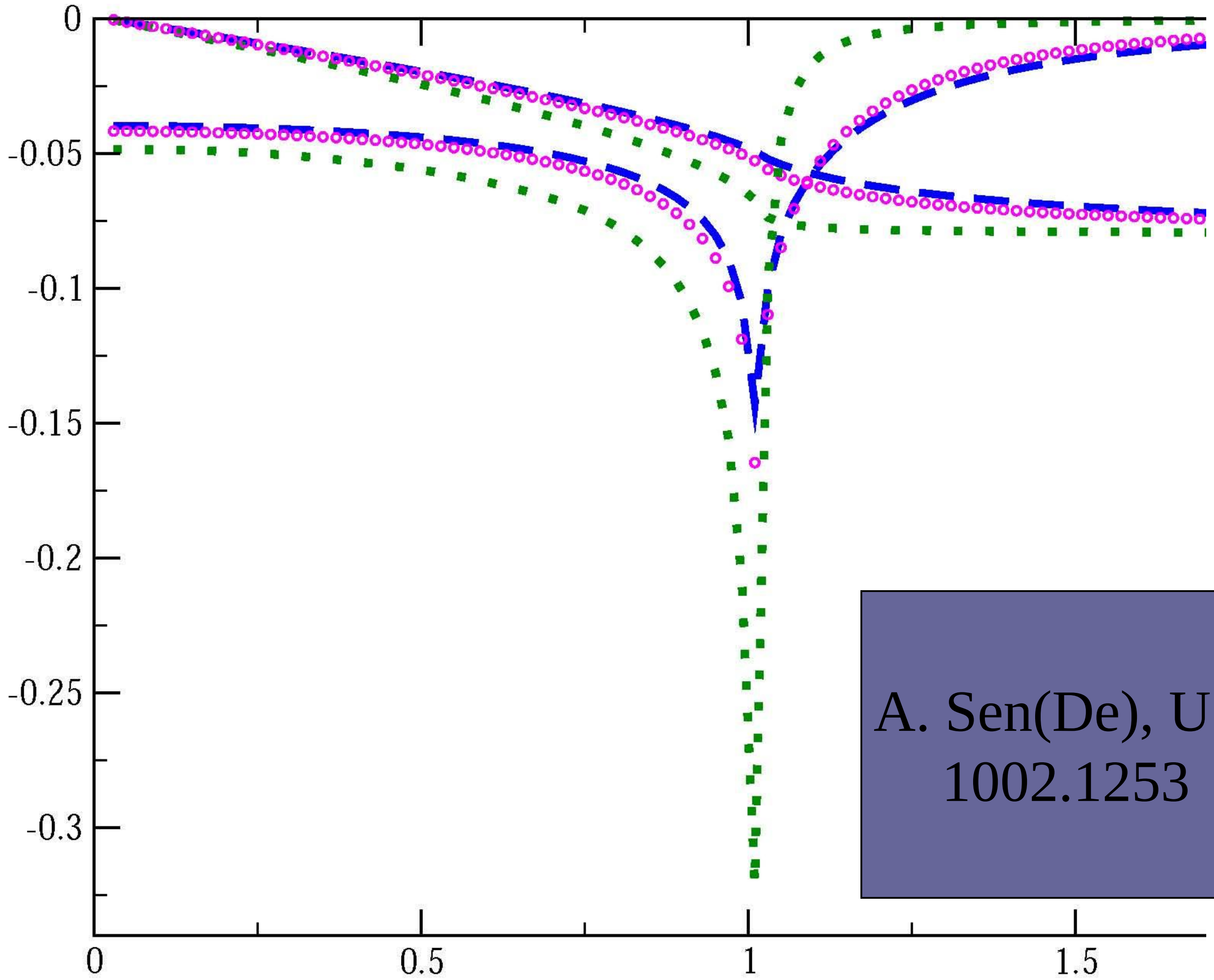
## Global measure of multipartite entanglement detects QPT




deOliviera, Rigolin, deOliviera, PRA’06

# Entanglement in many-body physics

# GGM detects QPT




A. Sen(De), US,
1002.1253

# Entanglement in many-body physics

GGM –1/2

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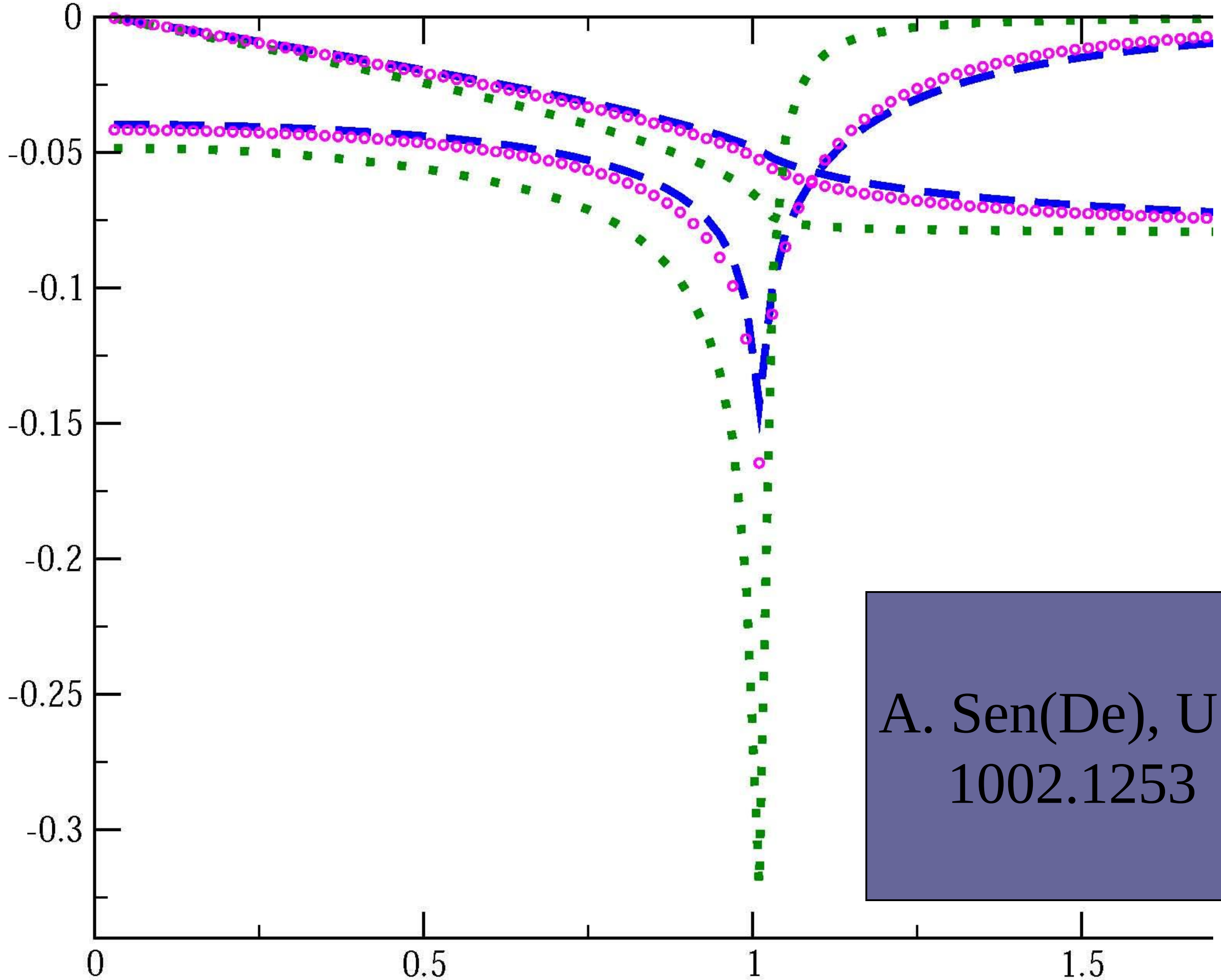


A. Sen(De), US, 1002.1253

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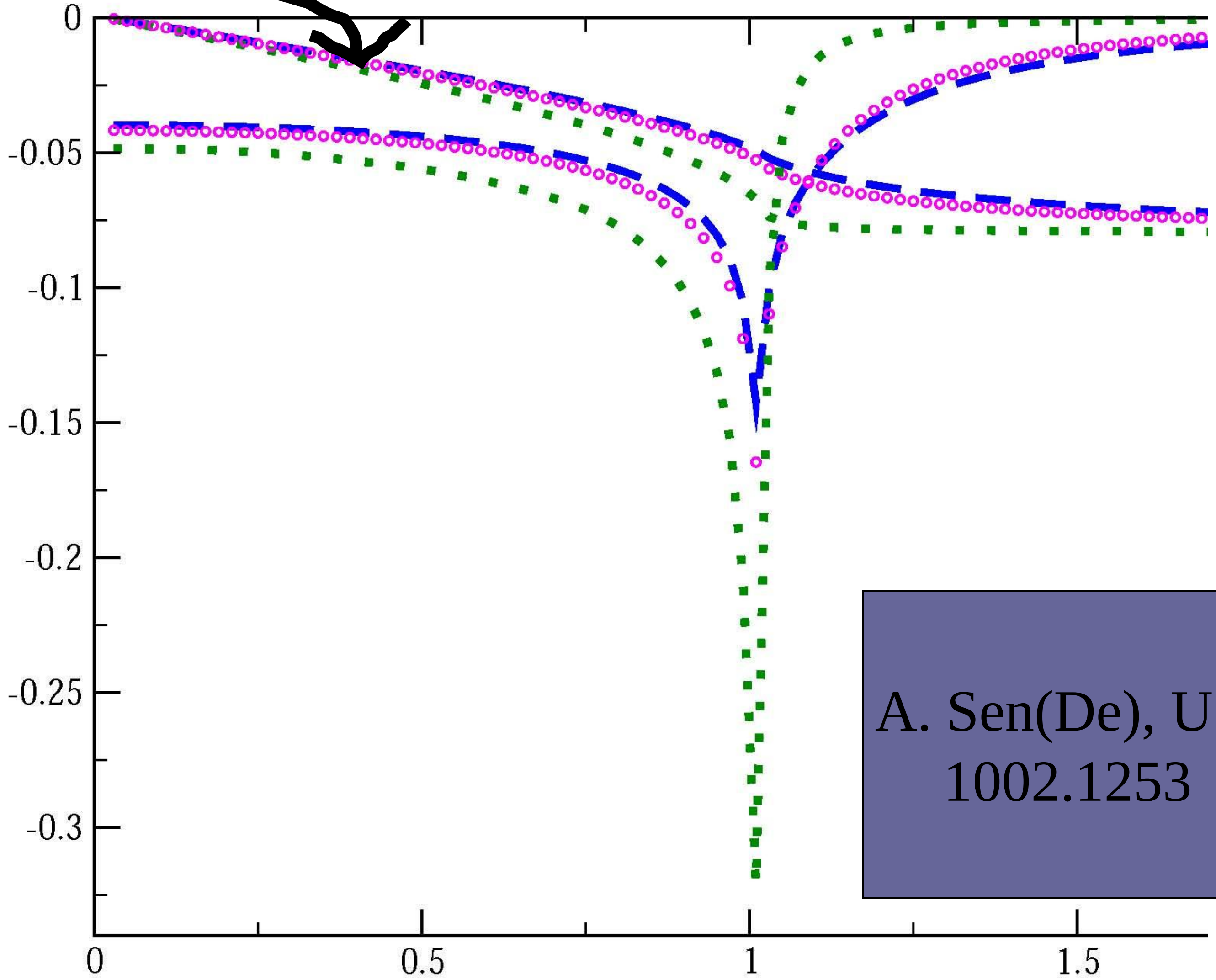


