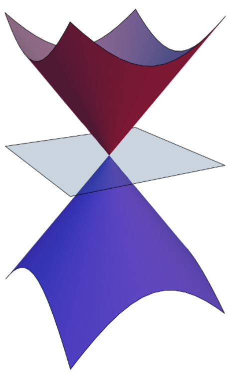
A Hall effect of triplons in the Shastry Sutherland Material



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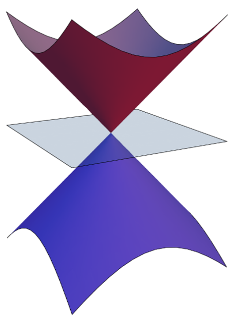
(to appear in *Nature Communications*)

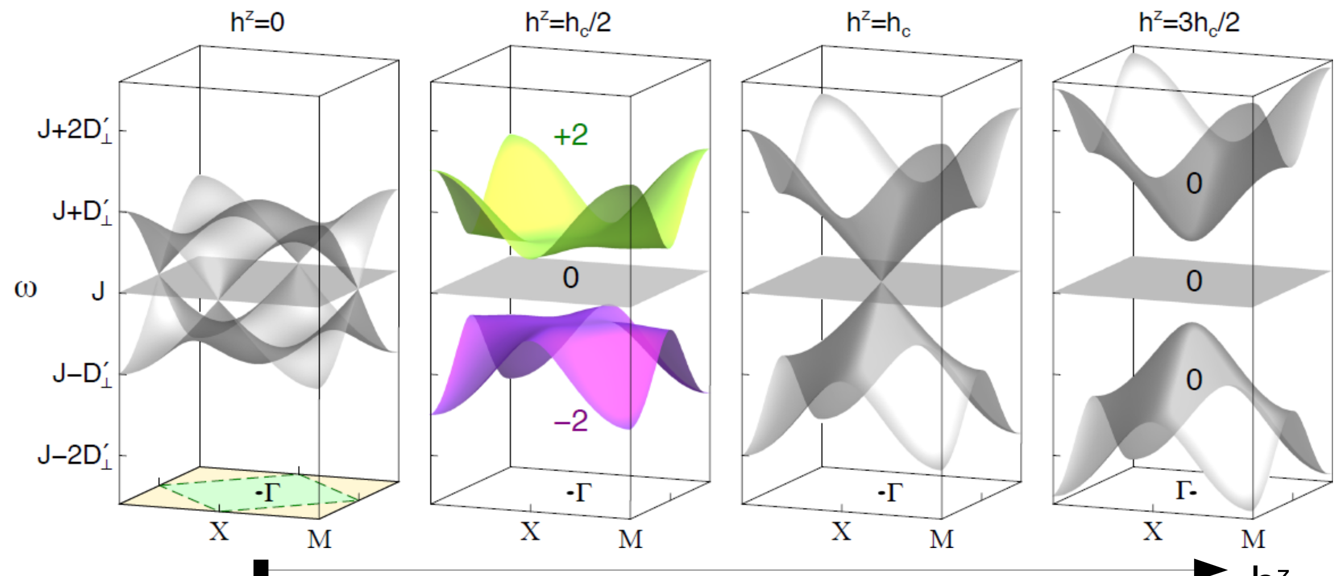
Leibniz Institute for Solid State and Materials Research Dresden

WIGNER

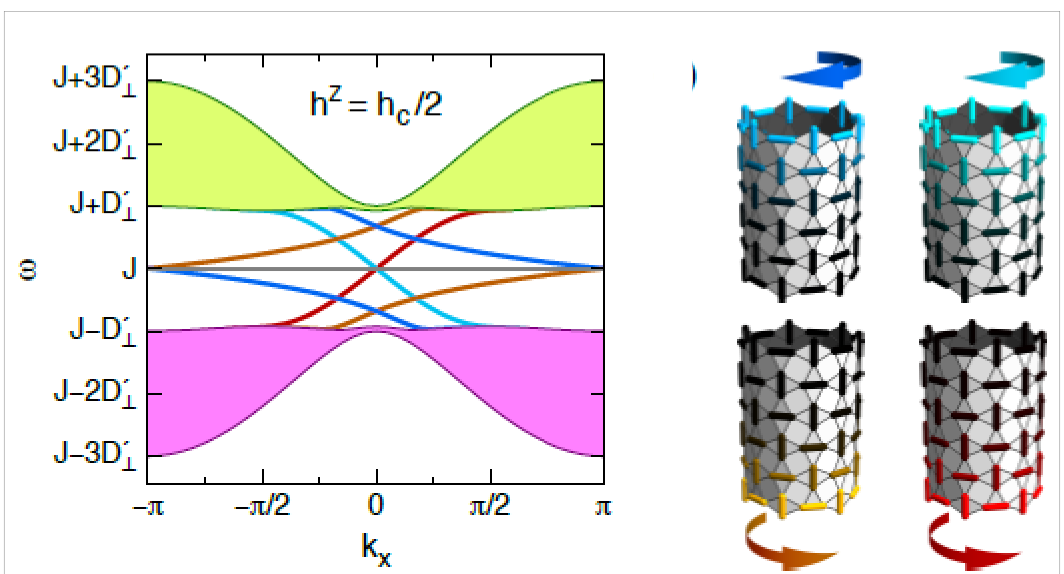
Topology with *triplons* in $SrCu_2(BO_3)_2$



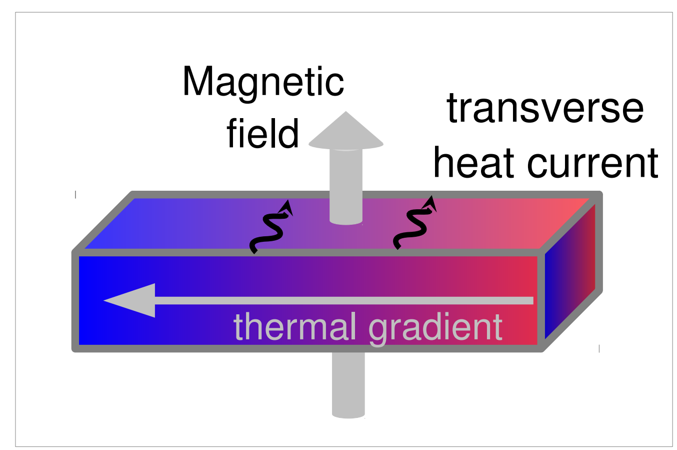
'spin-1 Dirac cone'



