Two-step melting of three-sublattice order

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Symmetry breaking transitions: Generalities

- Symmetry-breaking state characterized by long-range correlations of "order-parameter" $\hat{O}$
- phenomenological Landau free energy density $\mathcal{F}[\hat{O}]$
  Expanding $\mathcal{F}$ in powers of $\hat{O}$ (symmetry allowed terms)
- Neglecting derivatives (fluctuations):
  phase transition $\rightarrow$ change in minimum of $\mathcal{F}$

Fluctuation effects at continuous transitions:

- More complete description of long-wavelength physics: Include (symmetry allowed) gradient terms in $\mathcal{F}$
- In most cases: Corrections to mean-field exponents
- In rare cases: Fluctuation-induced first-order behaviour

Symmetries are (usually) decisive:

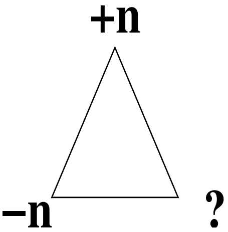
- Transformation properties of $\hat{O}$ determine nature of continuous transition

In this talk...

- Two well-known scenarios for *continuous* melting of three-sublattice order in frustrated triangular and Kagome-lattice easy-axis antiferromagnets:
  Two-step melting with intermediate power-law ordered phase with power-law exponent $\eta(T) \in (\frac{1}{9}, \frac{1}{4})$
  OR
  Three-state Potts transition
- Main message of talk—
  Thermodynamic signature of two-step melting process distinguishes between the two kinds of continuous transitions

Frustrated easy-axis antiferromagnets

- Easy-axis $\mathbf{n}$ and triangular motifs...



Wannier's triangular lattice Ising antiferromagnet

- $H_{\text{Ising}} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$ on the triangular lattice
- $T \to 0$ limit characterized by power-law correlations:
  $\langle \sigma_r^z \sigma_0^z \rangle \sim \frac{\cos(\mathbf{Q} \cdot \mathbf{r})}{r^{1/2}}$
  Incipient order at three-sublattice wavevector $\mathbf{Q} = (2\pi/3, 2\pi/3)$
  Stephenson (1964)
  Power-law spin-liquid in the $T \to 0$ limit

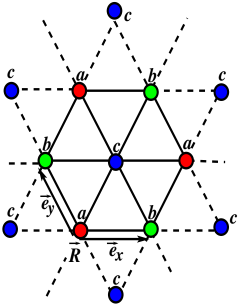
Lattice-gas models for monolayers on graphite

- Three-sublattice long-range order of noble-gas monolayers on graphite
  $H_{\mathrm{J_1J_2}} = J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - J_1\sum_{\langle\langle ij\rangle\rangle}\sigma_i^z\sigma_j^z - J_2\cdots - B\sum_i\sigma_i^z$
  Long-range three-sublattice ordering (wavevector $\mathbf{Q}$) at low temperature
  D. P. Landau (1983)

Prototypical example of order-by-quantum fluctuations

- $H_{\rm TFIM} = J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - \Gamma\sum_i\sigma_i^x$ on the triangular lattice
  Long-range order at three-sublattice wavevector $\mathbf{Q}$
- Equivalent: Plaquette-ordered valence-bond-solid state of honeycomb lattice quantum dimer model
  Moessner, Sondhi, Chandra (2001), Isakov & Moessner (2003)

Ferri vs antiferro three-sublattice order



θ	🔴	🟢	🔵
0	−S	+S	+S
$\frac{\pi}{6}$	−S	0	+S

$\psi = |\psi| e^{i\theta} = -\sum_{\vec{R}} e^{i\mathbf{Q}\cdot\vec{R}} S^z_{\vec{R}}$

Ferri vs antiferro order distinguished by the choice of phase θ

Ferri: $\theta = 2\pi m/6$, Antiferro: $\theta = (2m+1)\pi/6$ $(m = 0, 1, 2 \ldots 5)$

