Emergent Coherence From Field-Induced Instabilities of a Fractionalized Quantum Spin Liquid

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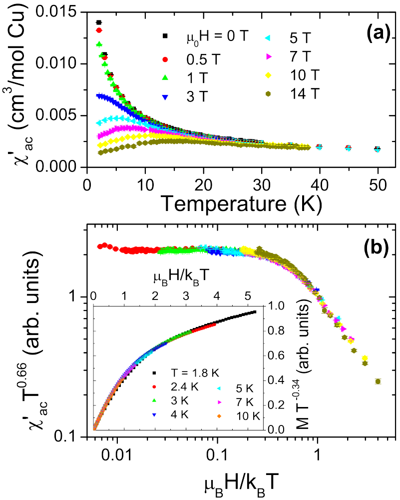
Outline

- Motivation.
- Kitaev Honeycomb Model: Fractionalization and Topological Order
- Our Model: Simplest Perturbed Kitaev Model
- Emergent Phenomena from a Perturbed Fractionalized Spin Liquid
- Applications
- Discussion and Open Issues.

Motivation

- Quantum Spin Liquids (QSL): Non-magnetic ground states of quantum spin models which do *not* spontaneously break *any* symmetries of the Hamiltonian.
- Elusive because (even quantum) spins generically "like to order".
- Exception(s) (Any-S) Heisenberg model on a Kagome lattice. Long-standing open problem. Quantum version (Herbertsmithite) shows finite-T signatures of a *critical* QSL (Helton et al, Mendels et al,...)
- nature of ground state (complex VBS, Z_2 QSL, $U(1)$-RVB) unsettled and controversial.

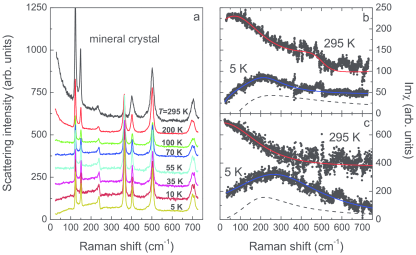
Susceptibility data

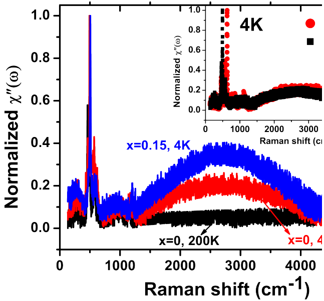


Motivation ....

- Increasing number of *real* Mott insulating TMOs with geometrically frustrated lattices (triangle, kagome) or with frustration induced by *orbital* degrees of freedom (Iridates).
- (Pseudo)spin frustration consequence of *directionality* of orbital hoppings in Mott-insulating TMO.
- Kugel-Khomskii spin-orbital Hamiltonian is frustrated in the orbital sector. However, care needed since crystal-field, spin-orbit, extended Heisenberg couplings can generically play spoilsport.
- But may it still be possible to consider these as perturbations over the idealized frustrated model???

Raman Shift data

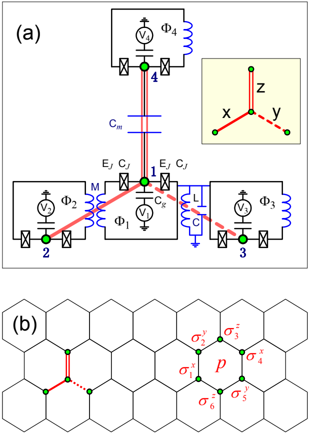




$$\mathcal{H} = \sum_a J_a \sum_{<i,j>_a} S_i^a S_j^a + J \sum_{<i,j>} \mathbf{S}_i.\mathbf{S}_j + (...)$$

- Artificially Engineered Kitaev Models with “Simple” Perturbations, e.g, Zeeman field!
- Explicit proposal of specially engineered Josephson Junction arrays (F. Nori’s group)
- For $J = 0$ rigorous topological order (TO).

Josephson Junction Array (Phys. Rev. B 81, 014505 (2010)).



Formalism

$$\mathcal{H} = \sum_a J_a \sum_{<i,j>_a} S_i^a S_j^a + J \sum_{<i,j>} \mathbf{S}_i . \mathbf{S}_j + (\dots)$$

- Deform honeycomb into brickwall lattice with "white" and "black" sites.
- Open BC: Consider JW transformation which threads the entire lattice by simple 1D path.

