Spontaneous time-reversal symmetry breaking (TRSB) in superconductors and related materials

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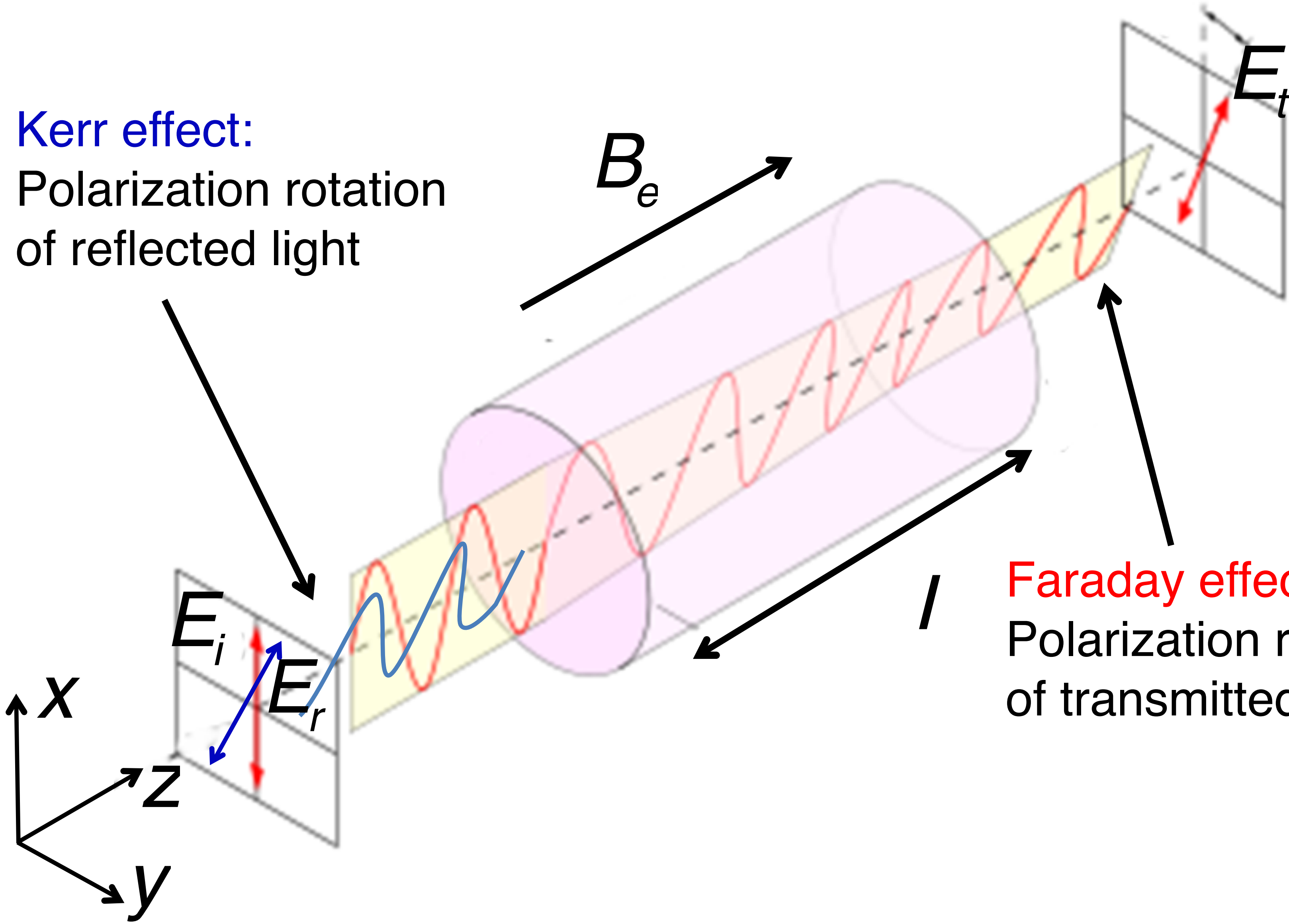
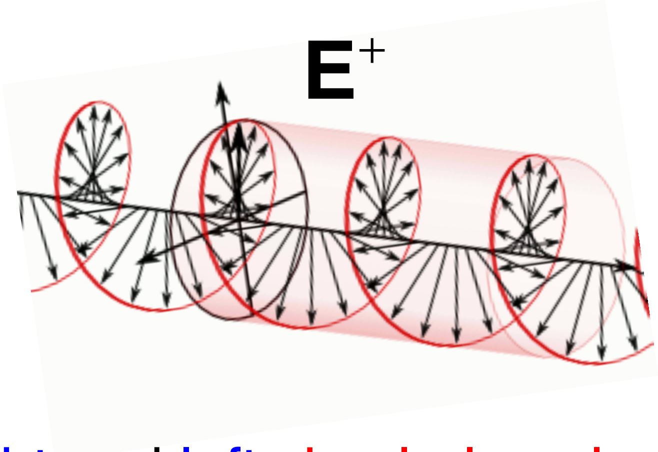
Faraday and Kerr effects in a magnetic field

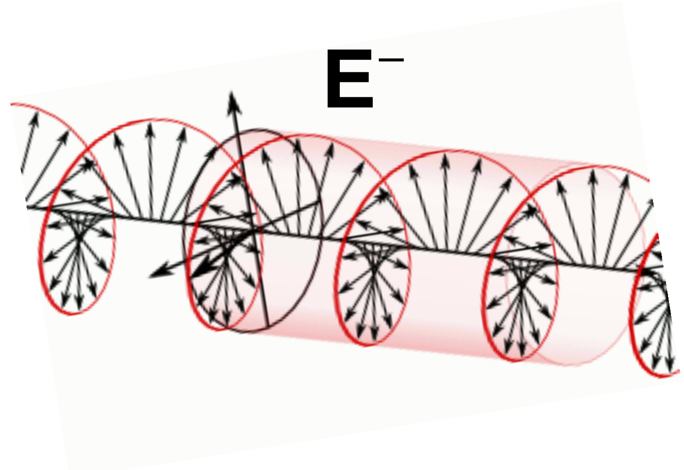
Kerr effect:
Polarization rotation
of reflected light



Faraday effect:
Polarization rotation
of transmitted light

Circular dichroism





Right and left circularly-polarized modes have different refraction indices due to Hall conductivity:

$$n^{+} - n^{-} \propto \sigma_{xy}(\omega)$$

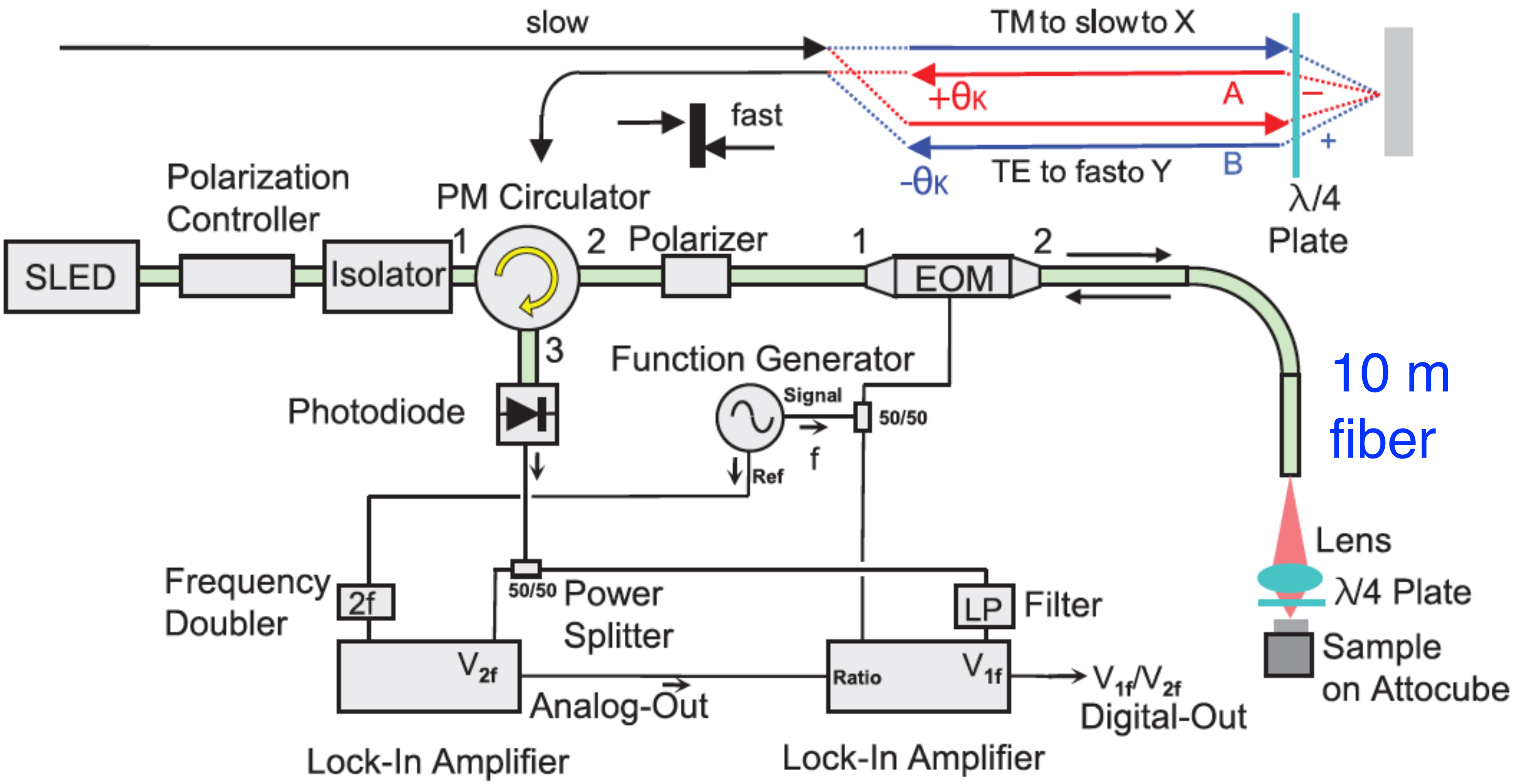
Kerr effect was observed in the PG phase of cuprates by Kapitulnik since 2008, interpreted as time-reversal symmetry breaking (TRSB).

Faraday rotation $\theta_F = \dfrac{\omega}{2c}\left(n^{+} - n^{-}\right) l$

→ Kerr rotation $\theta_K = \mathrm{Im}\left(\dfrac{n^{+} - n^{-}}{n^{+} n^{-} - 1}\right)$

(requires dissipation)

Ultrasensitive apparatus for optical detection of time-reversal symmetry breaking



Fried *et al*, *Rev Sci Instr* **85**, 103707 (2014)

Polar Kerr effect and TRSB in Sr_2RuO_4

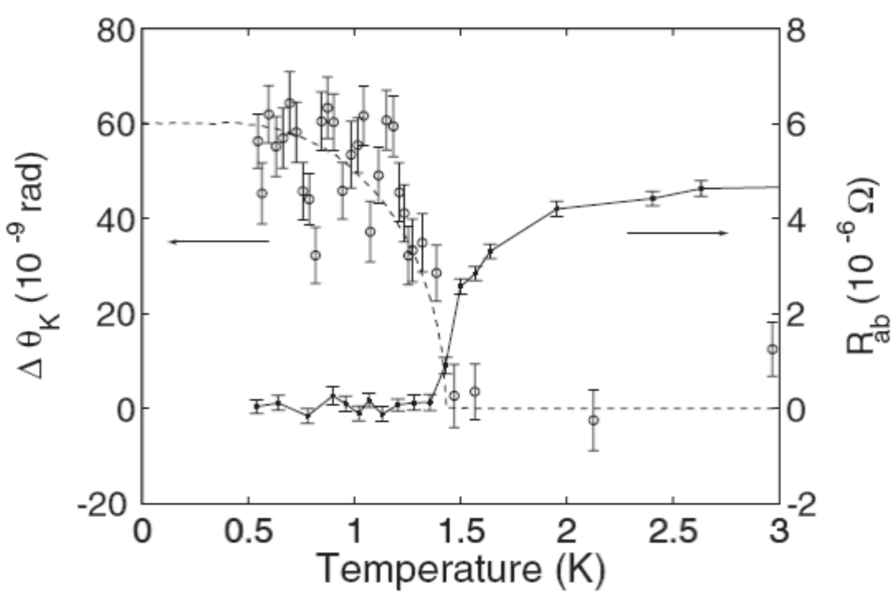


FIG. 2. Zero-field (earth field) measurement of Kerr effect (○) and *ab*-plane electrical resistance (dotted line). Dashed curve is a fit to a BCS gap temperature dependence.