Field tuned topological transitions h^z



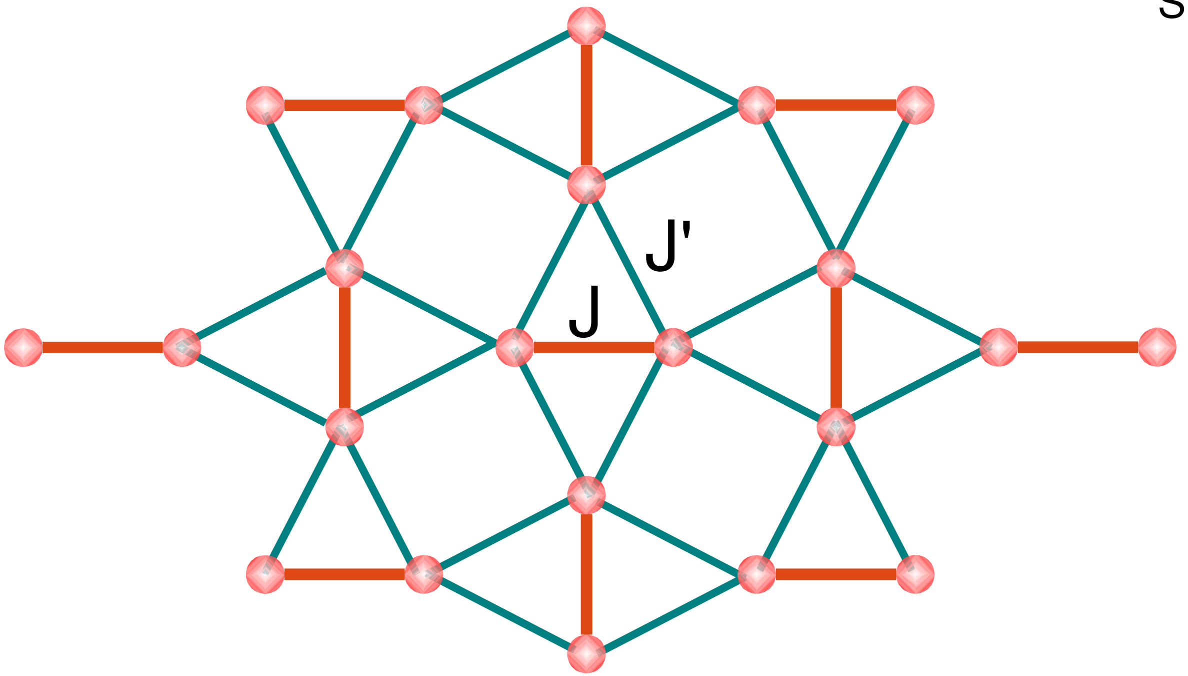
Triplonic edge modes

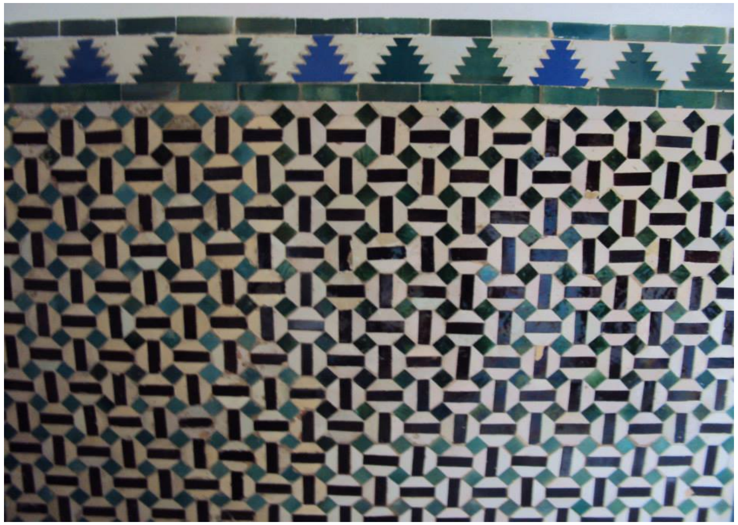


Thermal Hall signal

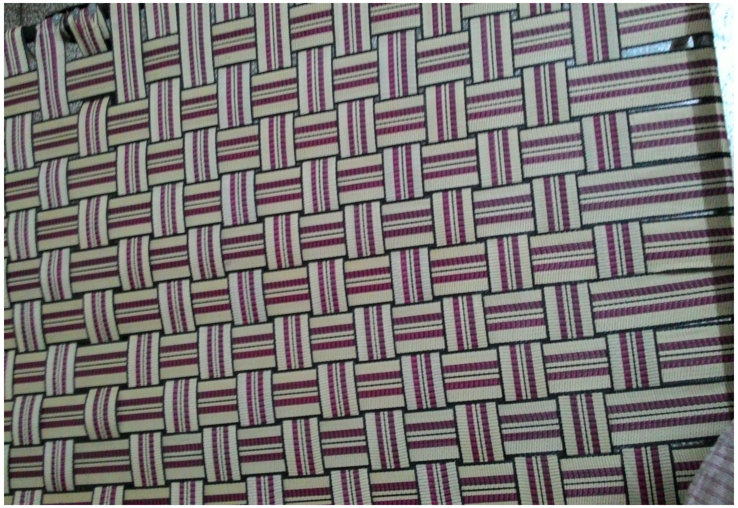
Shastry Sutherland Model

Shastry and Sutherland, Physica B+C 1981





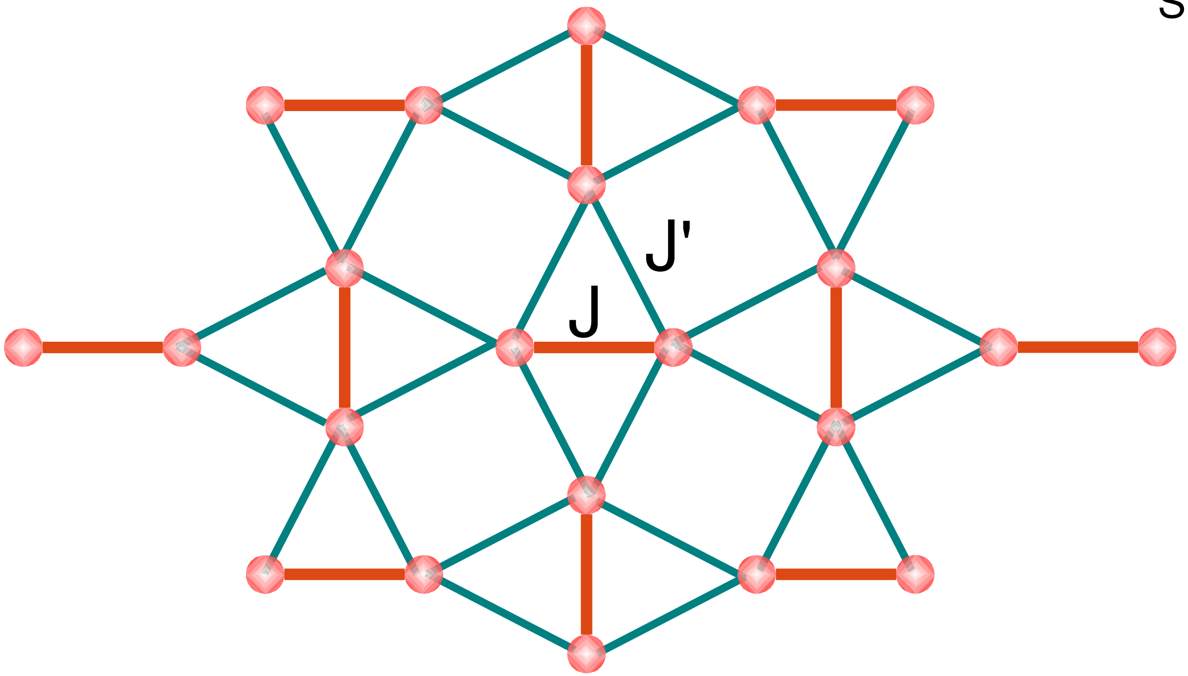
Seville



Delhi

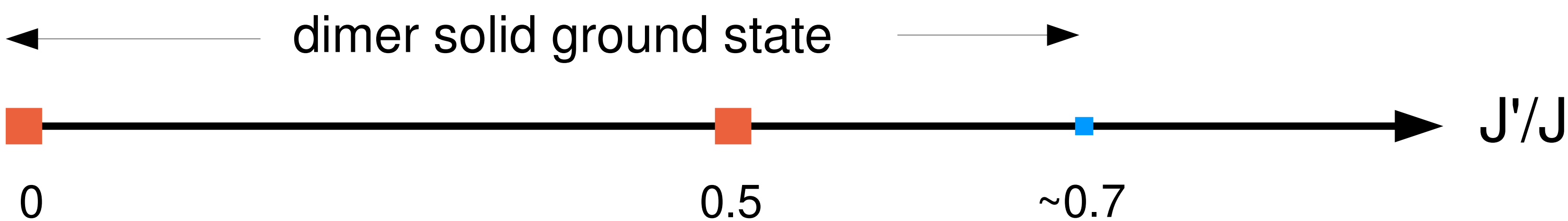
Shastry Sutherland Model

Shastry and Sutherland, Physica B+C 1981



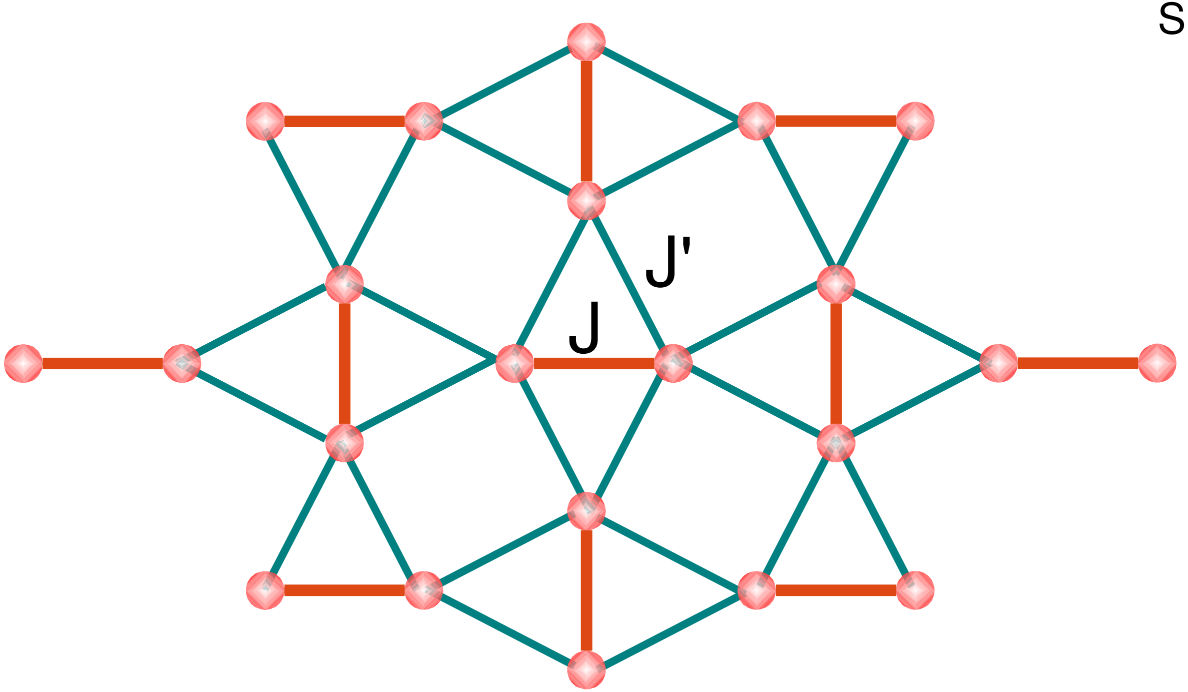
- J'=0 : isolated dimers
- J' = J/2 : exactly solvable limit

