THE HEISENBERG MODEL ON THE KAGOME LATTICE: RECENT DEVELOPMENTS

YASIR IQBAL
University of Würzburg, Würzburg, Germany

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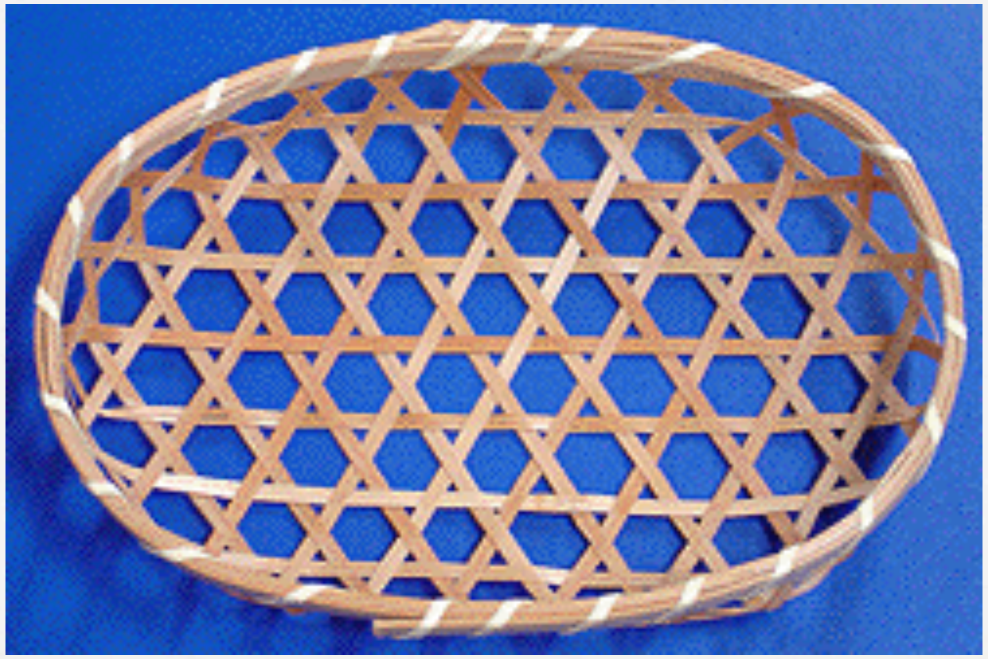
Collaborators:
Federico Becca (SISSA, Trieste, Italy)
Didier Poilblanc (LPT, Toulouse, France)

Ronny Thomale (Uni-Würzburg, Germany)
Johannes Reuther (Freie Universität, Berlin, Germany)

S=1/2 HEISENBERG ANTIFERROMAGNET ON THE KAGOME LATTICE

$$\hat{\mathcal{H}} = J_1 \sum_{\langle ij \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$



COMPETING PHASES

SPIN LIQUIDS

TOPOLOGICAL SPIN LIQUIDS

- ✶ Short range RVB states.
- ✶ Gapped Z2 spin liquids. — **Suttner' 14**
- ✶ Possibly chiral in nature? — **Messio' 12**
- ✶ Supported by DMRG. — **White' 11, Depenbrock' 12**

CRITICAL SPIN LIQUIDS

- ✶ Long range RVB states.
- ✶ Algebraic $U(1)$ Dirac spin liquid. — **Ran' 07**

VBC's

- CORE → *6-site UNIT CELL VBC's* — **Budnik' 04**
- QDM → *12-site UNIT CELL VBC's* — **Poilblanc' 11**
- Series Exp → *36-site UNIT CELL VBC's* — **Singh' 07**

MATHS FOR SPIN LIQUIDS

$$\hat{\mathbf{S}}_i = \frac{1}{2} \mathrm{c}^{\dagger}_{i,\alpha} \hat{\sigma}^{\alpha\beta} \mathrm{c}_{i,\beta}$$

Spinons $\mathrm{c}_{i,\alpha}$: S =1/2 charge neutral quasi-particles.
Mathematical trick + fractionalisation +
$SU(2)$ high energy gauge structure.

Baskaran' 87

Hubbard-Stratonovich transformation + Mean field approx.

$$\hat{\mathcal{H}}_{\mathrm{MF}} = \sum_{i,j,\alpha} (\chi_{ij} + \mu\delta_{ij}) c^{\dagger}_{i,\alpha} c_{j,\alpha} + \sum_{i,j} \{(\Delta_{ij} + \zeta\delta_{ij}) c^{\dagger}_{i,\uparrow} c^{\dagger}_{j,\downarrow} + h.c.\}$$

$$|\Psi_{\mathrm{VMC}}(\chi_{ij}, \Delta_{ij}, \mu, \zeta)\rangle = \mathcal{P}_{\mathbf{G}} |\Psi_{\mathrm{MF}}(\chi_{ij}, \Delta_{ij}, \mu, \zeta)\rangle$$

$$\mathcal{P}_{\mathbf{G}} = \prod_i (1 - n_{i,\uparrow} n_{i,\downarrow})$$

Enforcing one-fermion per site constraint, and hence including correlations into the system.