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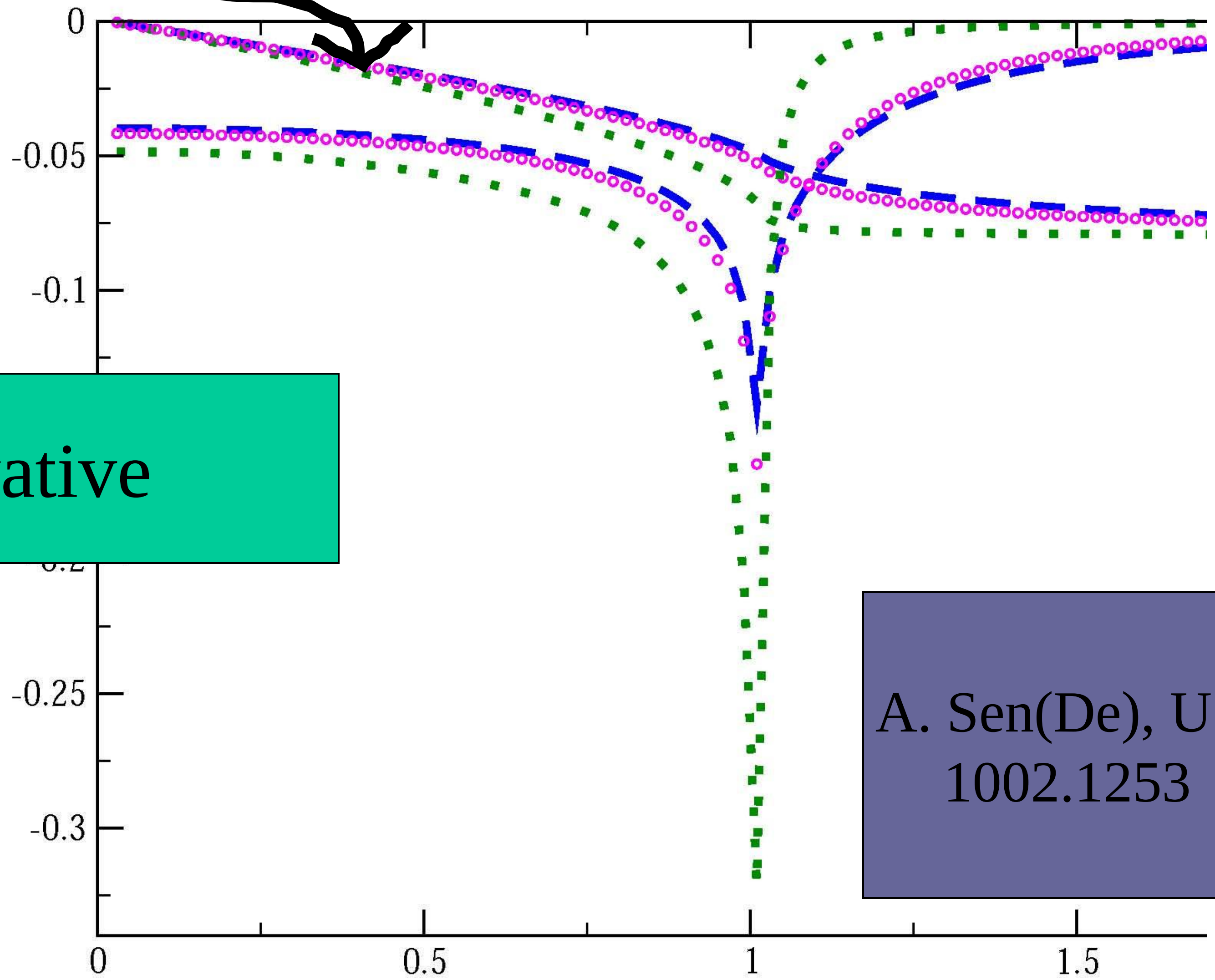


derivative

A. Sen(De), US, 1002.1253

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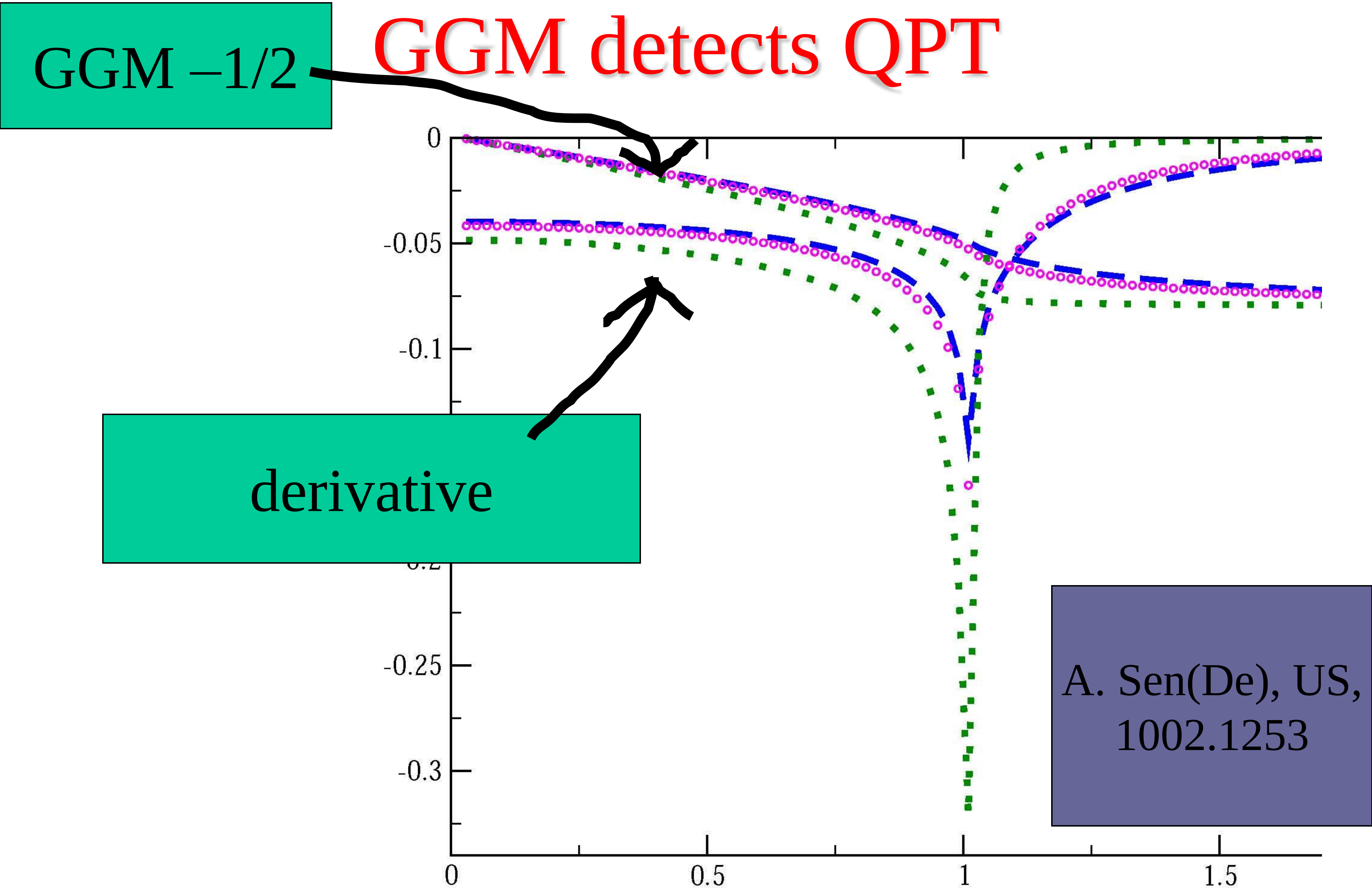

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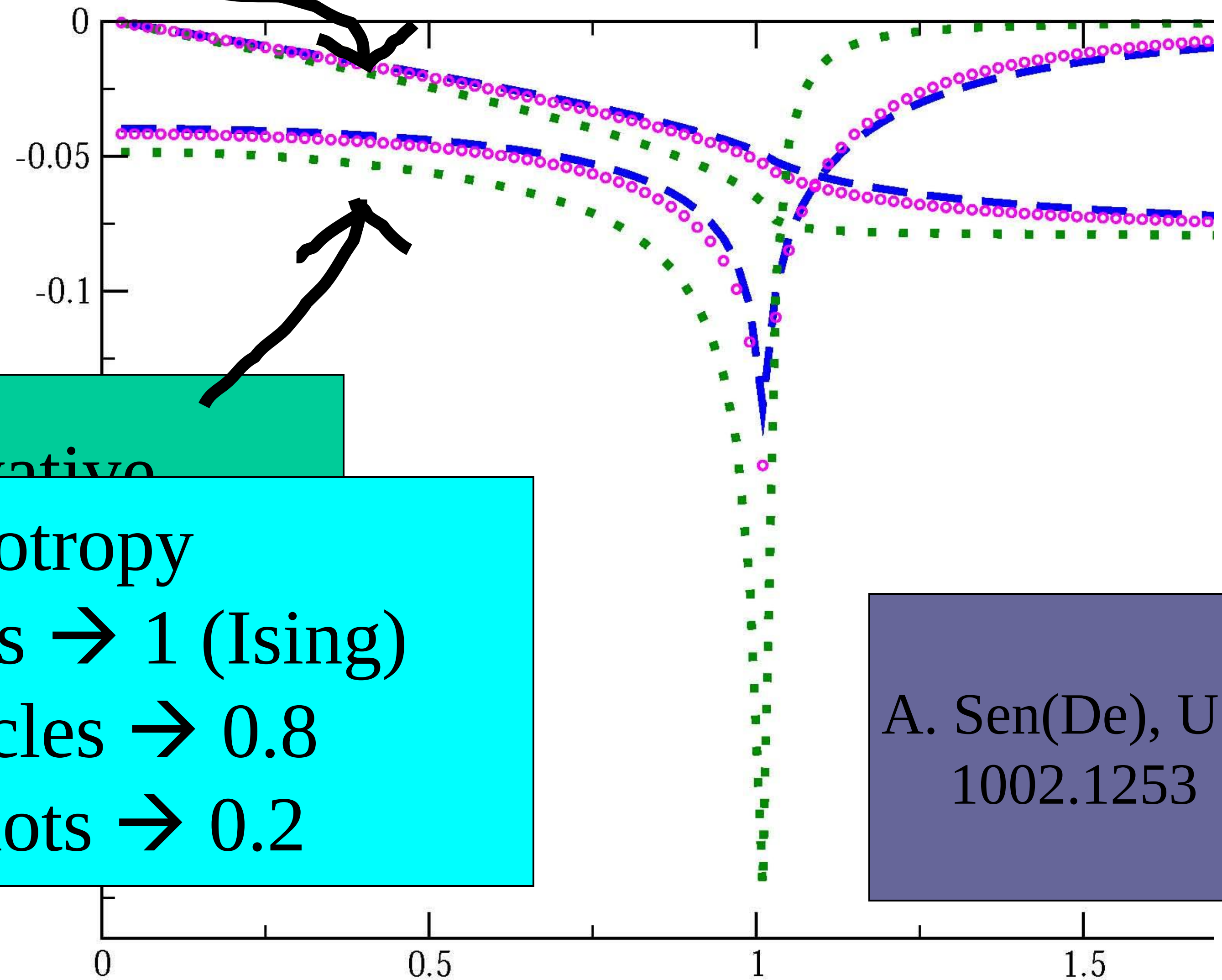