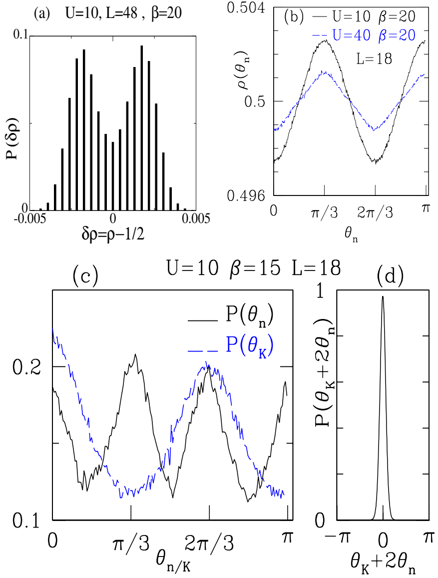
$S = 1$ antiferromagnets with single-ion anisotropy

- $H_{\rm AF} = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j - D\sum_i (S_i^z)^2$ on triangular lattice
- Low-energy physics for $D \gg J$:
  $H_b = -\frac{J^2}{D}\sum_{\langle ij\rangle}(b_i^\dagger b_j + h.c.) + J\sum_{\langle ij\rangle}(n_i - \frac{1}{2})(n_j - \frac{1}{2}) + \ldots$
- Low-temperature state for $D \gg J$: "supersolid" state of hard-core bosons at half-filling on triangular lattice with unfrustrated hopping $t = J^2/D$ and frustrating nearest-neighbour repulsion $U = J$
- Implies: Coexisting three-sublattice order in S^z and "ferro-nematic" order in $\vec{S}_\perp^2$ (KD & Senthil 2006)
  (Simple easy-axis version of Chandra-Coleman (1991) "spin-nematic" ideas)

Is three-sublattice ordering of S^z in H_{AF} ferri or antiferro?

- Natural expectation: Quantum fluctuations induce antiferro order
  $\rightarrow$
  Ordering will be antiferro three-sublattice order (like transverse field Ising antiferromagnet)
  e. g. Melko *et. al.* (2005)

QMC evidence: Ferri three-sublattice order of S^z



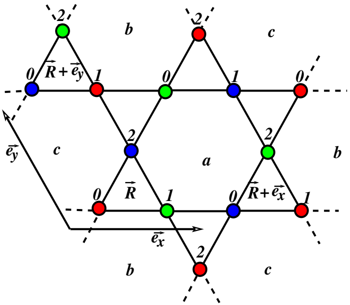
Heidarian and KD (2005)

Ising models for “Artificial Kagome-ice”

- $H_{\text{Kagome}} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - J_1 \sum_{\langle\langle ij \rangle\rangle} \sigma_i^z \sigma_j^z - J_2 \ldots$
- Only nearest-neighbour couplings $\rightarrow$ classical short-range spin liquid (Kano & Naya 1950)
- Further neighbour couplings destabilize spin liquid $\rightarrow$ three-sublattice order at low T (Wolff & Schotte 1988)
- “Artificial Kagome-ice: Moments $\mathbf{M}_i = \sigma_i^z \mathbf{n}_i$
  ($\mathbf{n}_i$ at different sites non-collinear)
  Expt: Tanaka *et. al.* (2006), Qi *et. al.* (2008), Ladak *et. al.* (2010,11)
  Theory: Moller, Moessner (2009), Chern, Mellado, Tchernyshyov (2011)

Three-sublattice order on the Kagome lattice

θ	● (red)	● (green)	● (blue)
0	−S	+S	+S
$\frac{\pi}{6}$	−S	0	+S



$$\psi = |\psi| e^{i\theta} = -\sum_{\vec{R}} \sum_{\alpha=0,1,2} e^{i\mathbf{Q}\cdot\vec{R} - 2\pi i \frac{\alpha}{3}} S^z_{\vec{R},\alpha}$$

Ferri vs antiferro distinguished by the choice of phase θ

Ferri: $\theta = 2\pi m/6$, Antiferro: $\theta = (2m+1)\pi/6$ ($m = 0, 1, 2 \ldots 5$)