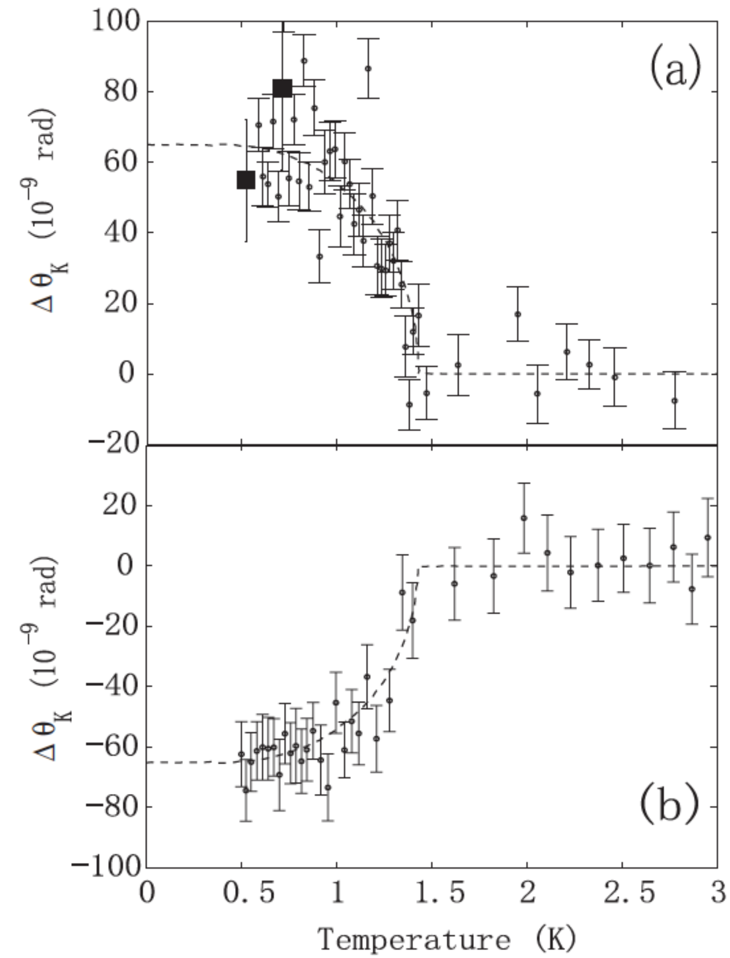
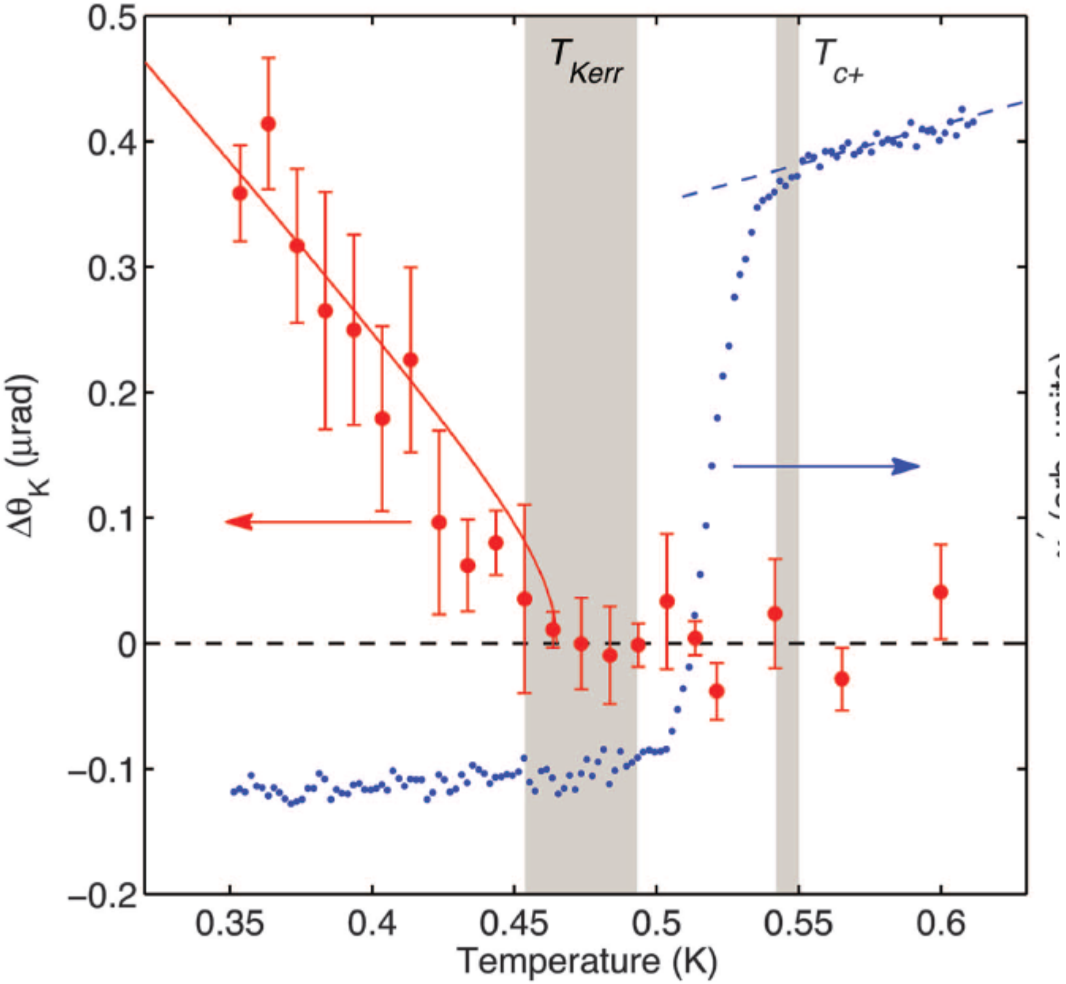
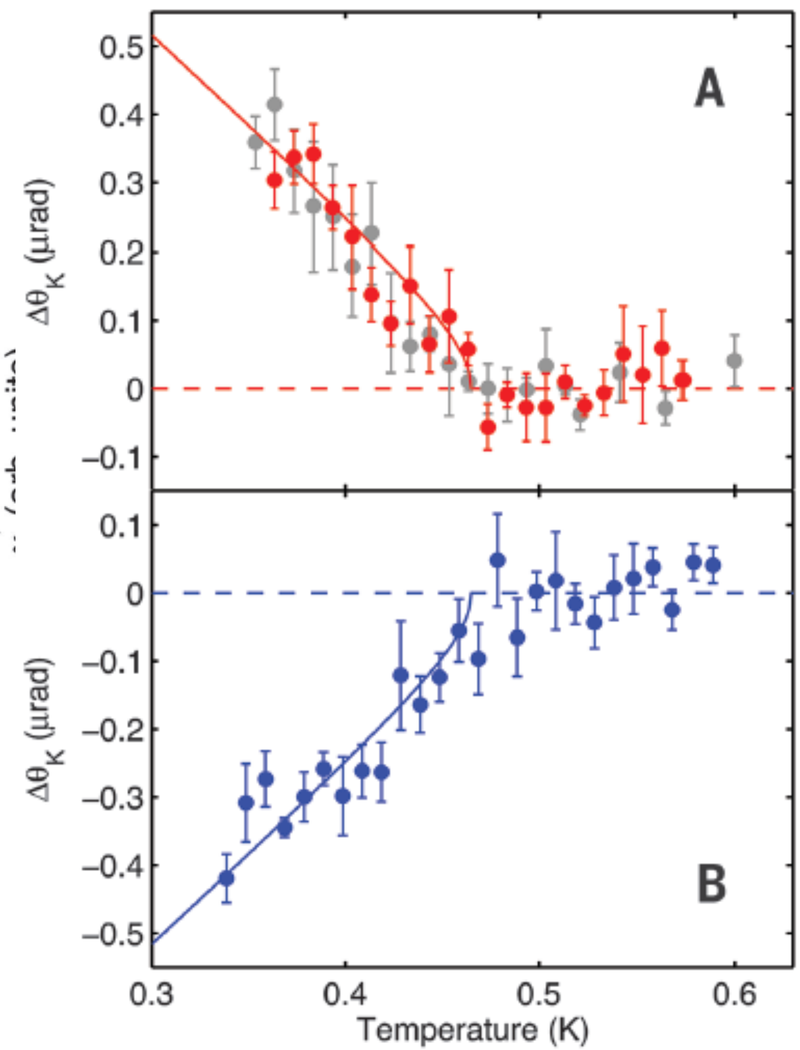


FIG. 3. Representative results of training the chirality with an applied field. (a) +93 Oe field cool, then zero-field warm-up (○). The two solid squares represent the last two points just before the field was turned off. (b) −47 Oe field cool, then zero-field warm-up (○). Dashed curves are fits to a BCS gap temperature dependence.

Xia, …, Kapitulnik,
PRL **97**, 167002 (2006)