derivative

anisotropy
Blue dashes → 1 (Ising)
Pink circles → 0.8
Green dots → 0.2

A. Sen(De), US, 1002.1253

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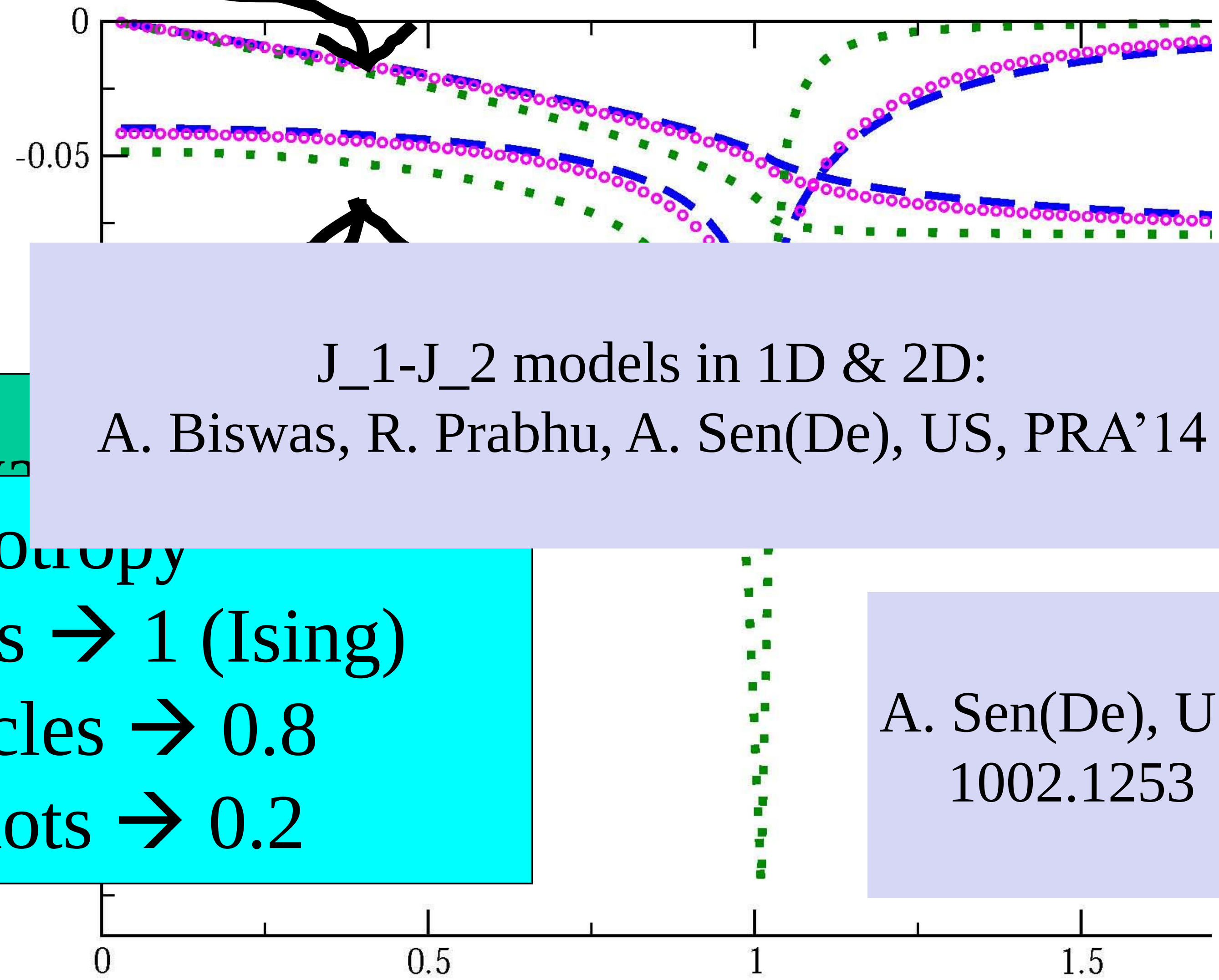


J_1-J_2 models in 1D & 2D:
A. Biswas, R. Prabhu, A. Sen(De), US, PRA’14

deriva

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# What is frustration?

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**From a classical perspective**

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From a classical perspective

Consider an Ising model:

$$\mathcal{H} = J \sum \sigma_i \sigma_j \, ; \quad J > 0$$

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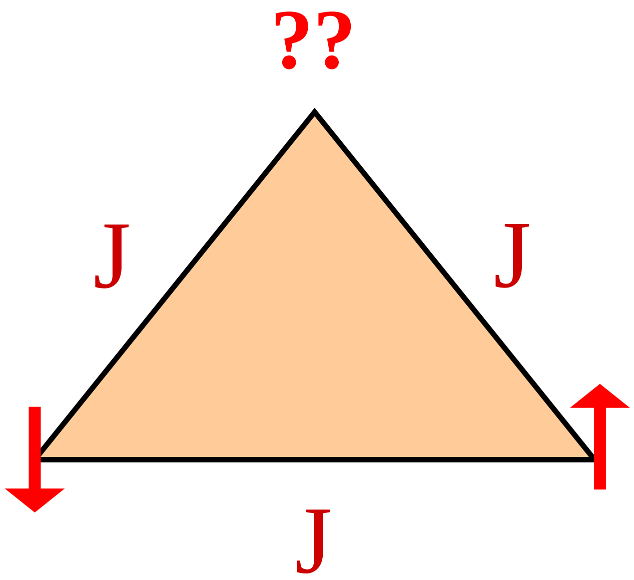
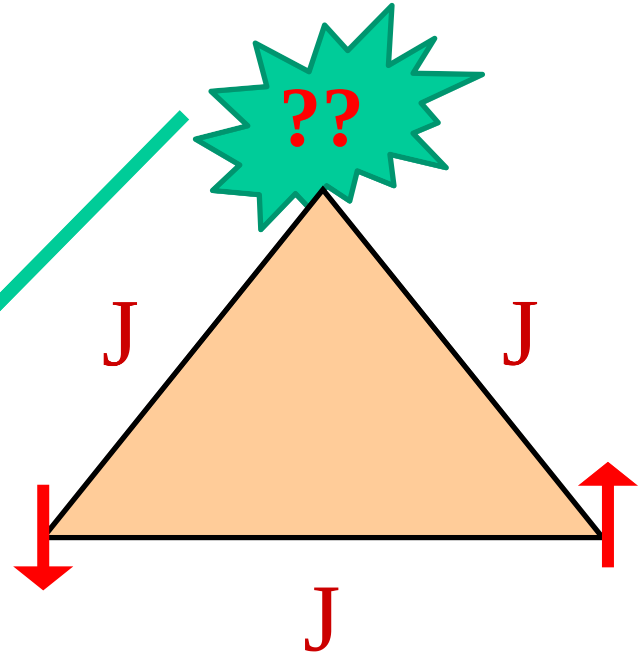


# What is frustration?

From a classical perspective

Failure to have spin configuration to minimize individual interaction terms



# What is frustration?

**From a *quantum* perspective**

# What is frustration?

From a quantum perspective

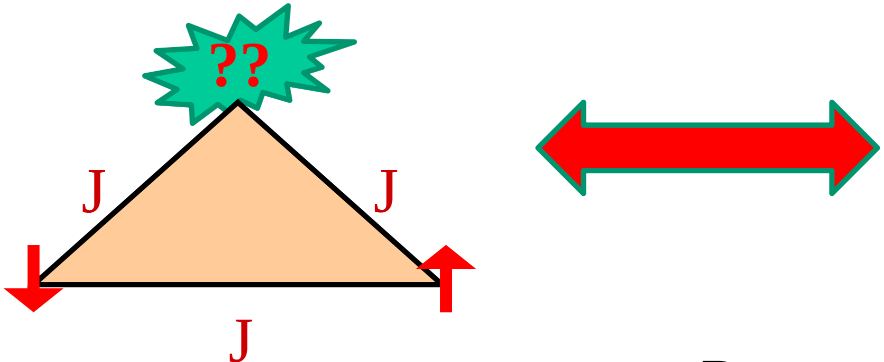
Draw a parallel

# What is frustration?

**From a quantum perspective**

**Classical frustration: spin configuration**

**Quantum frustration: GSs of two terms not same**

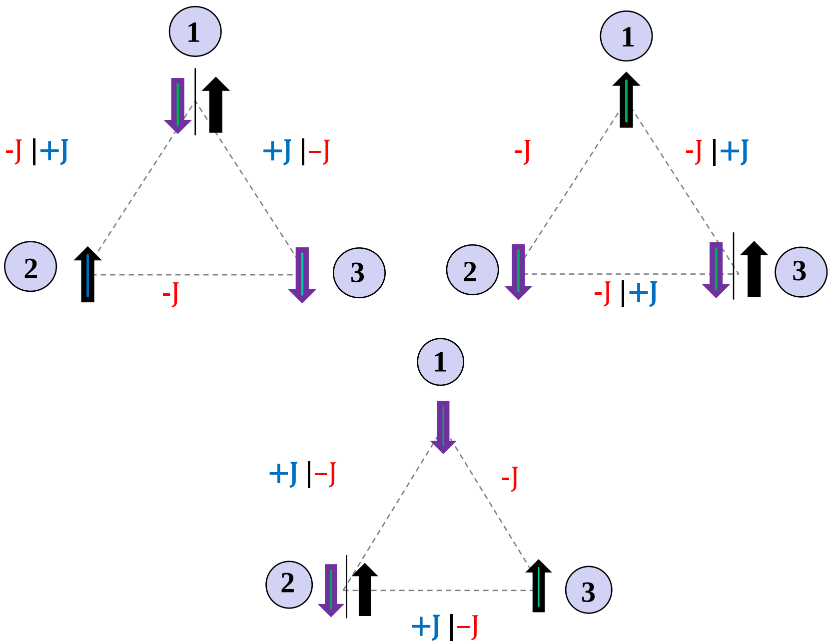



$$\mathcal{H} = \mathcal{H}_{loc} + \mathcal{H}_{int}$$

Dawson and Nielsen, PRA 69, 052316 (2004)

