Bosons in optical lattices

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Current trends in frustrated magnetism, JNU 2015

Collaborators

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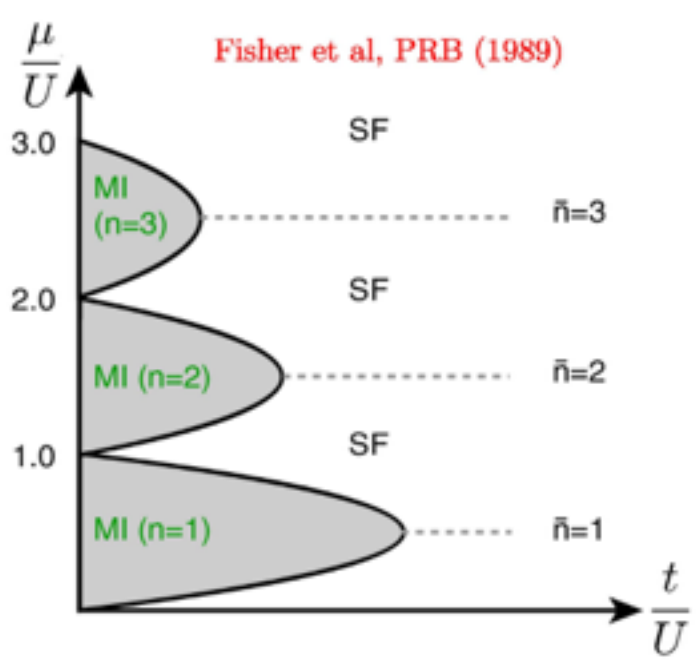
Outline

- Frustration and bosons
- Frustrated Bose-Hubbard ladder and the Chiral Mott phase
- Bond ordered and supersolid phases

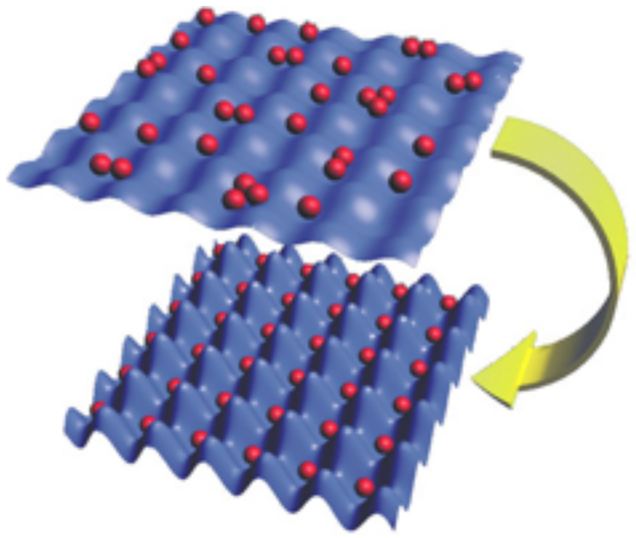
Bose-Hubbard model

$$H = -t\sum_{\langle ij\rangle} c_i^\dagger c_j + \mathrm{h.c.} + \frac{U}{2}\sum_i n_i\,(n_i - 1) - \mu\sum_i n_i$$

Interaction strength tunes quantum phase transition from a superfluid to a Mott insulator