} ground state is an arrangement of singlets on dimers



Corboz and Mila, PRB 2013 & references therein

Shastry Sutherland Model



Shastry and Sutherland, Physica B+C 1981

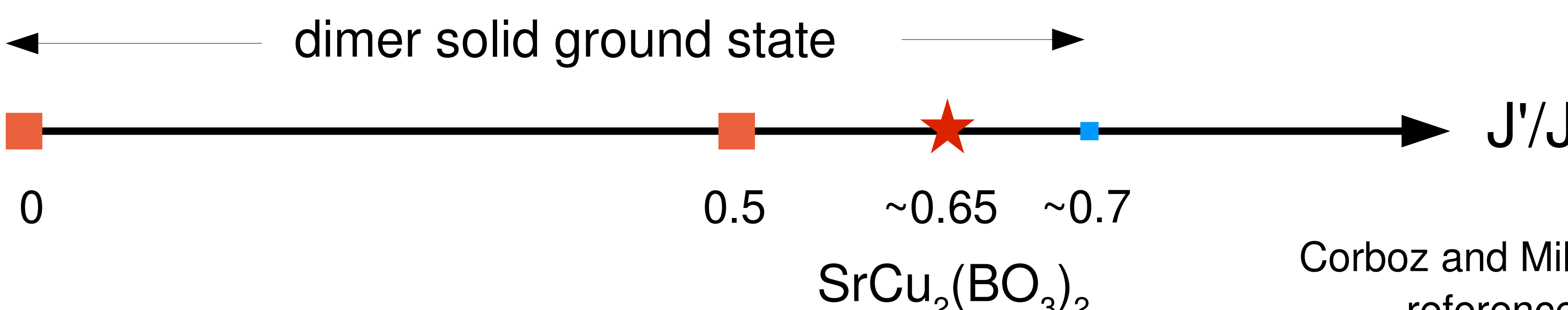
Realized in $SrCu_2(BO_3)_2$:

Cu^{3+} ($3d^9$) S=1/2 moments

Miyahara and Ueda, PRL 1999

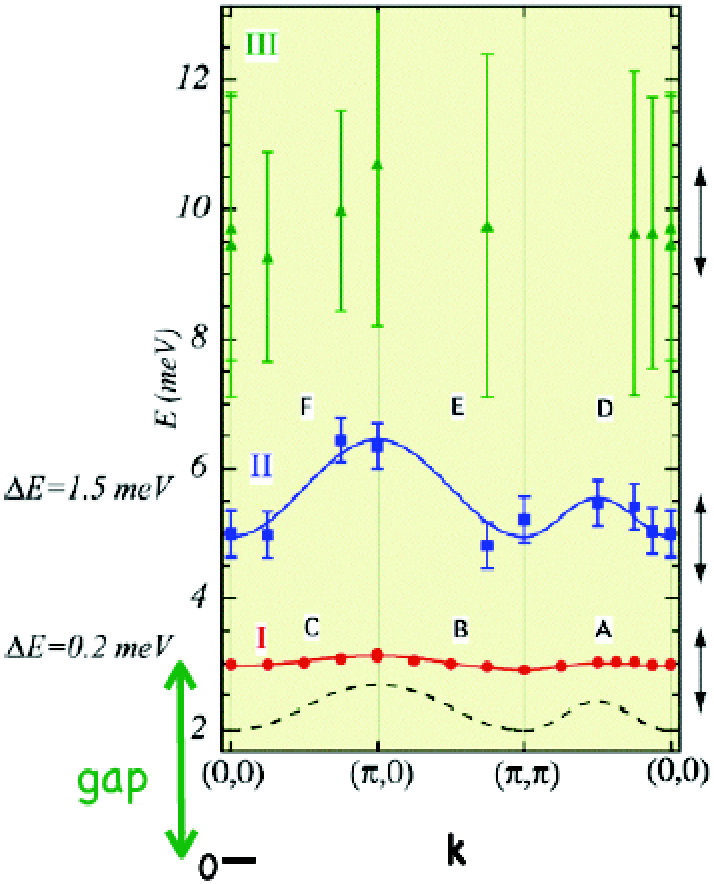
- J'=0 : isolated dimers
- J' = J/2 : exactly solvable limit

} ground state is an arrangement of singlets on dimers



Corboz and Mila, PRB 2013 & references therein

Excitations from neutron scattering

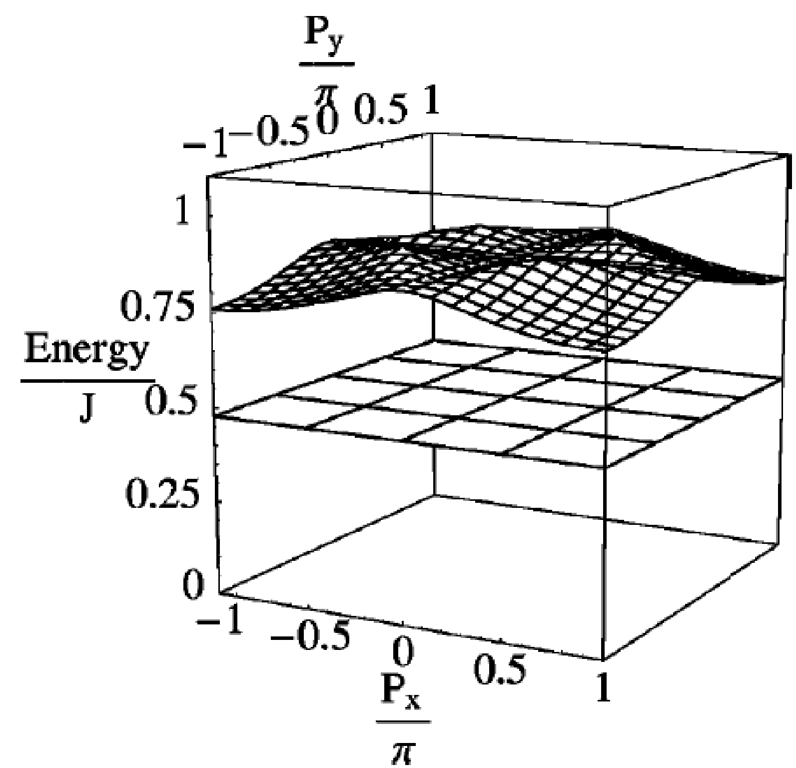


Two triplet excitations

Flat band of single triplet excitations

Kageyama et al, PRL (2000)

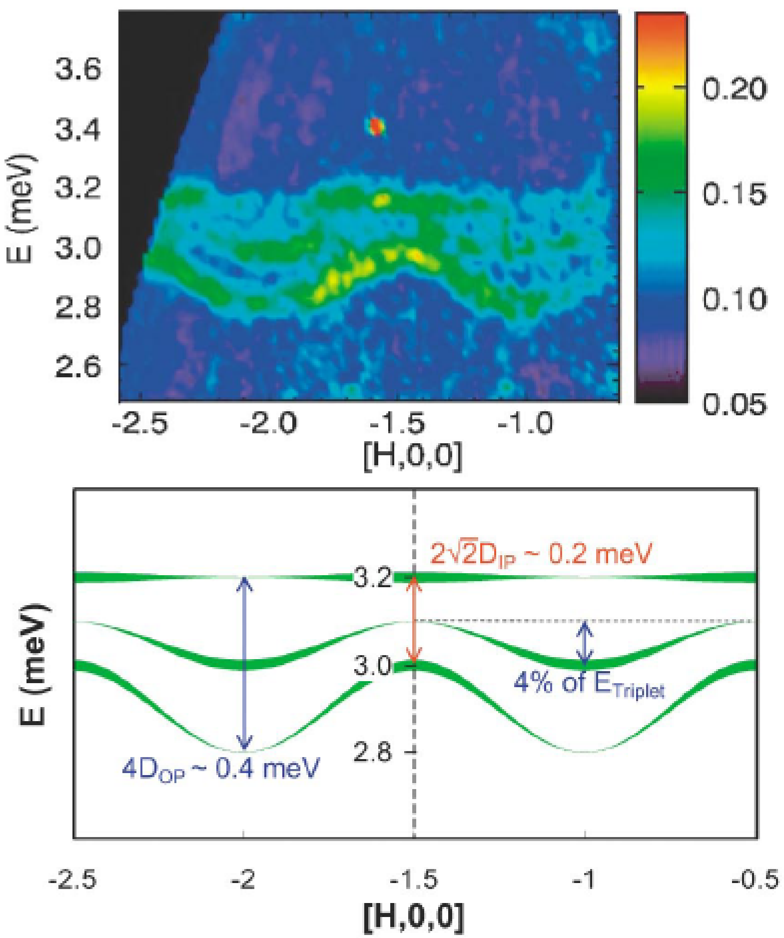
dimer Hilbert space



Momoi and Totsuka, PRB 2000

- Localized triplets ⇒ flat triplet band(s)
- Spin rotational symmetry ⇒ triply degenerate triplet band
  - Weak triplet hopping possible by 6[th] order process

Role of Anisotropies



Gaulin et al, PRL (2004)

- Precise measurements of triplon dispersion
  - neutron scattering
  - Electron Spin Resonance (ESR)
  - Infrared absorption