# What is frustration?

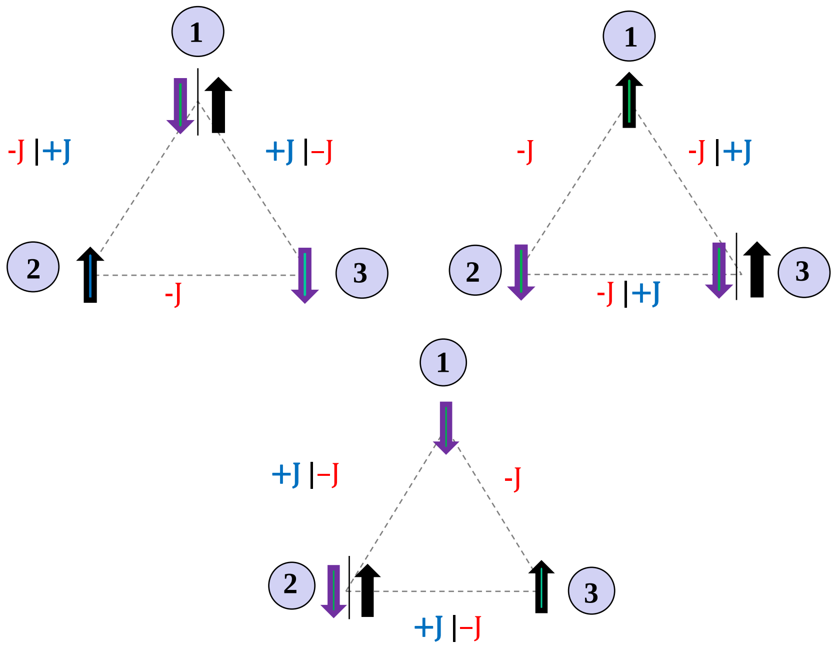
## Classical spin configuration




## Cannot get optimal spin configuration

# What is frustration?

## Classical spin configuration




Cannot get optimal spin configuration

## Quantum non-commutativity

$$\mathcal{H} = \mathcal{H}_{loc} + \mathcal{H}_{int}$$

$|\psi_{gs}\rangle_{loc}$ $|\psi_{gs}\rangle_{int}$

TRADE OFF

$|\psi_{gs}\rangle$

⟺ GSs of two terms not same

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# Characterizing "classical" frustration in q systems

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Classical frustration

# Characterizing "classical" frustration in q systems

Classical frustration ⟶ "Frustration degree"

*Sen(De), US, Dziarmaga, Sanpera, Lewenstein, PRL'08*
*Jindal, Rane, Dhar, Sen(De), US, PRA'14*

# Characterizing “classical” frustration in q systems

## *Frustration degree*

- Given H, $|\Gamma\rangle$,

# Characterizing "classical" frustration in q systems

## *Frustration degree*

- Given H, $|\Gamma\rangle$,

  replace one-body, two-body etc. in H by Ising ones,

  i.e. by $\sigma^z_i$ or $\sigma^z_i\sigma^z_j$ etc.

# Characterizing "classical" frustration in q systems

## ***Frustration degree***

- Given H, $|\Gamma\rangle$,

  replace one-body, two-body etc. in H by Ising ones,

  i.e. by $\sigma^z_i$ or $\sigma^z_i\sigma^z_j$ etc.

  Find $H^I$

# Characterizing "classical" frustration in q systems

## *Frustration degree*

- Given H, $|\Gamma\rangle$,

  replace one-body, two-body etc. in H by Ising ones,

  i.e. by $\sigma^z_i$ or $\sigma^z_i\sigma^z_j$ etc.

  Find $H^I$

Frustrated part — Non-Frustrated part

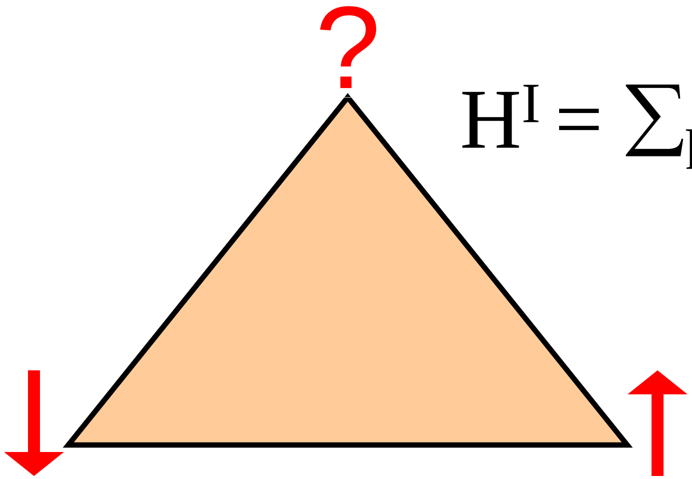
$$H^I = \sum_k H^k_f + \sum_l H^l_{nf}$$

$$\Phi = \text{avg} \; \frac{\sum_k \langle\Gamma|H^k_f|\Gamma\rangle}{\sum_l |\langle\Gamma|H^l_{nf}|\Gamma\rangle|}$$

# Characterizing "classical" frustration in q systems

## *Frustration degree*

Ising model: $H = J \sum \sigma^z_i \sigma^z_j$ with $J>0$

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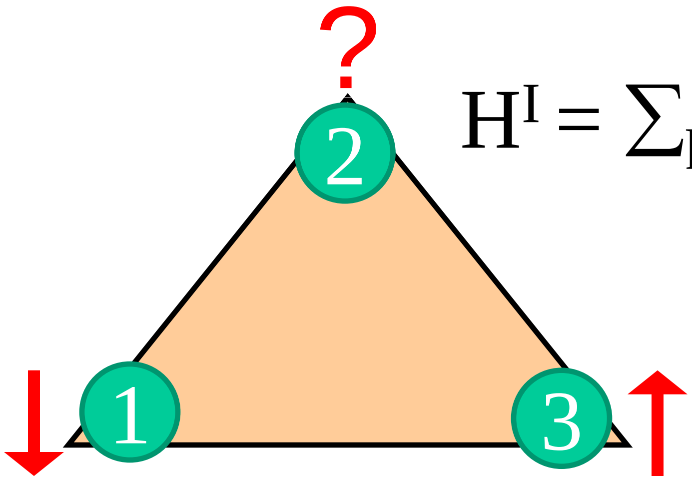


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$\sigma^z_1 \sigma^z_2 |\downarrow\rangle \rightarrow 1$

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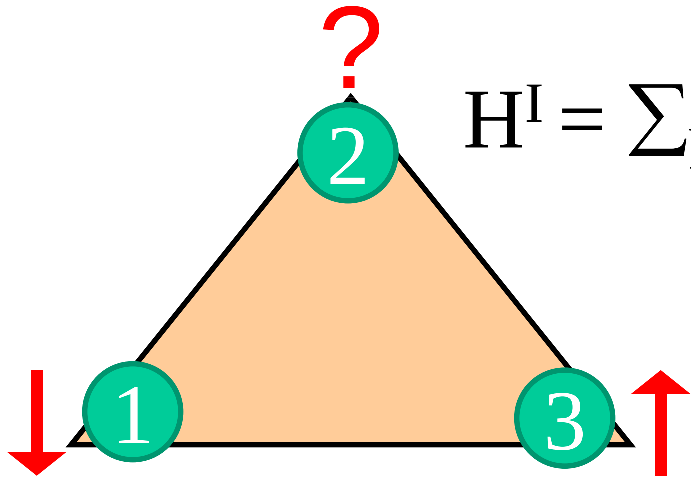


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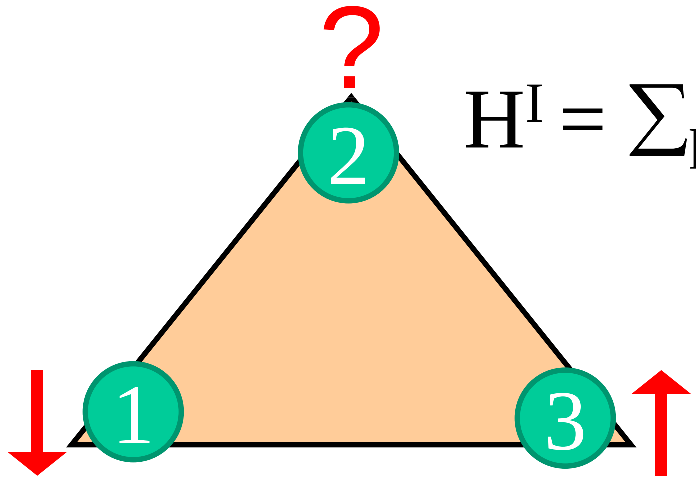


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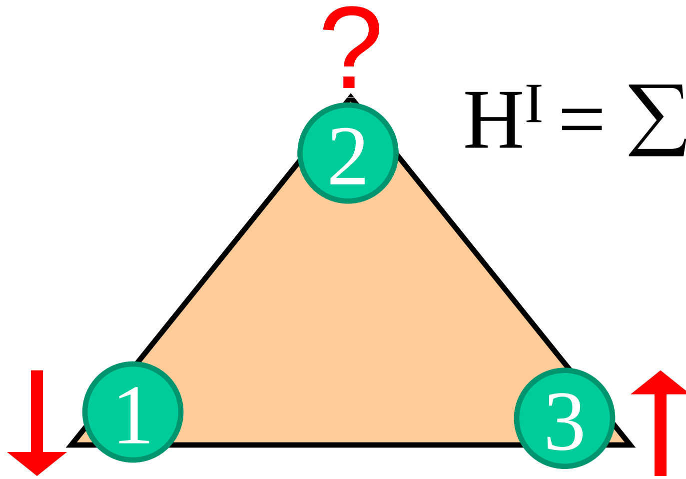


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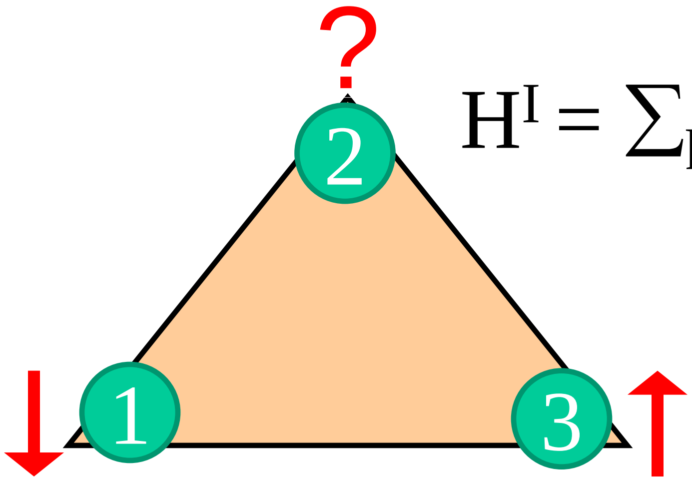


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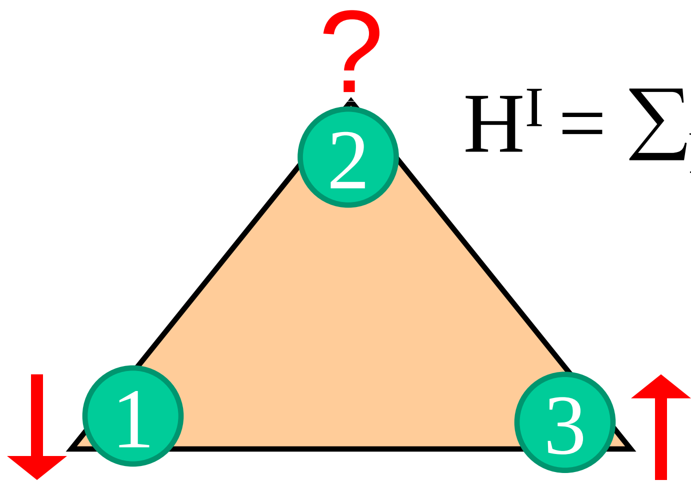