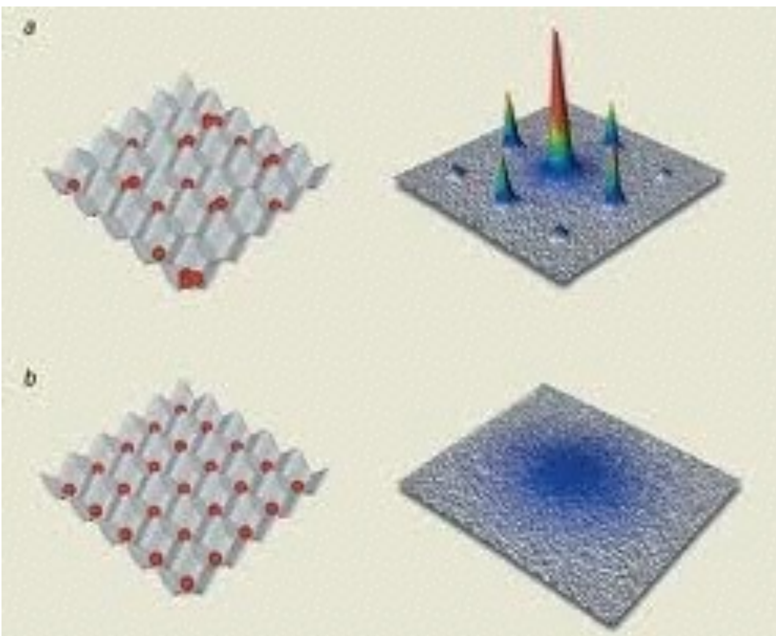


SF - superfluid
MI-Mott insulator

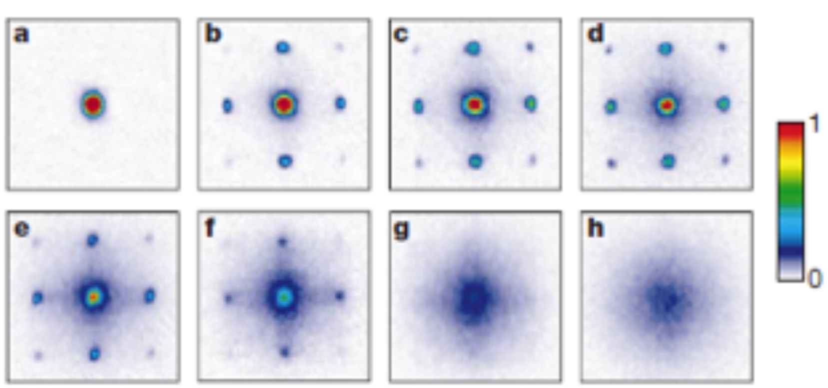


Experiments

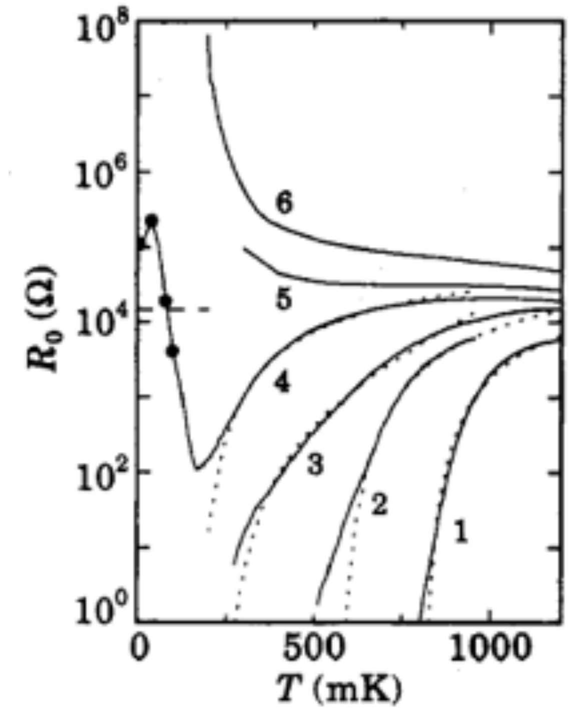
Cold atoms



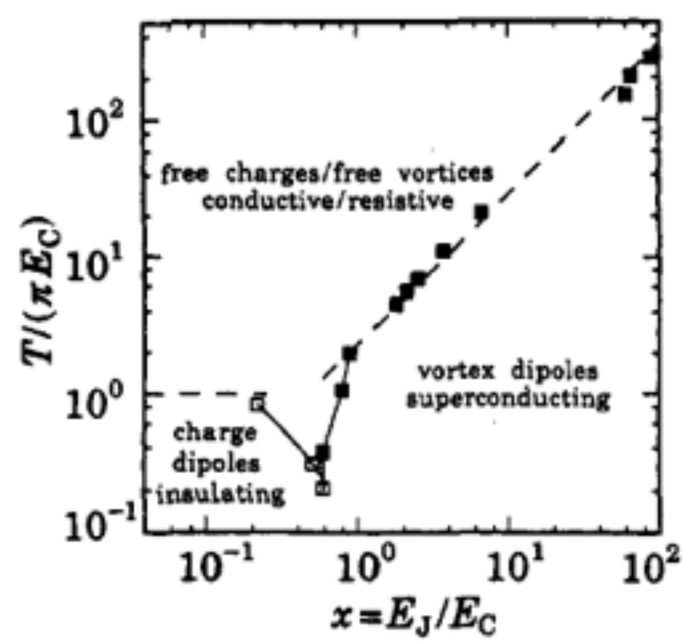
Greiner et. al. (2002)



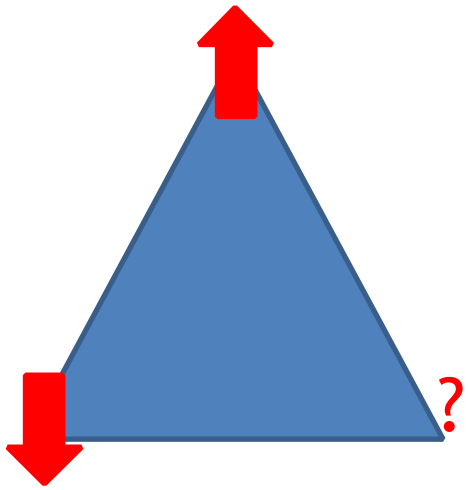
Josephson junction arrays



Mooij group (1992)



Frustration

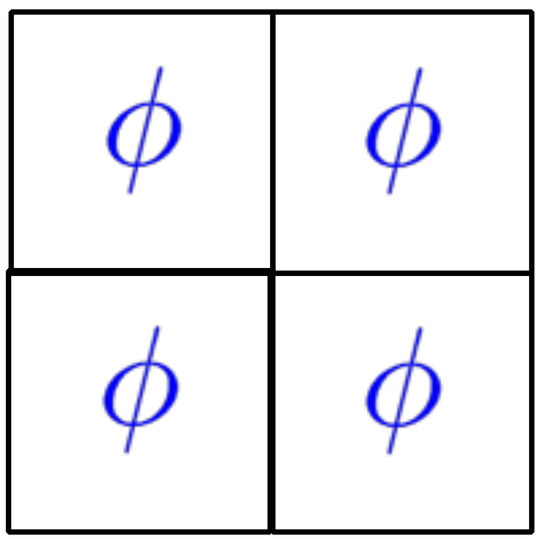


Why is it interesting?

- Many classical ground states
- Quantum effects pick particular states
- Interesting states like spin liquids

Bosons with frustration

$$H = -\sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + \text{h.c.} + \frac{U}{2} \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$



Flux $\phi = \frac{p}{q}$

q bands with degenerate minima

Kinetically frustrated bosons have many ways to condense

Interactions relieve frustration

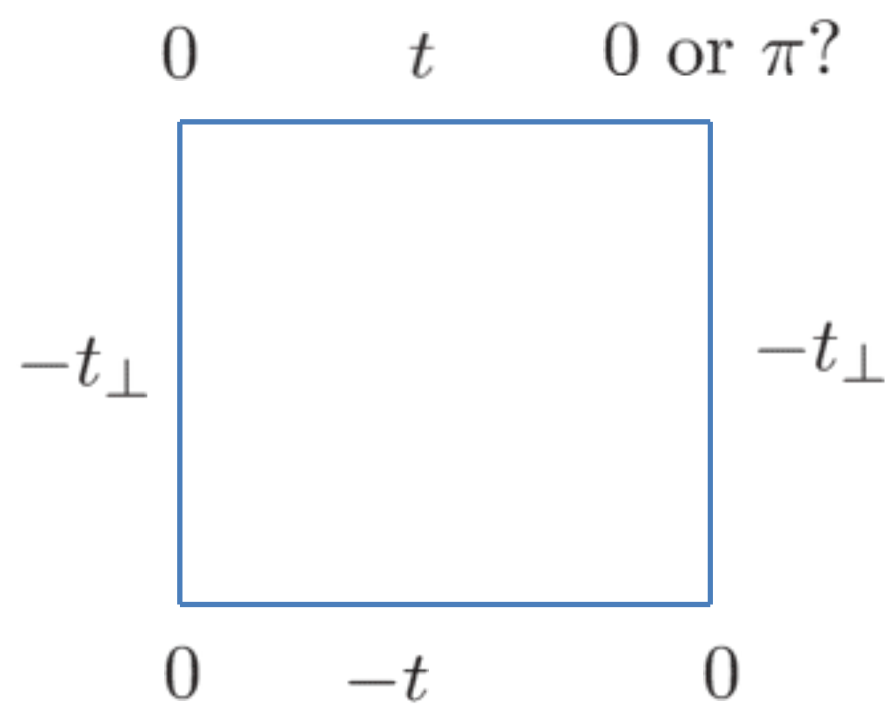
Phase diagram?

Frustrated bosons

$$\phi = 1/2$$

$$\langle a_j \rangle = |\psi_j| e^{-i\theta_j}$$

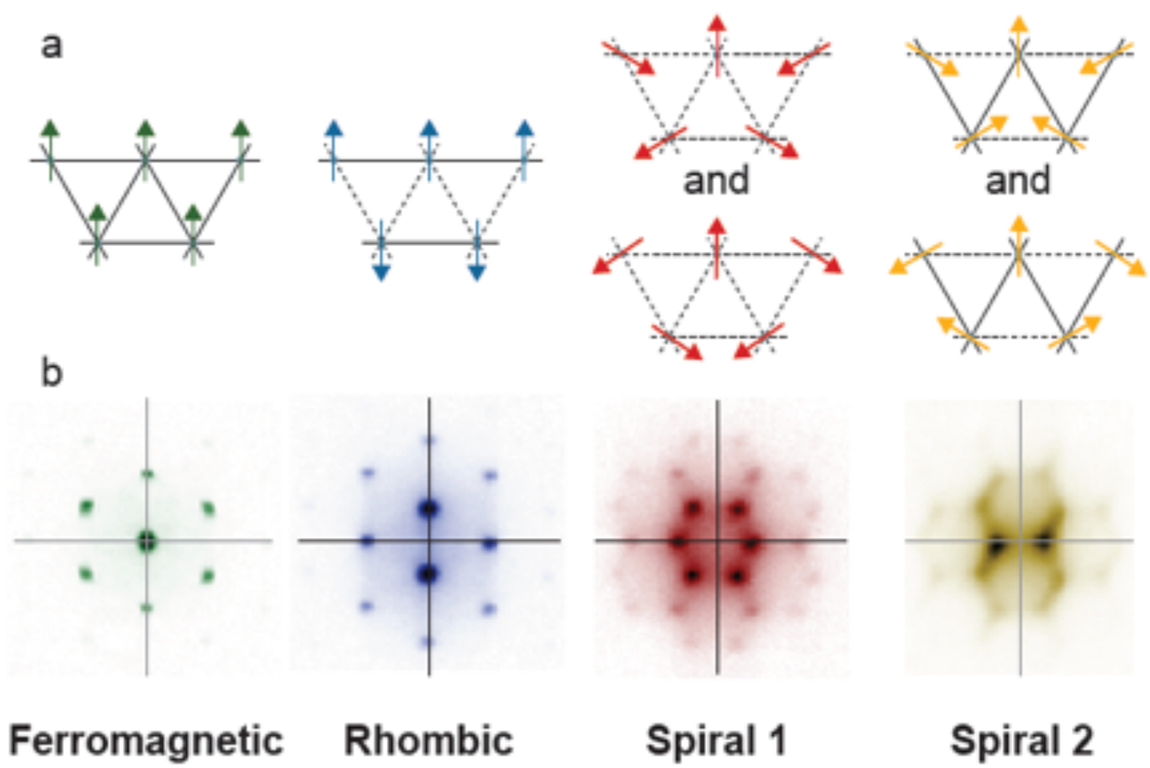
$$\langle a_j^\dagger a_k + a_k a_j^\dagger \rangle \sim \cos(\theta_k - \theta_j)$$