U(1) DIRAC SPIN LIQUID

$$\hat{\mathcal{H}}_{\mathrm{MF}} = \chi \sum_{\langle ij \rangle} s_{ij} \mathrm{c}^{\dagger}_{j,\alpha} \mathrm{c}_{i,\alpha} + \mathrm{h.c.}$$

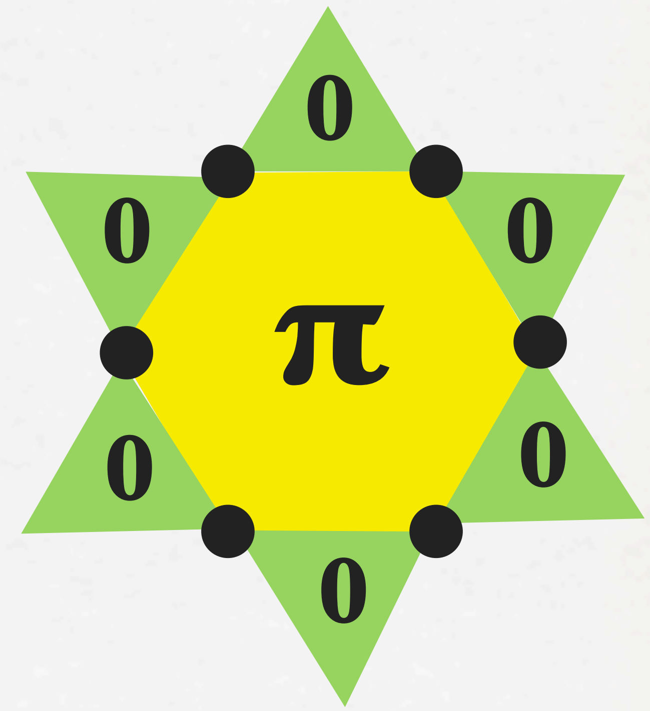
± 1

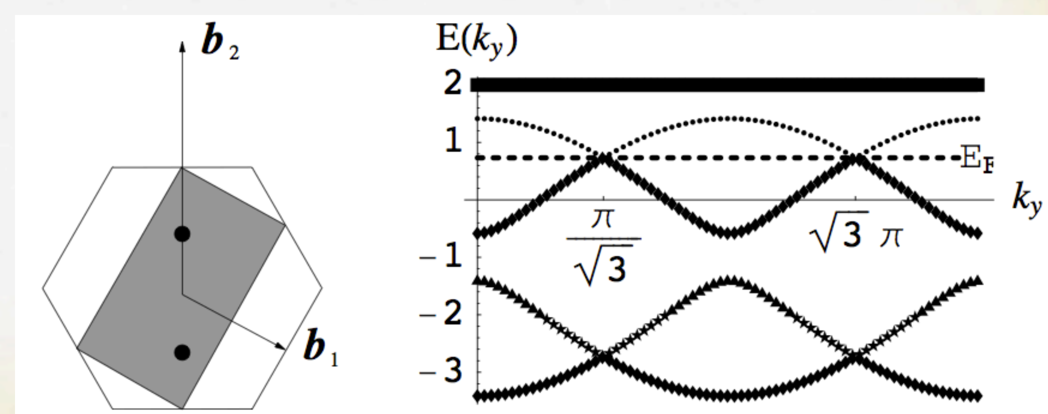
Ran' 07

$$e^{\imath\phi} = \prod_{\text{plaquette}} s_{ij}$$

Hermele' 08

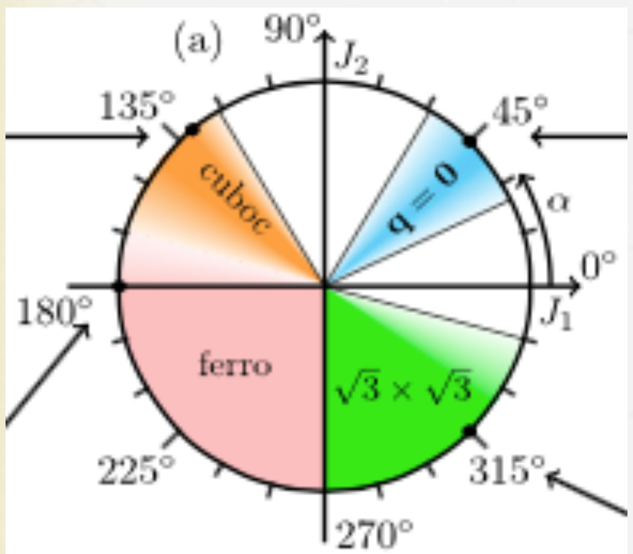
$$\langle S_i S_j \rangle \propto \frac{1}{r^4}$$ (Algebraic spin liquid)