Formalism..

$$\sigma_{ij}^{+} = 2[\Pi_{j'<j,i}\sigma_{i'j'^z}][\Pi_{i'<i}\sigma_{i'j}^{z}]c_{ij}^{\dagger}$$

$$\sigma_{ij}^{z} = (2c_{ij}^{\dagger}c_{ij} - 1)$$

- Majoranas: $A_w = (c - c^\dagger)_w/i, B_w = (c + c^\dagger)_w$ and $A_b = (c + c^\dagger)_b, B_b = (c - c^\dagger)_b/i$, followed by the introduction of fermions $c = (A_w + iA_b)/2, c^\dagger = (A_w - iA_b)/2$.

$$H_K = -\frac{i}{4}[\sum_{x-bonds} J_x A_w A_b - \sum_{y-bonds} J_y A_b A_w - \sum_{z-bonds} J_z \alpha_{bw} A_b A_w]$$

- Where, α_{bw}=iB_bB_w defined on each Z bond.
- With $[\alpha_{bw}, H_K]$=± 1

Formalism....

- Applying the transformations $c = \frac{1}{2}(A_w + iA_b)$, $c^\dagger = \frac{1}{2}(A_w - iA_b)$, we get

$$H_{K1} = \frac{1}{4}[J_x \sum_i (c_i^\dagger + c_i)(c_{i+e_x}^\dagger + c_{i+e_x}) + J_y \sum_i (c_i^\dagger + c_i)(c_{i+e_y}^\dagger - c_{i+e_y})]$$

$$H_{K2} = J_z \sum_i \alpha_i (2c_i^\dagger c_i - 1)$$

- Local order parameters!
- Consider $\sigma_{1w}^y \sigma_{2b}^z \sigma_{3w}^x = \frac{1}{i}(c^\dagger - c)_w \sigma_{2b}^z \sigma_{1w}^z \sigma_{2b}^z (c^\dagger - c)_w$ $= i(c^\dagger + c)_{1w}(c^\dagger + c)_{3w} = iB_{1w}B_{3w}$ and $\sigma_{6b}^x \sigma_{5w}^z \sigma_{4b}^y = iB_{4b}B_{6b}$
- $I_h = \sigma_{1w}^y \sigma_{2b}^z \sigma_{3w}^x \sigma_{4b}^y \sigma_{5w}^z \sigma_{6b}^x = \alpha_{34}\alpha_{16}$; $[I_h.H_K] = 0$.

Formalism.....

- Vortex variables products of z consecutive Ising bond variables α_r
- $[\alpha_i, c_i] = 0 = [\alpha_i, c_i^\dagger]$, G.S:- All $\alpha_i = 1(-1)$.
- After Fourier transformation we get,

$$H_K = \sum_q [\epsilon_q c_q^\dagger c_q + \frac{i\Delta_q}{2}(c_q^\dagger c_{-q}^\dagger + h.c)]$$

$$\epsilon_q = \frac{1}{4}[2J_z - 2J_x cosq_x - 2J_y cosq_y]$$

$$\Delta_q = 2J_x sinq_x + 2J_y sinq_y$$

- wave function: $|G >= \Pi_k(u_k + v_k c_k^\dagger c_{-k}^\dagger)|0\rangle)$

Perturbations

- Simplest perturbation: "External" Zeeman field, $H_z = -h_z \sum_i S_i^z$; $H = H_K + H_z$
- Naive expectation: field induced magnetization, perhaps metamagnetic transition.
- In Kitaev case, however, $S^z = ib^z c$, $[b_i^x b_j^x, H] = 0 = [b_i^y b_j^Y, H]$ $\forall (ij) \parallel xx, yy \implies$ emergent, local Z_2 symmetries.
- Topological order (TO) only partially lifted, as $[b_i^z b_j^z, H] \neq 0$

Nature of the remnant TO!