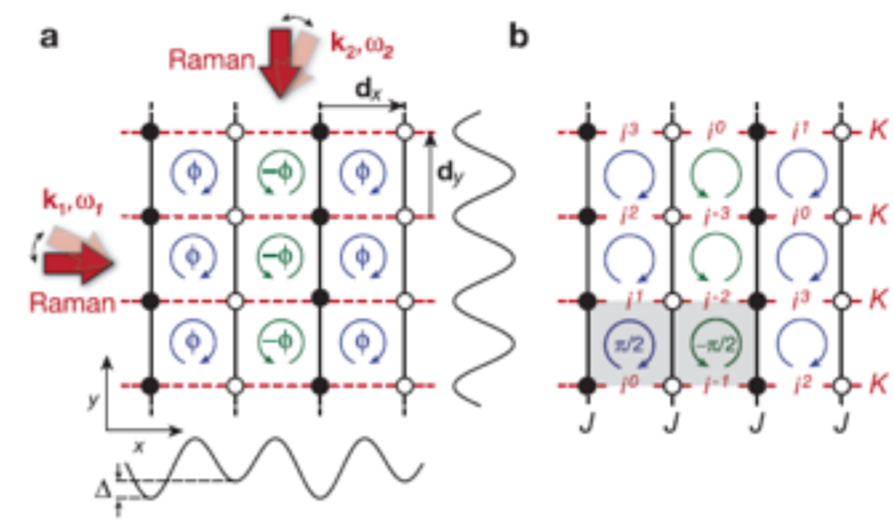
Frustration of the superfluid phase

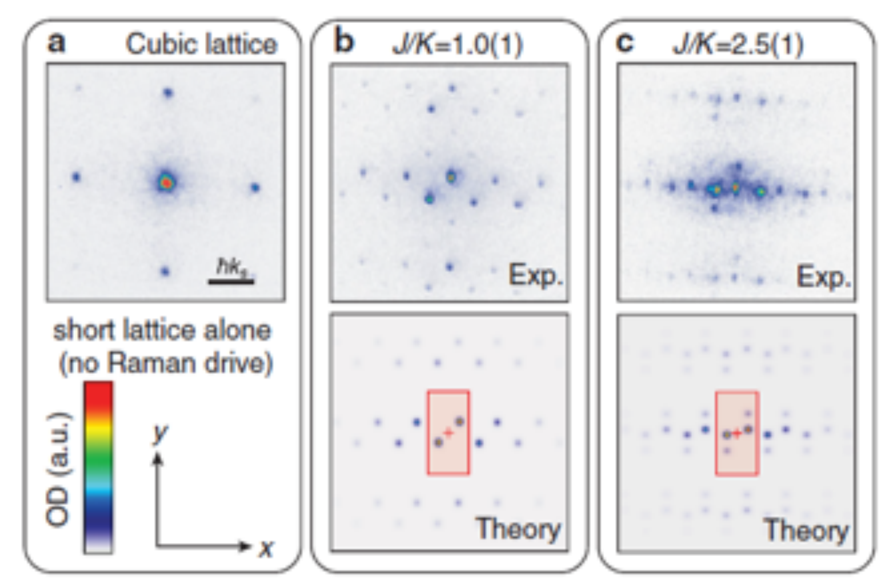
Experiments

Optical lattices with frustration using artificial gauge fields



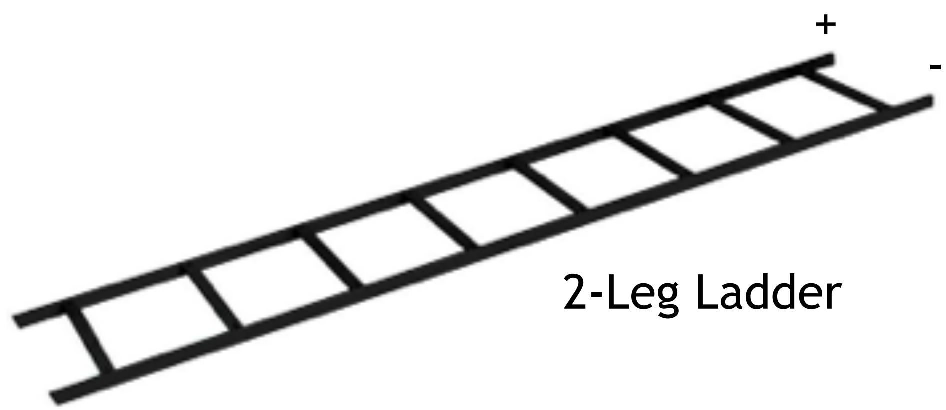
Struck et. al. (2011)





Aidelsberger et. al. (2011)

Frustrated Bose-Hubbard model

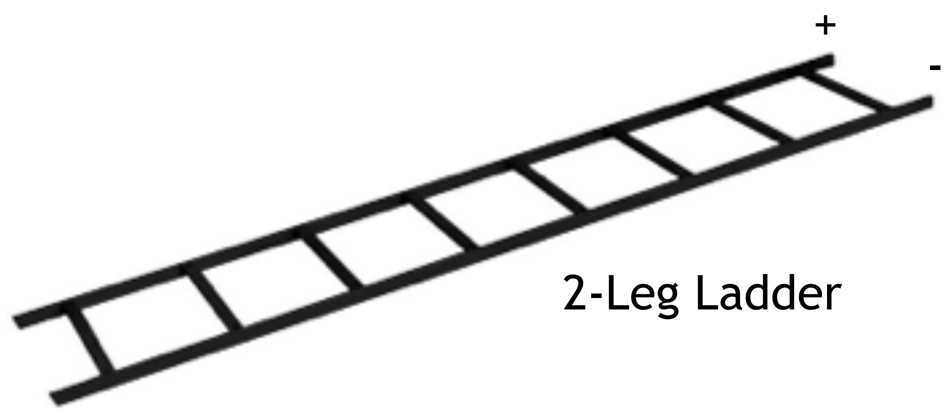


$$
\begin{aligned}
H &= -t\sum_i a_i^\dagger a_{i+1} + \text{h.c.} + \frac{U}{2}\sum_i n_i^{(a)}\left[n^{(a)} - 1\right] \\
&\quad +t\sum_i b_i^\dagger b_{i+1} + \text{h.c.} + \frac{U}{2}\sum_i n_i^{(b)}\left[n^{(b)} - 1\right]
\end{aligned}
$$