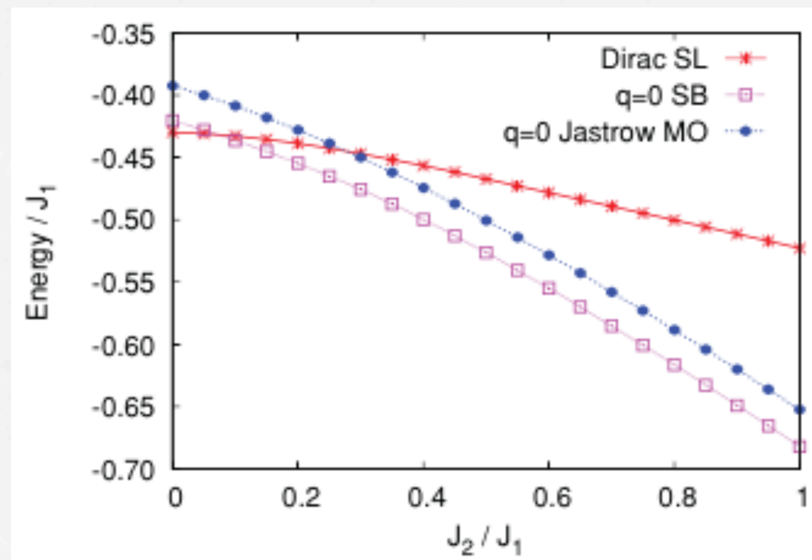


J1-J2 MODEL

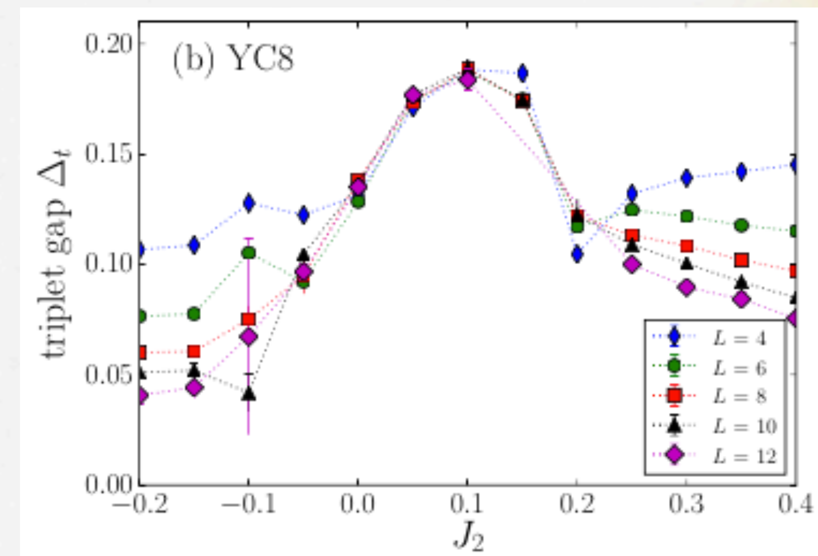
pf-fRG (Thomale, et al.)



SBMF (Motrunich, et al.)



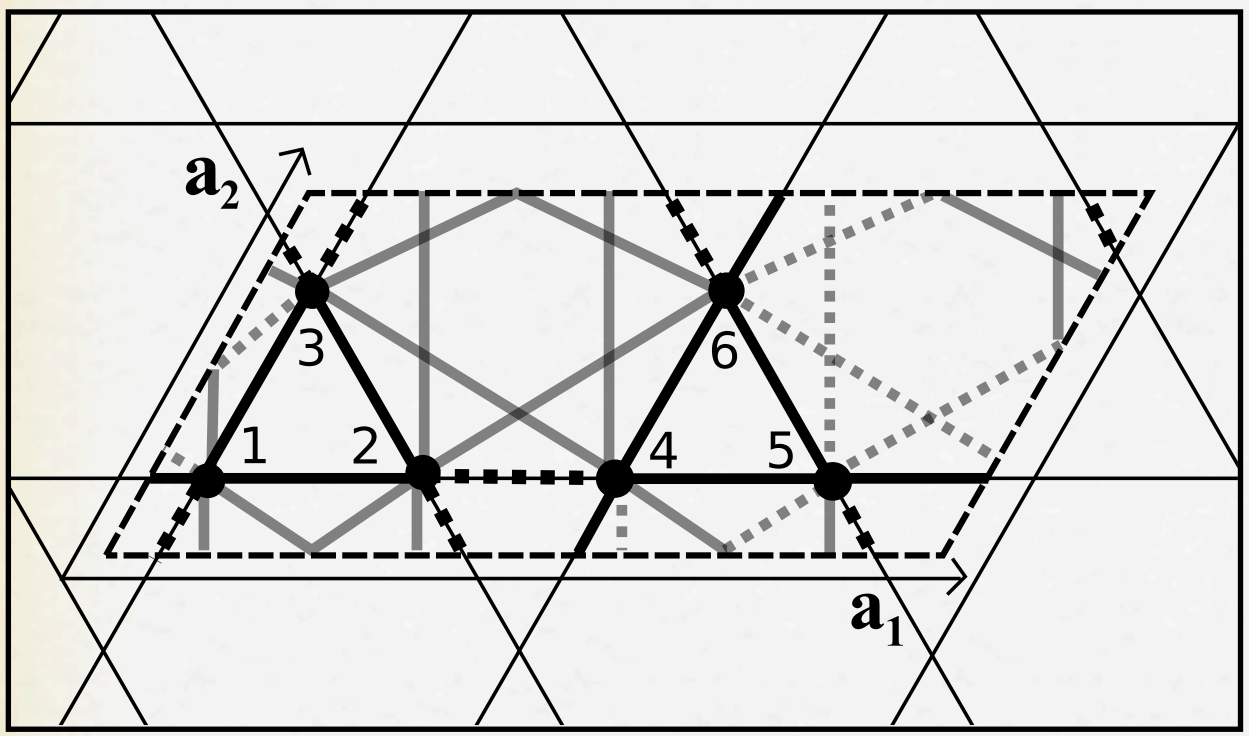
DMRG (Schollwöck, et al.)