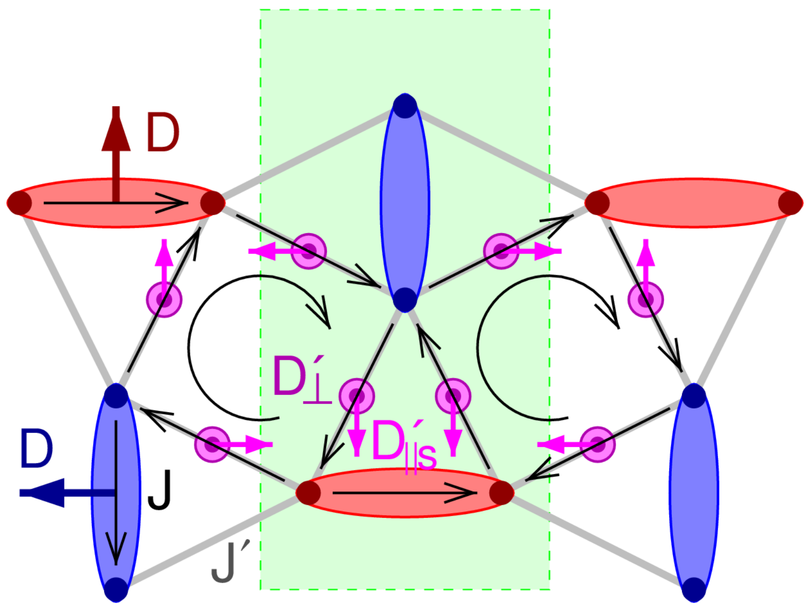
Nojiri et al, JPSJ (2003)

Rõõm et al, PRB (2004)

- Triplet degeneracy broken
- Weakly dispersing bands, bandwidth/gap ~ 10%

➔ Anisotropies arising from Dzyaloshinskii Moriya (DM) interactions

Minimal Hamiltonian



$$\mathcal{H} = J \sum_{n.n.} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{n.n.n.} \mathbf{S}_i \cdot \mathbf{S}_j - g_z h^z \sum_i S_i^z + \sum_{\text{n.n.}} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j) + \sum_{\text{n.n.n.}} \mathbf{D}'_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

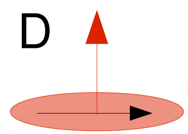
- DM coupling allowed by lattice symmetries

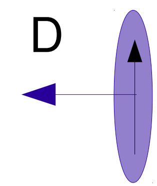
Cépas et al, PRL (2001)
Romhányi et al, PRB (2011)

- Intra-dimer coupling D is in-plane
- Inter-dimer couping D' is predominantly out of plane; only one in-plane component (as shown) enters in our treatment
- We use J=722 GHz, J' = 468 GHz, $D_{\parallel}^{*}$ = 20 GHz, $D'_{\perp}$ = -21 GHz
  - Reproduce ESR data within bond operator theory†
  - Minor corrections to parameters should not affect topological properties

Bond operator theory

$$|\psi_{var}\rangle \equiv \prod_{dimer} |\psi\rangle_{dimer} \qquad E_{var} = \langle \psi_{var}|\hat{H}|\psi_{var}\rangle$$





$$|\tilde{s}\rangle \sim |s\rangle - D/J|t_y\rangle \qquad |\tilde{s}\rangle \sim |s\rangle + D/J|t_x\rangle$$

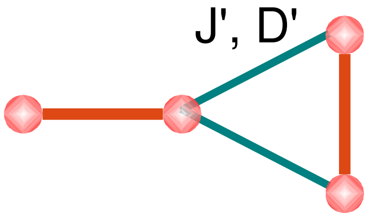
$$\begin{array}{c} |\tilde{t}_x\rangle \sim |t_x\rangle \\ |\tilde{t}_y\rangle \sim |t_y\rangle + D/J|s\rangle \\ |\tilde{t}_z\rangle \sim |t_z\rangle \end{array} \qquad \begin{array}{c} |\tilde{t}_x\rangle \sim |t_x\rangle - D/J|s\rangle \\ |\tilde{t}_y\rangle \sim |t_y\rangle \\ |\tilde{t}_z\rangle \sim |t_z\rangle \end{array}$$

- D, D', $h^z \ll$ J, J'
  - Keep up to linear order
- Small O(D^2) magnetic moments on each dimer
- Three 'triplon' excitations: use a bosonic representation

Dynamics of triplons

- Hopping like processes

$$t_{i\alpha}^{\dagger} t_{j\beta} \sim \langle \tilde{t}_{i,\alpha}, \tilde{s}_j | \hat{H}_{ij} | \tilde{s}_i, \tilde{t}_{j,\beta} \rangle$$



- Pairing like processes - neglect*

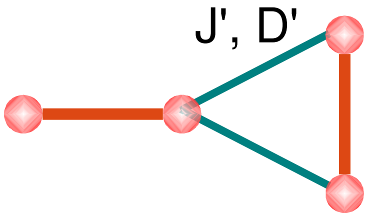
$$t_{i\alpha}^{\dagger} t_{j\beta}^{\dagger} \sim \langle \tilde{t}_{i,\alpha}, \tilde{t}_{j,\beta} | \hat{H}_{ij} | \tilde{s}_i, \tilde{s}_j \rangle$$

  - Involve two triplet excitations
  - Do not affect triplon energy to O(D,D')
  - Negligible in the dilute triplon limit when $T \ll J$
- Neglect 3-particle and 4-particle interactions, assuming dilute triplons

Dynamics of triplons

- Hopping like processes

$$t_{i\alpha}^{\dagger} t_{j\beta} \sim \langle \tilde{t}_{i,\alpha}, \tilde{s}_j | \hat{H}_{ij} | \tilde{s}_i, \tilde{t}_{j,\beta} \rangle$$



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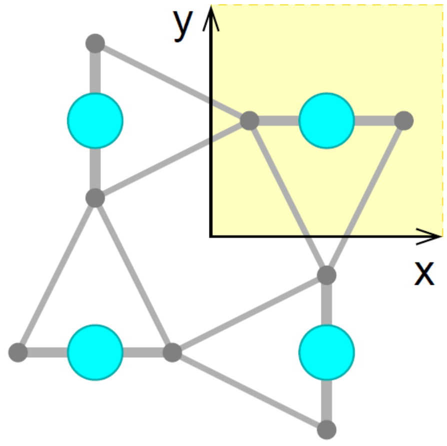
  - Involve two triplet excitations
  - Do not affect triplon energy to O(D,D')
  - Negligible in the dilute triplon limit when T ≪ J
- Neglect 3-particle and 4-particle interactions, assuming dilute triplons
- Unitary transformation renders the two dimers equivalent
  - Square lattice: each site hosts three flavours of bosons



Hopping Hamiltonian

$$H = \sum_{\mathbf{k}} \begin{pmatrix} t^\dagger_{x,\mathbf{k}} & t^\dagger_{y,\mathbf{k}} & t^\dagger_{z,\mathbf{k}} \end{pmatrix} (J\mathbf{1} + \mathbf{d_k} \cdot \mathbf{L}) \begin{pmatrix} t_{x,\mathbf{k}} \\ t_{y,\mathbf{k}} \\ t_{z,\mathbf{k}} \end{pmatrix}$$

- L is a vector of spin-1 (3x3) matrices

$$\mathrm{L} = \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

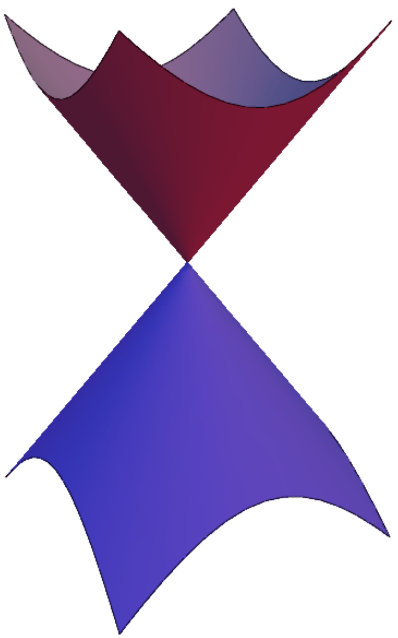
satisfying $[L_\alpha, L_\beta] = i\, \varepsilon_{\alpha\beta\gamma} L_\gamma$ and

$$\mathbf{d}(\mathbf{k}) = \left[D^*_\parallel \sin k_x, D^*_\parallel \sin k_y, -h^z g_z - D'_\perp (\cos k_x + \cos k_y) \right]$$

- $\mathbf{d}_\mathbf{k}$ is a three dimensional vector, a function of momentum
- Three eigenvalues for every **k**: J - $|\mathbf{d}_\mathbf{k}|$, J, J + $|\mathbf{d}_\mathbf{k}|$
- Reproduces ESR peaks with our parameters