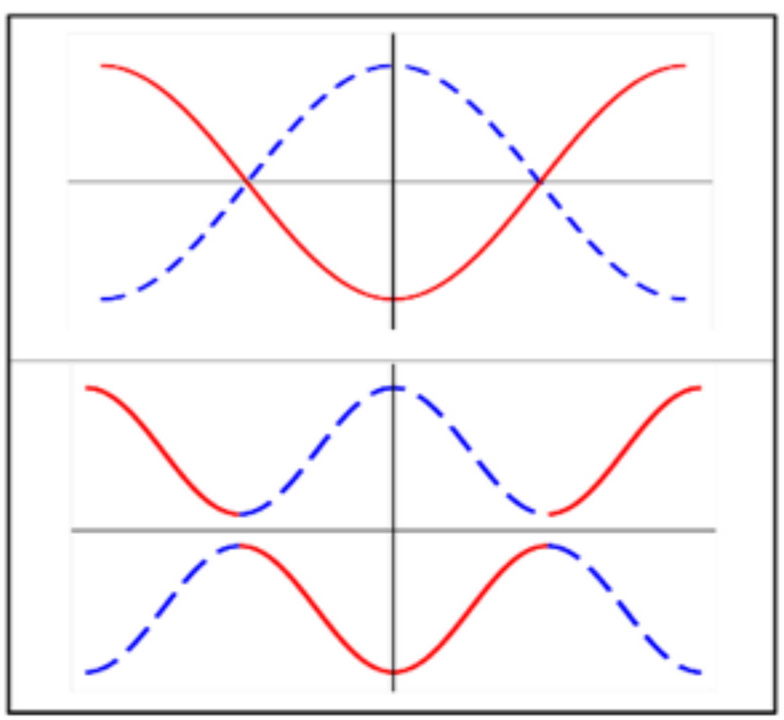
Filling: one boson per site

$\phi = 1/2$ per plaquette. Frustration

Band structure



$U = 0$ band structure

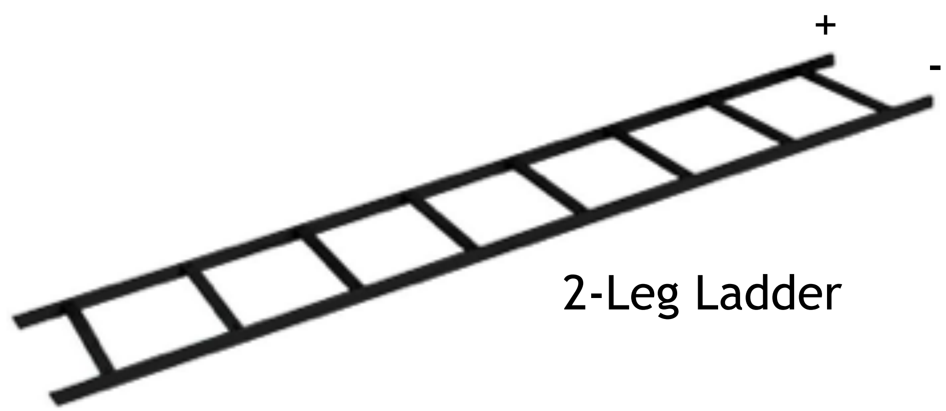


$t_\perp = 0$ Multiple minima

$t_\perp \neq 0$ Single boson condensate wavefunction

$|\psi\rangle = a_0|0\rangle + a_\pi|\pi\rangle$

Weak interactions



$$|\psi\rangle = a_0|0\rangle + a_\pi|\pi\rangle$$

$|0\rangle(|\pi\rangle)$ has the character of the $a(b)$ leg

Avg. density in $a(b)$ leg $\propto |a_0|^2(|a_\pi|^2)$

Filling $n = 1$ and $U > 0 \Rightarrow |a_0| = |a_\pi|$

Chiral order

$$|\psi\rangle = Ae^{i\theta}\left(|0\rangle + e^{i\phi}|\pi\rangle\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{N!}}\left[e^{i\theta}\left(a_0^\dagger + e^{i\phi}a_\pi^\dagger\right)\right]^N |0\rangle$$

Ginzburg-Landau theory

$$E_{\mathrm{low}}^{\mathrm{mft}} = (-E_0 - \mu)\sum_{i=0,\pi}|\varphi_i|^2 + U\left(u_0^4 + v_0^4\right)\sum_{i=0,\pi}|\varphi_i|^4 + 8Uu_0^2v_0^2|\varphi_0|^2|\varphi_\pi|^2 + \boxed{2Uu_0^2v_0^2\left(\varphi_0^{*2}\varphi_\pi^2 + \varphi_\pi^{*2}\varphi_0^2\right)}$$

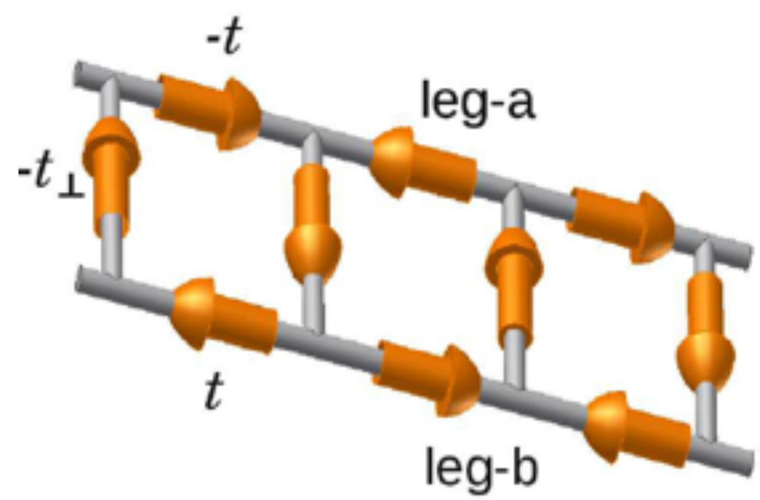
↓

Favours $\phi = \pm\pi/2$ Z_2 symmetry

$|\varphi_0| = |\varphi_\pi|$

Dhar, Maji, Mishra, Pai, Mukerjee & Paramekanti (2013)

Chiral SuperFluid (CSF)



Breaks $U(1) \times Z_2$ symmetry

$$|\psi\rangle = Ae^{i\theta}\left(|0\rangle + e^{\pm i\pi/2}|\pi\rangle\right)$$

Polini et. al. (2005), Lim et. al. (2008), Powell et. al. (2010), Moller and Cooper (2010), Sinha and Sengupta (2011)

Increasing interaction strength

Single site mean-field theory gives single transition from CSF to MI

$$\frac{1}{\sqrt{4t^2+t_\perp^2}} = \frac{n}{\mu - U(n-1)} + \frac{n+1}{Un-\mu}$$