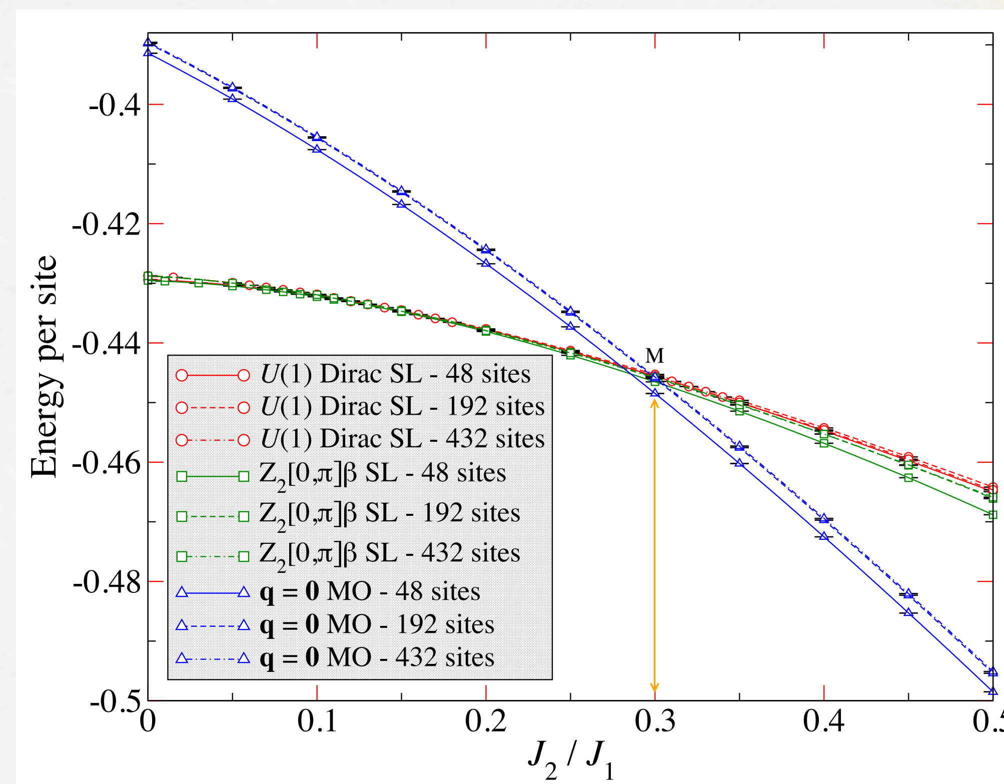
Existence of magnetic order in the j1-j2 model?
If not, what is the nature of the quantum paramagnetic phase?