- Focus on the XX-YY part. For a single chain, can solve exactly!
- For $J_x \neq J_y$, $\epsilon_q^{\pm} = \pm\sqrt{(}J_x^2 + J_y^2 + 2J_xJ_y cosq_x)$; lower band full, energy gap.
- For $J_x = J_y$; gap closes continuously. Transition does not involve change of symmetry, but of TO.
- We can write, $S_i^x = \tau_{i-1}^x\tau_i^x$, $S_i^y = \Pi_{l=i}^{2N}\tau_l^y$
- $H_K = \sum_{i=1}^{N}(J_x\tau_{2i-2}^x\tau_{2i}^x + J_y\tau_{2i}^y)$, 1D QIM!
- For $J_x > J_y$:- $Lim_{i\to\infty} < \tau_o^x\tau_{2i}^x > \sim [1-(\frac{J_y}{J_x})^2]^{1/4}$ $= Lim_{i\to\infty} < \Pi_{l=2}^{2i+1}S_l^Y \neq 0 >$

- Hence string orders both melt at QCP ($J_x = J_y$).
- Due to emergent d=1 GLS partial topological order survives.
- The QCP is easy to characterize in dual variables, where two spin nematic ordered states $< S_i^x S_j^x - S_i^y S_j^y > = \pm < n >$, simultaneously vanish at $J_x = J_y$ ("spin liquid"!)
- How does the field induced magnetization along ZZ-bonds "interplay" with remnant TO above? Consequences?

Our work starts here

- Clearer picture from JW fermion language!
- $H_z = 2h_z \sum_i (c_i^\dagger \alpha_i + h.c)i$

$$H_K = \sum_q [\epsilon_q c_q^\dagger c_q + \frac{i\Delta_q}{2}(c_q^\dagger c_{-q}^\dagger + h.c)] + \frac{J_z}{4}\sum_i (2n_{\alpha,i} - 1)(2c_i^\dagger c_i - 1)$$

- "Hubbard like" model of JW fermions.
- p-wave BCS pairing.
- onsite "Hubbard" $U = J_z$.
- local "spin-flip" or hybridization.
- $\Longrightarrow$ orbital selectivity on a 2-D square lattice.

JW ..

- However, as $[n_{i\alpha}, H] \neq 0$, local Z_2 gauge symmetry is lost.
- Gauge field becomes truly dynamical.
- Now, no exact solution.
- However can still be solved almost exactly as:
- xx,yy spin correlations exactly subsumed into bilinears of JW fermions.
- At $h_z = 0$ spin correlations rogorously only 1 lattice spacing long.
- Problem is similar to **Anderson OC**; however, singular behavior cut off by "Dirac" JW fermion spectrum, and by non-zero $(J_z/2)$ α fermion energy (Baskaran et al. 2007, Knolle et al. 2014).
- For $h_z \neq 0$, an approximation, however is expected to be adequate.

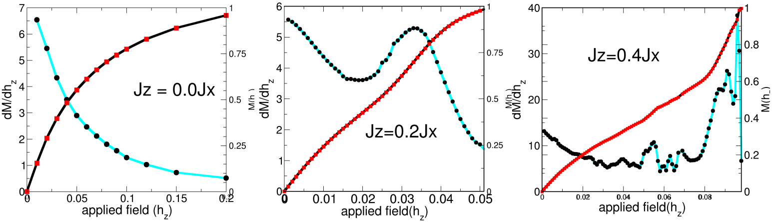
JW ...

- Impurity solver: Two-band IPT.
- Works quantitatively for related spinful Anderson Lattice model.
- Expect p-wave BCS+ field induced magnetizaton, **perhaps metamagnetic quantum criticality.**

- **Surprises in Store!!**

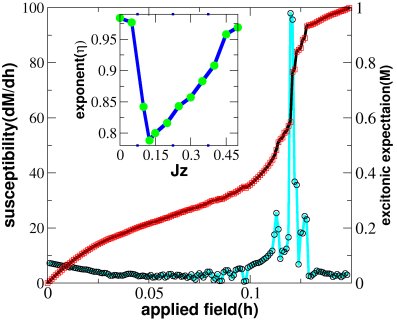
Susceptibility : anisotropic Kitaev limit

- Small h_z: spin liquid remains stable (symmetry protected TO).
- $J_x = J_y > J_z$, For $J_z < 0.25 J_x$ $m(h_z)$ smooth function of h_z.



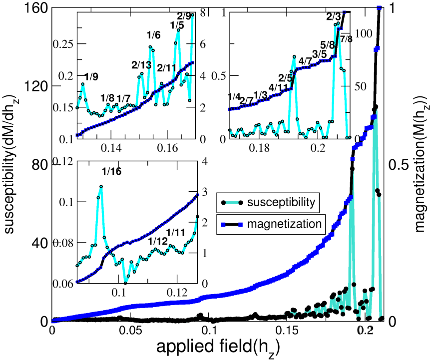
Susceptibility : anisotropic Kitaev limit

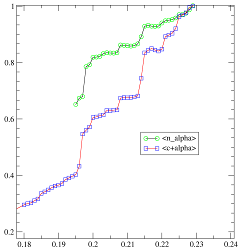
- The nature is similar to conventional field-induced magnetization in a "free e^-" paramagnet.
- However, $m(h_z) \propto h^{\alpha}$, where $0.78 < \alpha < 1.0$.
- Using exact GFs of KM, can show that $< S_i^z ; S_j^z > \propto (i-j)^{-4}$ (Feigelman et al.).