Beyond mean-field theory

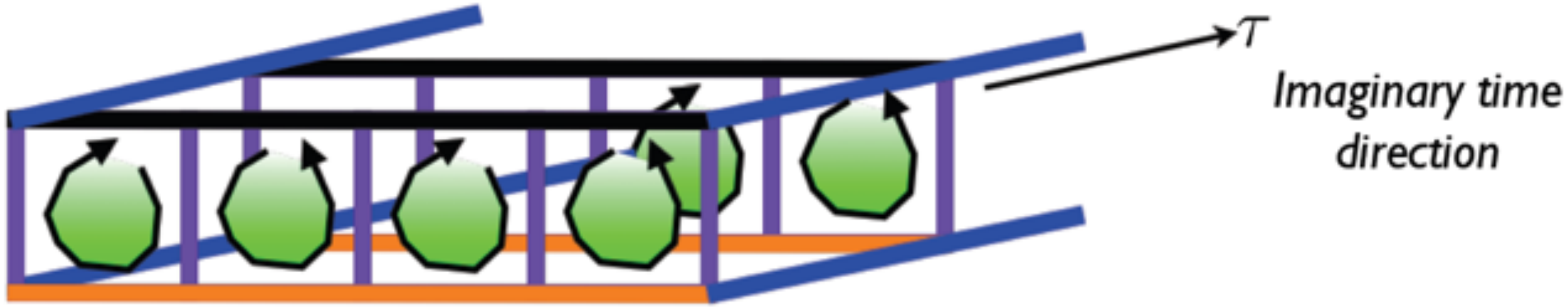
- Monte Carlo on classical 1+1D model
- DMRG on quantum model

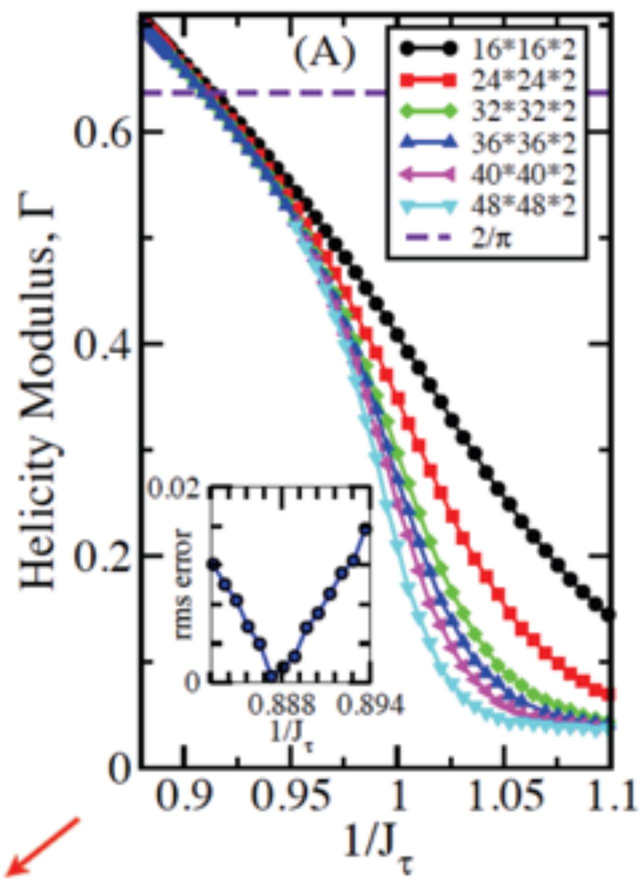
Cut to the chase

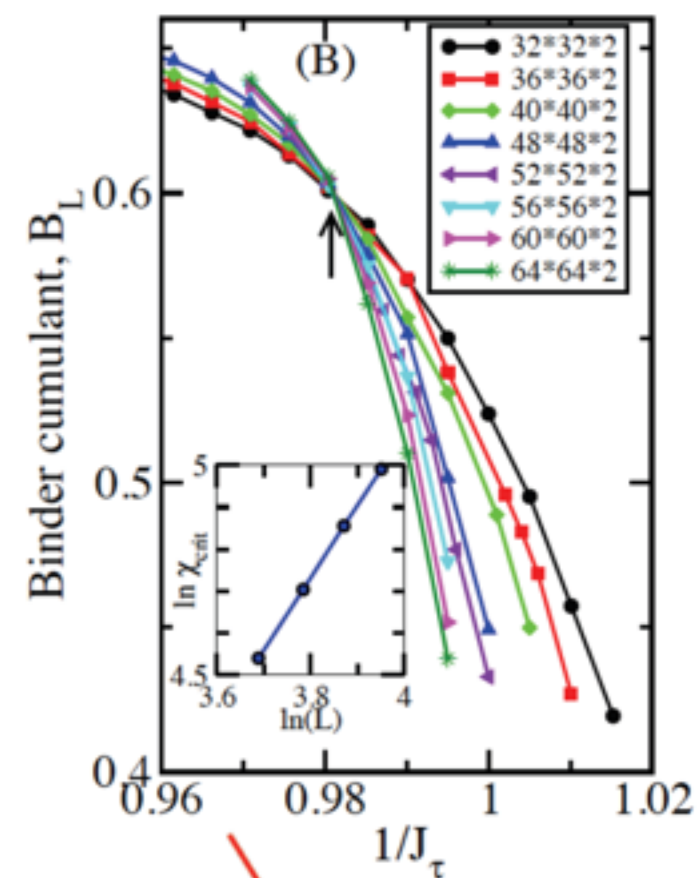
As U is increased a novel Chiral Mott Insulator (CMI) forms which has a charge gap but retains the chiral order of the CSF. Upon further increase of U, a regular MI develops.

Dhar, Maji, Mishra, Pai, Mukerjee & Paramekanti (2013)