ENERGY VS J2

Z2 spin liquid Ansatz



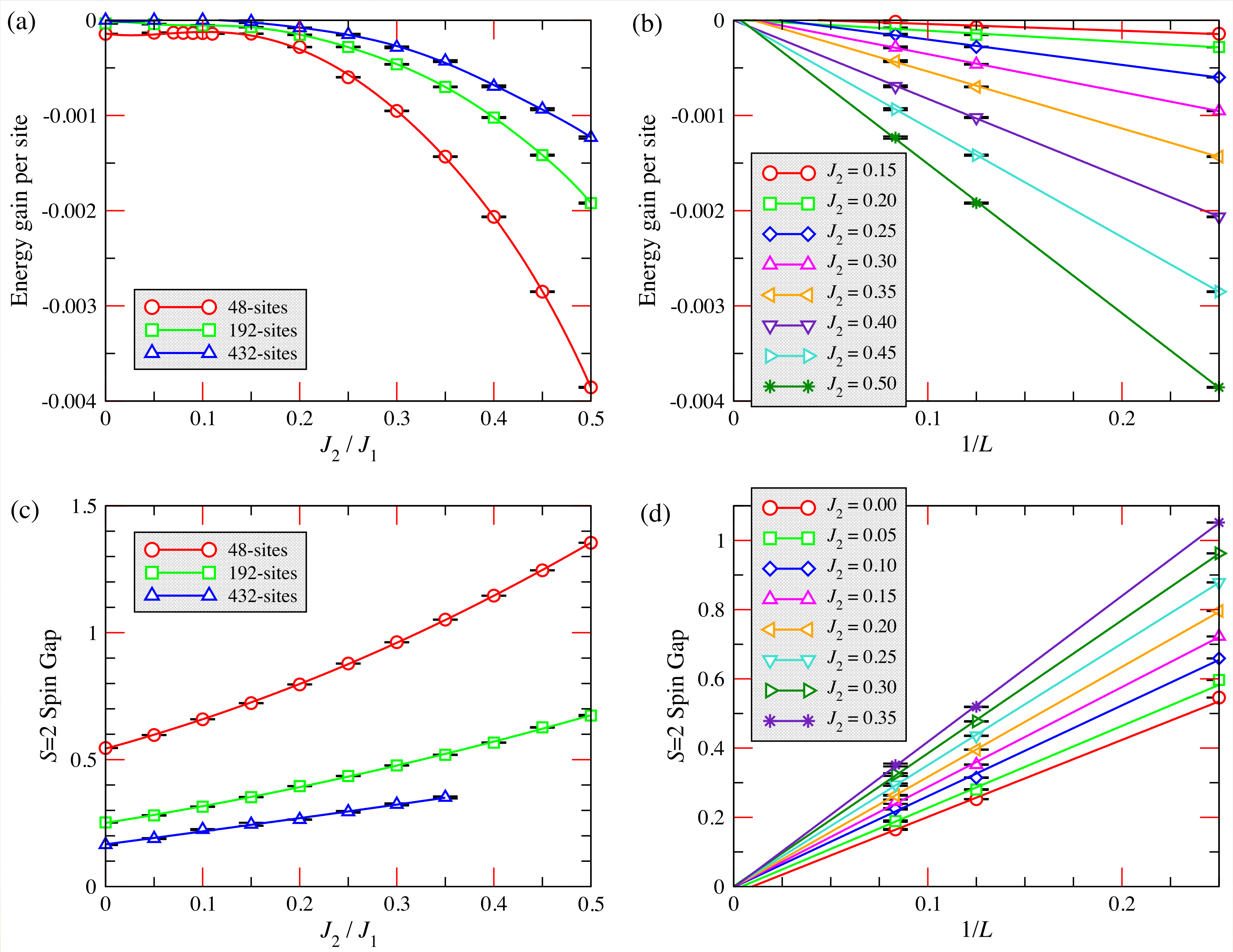
Energy vs j2



Onset of q=0 magnetic order for j2/j1>0.3

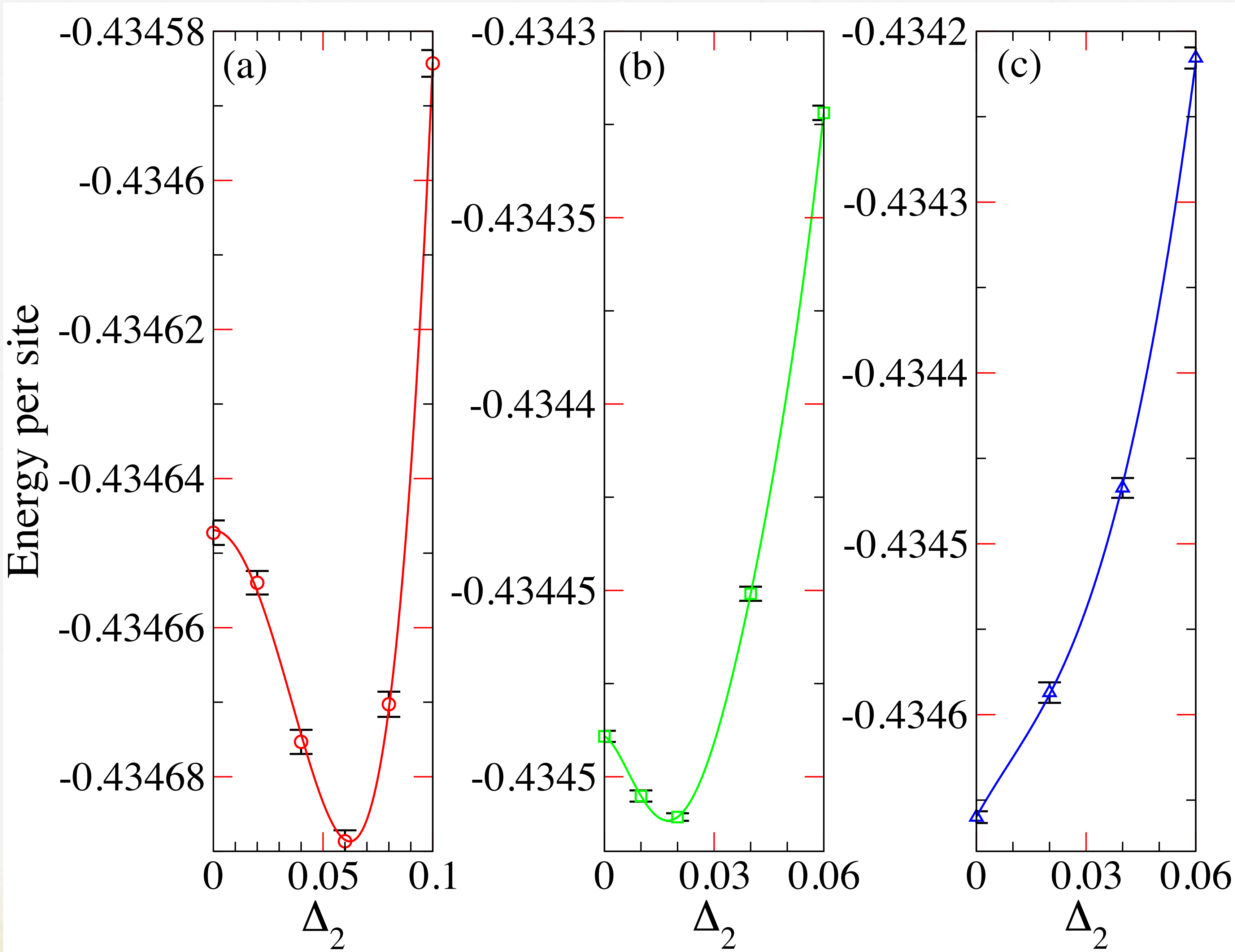
Stabilization of a Z2 spin liquid for finite j2 coupling

SPIN GAP



(a)
Energy gain per site
J_2 / J_1
48-sites
192-sites
432-sites
(b)
Energy gain per site
1/L
$J_2 = 0.15$
$J_2 = 0.20$
$J_2 = 0.25$
$J_2 = 0.30$
$J_2 = 0.35$
$J_2 = 0.40$
$J_2 = 0.45$
$J_2 = 0.50$
(c)
S=2 Spin Gap
J_2 / J_1
48-sites
192-sites
432-sites
(d)
S=2 Spin Gap
1/L
$J_2 = 0.00$
$J_2 = 0.05$
$J_2 = 0.10$
$J_2 = 0.15$
$J_2 = 0.20$
$J_2 = 0.25$
$J_2 = 0.30$
$J_2 = 0.35$

GL-FUNCTIONAL



PF-FRG

$$H = \sum_{ij} \sum_{\mu} J_{ij}^{\mu} \mathbf{S}_i^{\mu} \mathbf{S}_j^{\mu} \longrightarrow \frac{1}{4} \sum_{ij} \sum_{\mu} J_{ij}^{\mu} \left(f_i^{\dagger} \sigma^{\mu} f_i \right) \left(f_j^{\dagger} \sigma^{\mu} f_j \right)$$

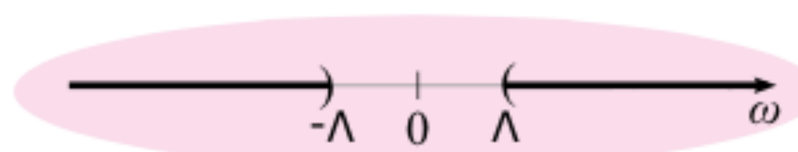
Mean-Field Decoupling

$\langle S_i^{\mu} \rangle = \frac{1}{2} \langle f_i^{\dagger} \sigma^{\mu} f_i \rangle$	$\langle f_{i\alpha}^{\dagger} f_{j\alpha} \rangle$	$\langle f_{i\uparrow}^{\dagger} f_{j\downarrow}^{\dagger} \rangle$
Magnetic order	Hopping	Pairing