Plateaus and jumps: isotropic Kitaev limit

- However for $h > h_z^c$, we find a remarkable series of magnetization plateaus in m_{h_z} vs h_z at
- $\frac{m}{m_{sat}} = \frac{1}{16}, \frac{1}{12}, \frac{1}{11}, \frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{2}{13}, \frac{1}{6}, \frac{2}{11}, \frac{1}{5}, \frac{2}{9}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{3}{8}, \frac{3}{7}, \frac{2}{5}, \frac{4}{7}, \frac{5}{8}, \frac{3}{4}$.





- Both even and odd denominator plateaus.
- Odd denominator plateaus in σ_{xy} well-known in FQHE, which is also the odd only other example of a real system showing rigorous TO.
- Also the possibility of even denominators is observed in Shastry-sutherland models (Mila's talk): crystals of two triplon bound states. due to competition between frustration and field induced magnetization.

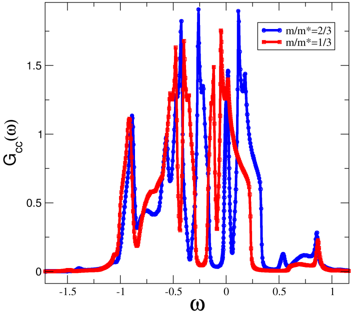
- So no relation to FQHE but to “incompressible” solids of kink-dipole crystals (excitonic solids), sandwiched between BECs of kink-dipoles.

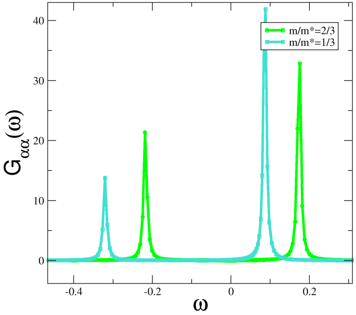
- Oscillations in χ_{h_z} as *dHvA* or *SdH* quantum oscillations of JW fermions in partially magnetized spin liquid phase. Hidden coherence in a spin liquid (Anderson, 1973).

- Here due to nodal Bogoliubov (p-wave) fermions in H_K the TO phase of KM unstable to an intricate sequence of partial ordered “solids” co-existing with remnant of TO state (before reaching saturation).

Spectral functions : different plateaus

- Clear orbital selectivity: $G_{\alpha\alpha}(\omega) \Longrightarrow$ Kondo Insulator.
- $G_{cc}(\omega)$: "spin-metal" of "nodal" JW-Bogoliubov qps.

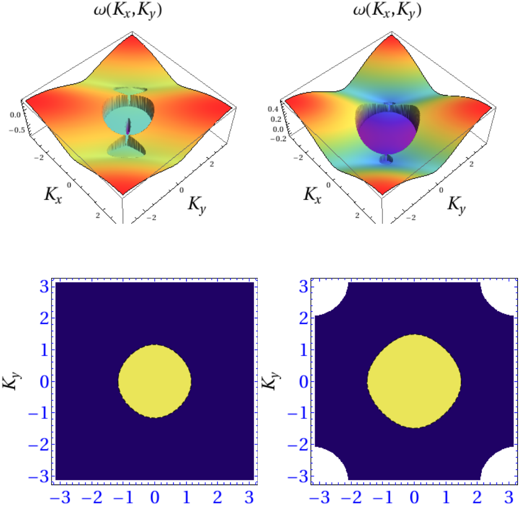




- Large-scale spectral weight reshuffling across energy scales O($2J_x$) occurs. Thus the plateaus originate from between Mott localization (J_z) and hybridization + hopping (h,J_x).
- Can be mssed by static HF.
- Alike FL* (c.f f electron QCP, Senthil et al.; PRL 2003)
- Topological FL^* state!