Polar Kerr effect and TRSB in UPt_3

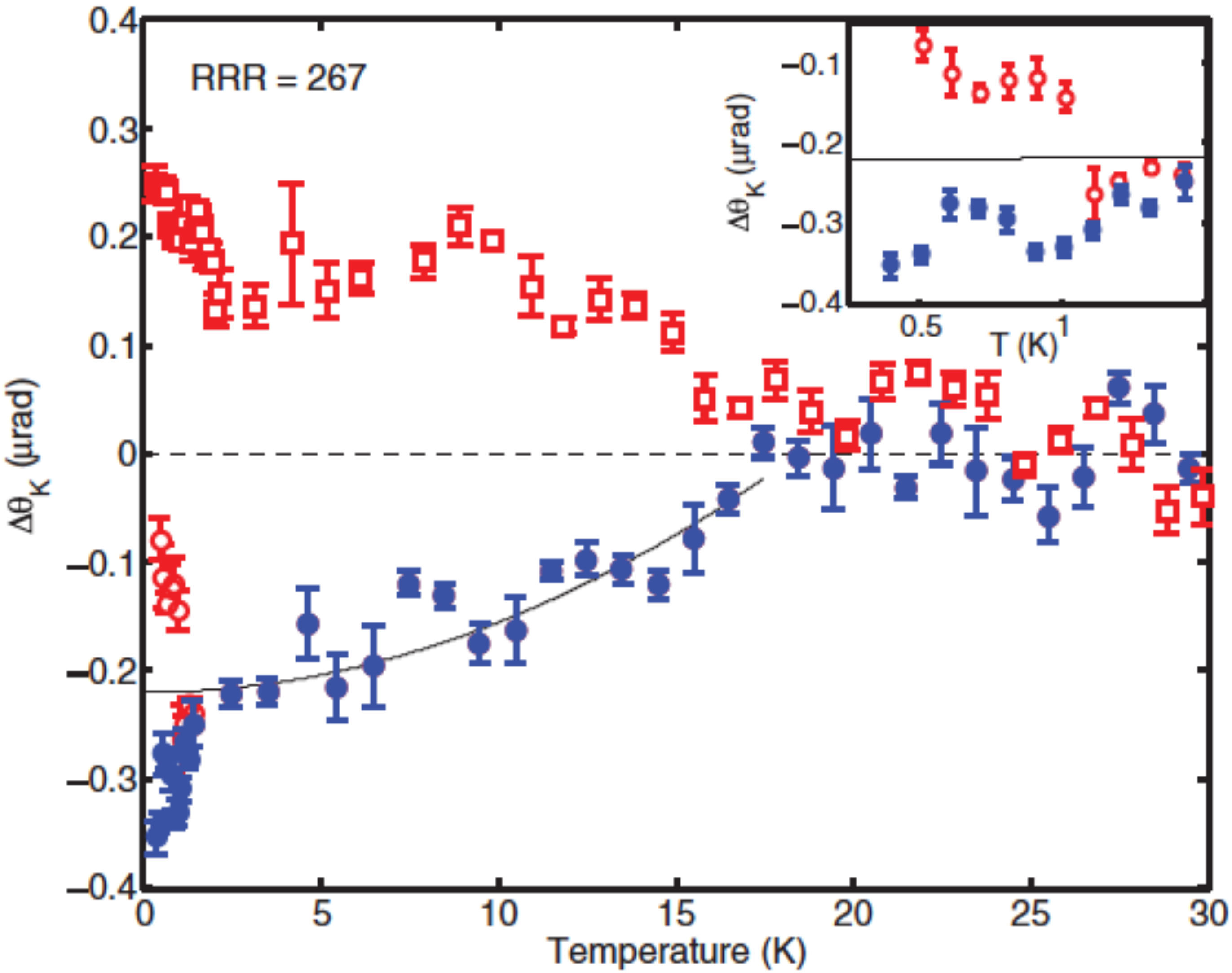




Two superconducting transitions

Schemm, …, Kapitulnik, *Science* **345**, 190 (2014)

Polar Kerr effect and TRSB in URu_2Si_2



Hidden order

T_{HO}=17.5 K

Super-
conductivity

T_c=1.5 K

Schemm, …, Kapitulnik, *PRB* **91**, 140506 (2015)

Theory of the high-frequency chiral optical response of a p_x+ip_y superconductor

Victor Yakovenko

with **Roman Lutchyn** and **Pavel Nagornykh**

Joint Quantum Institute (JQI) at
the University of Maryland

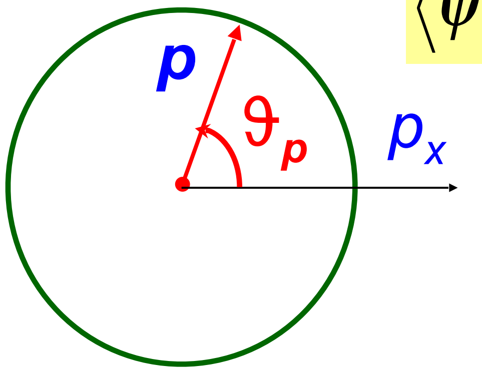
PRL **98**, 087003 (2007)

PRB **77**, 144516 (2008)

PRB **80**, 104508 (2009)

The experiment shows that the superconducting order parameter in Sr_2RuO_4 must have intrinsic vorticity. The natural candidate is the triplet p_x+ip_y pairing (Rice & Sigrist, 1995):

$$\langle\psi(\boldsymbol{p})\psi(-\boldsymbol{p})\rangle \propto \Delta(\boldsymbol{p}) = \Delta_0\left(p_x + ip_y\right)/\,p_F = \Delta_0 e^{i\vartheta_{\boldsymbol{p}}}$$



$\Delta(\boldsymbol{p})$ accumulates phase 2π around the Fermi surface – a vortex in momentum space.

It represents the Cooper pairing between electrons with a non-zero angular momentum $L_z=1$.

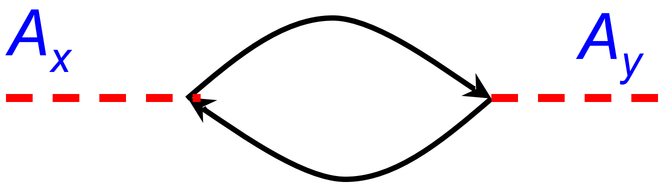
Kerr rotation is permitted by symmetry in this case, but we need to calculate the magnitude of the Kerr angle θ_K.

The experiment gives θ_K = 65 nanorad at $T=0$.

According to the textbook, $\theta_K = \mathrm{Im}\dfrac{4\pi\sigma_{xy}(\omega)}{n(n^2-1)\omega d}$

where n is the refraction coefficient, d is the interlayer distance.

The ac Hall conductivity $\sigma_{xy}(\omega)$ can be obtained by calculating the one-loop current-current response function

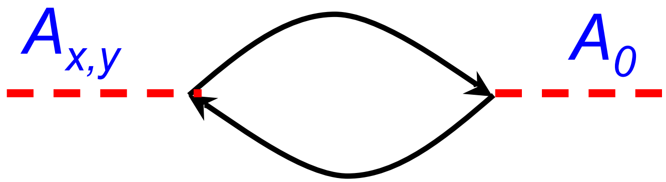


using the Nambu Green function for a chiral superconductor

$$G = -\frac{i\omega + \varepsilon(\boldsymbol{p})\hat{\tau}_3 + p_x\Delta_x\hat{\tau}_1 - \Delta_y p_y\hat{\tau}_2}{\omega^2 + E^2(\boldsymbol{p})}$$

However, this diagram vanishes identically (by taking traces, before integration), even though it is permitted by symmetry.

On the other hand, the one-loop current-charge response function does not vanish for a chiral superconductor



It was calculated by Volovik (JETP 1988) for ω=0 (dc response) and by Yakovenko (PRL 2007) for $\omega\neq 0$ (ac response).

This diagram generates the Chern-Simons-like term in the effective action of the system:

$$S = \sigma_{xy}\int dxdydt\left(A_0 + \partial_t\varphi / 2e\right)\left(\partial_y A_x - \partial_x A_y\right)/ c,$$

where φ is the superconducting phase.

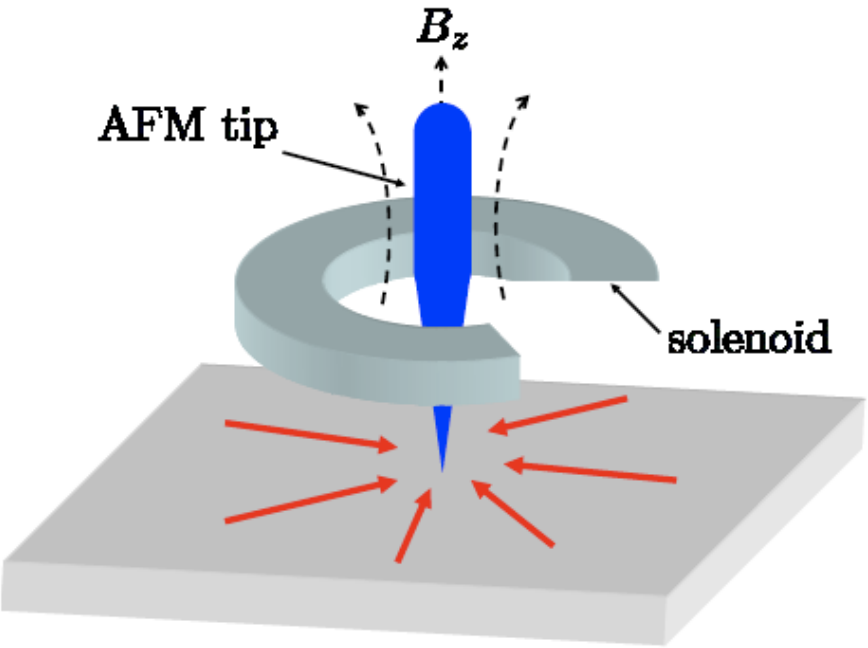
It describes a change of the electron charge density ρ in response to an applied magnetic field B_z:

$$\rho = \frac{\delta S}{\delta A_0} = \frac{1}{2}\frac{e^2}{hc}B_z$$

Magnetoelectric response

The electric charge of a standard vortex with the flux quantum $\phi_0 = hc/2e$ is $Q=e/4$ (Goryo 2000, Stone & Roy 2004).