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# Characterizing "classical" frustration in q systems

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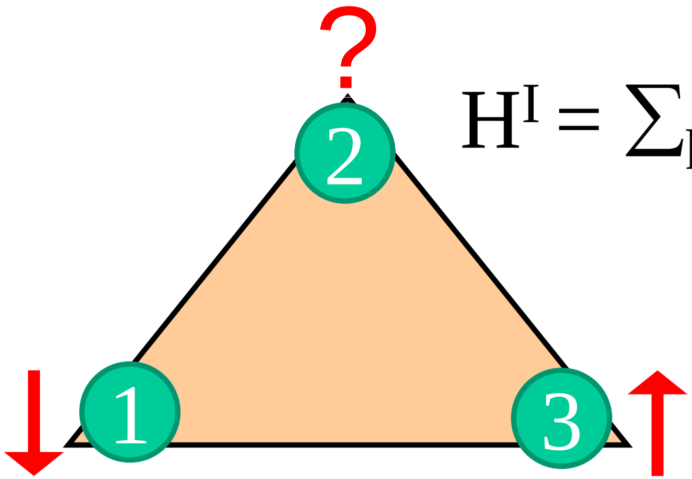


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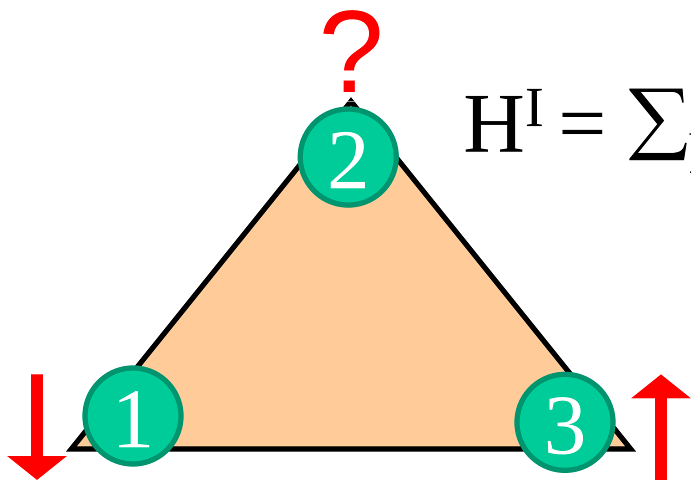


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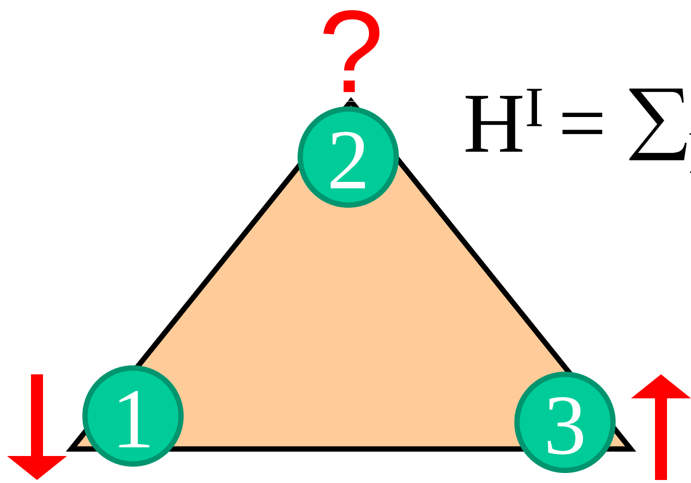


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$$\sum_l |\langle\Gamma|H^l_{nf}|\Gamma\rangle|$$

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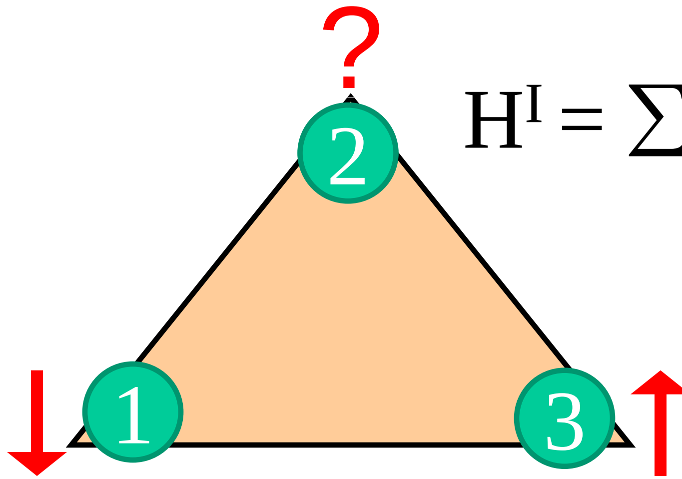


$$H^I = \sum_k H^k_f + \sum_l H^l_{nf}$$

$$\sum_l |\langle \Gamma | H^l_{nf} | \Gamma \rangle|$$

$$\Phi = 1/2$$

# Outline

1. Understanding entanglement

2. Entanglement in many-body physics

3. What is frustration?

4. Characterizing "classical" frustration in q systems

5. Frustration and Entanglement

    I. Area Law

    II. Genuine multipartite entanglement

6. End remarks

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1. Understanding entanglement

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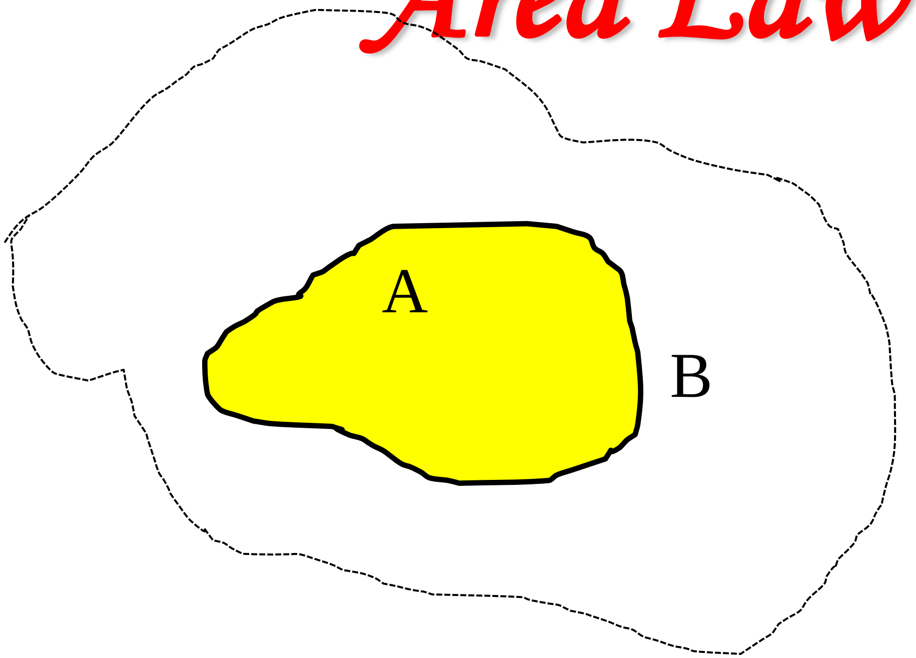
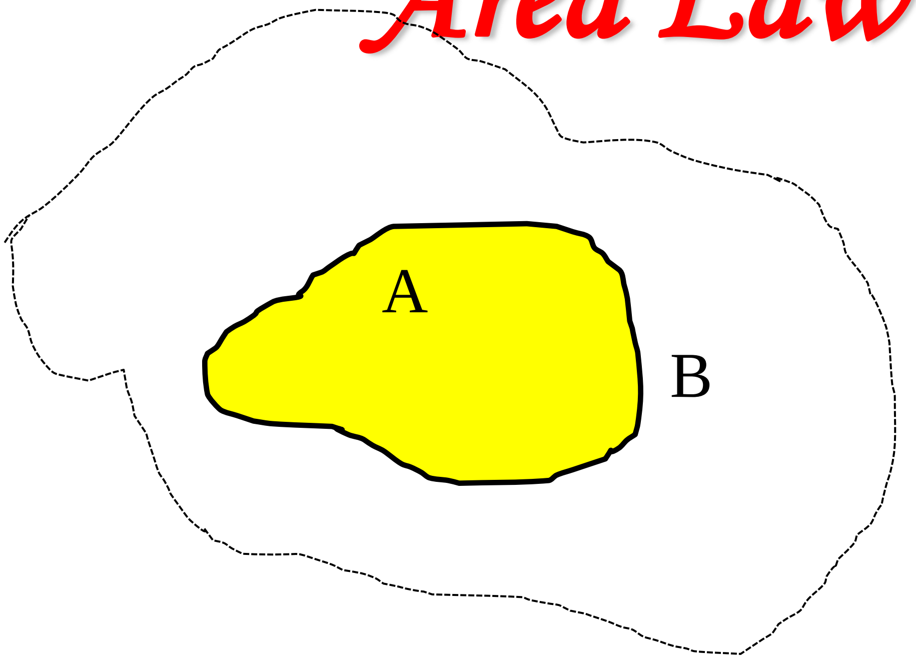
# Area Law




Reduced entropy S would depend on the surface of separation between A and B.

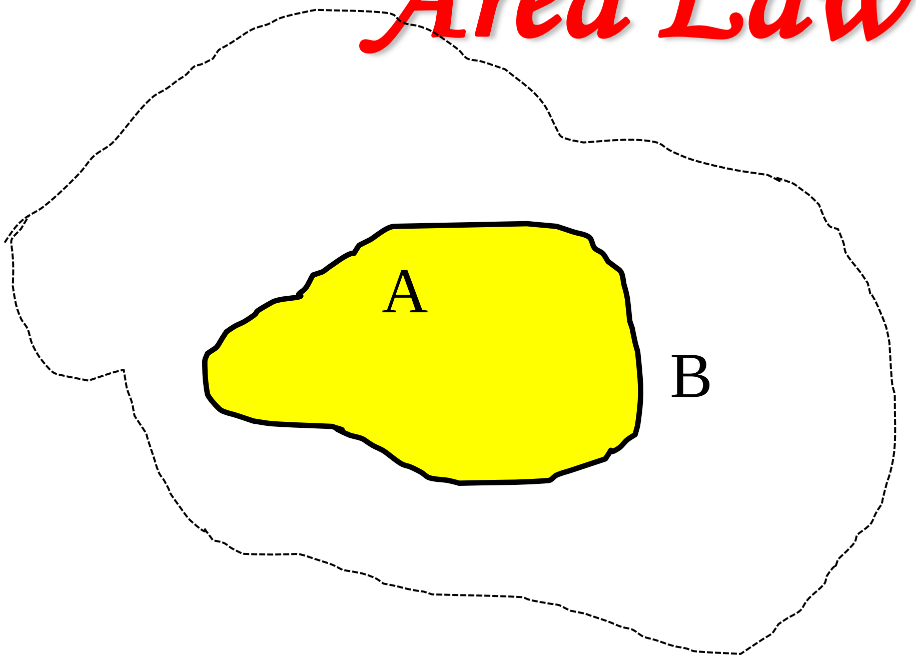
# Area Law




Reduced entropy S would depend on the surface of separation between A and B.