Dispersion and “Fermi surface”

- $G_{cc}^{-1}(k,\omega) = 0$



- Surface of zeroes of $G_{cc}(k,\omega)$, rather than of poles.
- Evident via $\Sigma_{an}(k,\omega) = \frac{\Delta_k^2}{\omega+\epsilon_k+\Sigma_{cc}(-\omega)-\frac{h_z^2}{\omega-\Sigma_{\alpha\alpha}(\omega)}}$
- So poles of Σ_{an} appear as zeroes of $G_{cc}(k,\omega)$
- again, exactly alike FL^* in OSMT.
- Topological change of FS across each plateau.
- Explicit realization of YRZ ansatz (cf. underdoped cuprates).
- Remarkably, all this caused by field-induced spectral-weight transfer from QSL to the magnetized component.

Kitaev Toric Code Model (TCM)

- In JW fermion language, $J_z \gg J_x \implies$ no double occupancy constraint.
- Implement by Gutzwiller Projection: $P_G = \Pi_i(1 - n_{ic}n_{i\alpha})$ acting on $|\psi_{PBCS}>$.
- $|\Psi_{TC}> = P_G \Pi_k(u_k + v_k c_k^\dagger c_{-k}^\dagger)|0>$
- Precisely the Gutzwiller-projected p-wave BCS state or p-wave RVB state! (F. Becca et al.)
- Can investigate TCM and it's non-abelian excitations in terms of variational wavefunctions/DMFT.

JJA

- Josephson charge qubits joined together together in a special way along three bonds.
- Capacitive coupling along XX bonds.
- Inductive coupling along YY bonds.
- Charge coupling along ZZ bonds.
- Identify Kitaev spins with $n_i = \frac{1-S_i^z}{2}$, $cos\phi_i = \frac{S_i^x}{2}$, $sin\phi_i = -\frac{S_i^y}{2}$
- This is written in the Cooper pair number basis, $n_i = 0, 1 = |\uparrow\rangle_i, |\downarrow\rangle_i$

JJA

- If each charge qubit placed at $n_i = \frac{1}{2} = n_g$, and $J_x (= J_y) < J_z$
- We get $H = H_K + h_i S_i^x$ (not S_i^z!)
- But $S_i^x \rightarrow S_i^z$, $S_i^z \rightarrow -S_i^x$
- $\implies H = H_K + h_i S_i^z$, $J_z < J_x = (J_y)$ $h = E_{J_i}(\phi_i) = 2E_{J_i} cos(\frac{\pi \phi_i}{\phi_0})$ where $\phi_0 = \frac{\hbar \pi}{e}$.
- **Exactly our model!**

Relevances

- Upon varying flux by changing real magnetic field or/and adding non-magnetic impurities we have a host of “JW excitonic solid” criystals.
- Can be realised as competition between material parameters and flux.
- Kink-dipole crystals!
- Co-exist with remnant of TO state of H_K (critical topological supersolid).
- Excitonic Josephson Effect as in e-h bilayers (Y Joglekar et al., PRB 72, 205313 (2005)).

Relevances..

- Critical current $J_c \simeq h_z^2 < c_i^\dagger \alpha_i; c_j^\dagger \alpha_j > \simeq h_z^2 < c_i^\dagger \alpha_i >< c_j^\dagger \alpha_j >$
- Also have direct "fermionic" current from p-wave Bogoliubov quasiparticles.
- Critical current shows fractional oscillations as flux (ϕ_i) ramped up.
- Also fractional Shapiro steps, generation of GHz (sub GHz) radiation (Topological plasmonics?)

Open Questions ..

- Search for suitable TM oxide-based materials?
- Orbital Kondo effect (topological version) and its breakdown?
- QCPs due to Kondo-breakdown?

More Open Questions ....

- Whither Kitaev-Heisenberg model(s), Kugel-Khomskii models?
- Hole-doping: is unconventional metallicity/superconductivity cleanly demonstrable?
- Real TMO based materials?
- Generalizations to 3d hyper honeycomb lattice?
- Clean approximation-free demonstration of (some) observed plateaus.

Conclusions

- Rigorous TO (spin-liquid) phase of H_K with Zeeman field.
- unstable to novel orders exemplifying emergent coherence.
- "magnetization steps" $\Longrightarrow$ "JW excitonic solid" crystals.
- "Barkhausen" steps in a field-perturbed spin liquid.
- Hidden coherence $\Longrightarrow$ remnant TO (emergent d=1 GLS).
- Novel applications to QIP, plasmonics, TQC.
- Maybe to unconventional orders in TMO.