Proposed experiment to detect electric charge induced by magnetic field (Lutchyn, Nagornykh, Yakovenko 2008)



However, the anomalous charge response does not imply the Hall effect. The Štreda formula does not apply to superconductors!

The effective action gives the Hall-like current

$$\boldsymbol{j} = c\frac{\delta S}{\delta \boldsymbol{A}} = \sigma_{xy}\left[\boldsymbol{E} - \tfrac{1}{2e}\tfrac{\partial}{\partial t}\left(\nabla\varphi - \tfrac{2e}{c}\boldsymbol{A}\right)\right]\times\hat{z}$$

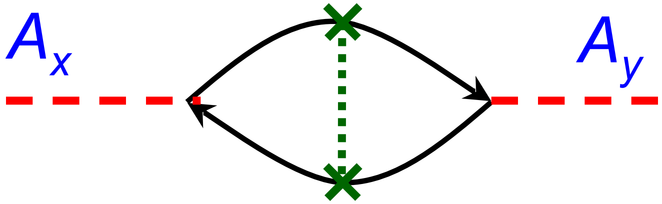
When the effective action for the superconducting phase φ is added, and φ is integrated out, the Hall current vanishes

- Goryo and Ishikawa, Phys. Lett. A 1998, 1999
- Horovitz and Golub, Europhys Lett 2002, PRB 2003
- Lutchyn, Nagornykh, and Yakovenko, PRB 2008
- Roy and Kallin, PRB 2008

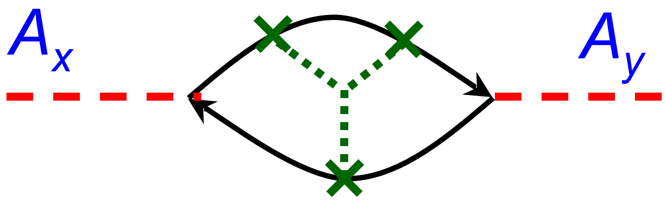
Vanishing of the Hall current for a chiral superconductor can be understood as a consequence of the Galileo invariance (Read and Green, PRB 2000). In the absence of an external magnetic field, an electric field cannot cause a sideways deflection of an electron cloud, even if electrons have the chiral Cooper pairing.

So, the only way to produce a non-zero Hall effect in a chiral superconductor is by taking into account impurity scattering. It produces "friction" between the Cooper pairs and the lattice, which results in a sideways Magnus force.

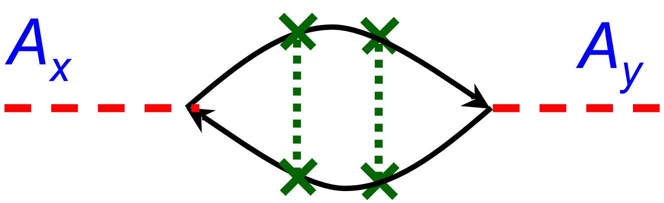
However, the lowest-order diagram with one impurity line vanishes identically (by taking traces):

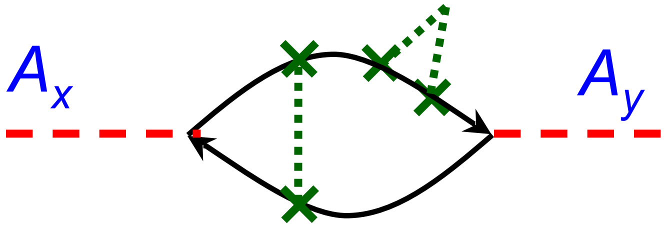


A non-zero Hall effect comes from the following diagrams:



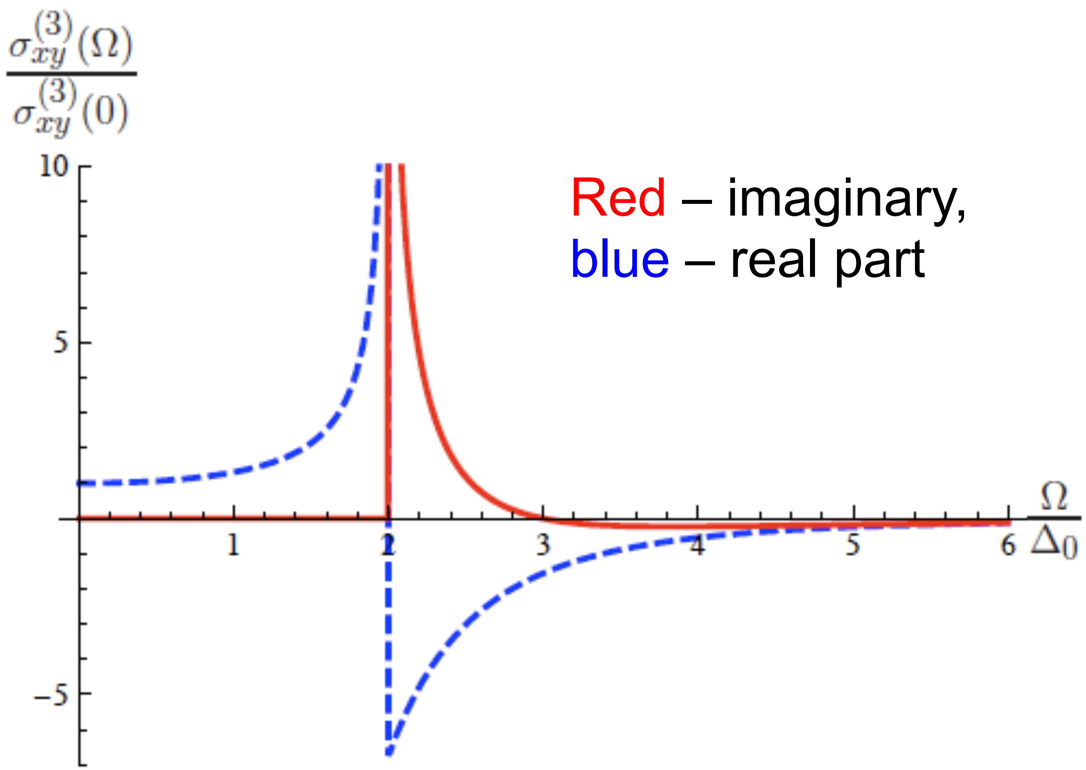
Goryo PRB 2008: In this diagram, the sign of the Hall effect is determined by the sign of the impurity potential.





Lutchyn, Nagornykh, Yakovenko, *PRB* **80**, 104508 (2009): The Hall effect sign depends on the electron-hole asymmetry.