1+1D classical model



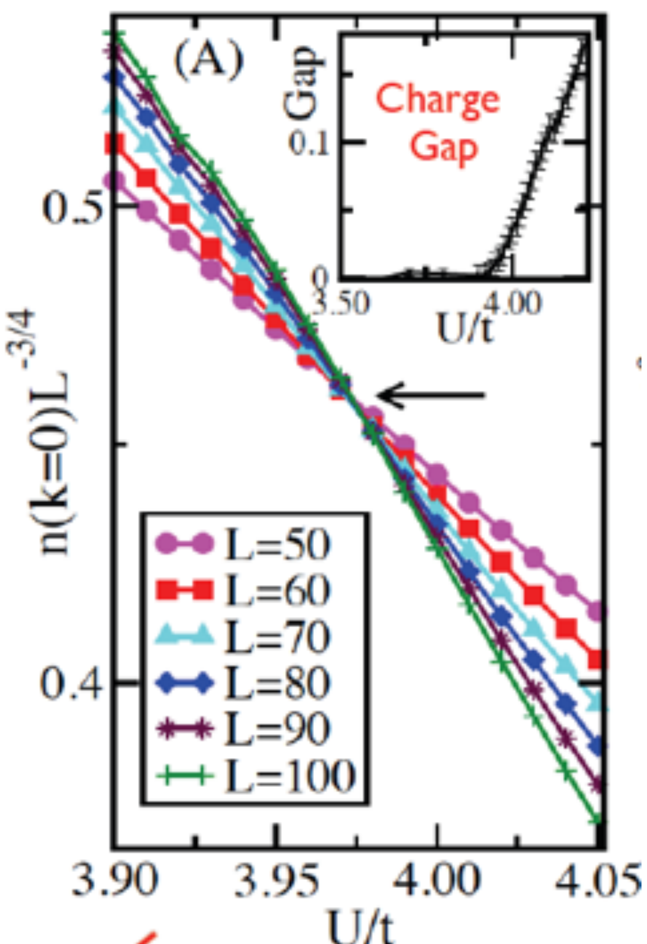




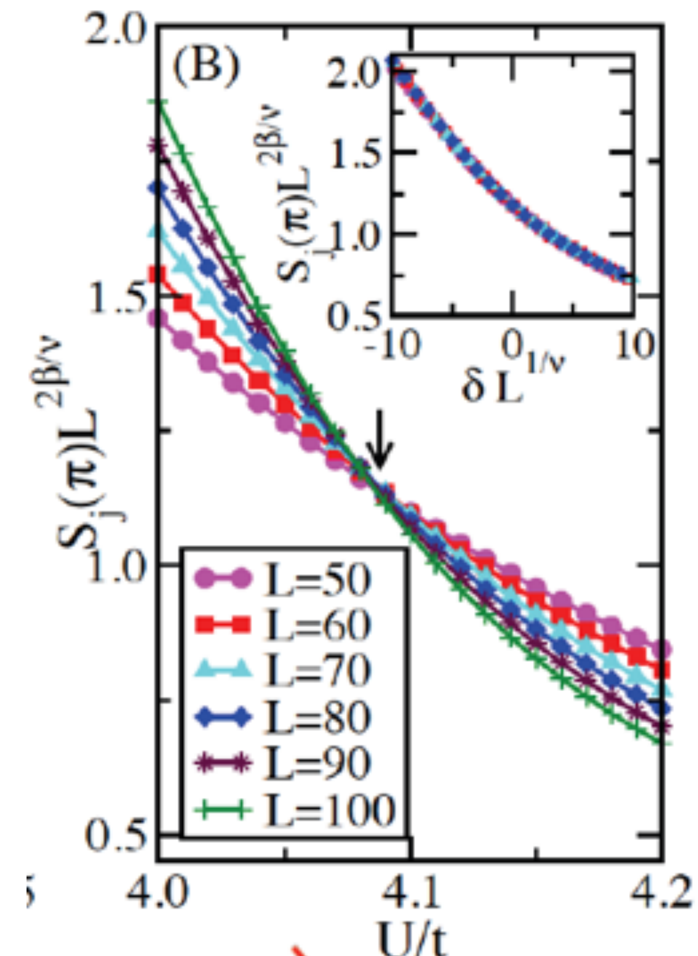
BKT transition from CSF to CMI

Ising transition from CMI to MI

DMRG on the quantum model



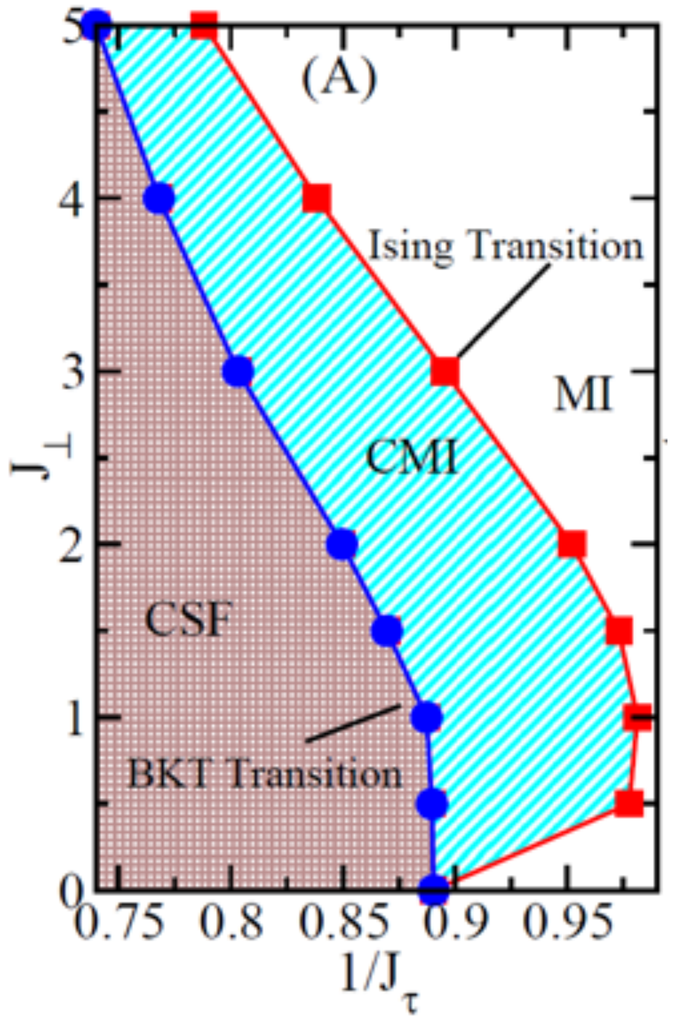
BKT transition from CSF to CMI



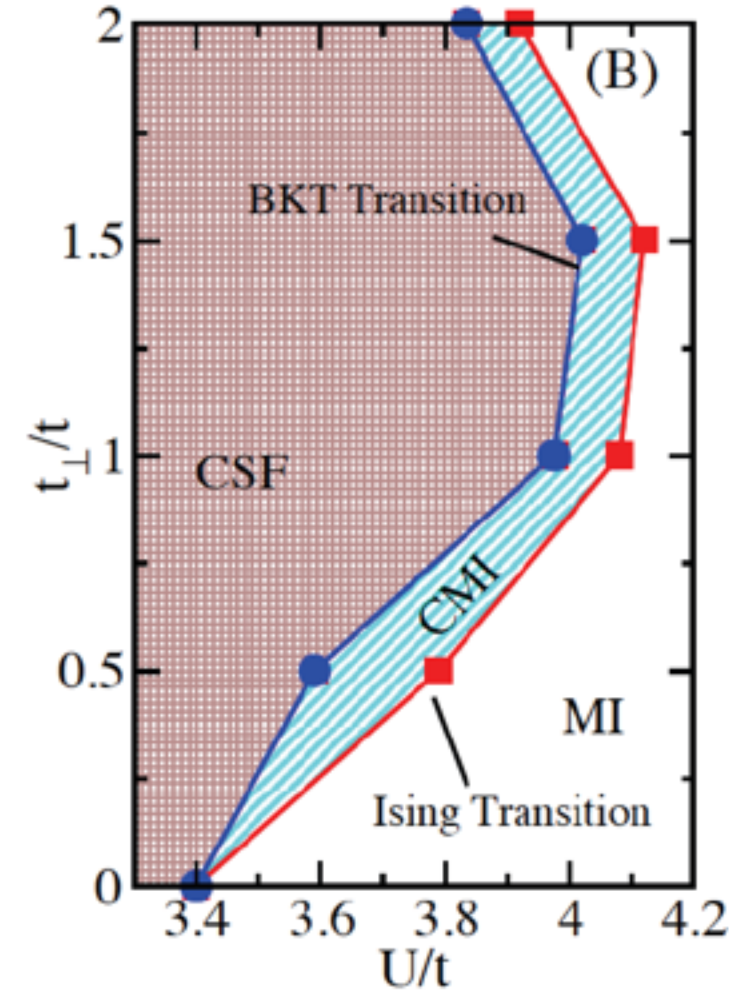
Ising transition from CMI to MI

Phase diagram

Monte-Carlo



DMRG



- CSF- Algebraic SF, long-ranged loop current order
- CMI - Charge gap, long-ranged loop current order
- MI - Charge gap, no loop current order