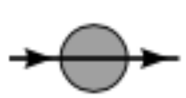
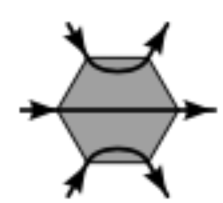
We want to treat the fermionic Hamiltonian in its full complexity

Introduce infrared frequency cutoff in the propagator:

$$G_0(i\omega) = \frac{1}{i\omega} \longrightarrow G_0^{\Lambda}(i\omega) = \frac{\Theta(|\omega| - \Lambda)}{i\omega}$$

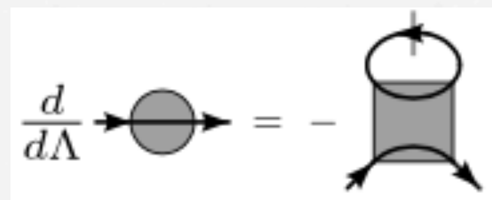


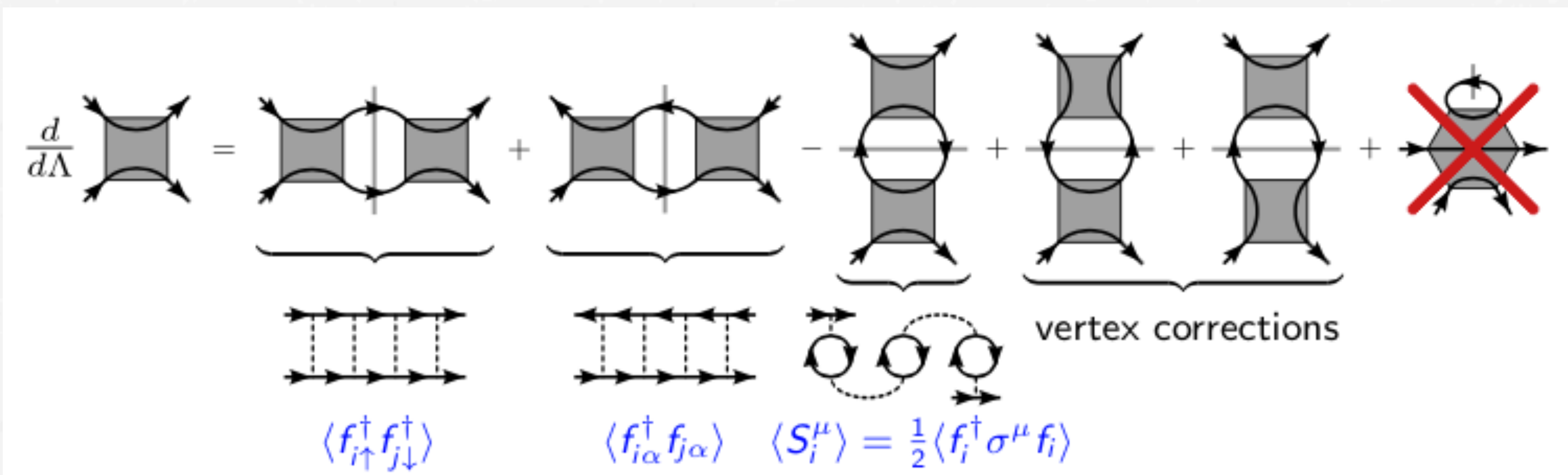
Then:

$\Sigma =$  $\longrightarrow \Sigma^{\Lambda}$, $\Gamma =$  $\longrightarrow \Gamma^{\Lambda}$, $\Gamma_3 =$  $\longrightarrow \Gamma_3^{\Lambda}$

FRG EQUATIONS

FRG formulates differential equations for all m-particle vertices

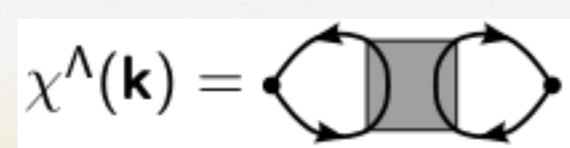




Leading contribution in $1/N$ expansion

Leading contribution in $1/S$ expansion

Susceptibility obtained from the two-particle vertex:



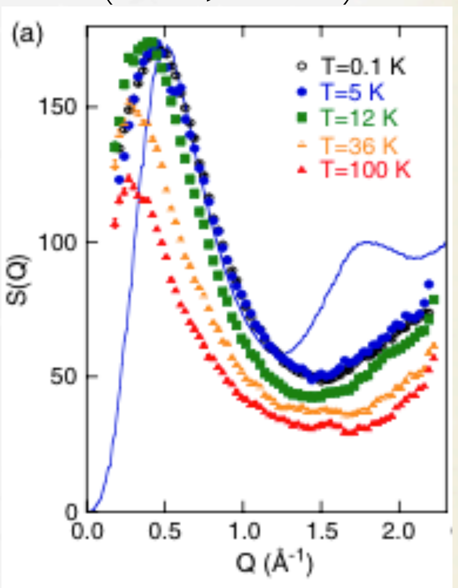
KAPELLASITE

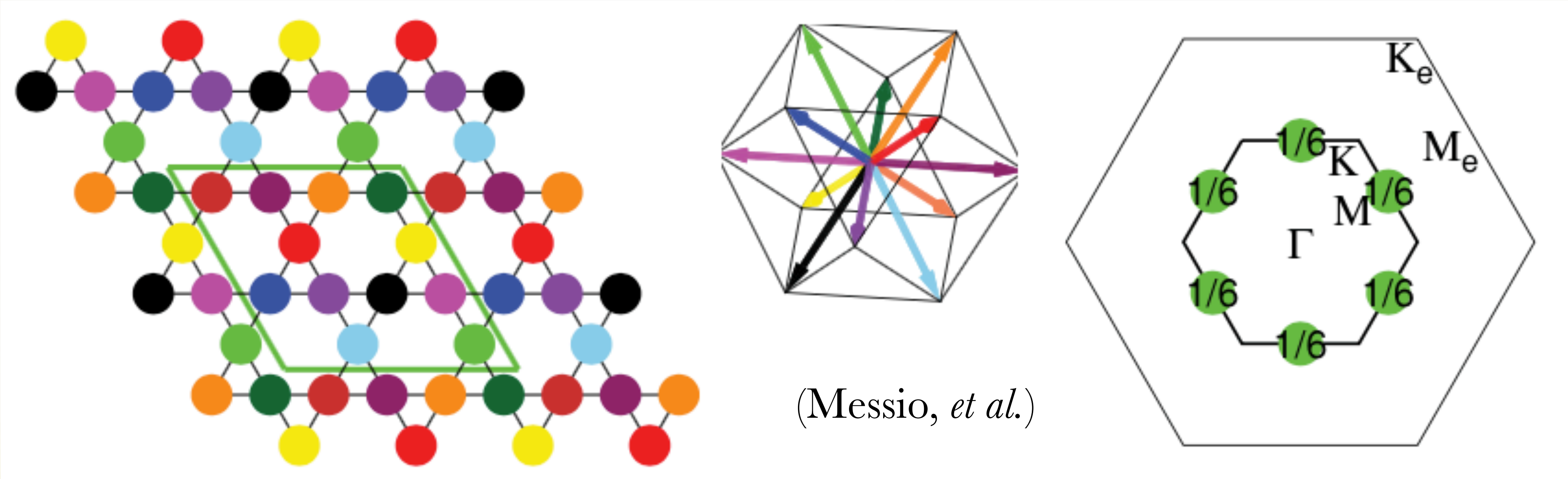
Inelastic Neutron Scattering and Muon-Spin relaxation show absence of long-range magnetic order down to 20 mK

Expected to be a gapless spin liquid with dynamic short-range correlations of cuboc-2 type

J1=-12K, J2=-4K, and Jd=+15.6K (Bernu, *et al.*)

(Fåk, *et al.*)