We r talking abt interacting systems.

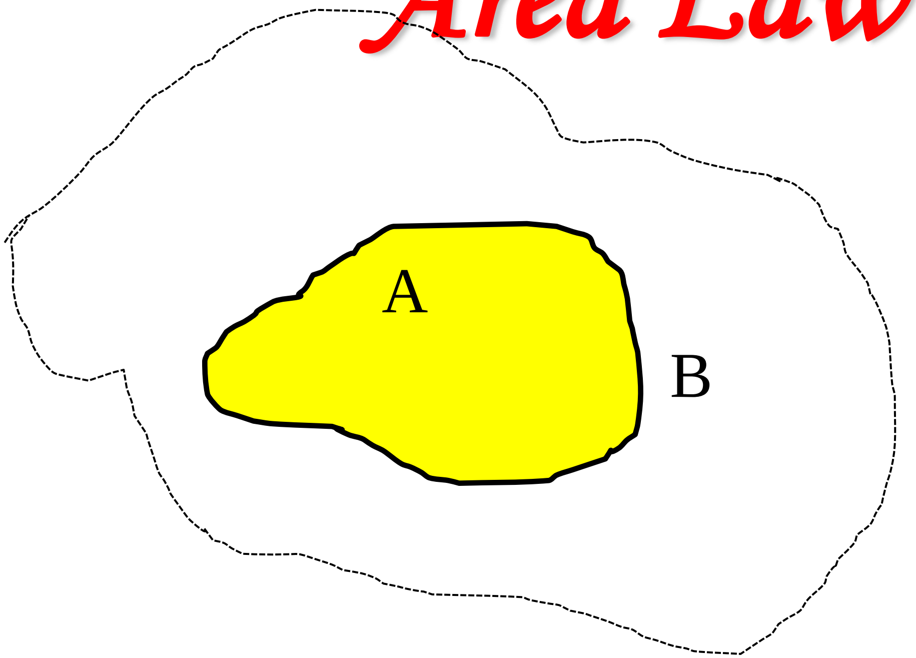
# Area Law




Reduced entropy S would depend on the surface of separation between A and B.

Would be true (trivially) if …

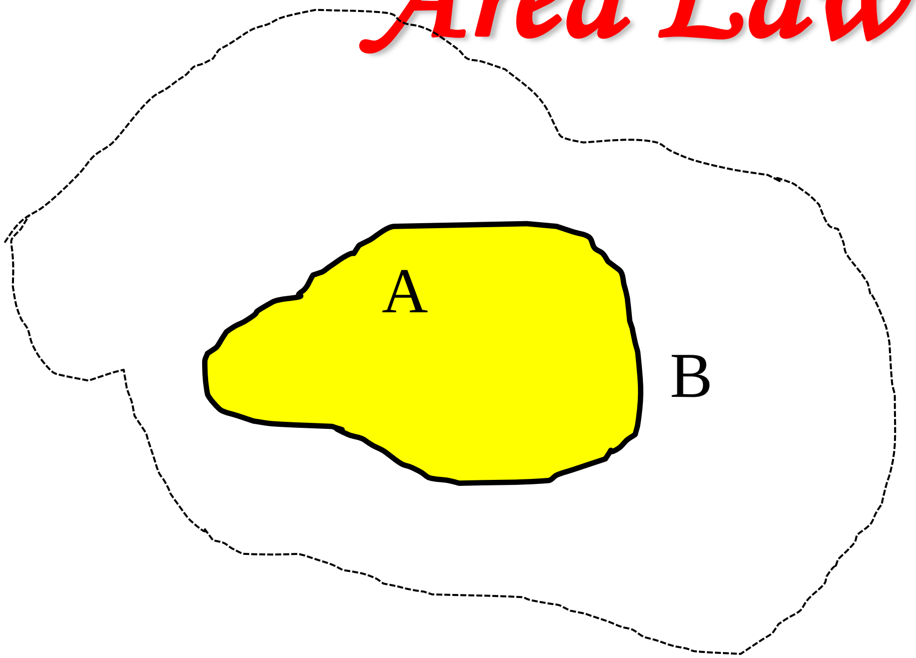
# *Area Law*




Reduced entropy S would depend on the surface of separation between A and B.

Boundary particles are *pure* entangled states.

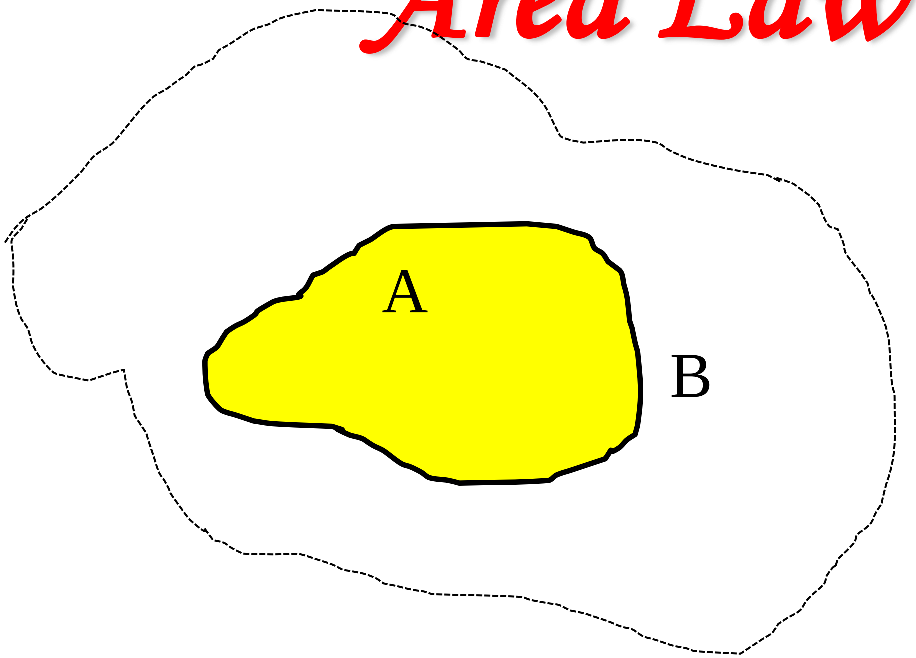
# Area Law




Reduced entropy S would depend on the surface of separation between A and B.

Boundary particles are *pure* entangled states.
Plus no long-range entangled pairs.

# Area Law




Reduced entropy S would depend on the surface of separation between A and B.

Typical situation is far from being such.

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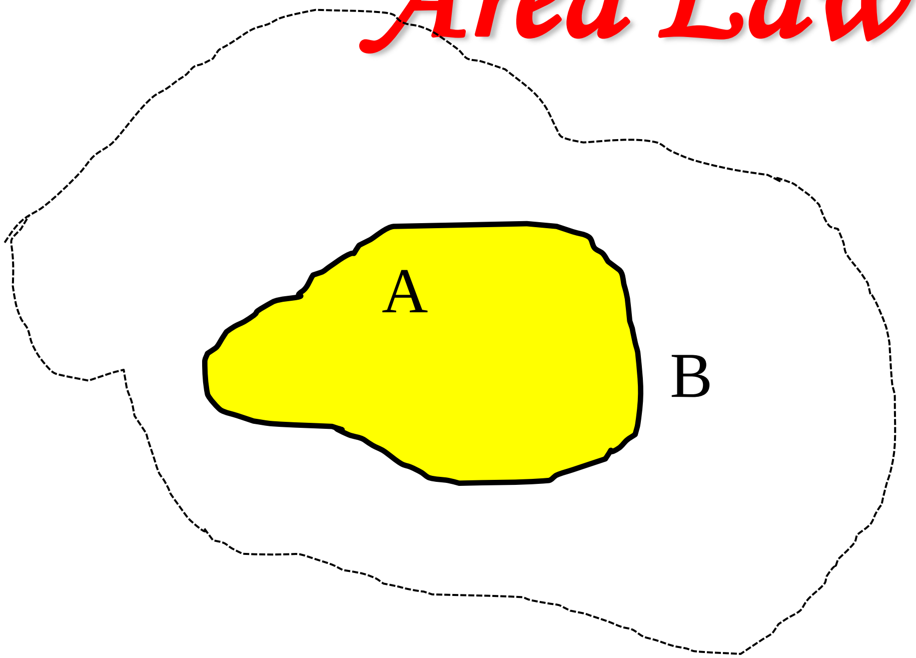
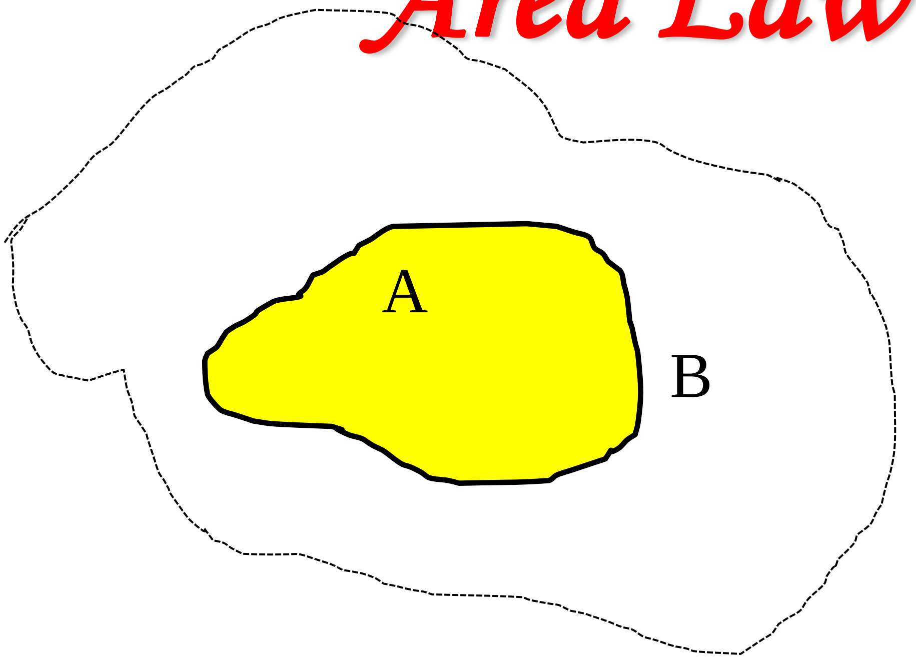



Reduced entropy S would depend on the surface of separation between A and B.

Typical situation is far from being such.
Usually intricately multiparty quantum correlated.