Chiral Mott Insulator

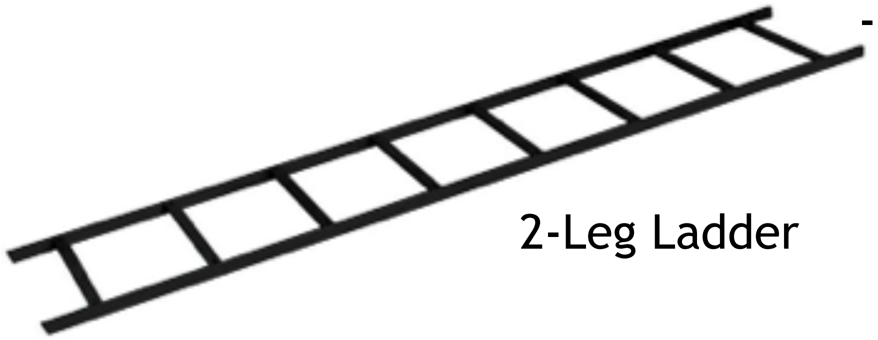
Physical Pictures

- Vortex-antivortex supersolid
- Indirect excitonic condensate

Variational wavefunction

$$\psi(r_1, r_2, \ldots r_n) = e^{-\sum_{ij} v(r_i - r_j)} \psi_{\mathrm{CSF}}(r_1, r_2, \ldots r_n)$$

Sine-Gordon model



$$\phi_{s(a)} = \phi_+ \pm \phi_-;\ \theta_{s(a)} = \phi_+ \pm \phi_-$$

$$\begin{aligned} H \quad = \quad & H_s^0 + H_a^0 + g_1 \int dx \cos\left[\sqrt{8}\theta_a(x)\right] + g_2 \int dx \cos\left[\sqrt{8}\phi_s(x)\right] \\ & + g_3 \int dx \cos\left[\sqrt{8}\phi_a(x)\right] + g_4 \int dx \cos\left[\sqrt{2}\phi_s(x)\right] \cos\left[\sqrt{2}\phi_s(x)\right] \end{aligned}$$

$$H^0 = \frac{u}{2\pi} \int dx \left[K \left(\nabla\theta(x)\right)^2 + \frac{1}{K} \left(\nabla\phi(x)\right)^2\right]$$

Also Tokuno & Georges (2014)

Sine-Gordon model

$$H = H_s^0 + H_a^0 + g_1 \int dx \cos\left[\sqrt{8}\theta_a(x)\right] + g_2 \int dx \cos\left[\sqrt{8}\phi_s(x)\right] + g_3 \int dx \cos\left[\sqrt{8}\phi_a(x)\right] + g_4 \int dx \cos\left[\sqrt{2}\phi_s(x)\right] \cos\left[\sqrt{2}\phi_a(x)\right]$$

Phase	Relevant	Irrelevant
SF	g_3	$g_1,\ g_2,\ g_4$
CSF	g_1	$g_2,\ g_3,\ g_4$
MI	g_4	g_1
CMI	$g_1,\ g_2$	$g_3,\ g_4$
??	$g_2,\ g_3$	$g_1,\ g_4$

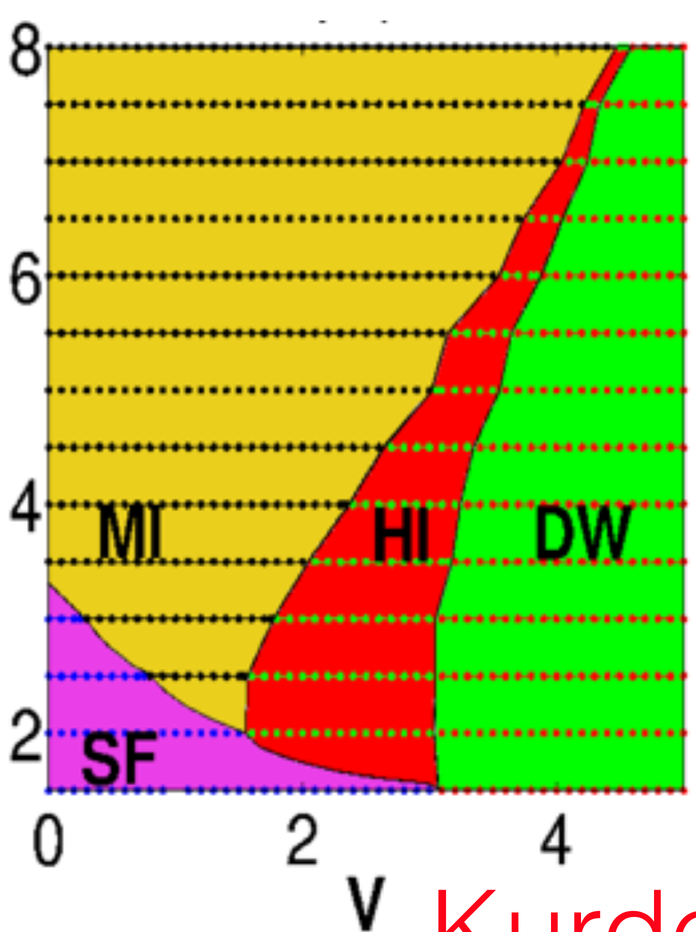
Chiral Mott states elsewhere

- Two component boson system, Petrescu & Le Hur (2013)
- Frustrated triangular lattice, Zalatel, Parameswaran, Ruegg & Altman (2014)
- Chiral Bose liquid at finite temperature, Li, Paramekanti, Hemmerich & Liu (2014)
- Field theoretical study of ladders with flux, Tokuno & Georges (2014)

Extended Bose-Hubbard model

Extended Bose Hubbard model

$$H = -t \sum_j \left[b_j^\dagger b_{j+1} + \mathrm{h.c.} + \frac{U}{2} n_j \left(n_j - 1 \right) + V n_j n_{j+1} \right]$$



MI - Mott Insulator
SF - Superfluid
DW - Density Wave
HI - Haldane Insulator

Kurdestany, Pai, Mukerjee & Pandit (2014)

Other types of chiral states

Unfrustrated coupled extended Bose-Hubbard ladders

Essentially the same phases

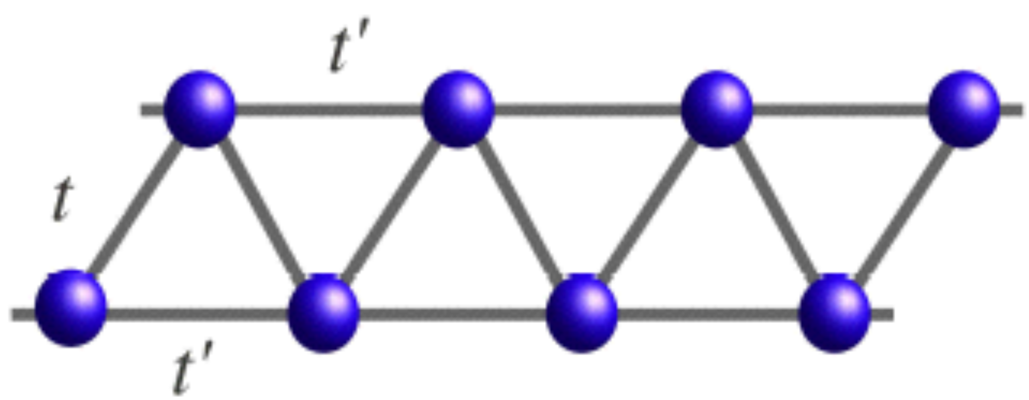
Dela Torre, Berg, Altman and Giamarchi (2011)

Frustrated ladders, chiral version of DW and HI phase?