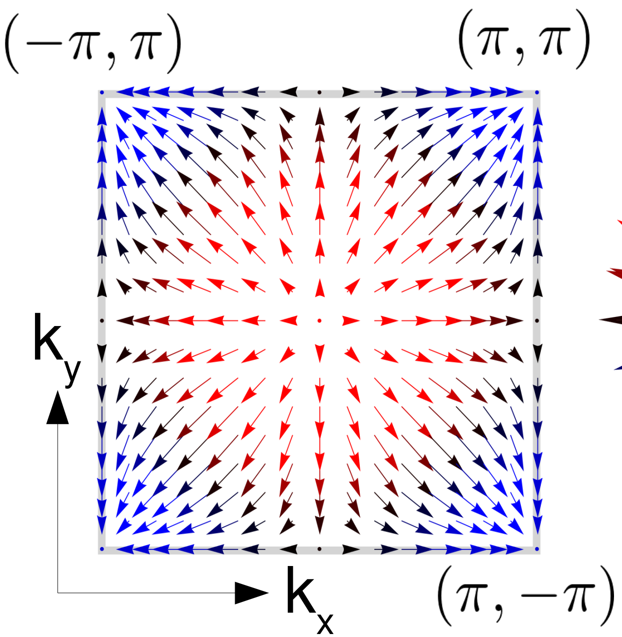
Spin-1/2 analogy: two band problem

- Any 2x2 Hermitian matrix is of the form: $H_{\mathbf{k}} = (J\mathbf{1} + \mathbf{d}_{\mathbf{k}}.\sigma)$
- **σ** are spin-1/2 Pauli matrices; $\mathbf{d}_k$ is a 3 dimensional vector
- Eigenvalues: $J+|\mathbf{d}_k|/2$, $J-|\mathbf{d}_k|/2$
  - Each eigenvalue forms a band over the Brillouin zone
- If $\mathbf{d}_k$ =0 at a point, both bands touch ⇒ Dirac point
- If $\mathbf{d}_k$ is never zero, bands are well separated
  - Topology characterized by Chern number
  - Chern numbers are $+N_{skyrmion}$, $-N_{skyrmion}$

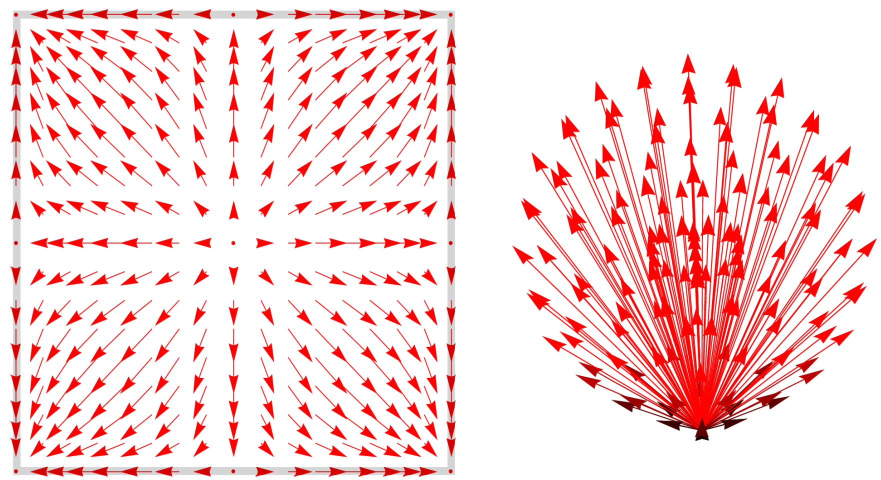


Spin-1/2 analogy: Topology in k-space

- Brillouin zone (BZ) is a 2D torus
- $\mathbf{d}_k$: 3D vector field defined at each point in the BZ
- Topology classified by skyrmion number – maps to Chern number of bands



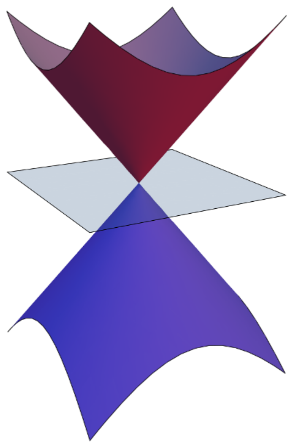
One skyrmion
Chern numbers +1, -1



No skyrmion
Chern numbers 0, 0

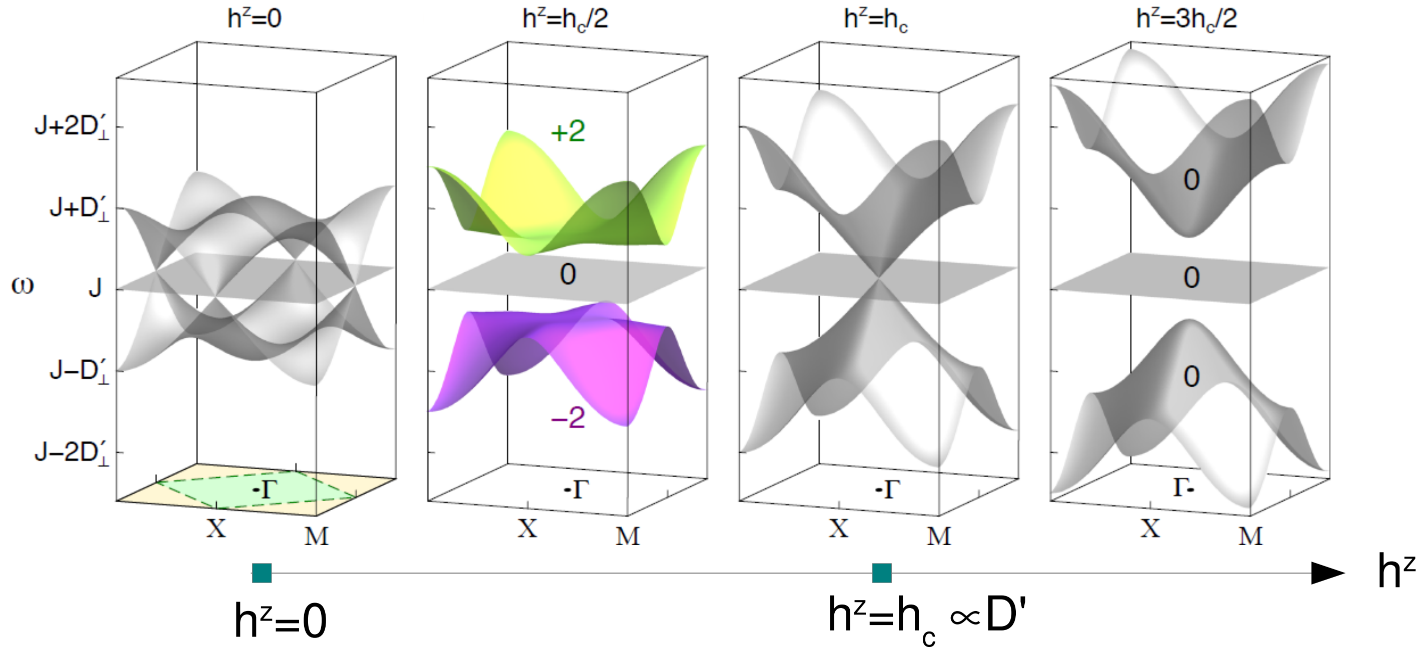
Spin-1 realization: triplons in $SrCu_2(BO_3)_2$

$$H = \sum_{\mathbf{k}} \begin{pmatrix} t^\dagger_{x,\mathbf{k}} & t^\dagger_{y,\mathbf{k}} & t^\dagger_{z,\mathbf{k}} \end{pmatrix} (J\mathbf{1} + \mathbf{d}_\mathbf{k} \cdot \mathbf{L}) \begin{pmatrix} t_{x,\mathbf{k}} \\ t_{y,\mathbf{k}} \\ t_{z,\mathbf{k}} \end{pmatrix}$$