Goryo's diagram for $\sigma_{xy}(\omega)$ (non-Gaussian disorder)



Red – imaginary,
blue – real part

$$\sigma_{xy}^{(3)}(\omega=0)=\frac{8}{105\pi}\frac{e^2}{\hbar}\frac{W}{\Delta_0^2}$$

Divergence

$$\frac{1}{\sqrt{\omega-2\Delta_0}} \quad \text{at} \quad \omega\to 2\Delta_0$$

For $\omega\gg\Delta_0$:

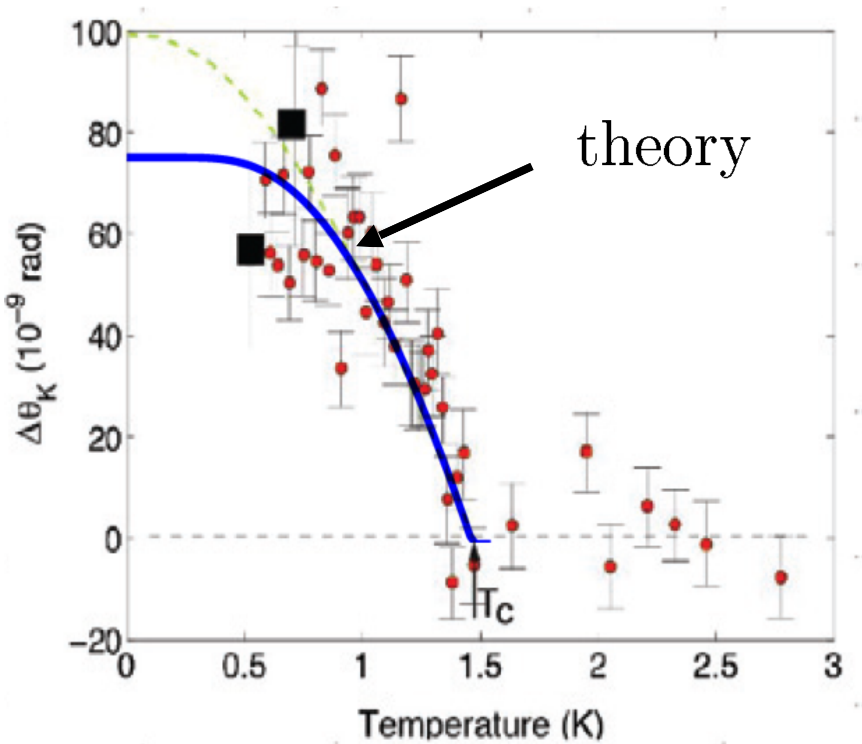
$$\sigma_{xy}^{(3)}(\omega)=-\frac{e^2}{\hbar}\frac{W\Delta_0}{\omega^3}\left[i+\frac{4}{\pi}\frac{\Delta_0}{\omega}\ln\left(\frac{\omega}{\Delta_0}\right)\right] \quad \text{at} \quad T=0$$

$$\sigma_{xy}^{(3)}(\omega)=-\frac{e^2}{\hbar}\frac{W\Delta_0}{\omega^3}\frac{\Delta_0}{T_c}\left[\frac{i}{2}+\frac{4}{\pi}\frac{T_c}{\omega}\ln\left(\frac{\omega}{T_c}\right)\right] \quad \text{at} \quad T\to T_c$$

General property:
$\sigma(\omega)=\sigma^*(-\omega)$

Temperature dependence of the high-frequency $\sigma_{xy}(\omega)$ (for non-Gaussian disorder)

Our theory: *PRB* **80**, 104508 (2009)



$$\frac{\sigma''^{(3)}_{xy}(\omega,T)}{\sigma''^{(3)}_{xy}(\omega,0)} = \frac{\Delta_0(T)}{\Delta_0(0)} \tanh\left(\frac{\Delta_0(T)}{2T}\right)$$

for $\quad \omega >> T_c$

Experimental data:
Kapitulnik *et al.*,
New J Phys **11**, 055060 (2009)

For T near T_c, we find $\sigma_{xy} \propto \Delta_x \Delta_y \propto \Delta_0^2(T) \propto (T_c - T)$

Anomalous $\sigma_{xy}(\omega)$ for multi-band superconductivity

- Taylor and Kallin, *PRL* **108**, 157001 (2012); *J Phys Conf Ser* **449** 012036 (2013), 3-band model for Sr_2RuO_4
- Gradhand, Wysokiński, Annett, Györffy, *PRL* **108**, 077004 (2012); *PRB* **88**, 094504 (2013); *J Phys Cond Mat* **26** 274205 (2014); *Phil Mag* **95**, 525 (2015)
- Brydon and Yakovenko, in preparation, 2-band superconducting pairing on a hexagonal lattice for TMD

$$\Psi_{\mathbf{k}} = \left(a_{\mathbf{k},\uparrow},\ b_{\mathbf{k},\uparrow},\ a^{\dagger}_{-\mathbf{k},\downarrow},\ b^{\dagger}_{-\mathbf{k},\downarrow} \right)^T ,$$

$$\check{H}_{\mathbf{k}} = \begin{pmatrix} -\mu & \epsilon_{\mathbf{k}} & 0 & \Delta_1(\mathbf{k}) + i\Delta_2(\mathbf{k}) \\ \epsilon^*_{\mathbf{k}} & -\mu & \Delta_1(\mathbf{k}) - i\Delta_2(\mathbf{k}) & 0 \\ 0 & \Delta^*_1(\mathbf{k}) + i\Delta^*_2(\mathbf{k}) & \mu & -\epsilon_{\mathbf{k}} \\ \Delta^*_1(\mathbf{k}) - i\Delta^*_2(\mathbf{k}) & 0 & -\epsilon^*_{\mathbf{k}} & \mu \end{pmatrix}$$

$$\sigma_H(\omega) = \frac{4e^2}{i\omega}\frac{1}{N}\sum_{\mathbf{k}} \frac{(\mathbf{v}_{\mathbf{k}} \times \mathbf{v}^*_{\mathbf{k}}) \cdot \mathbf{e}_{\mathbf{z}}\, \mu \operatorname{Im}\{\Delta_1(\mathbf{k})\Delta^*_2(\mathbf{k})\}}{E_{\mathbf{k},+}E_{\mathbf{k},-}(E^2_{\mathbf{k},+} + E^2_{\mathbf{k},-})} \left(\frac{1}{\omega + i0^+ + E_{\mathbf{k},+} + E_{\mathbf{k},-}} + \frac{1}{\omega + i0^+ - E_{\mathbf{k},+} - E_{\mathbf{k},-}} \right)$$

Conclusions

- The Hall effect for a clean one-band chiral superconductor (without impurities) vanishes identically.
- A non-zero Hall effect can be obtained only if we take into account impurity scattering or multi-band superconductivity.
- Magnitude of the Hall effect depends on the concentration of impurities and strength of the scattering potential.
- There is a preliminary experimental indication that the Kerr effect is stronger in the Sr_2RuO_4 samples of the lower quality, i.e. with the higher concentration of defects.
- Temperature dependence of the Kerr angle follows $\theta_K \sim \Delta^2(T) \sim (T_c - T)$ neat T_c.