# Area Law

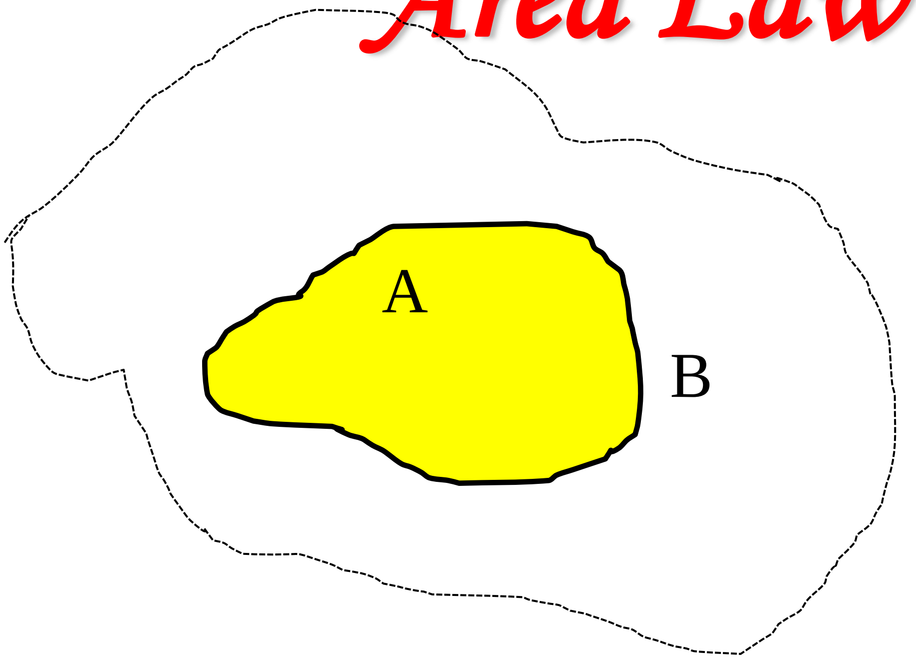



Reduced entropy S would depend on the surface of separation between A and B.

$$S(\rho_L) \sim L^{d-1}$$

L: characteristic length of A

# Area Law

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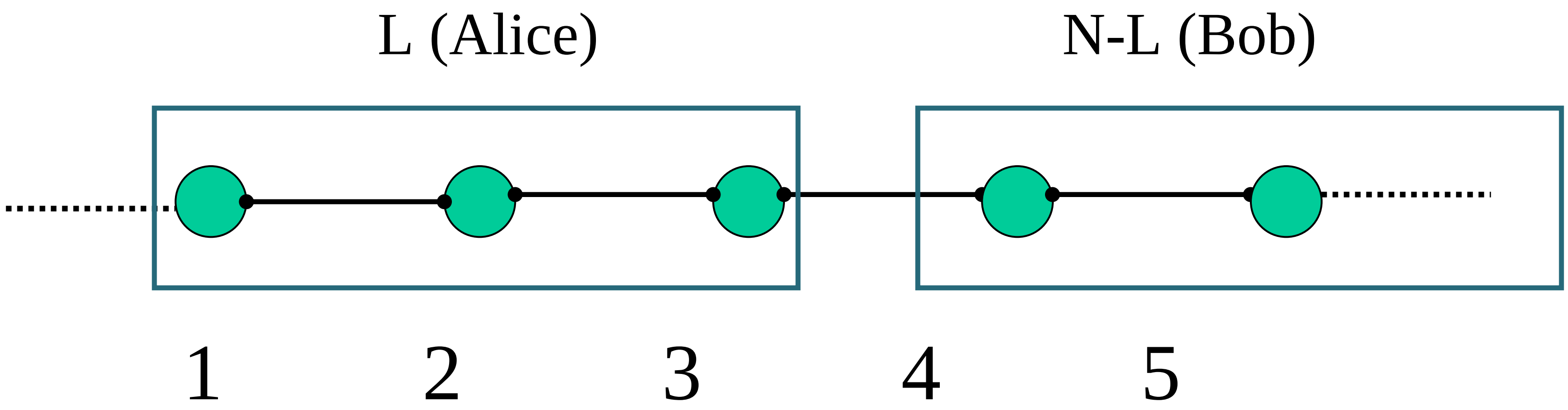
**AREA LAW**

L: characteristic length of A

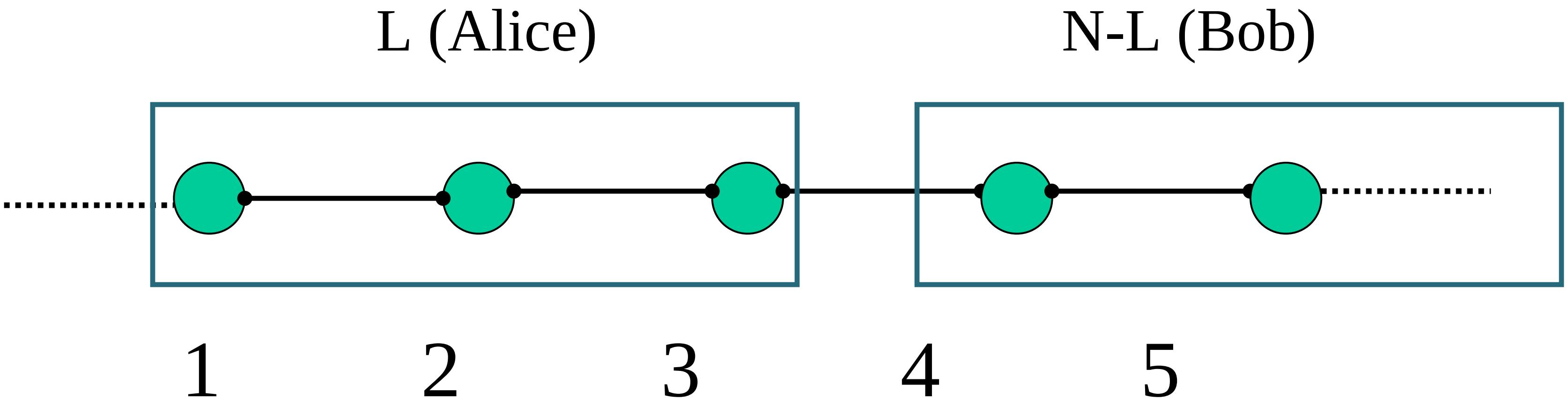
# Area Law:1D




Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$E(|\Psi\rangle_{L:N-L}) = S(\rho_L)$$

# Area Law:1D

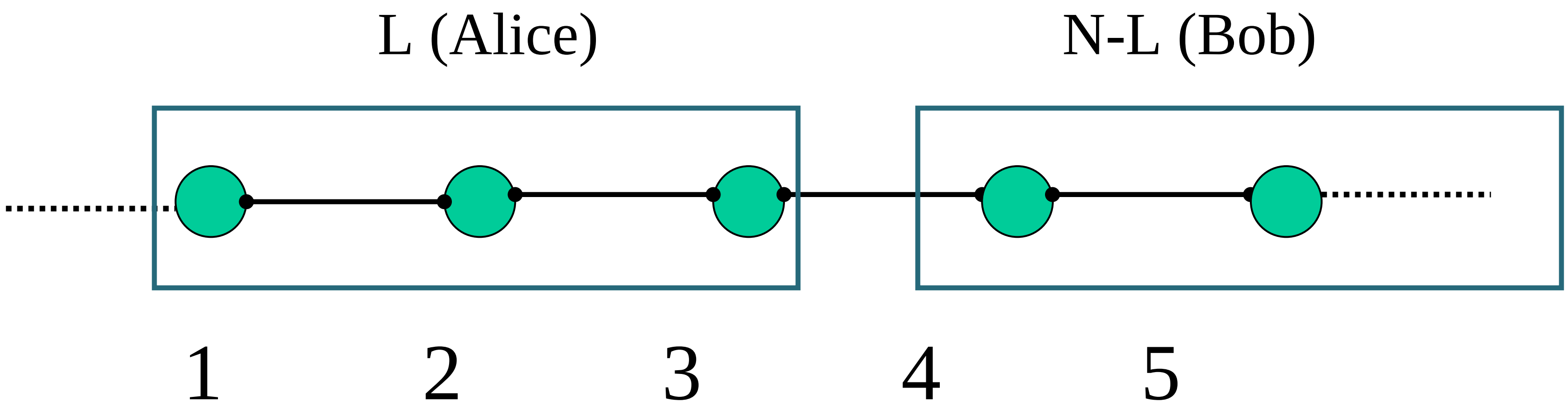



Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$E(|\Psi\rangle_{L:N-L}) = S(\rho_L) \sim L^{d-1} \equiv \text{constant}$$

away from criticality

# Area Law: 1D




Block entanglement: $E(|\Psi\rangle_{L:N-L})$

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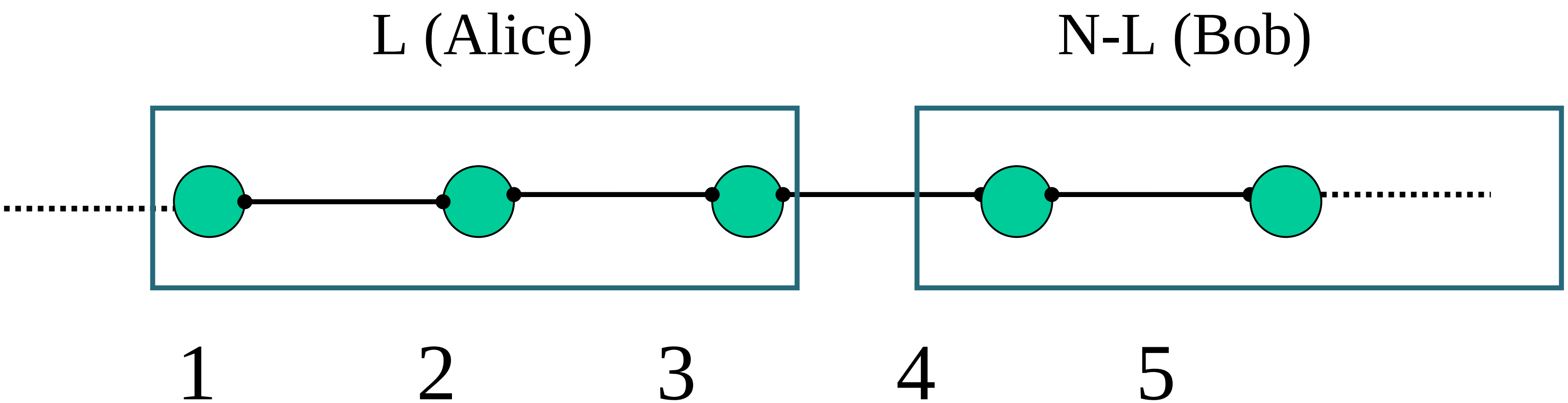
independent of block-size

# *Area Law: 1D*

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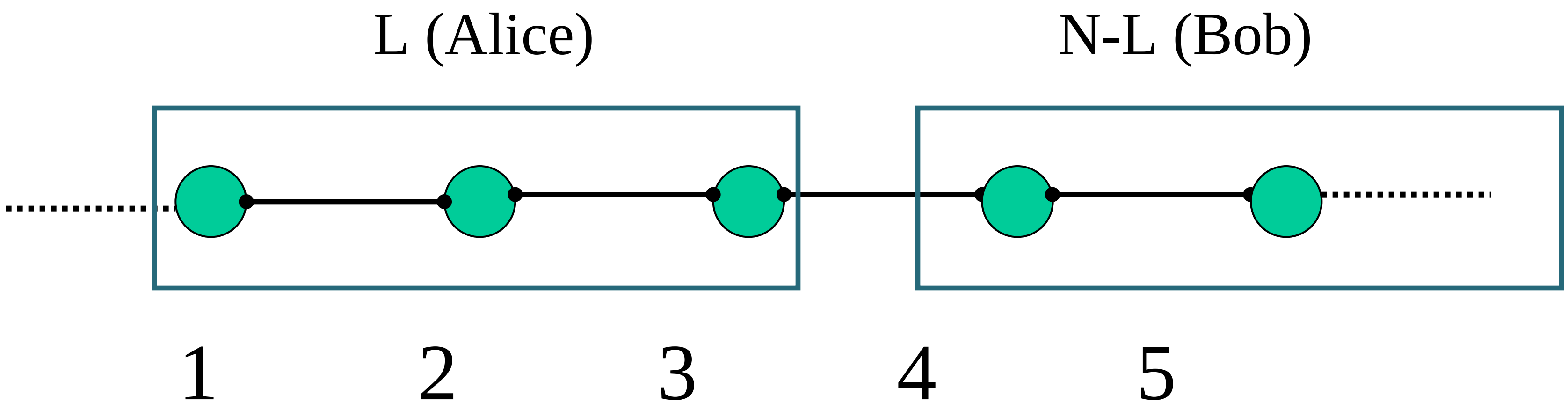
at criticality

# Area Law:1D




Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$E(|\Psi\rangle_{L:N-L}) = S(\rho_L) \sim \ln L$$

at cr

log divergence

Lot of progress in different directions.