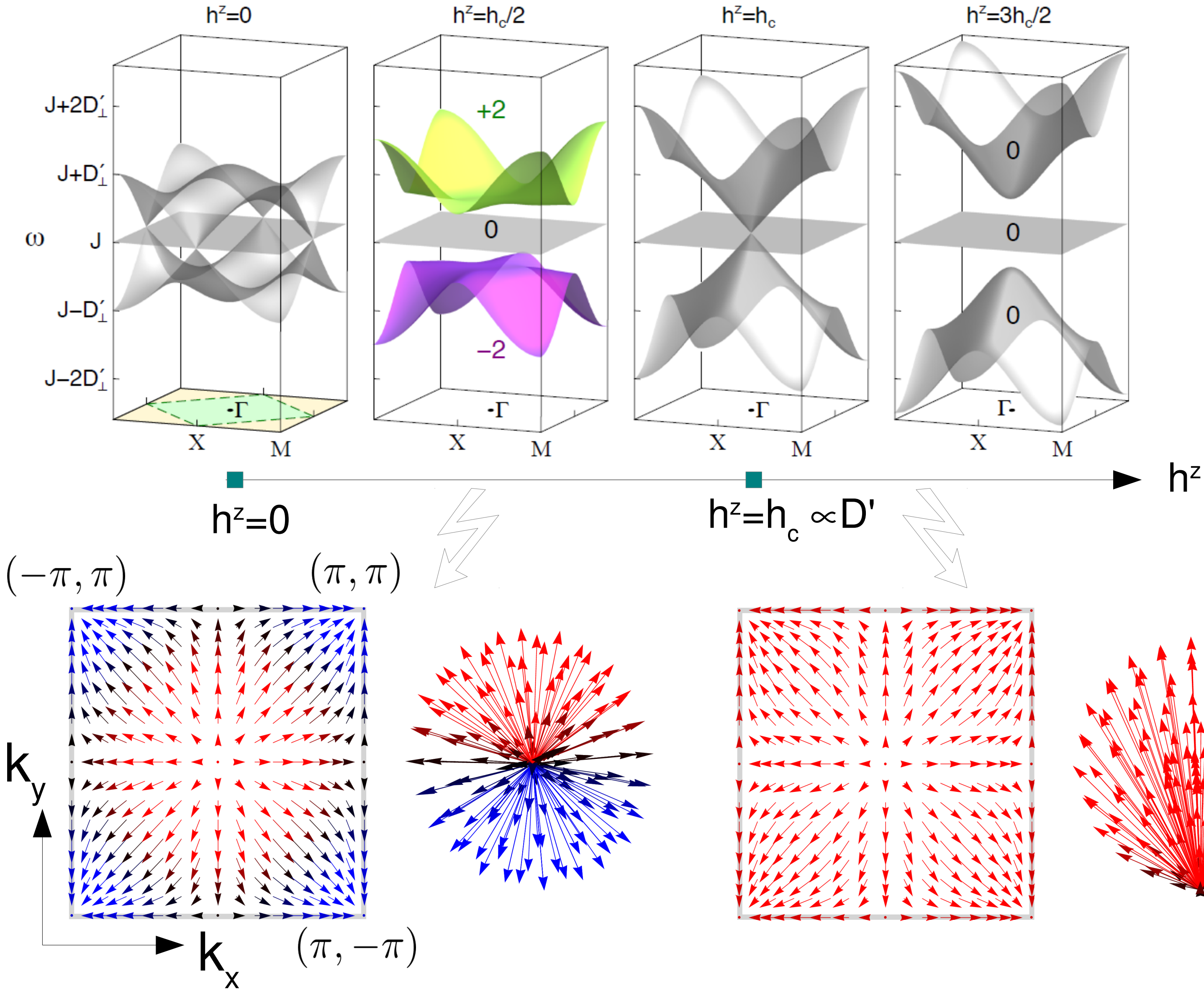


- Not the most general 3x3 unitary Hamiltonian!
- Eigenvalues: J - $|\mathbf{d}_k|$, J, J + $|\mathbf{d}_k|$
  - One flat band with energy J
- When $\mathbf{d}_k$ is zero, three bands touch and form a spin-1 Dirac cone
- If $\mathbf{d}_k$ never vanishes on the BZ, we have three well-separated bands
  - Chern numbers are $-2N_{skyrmion}$, 0, $2N_{skyrmion}$
  - Spin-1 nature of Hamiltonian naturally gives Chern numbers ∓2

Magnetic field tuned topological transitions



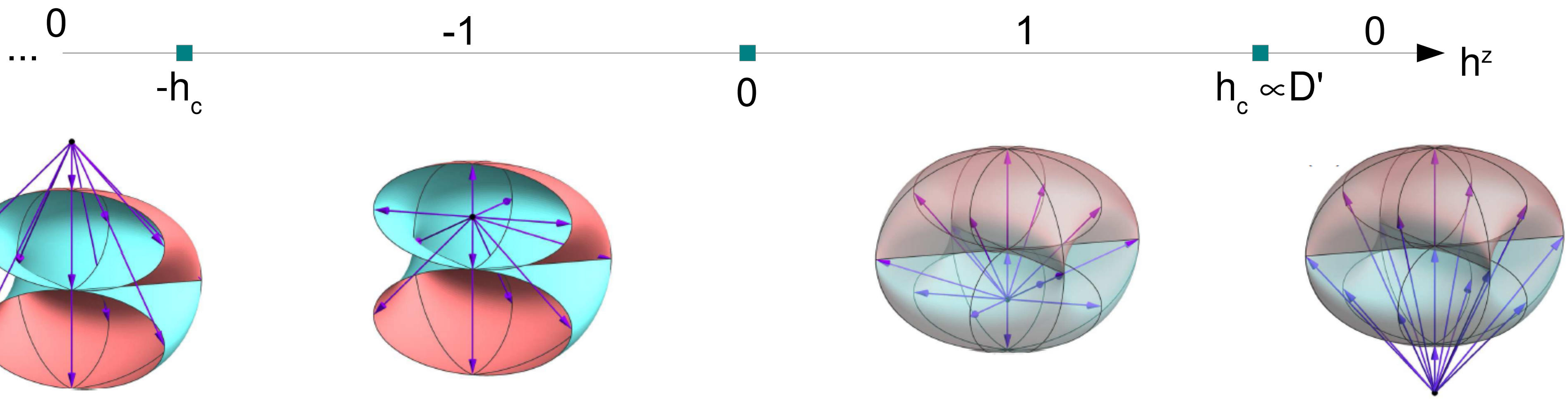
Magnetic field tuned topological transitions



$$\mathbf{d}(\mathbf{k}) = \left[D_{\parallel}^{*}\sin k_x, D_{\parallel}^{*}\sin k_y, -h^z g_z - D'_{\perp}(\cos k_x + \cos k_y)\right]$$

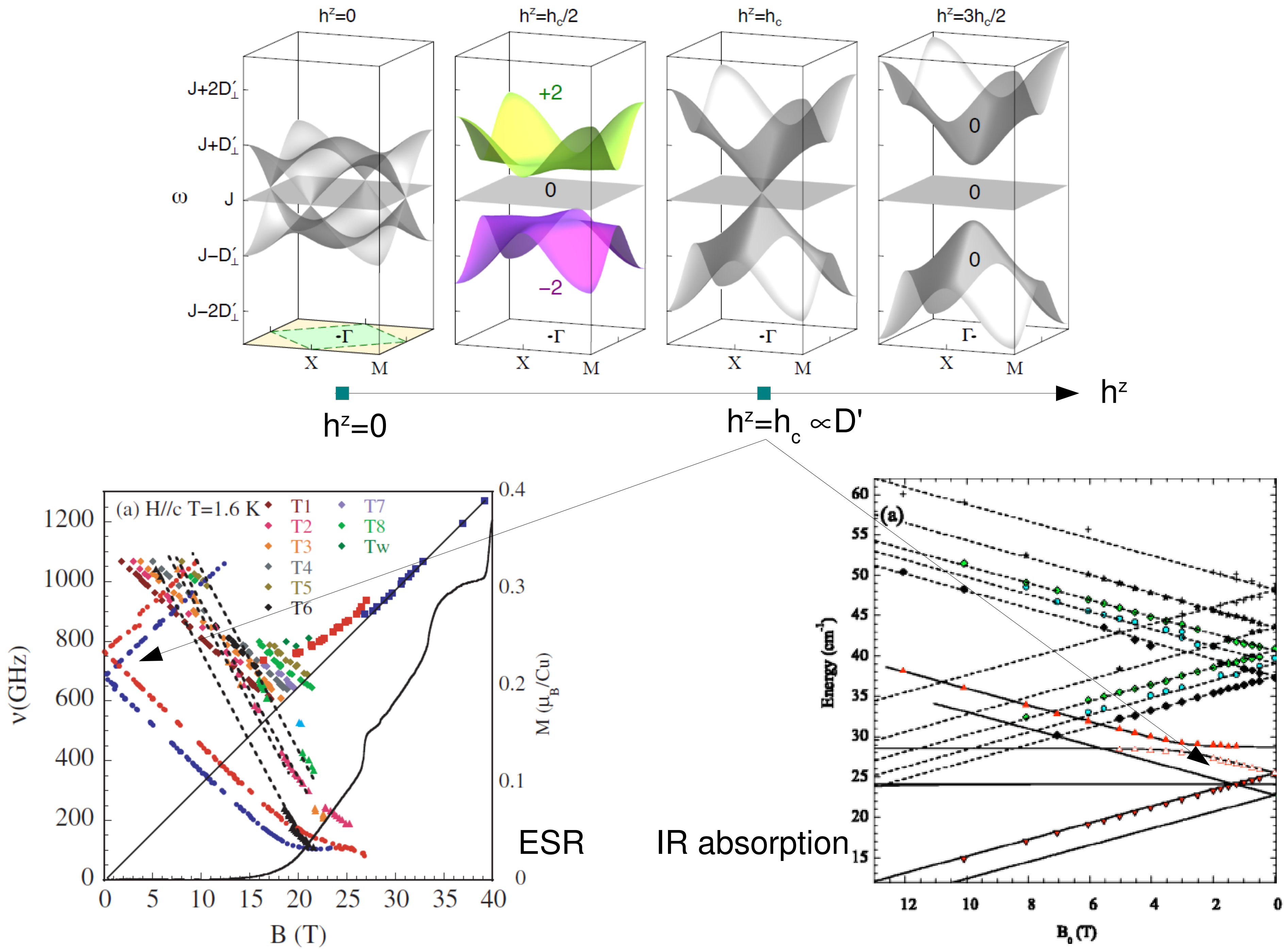
Skyrmions in momentum space

$$\mathbf{d}(\mathbf{k}) = \left[\frac{DJ'}{2J}\sin k_x, \frac{DJ'}{2J}\sin k_y, h^z g_z - 2D'_\perp \frac{1}{2}(\cos k_x + \cos k_y)\right]$$