Roy, Mukerjee & Paramekanti (in progress)

t-*t'*-*V* model

Hard core bosons



$$H = -t \sum_i b_i^\dagger b_{i+1} + \text{h.c.} - t' \sum_i b_i^\dagger b_{i+2} + \text{h.c.} + V \sum_i n_i n_{i+1}$$

Frustration $t > 0, t' < 0$

Mishra, Pai, Mukerjee & Paramekanti (2013 & 2014)

t-t'-V model

Effective spin 1/2 model

$$H = -2t\sum_i S_i^+ S_{i+1}^- + \text{h.c.} - 2t'\sum_i S_i^+ S_{i+2}^- + \text{h.c.} + V\sum_i n_i n_{i+1}$$

$V = 0$ & $t' = t/2$: Easy plane Majumdar Ghosh model

Ground state: $|\psi\rangle_G \sim \prod_{j\in even} (|\uparrow\rangle_j|\downarrow\rangle_{j+1} + |\downarrow\rangle_j|\uparrow\rangle_{j+1})$

Bond ordering

t-*t*'-*V* model

$t' = 0$ can be mapped on the XXZ model

$$H = -2t \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y \right) + V \sum_j S_j^z S_{j+1}^z$$

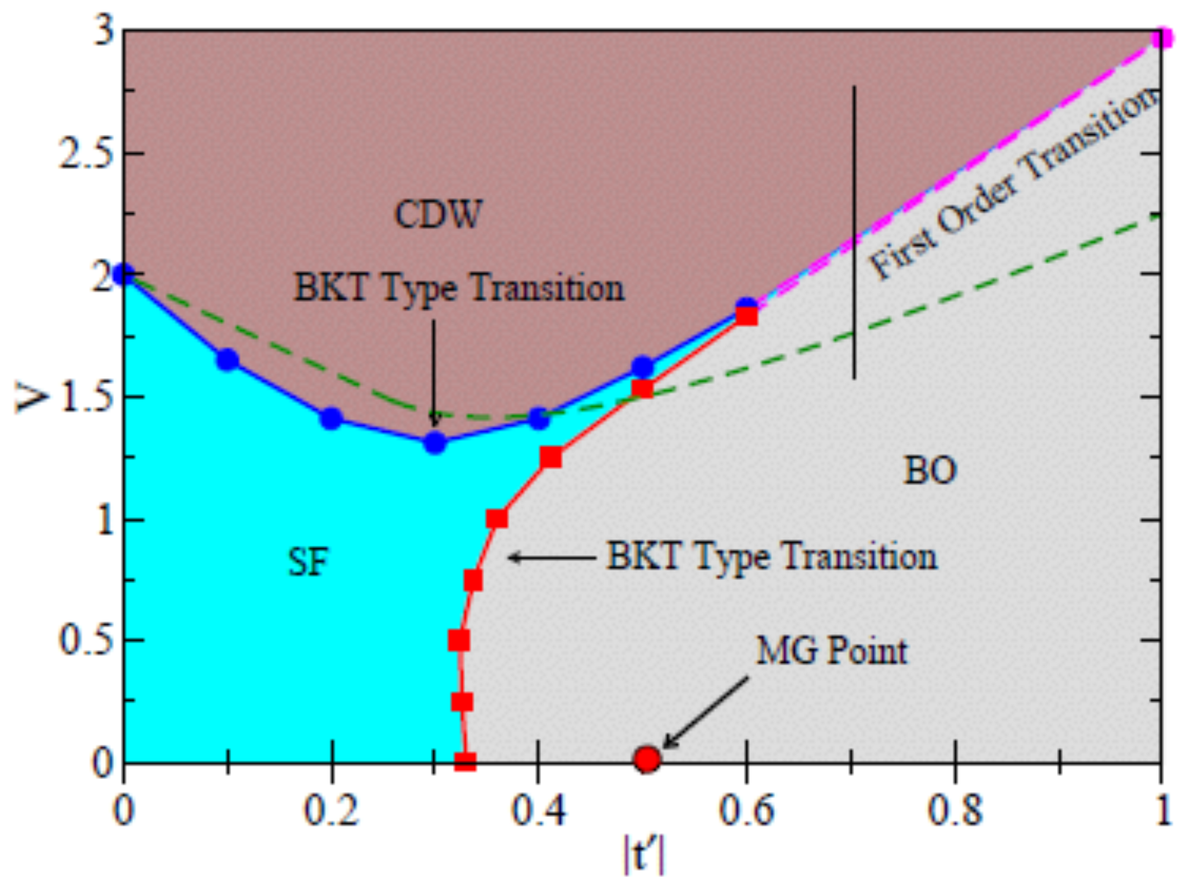
Bethe anstaz solvable

$V \leq 2t$ spin Luttinger liquid (superfluid)

$V > 2t$ gapped Ising antiferromagnet (CDW)

$$|\psi\rangle_G \sim |\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle \cdots$$

t-t'-V model

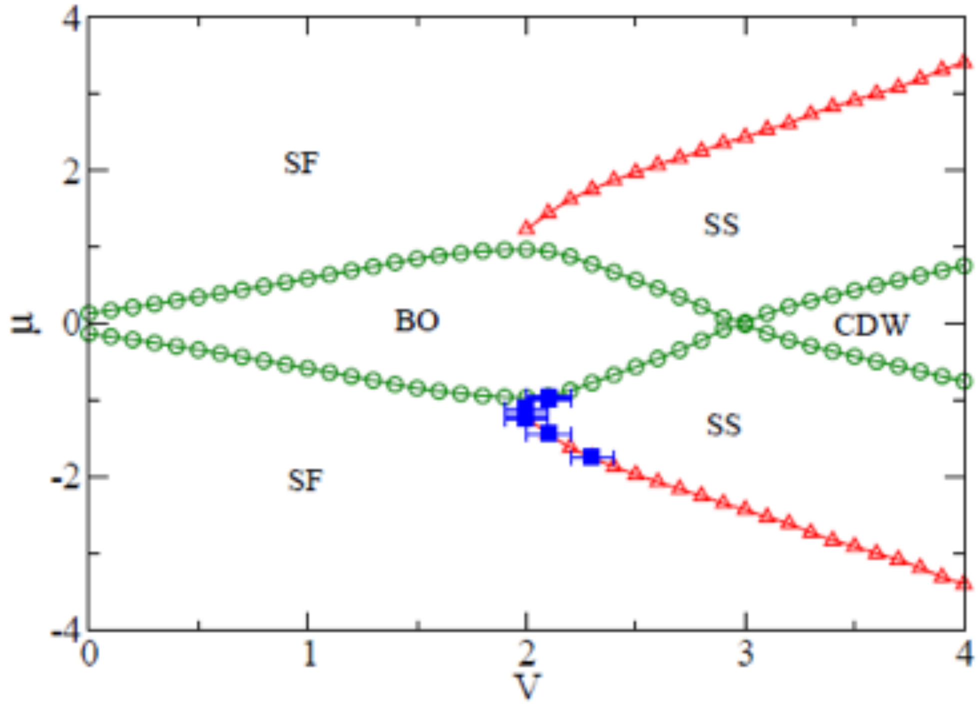


DMRG calculation

Mishra, Pai, Mukerjee & Paramekanti (2013)

- Re-entrant phase transition
- Continuously varying Luttinger parameter along phase boundaries

t-t'-V model



DMRG calculation

Incommensurate filling

- SS - supersolid phase
- No bond-ordered supersolid found

Summary

Frustration in bosonic systems can produce interesting phases like the Chiral Mott state, bond ordered solid, supersolid etc.