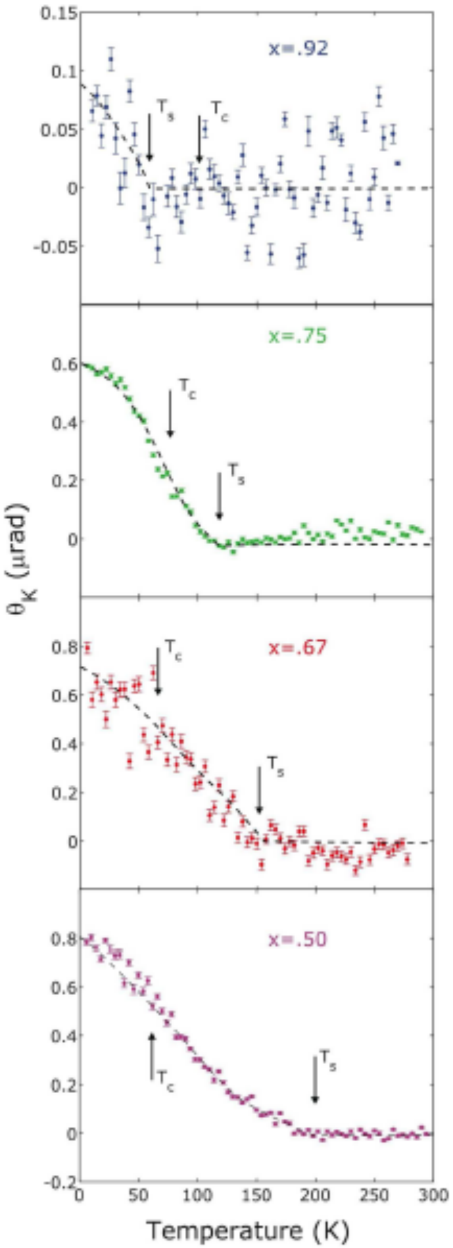
Connection with the quantum field theory

"Pseudogap and time reversal breaking in a holographic superconductor"
M. M. Roberts and S. A. Hartnoll
Journal of High-Energy Physics **8**, 035 (2008)

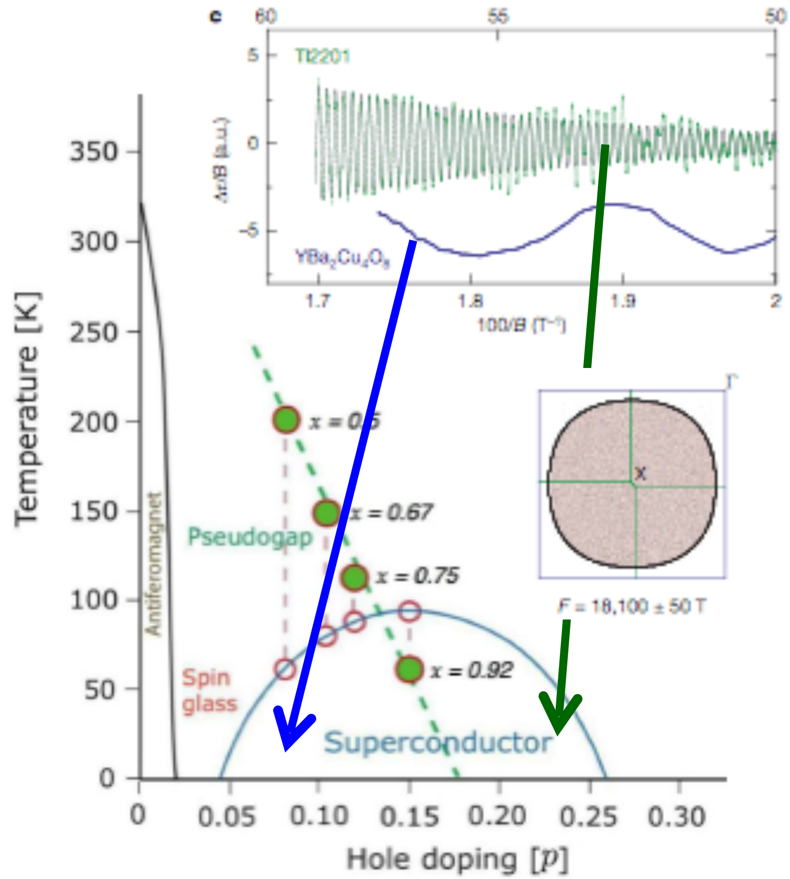
Abstract: Classical SU(2) Yang-Mills theory in 3+1 dimensional anti-de Sitter space is known to provide a holographic dual to a 2+1 system that undergoes a superconducting phase transition. We study the electrical conductivity and spectral density of an isotropic superconducting phase. We show that the theory exhibits a pseudogap at low temperatures and a nonzero Hall conductivity. The Hall conductivity is possible because of spontaneous breaking of time reversal symmetry.

Time-reversal symmetry breaking in underdoped cuprates

Experimental observation of the polar Kerr effect in underdoped cuprates $YBa_2Cu_3O_{6+x}$



Quantum magnetic oscillations



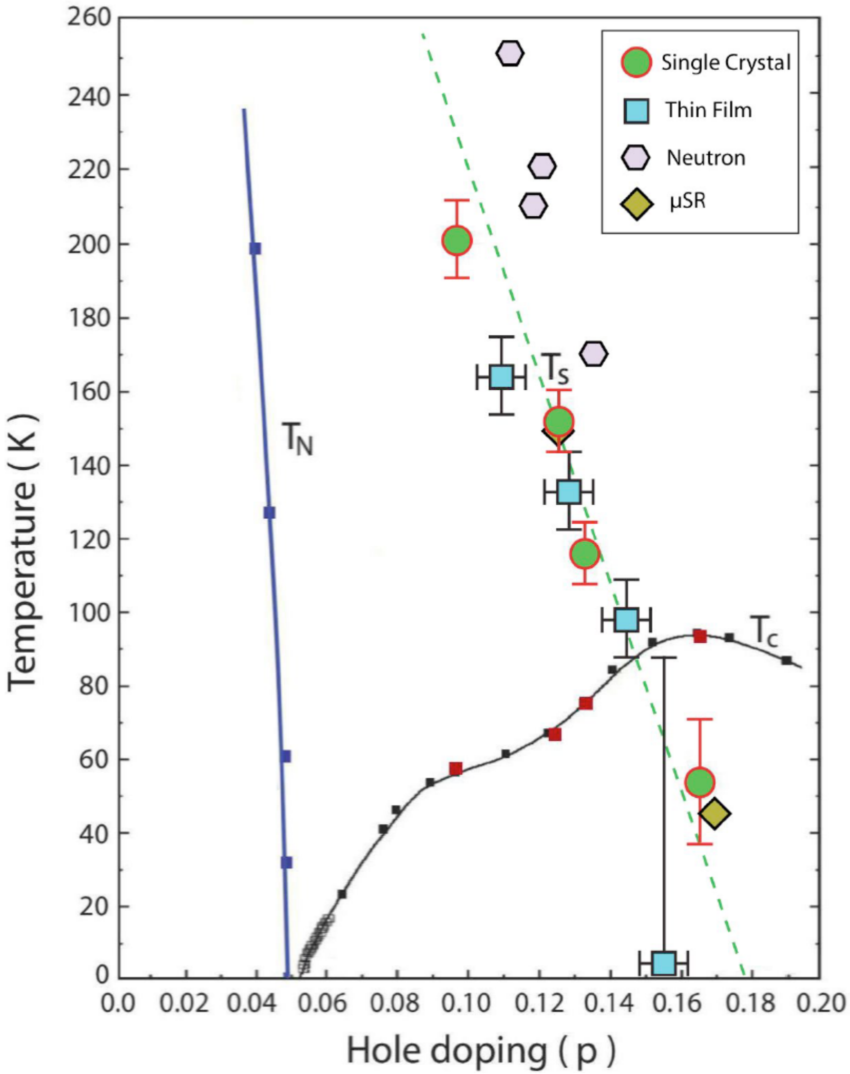
Xia, ..., Kapitulnik,
PRL **100**, 127002 (2008)

Observation of the polar Kerr effect demonstrates spontaneous time-reversal-symmetry breaking (TRSB) in underdoped cuprates.

TRSB is unrelated to superconductivity.

TRSB represents a different phase transition, which seems to be related to the pseudogap.