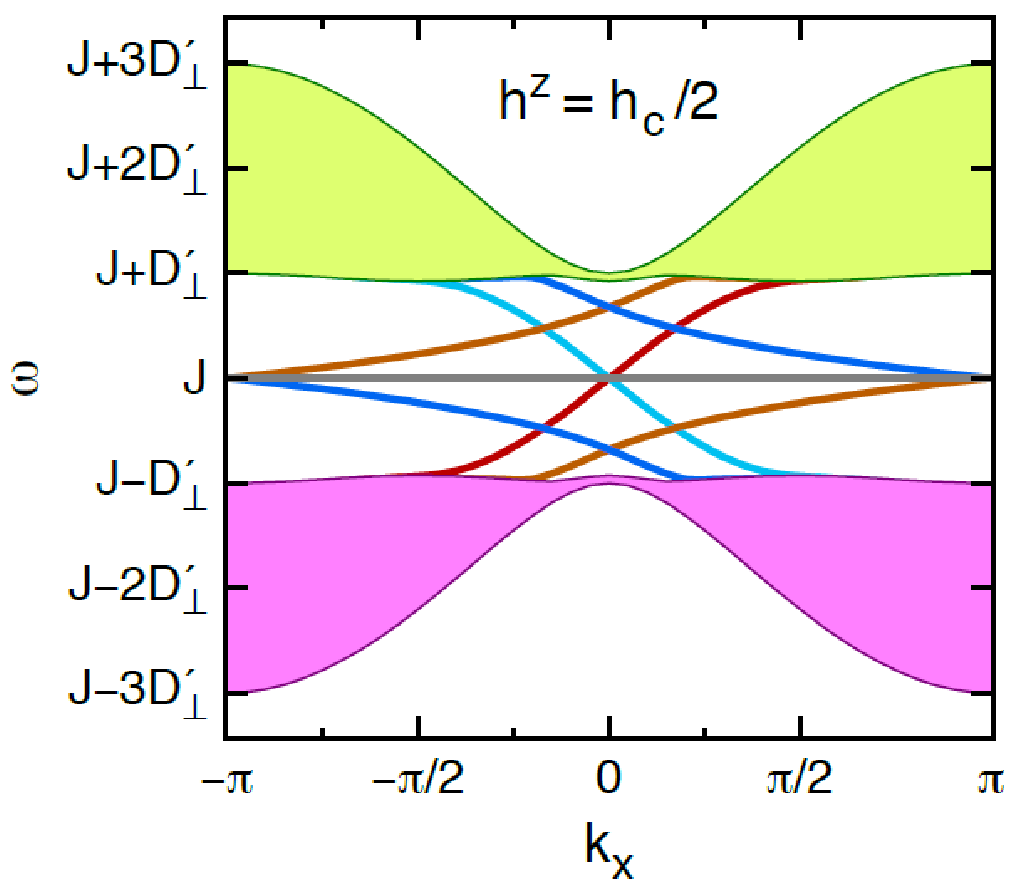


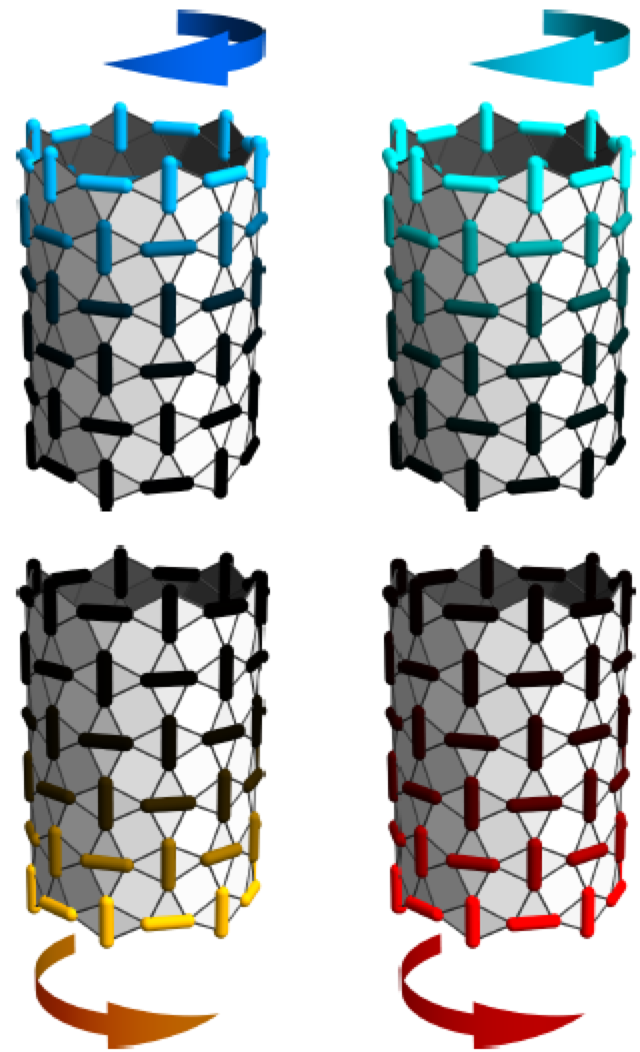
- Associate each momentum in the BZ with a 3D $\mathbf{d}_k$ vector
  - a closed, orientable 2D surface embedded in 3D
  - Composed of two disconnected chambers touching along line nodes
  - Inner surface of upper chamber smoothly connects to outer surface of lower chamber
- If surface passes through origin, $\mathbf{d}_k = 0 \Rightarrow$ gap closes in a spin-1 Dirac point
- Origin is monopole of Berry flux; Chern number is total flux through surface

Spin-1 Dirac point



Protected edge states





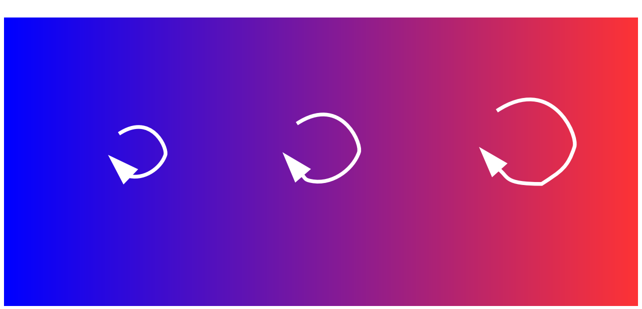
- Edge states are protected by topology
- Even with interactions, protected against damping by energy conservation

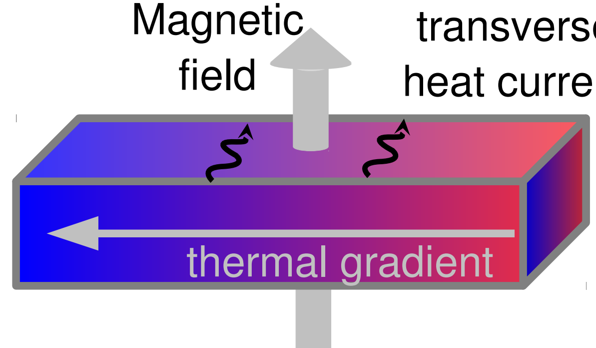
Zhitomirsky and Chernyshev, RMP (2013)

Thermal Hall effect

- Chern bands possible when time reversal is broken
- Electronic systems → integer Hall effect
  - Doping places Fermi level in gap
  - Transverse current carried by edge states
- Bosonic systems: no Fermi level, cannot fully populate a band
  - Not electrical, but heat currents
- Chern bands can be populated thermally
  - Wavepacket in a Chern band has rotational motion Sundaram and Niu, PRB 1999
  - Magnon Hall effect in ferromagnets: DM coupling/dipolar interactions