Landau-theory for melting of three-sublattice order

- $\mathcal{F} = K|\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 + \lambda_6(\psi^6 + \psi^{*6}) + \dots$
  Connection to physics of six-state clock models
  $Z = \sum_{\{p_i\}} \exp[\sum_{\langle ij\rangle} V(\frac{2\pi}{6}(p_i - p_j))]$
  Each $p_i = 0, 1, 2, ...5$
  $V(x) = K_1\cos(x) + K_2\cos(2x) + K_3\cos(3x)$
  Cardy (1980)

Melting scenarios for three-sublattice order

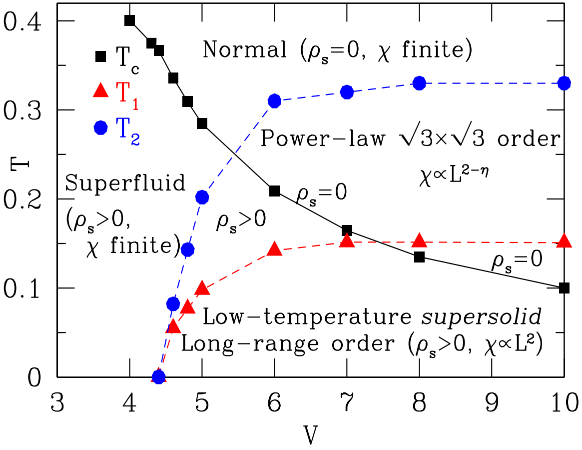
- Analysis (Cardy 1980) of generalized six-state clock models
  $\rightarrow$ Three generic possibilities of relevance here:
  Two-step melting, with power-law ordered intermediate phase
  OR
  3-state Potts transition
  OR
  First-order transition (always possible!)

Both these continuous melting scenarios realized in one or more examples on triangular and kagome lattices

Nature of melting transition in triangular lattice supersolid?

- Clearly: Nature of melting transition not *a priori* obvious
- Prediction of Boninsegni & Prokofiev (2005)
  Three-state Potts transition
  Prediction based on argument about relative energies of different kinds of domain walls
  hard to get right at quantitative level

Our answer from large-scale QMC simulations



KD & Heidarian (*in preparation*)

Detecting power-law order?

Need extremely sensitive scattering experiment to detect power-law version of Bragg peaks

Or

High resolution *real-space* data by scanning some local probe $+$ Lots of image-processing

difficult!

Alternate thermodynamic signature(!)

- Singular thermodynamic susceptibility to *uniform* easy-axis field B:
  $\chi_u(B) \sim \frac{1}{|B|^{p(T)}}$
- $p(T) = \frac{4-18\eta(T)}{4-9\eta(T)}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$
  So $p(T)$ varies from $1/3$ to 0 as T increases from T_1 to just below T_2

(KD 2014, *with referees*)

Review: picture for power-law ordered phase

- In state with long-range three-sublattice order, θ feels $\lambda_6 \cos(6\theta)$ potential.
  Locks into values $2\pi m/6$ (resp. $(2m+1)\pi/6$) in ferri (resp. antiferro) three-sublattice ordered state for $T < T_1$
- In power-law three-sublattice ordered state for $T \in (T_1, T_2)$, λ_6 does not pin phase θ
  θ spread uniformly $(0, 2\pi)$
  Distinction between ferri and antiferro three-sublattice order lost for $T \in (T_1, T_2)$

Review: more formal RG description

- Fixed point free-energy density: $\frac{\mathcal{F}_{\text{KT}}}{k_B T} = \frac{1}{4\pi g}(\nabla\theta)^2$
  with $g(T) \in (\frac{1}{9}, \frac{1}{4})$ corresponding to $T \in (T_1, T_2)$
- $\lambda_6 \cos(6\theta)$ *irrelevant* along fixed line
- $\langle \psi^*(r)\psi(0)\rangle \sim \frac{1}{r^{\eta(T)}}$
  with $\eta(T) = g(T)$

Jose, Kadanoff, Kirkpatrick, Nelson (1977)

General argument for result—I

- Landau theory admits term $\lambda_3 m(\psi^3 + \psi^{*3})$
  m is uniform magnetization mode
- Formally irrelevant along fixed line $\mathcal{F}_{\mathrm{KT}}$
  $\rightarrow$
  Physics of two-step melting unaffected—m "goes for a ride..."