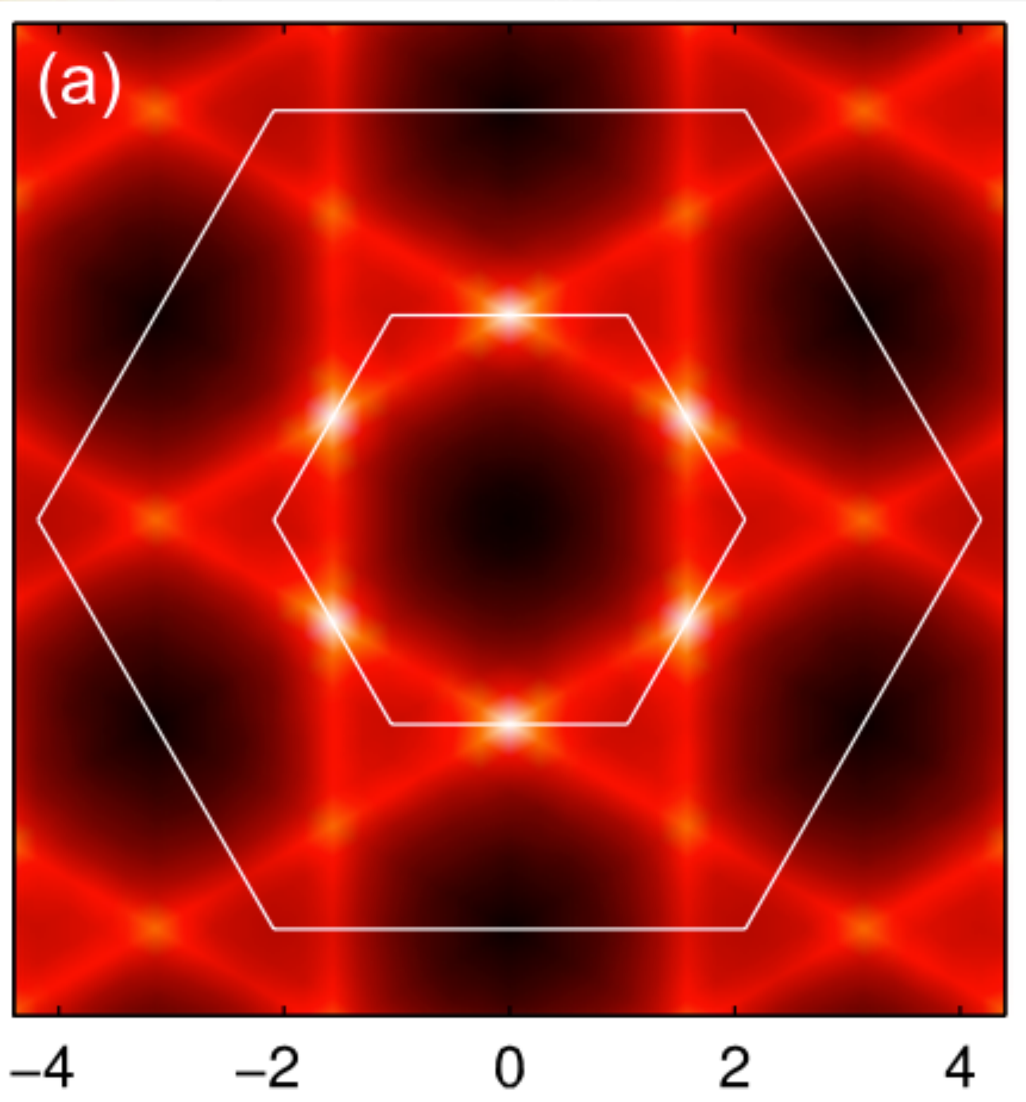




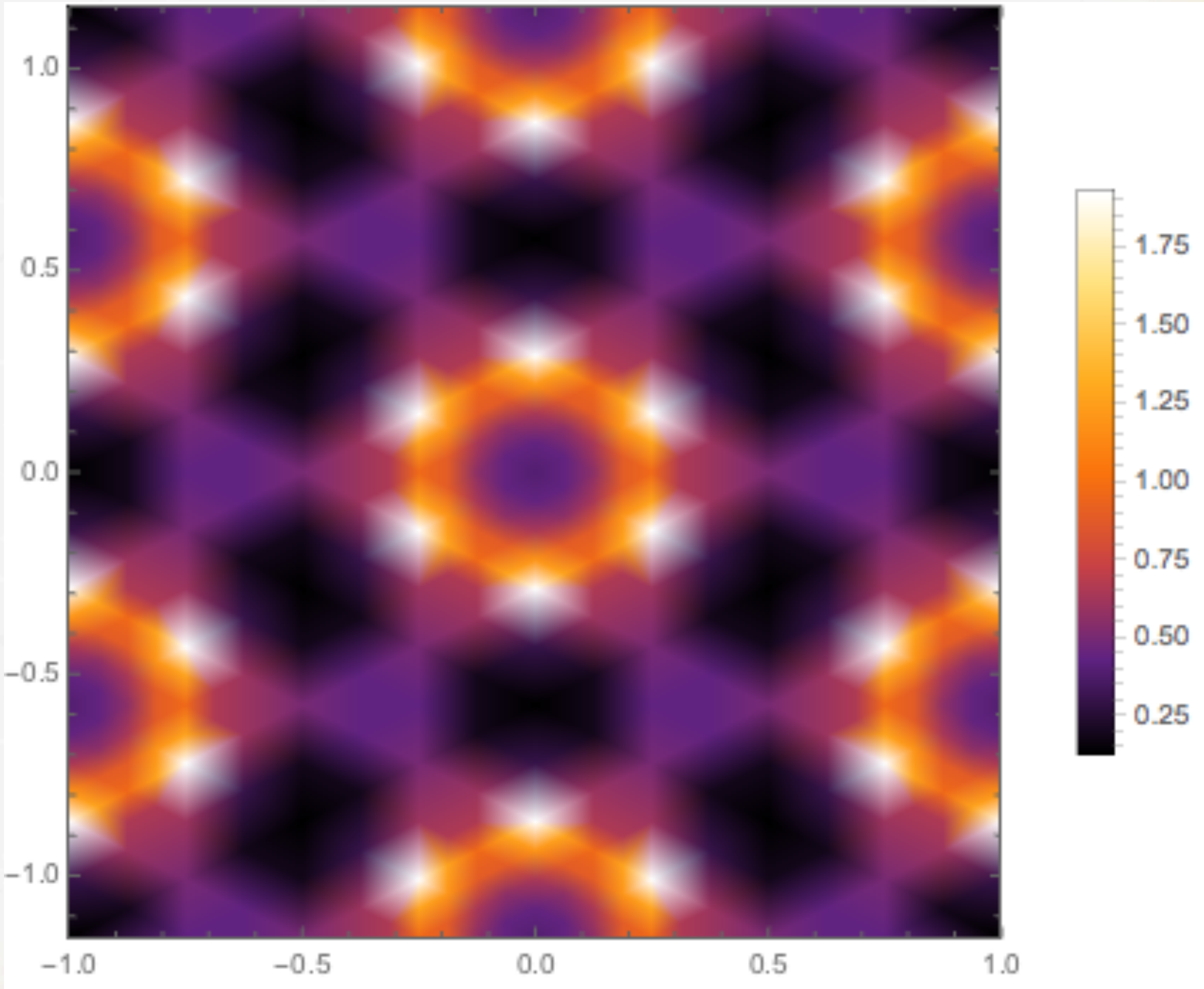
(Messio, *et al.*)

S(Q)

Projected *f*-variational wf (Bieri, *et al.*)



pf - fRG



CONCLUSIONS

- The stabilisation of a Z2 spin liquid in the spin-1/2 KHAF is likely to be a finite-size effect which disappears in the thermodynamic limit.
- The algebraic U(1) Dirac spin liquid is remarkably stable over relatively large region of the phase diagram.
- The unbiased spin-fRG method estimates the spin susceptibility profile in excellent agreement with experiment.

Thank You for your attention

The kagome lattice is lovely, dark, and deep
And there are miles to go before we sleep.

REFERENCES

- Phys. Rev. B 91, 020402(R) (2015), Y. Iqbal, D. Poilblanc, and F. Becca.
- Phys. Rev. B 89, 020407(R) (2014), Y. Iqbal, D. Poilblanc, and F. Becca.
- Phys. Rev. B 87, 060405(R) (2013), Y. Iqbal, F. Becca, S. Sorella, and D. Poilblanc.
- New J. Phys. 14 115031(2012), Y. Iqbal, F. Becca, and D. Poilblanc.
- Phys. Rev. B 84, 020407(R) (2011), Y. Iqbal, F. Becca, and D. Poilblanc.
- Phys. Rev. B 83, 100404(R) (2011), Y. Iqbal, F. Becca, and D. Poilblanc.