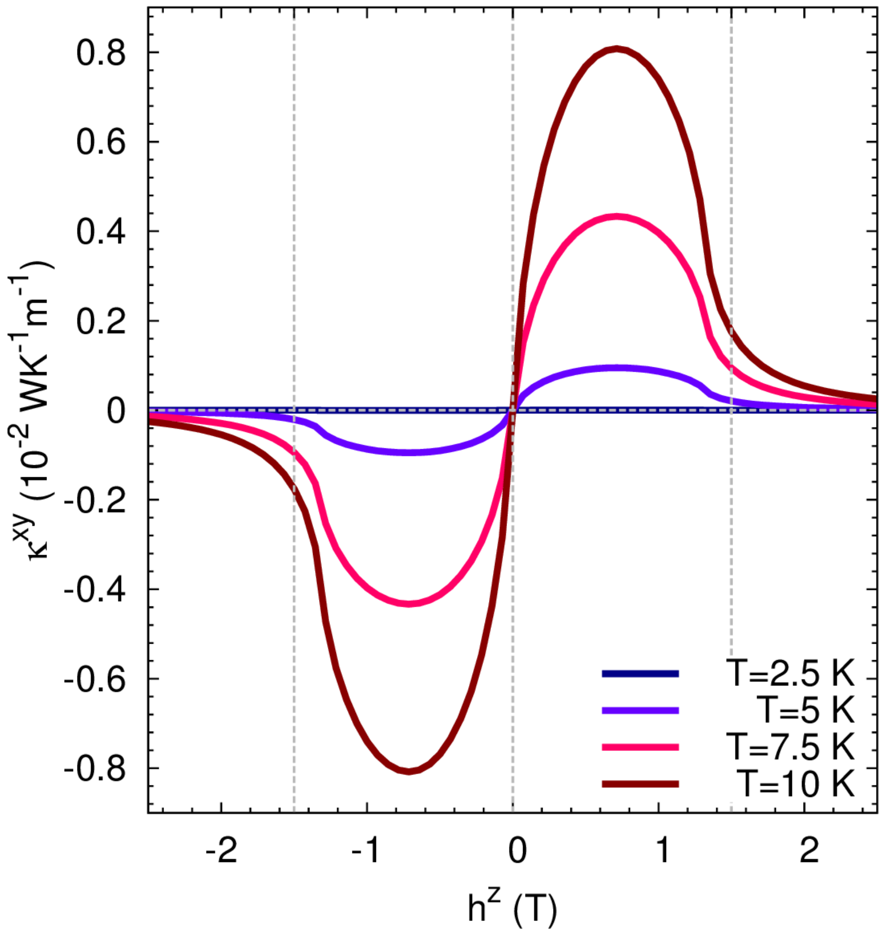
Matsumoto and Murakami, PRB, PRL 2011

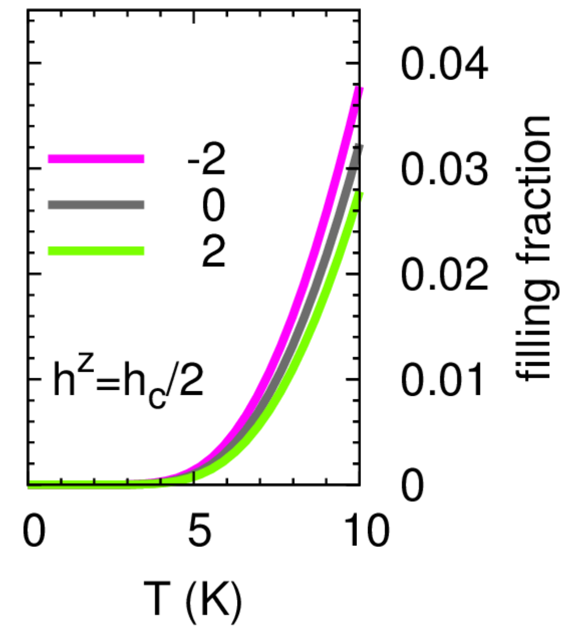




Thermal Hall signal

Matsumoto and Murakami, PRB & PRL 2011





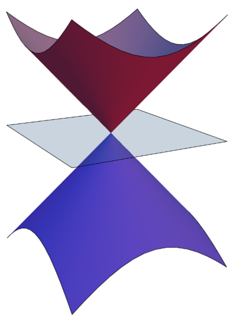
- Within our assumptions, Hall signal increases monotonically with temperature
- At 5 K, neutron scattering shows very little broadening of triplon mode → interactions can be ignored
- Even at ~10 K, band occupation ~ 5% → justifies our quadratic treatment

Bosonic Analogues of IQHE

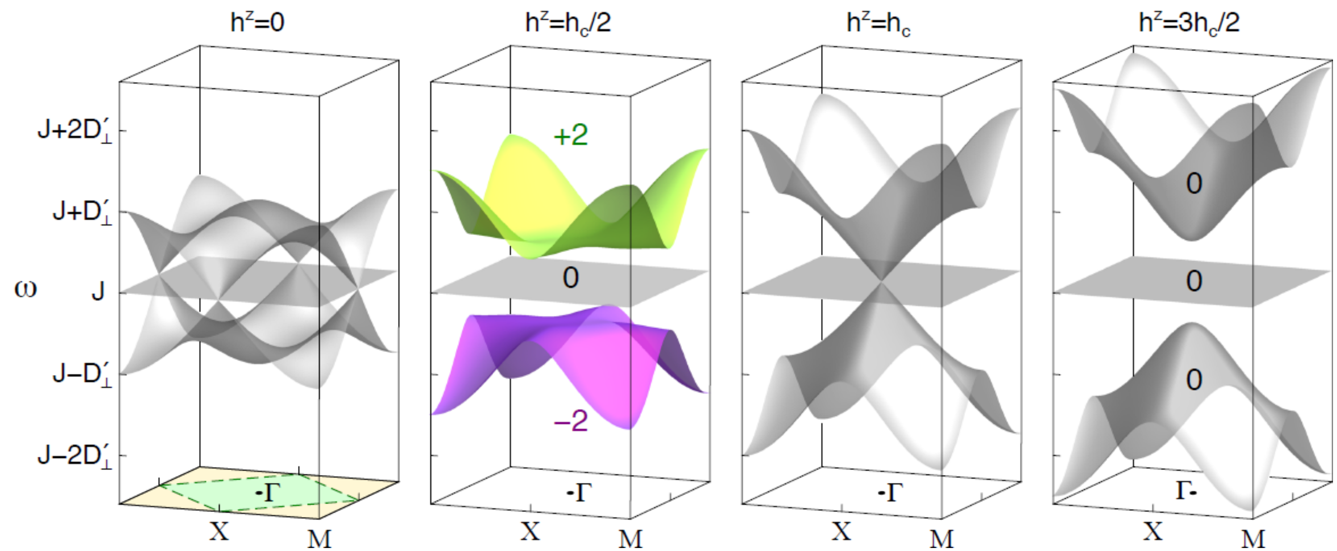
Photons	Photonic crystals with Faraday effect	Raghu et al., PRA 2008
Phonons	Raman spin-phonon coupling	Zhang et al., PRL 2010
Magnons	Kagome ferromagnets with DM	Katsura et al., PRL 2010

- $SrCu_2(BO_3)_2$ is the first quantum magnet to show this physics
- Key ingredient is Dzyaloshinskii Moriya interaction

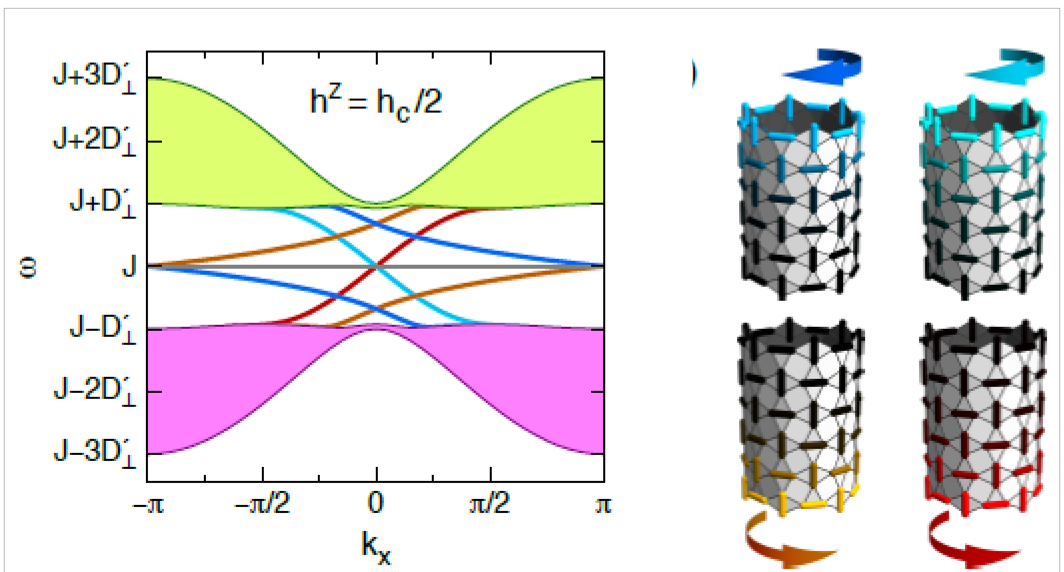
Topology with triplons in $SrCu_2(BO_3)_2$



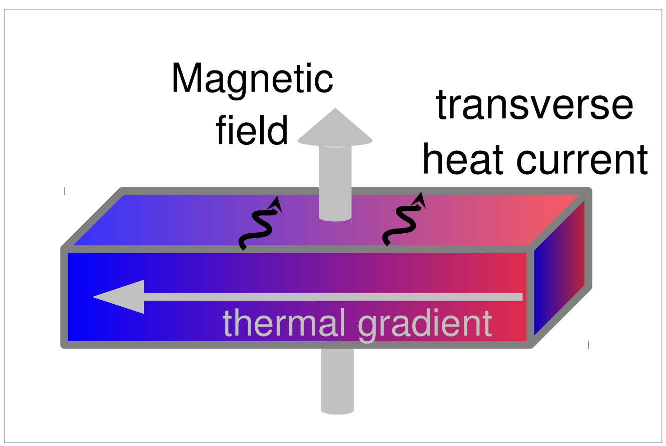
spin-1 Dirac cones



Field tuned topological transitions



Triplonic edge modes



Thermal Hall signal

Effect of next nearest neighbour triplet hopping