But ...

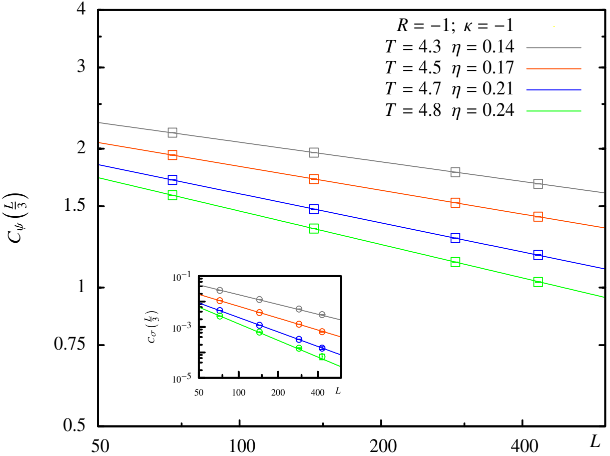
General argument for result—II

- m "inherits" power-law correlations of $\cos(3\theta)$:
  $C_m(r) = \langle m(r)m(0)\rangle \sim \frac{1}{r^{9\eta(T)}}$
- $\chi_L \sim \int^L d^2 r C_m(r)$ in a finite-size system at $B = 0$
- $\chi_L \sim L^{2-9\eta(T)}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$
  Diverges with system size at $B = 0$

General argument for result—III

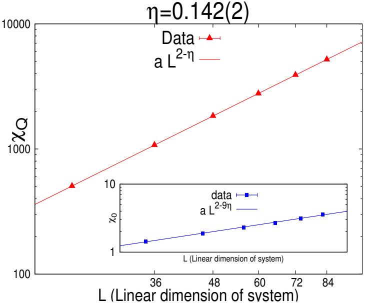
- Uniform field $B > 0 \rightarrow$ additional term $h_3 \cos(3\theta)$ in $\mathcal{F}_{\mathrm{KT}}$
- Strongly relevant along fixed line, with RG eigenvalue $2 - 9g/2$
- Implies finite correlation length $\xi(B) \sim |B|^{-\frac{2}{4-9\eta}}$
- $\chi_u(B) \sim |B|^{-\frac{4-18\eta}{4-9\eta}}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$

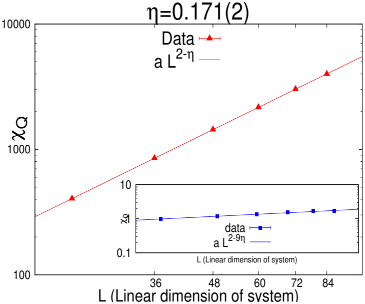
The proof of the pudding...I



In power-law ordered phase of $H_{\mathrm{J_1J_2}}$ ($R = -(J_1 + J_2)/J$ and $\kappa = (J_2 - J_1)/J$) (Ghanshyam, KD (*in preparation*))

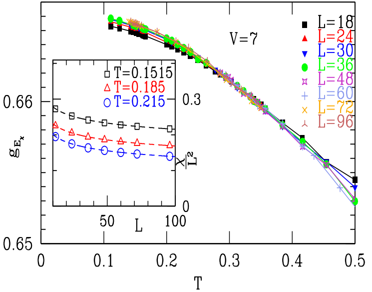
The proof of the pudding...II

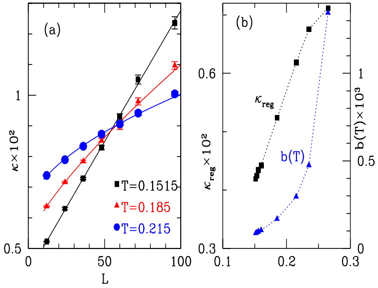




In power-law ordered phase of H_{TFIM} (Biswas, KD (*in preparation*))

The proof of the pudding...III





In power-law ordered phase of H_b (KD, Heidarian (*in preparation*))

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