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A case study:

Frustrated systems

# Main Thesis

- Highly frustrated systems do not follow area law

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- Highly frustrated systems do not follow area law

*while*

- Weakly frustrated systems follow same area law as nonfrustrated systems away from criticality

*Sen(De), US, Dziarmaga, Sanpera, Lewenstein, PRL'08*
*Jindal, Rane, Dhar, Sen(De), US, PRA'14*

# *Area Law for frustrated systems*

1. *Long range Ising model*
2. *Majumdar Ghosh model*
3. *Shastry-Sutherland model*
4. *Ising chain with NN interactions*

# Cooling/Quenching Method

- Initial state:

$$|\Phi\rangle_{\mathbf{in}} \equiv |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes |\psi\rangle_3 \otimes \ldots \otimes |\psi\rangle_N$$

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- Maximize $E_{N/2:N/2}(|\Phi\rangle_f)$ over all choices of the initial state.

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$\Phi \approx 1$

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$H = J \sum \sigma^z_i \sigma^z_j$ with $J > 0$

After quenching:

$|\psi\rangle =$ superposition of all vectors with $m|0\rangle$s and $m|1\rangle$s

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**Note:** Effect due to frustration.
Not due to long-range interactions.

# Area law

## Clear departure from area law

- Long range Ising model: “Infinite” dimensions
- Possible area law: $k^{1-1/d}$ with $d \to \infty$

**Note:** Effect due to frustration.

Not due to long-range interactions.

Ising with $J<0$ : constant block entanglement.

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# Majumdar-Ghosh model

$H = J_1 \sum \sigma_i \sigma_{i+1} + J_2 \sum \sigma_i \sigma_{i+2}$ with $J_1, J_2 > 0$; $J_2 = J_1/2$

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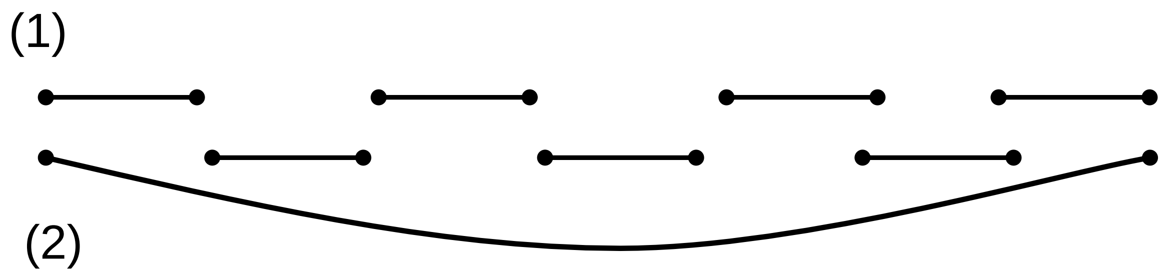

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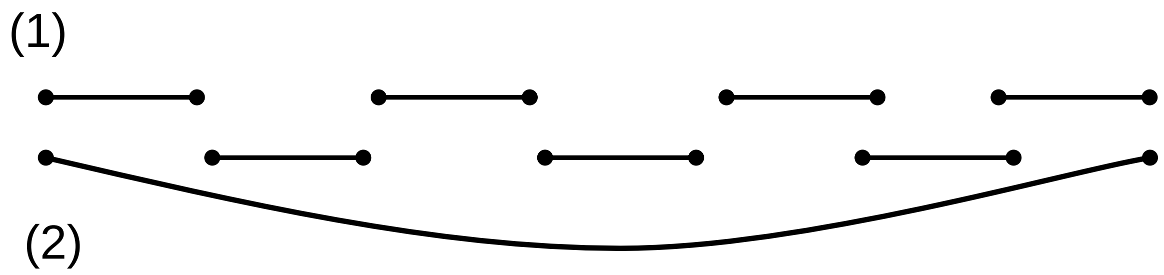


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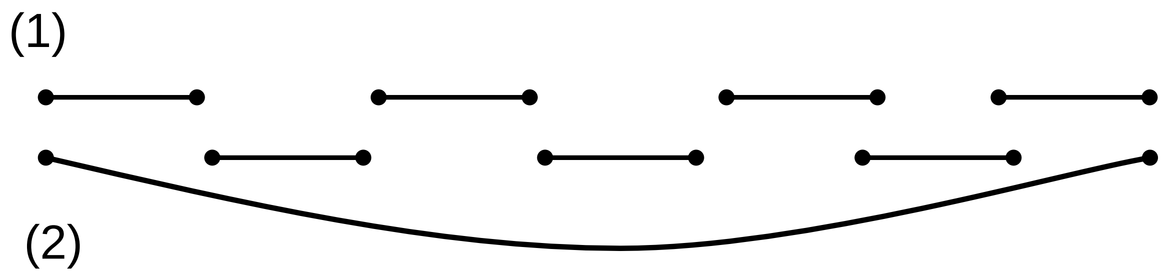


After quenching:

$E \geq 2$ (even) or 1 (odd)

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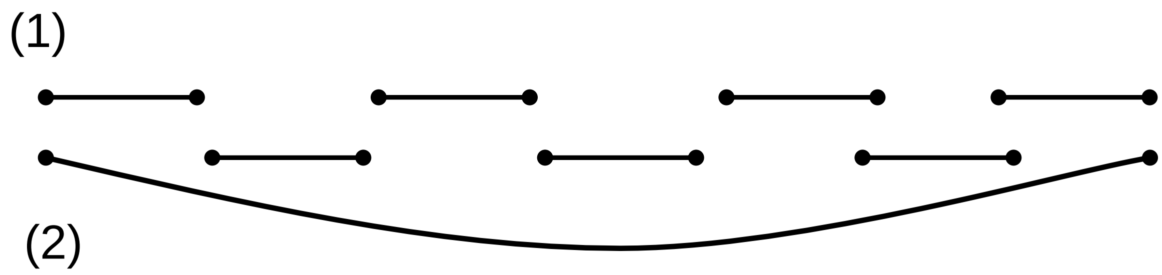


After quenching:

$E \geq 2$ (even) or 1 (odd)

$E \leq \log 5$ (even) or $\log 3$ (odd)

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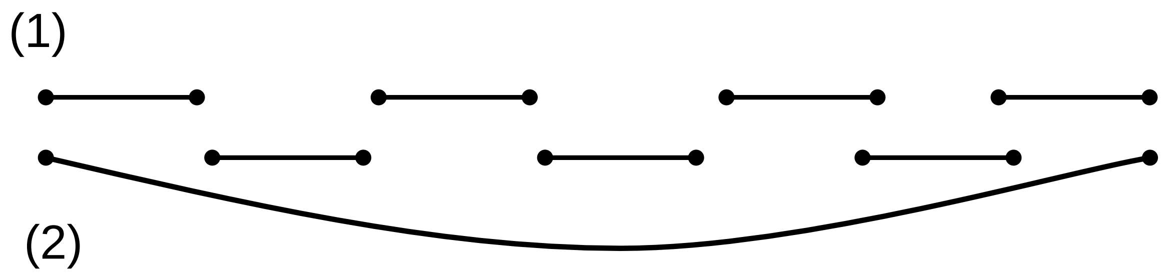


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Numerically, $E = 2.3$ for 8 spins

# Majumdar-Ghosh model

$H = J_1 \sum \sigma_i \sigma_{i+1} + J_2 \sum \sigma_i \sigma_{i+2}$ with $J_1, J_2 > 0$; $J_2 = J_1/2$

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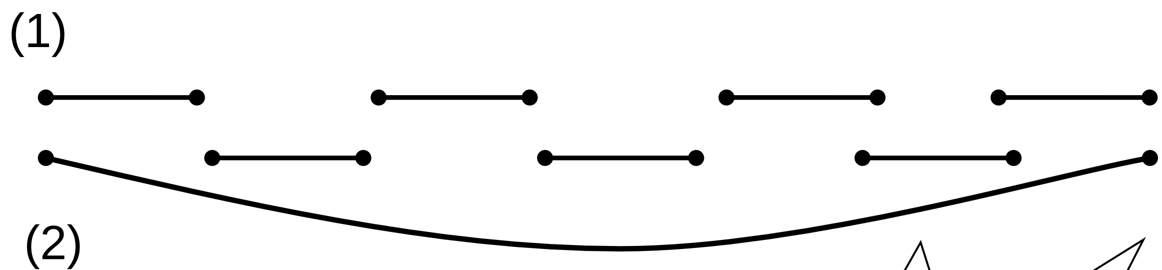


After quenching:

$E \geq 2$ (even) c

$E \leq \log 5$ (e l)

Constant

Numerically, $E = 2.3$ for 8 spins

# *Area Law*
# *for frustrated systems*

→ NO Area law

# Area Law for frustrated systems

NO Area law

Area law

# Outline

1. Understanding entanglement

2. Entanglement in many-body physics

3. What is frustration?

4. Characterizing “classical” frustration in q systems

5. Frustration and Entanglement

    I. Area Law

    II. Genuine multipartite entanglement

6. End remarks

# *Main Thesis*

- Highly frustrated systems do not follow any area law

Highly frustrated systems r near-maximally genuine multi-party entangled

*While*

Weakly frustrated systems do not have a similar definite behavior regarding genuine multi-party entanglement.

- Weakly frustrated systems follow the same area law as nonfrustrated systems away from criticality.

*Sen(De), US, Dziarmaga, Sanpera, Lewenstein, PRL'08*

*Jindal, Rane, Dhar, Sen(De), US, PRA'14*

# Frustrated systems: Area law and Genuine multiparty entanglement

In CONCLUSION

NO Area law

High genuine multiparty entanglement

Area law

No definite genuine multiparty entanglement

# More work done

- Adv. Phys. **56**, 243 (2007)
- Rev. Mod. Phys. **80**, 517 (2008)


Thank you!

QIC Group @ HRI, 2013

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References r incomplete!