More evidence for a phase transition line



Kapitulnik *et al.*,
New J Phys **11**, 055060 (2009)

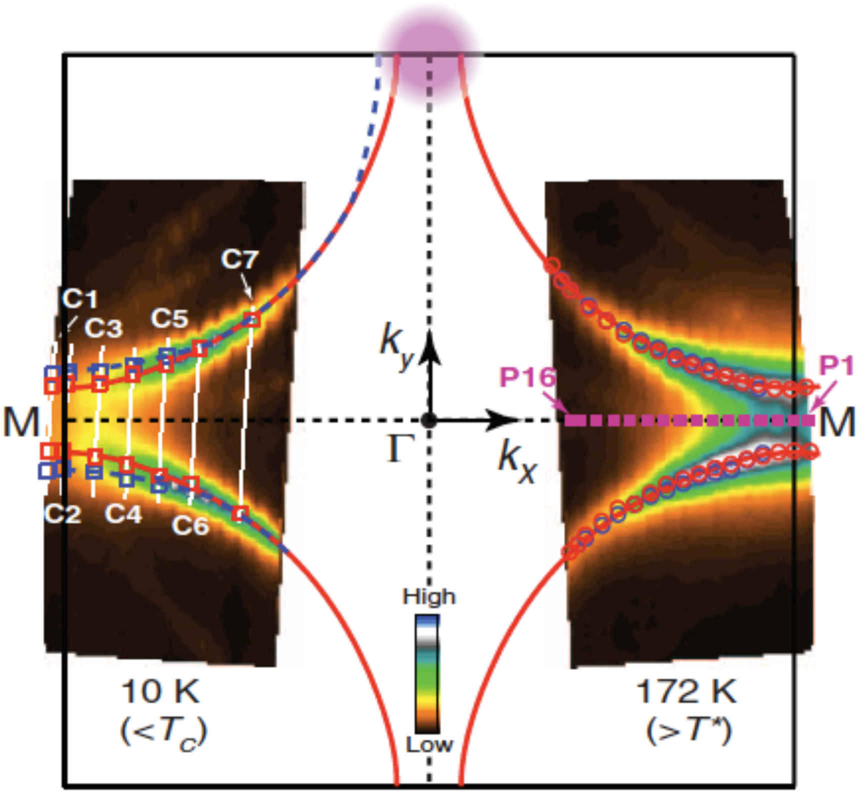
More evidence for the time-reversal symmetry breaking at "pseudogap" line

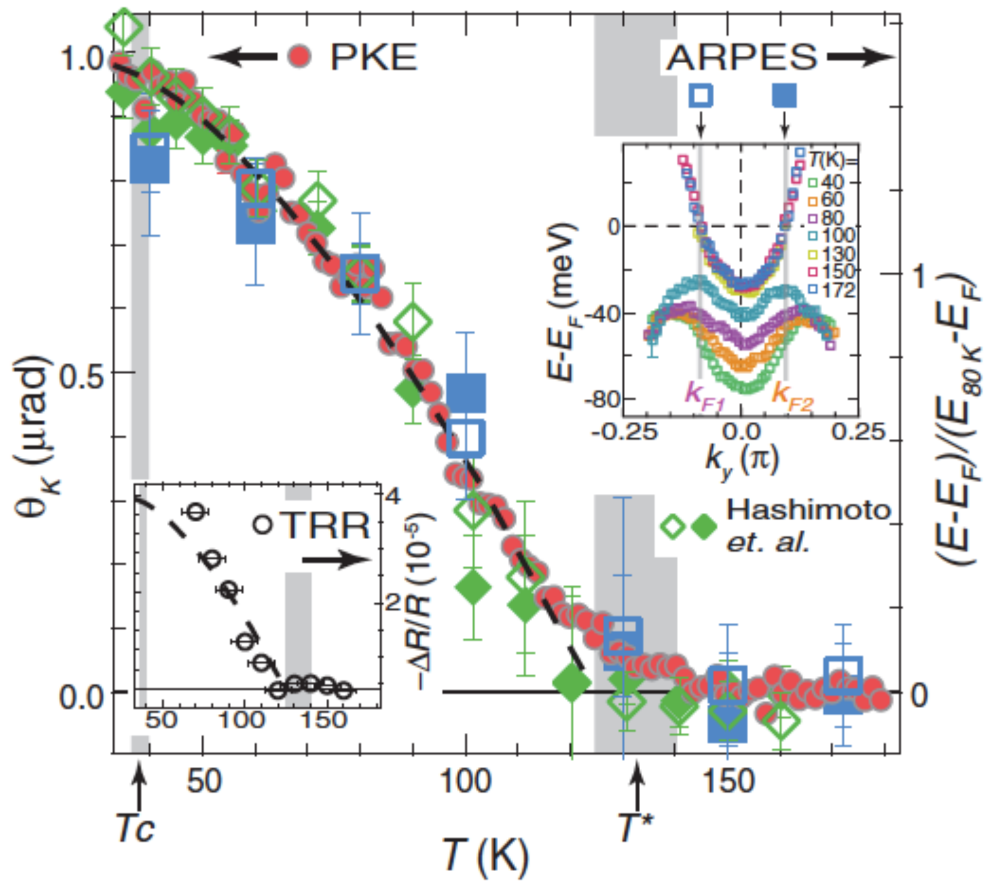
- μSR: Sonier *et al.*, *Science* **292**, 1692 (2001)
- Neutron scattering experiments: Fauqué *et al.*, *PRL* **96**, 197001 (2006) Mook *et al.*, *PRB* **78**, 020506 (2008)
- Noise dynamics: Van Harlingen, *PRL* **104**, 177001 (2010)

Telegraph noise develops below the "pseudogap" line. Domain motion? Domains imply symmetry breaking due to a phase transition. The noise is sensitive to a magnetic field. TRSB?

Growing experimental evidence shows that the "pseudogap" line is a phase transition line with symmetry breaking.

Pseudogap transition in Bi2201 in ARPES, Kerr effect, and time-resolved reflectivity





Rui-Hua He,[1,2,3]* M. Hashimoto,[1,2,3]* H. Karapetyan,[1,2] J. D. Koralek,[3,4] J. P. Hinton,[3,4] J. P. Testaud,[1,2,3] V. Nathan,[1,2] Y. Yoshida,[5] Hong Yao,[1,3,4] K. Tanaka,[1,2,3,6] W. Meevasana,[1,2,7] R. G. Moore,[1,2] D. H. Lu,[1,2] S.-K. Mo,[3] M. Ishikado,[8] H. Eisaki,[5] Z. Hussain,[3] T. P. Devereaux,[1,2]† S. A. Kivelson,[1]† J. Orenstein,[3,4]† A. Kapitulnik,[1,2]† Z.-X. Shen[1,2]†

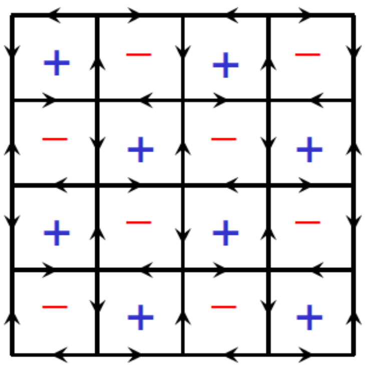
Science 331, 1579 (2011)

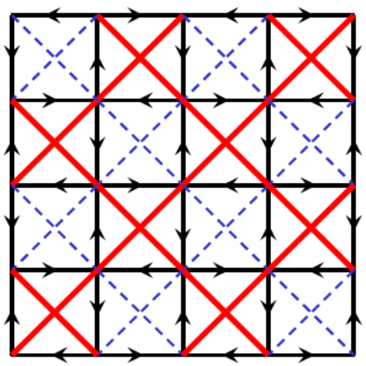
Initial interpretation:
Kerr effect = macroscopic TRSB

Theoretical scenario by Tewari, Zhang, Yakovenko, & Das Sarma, *PRL* **100**, 217004 (2008), based on Yakovenko, *PRL* **65**, 251 (1990):

$d_{xy}+id_{x^2-y^2}$ density wave with $\boldsymbol{Q}=(1/2,1/2)$

$id_{x^2-y^2}$ density wave: staggered currents along bonds



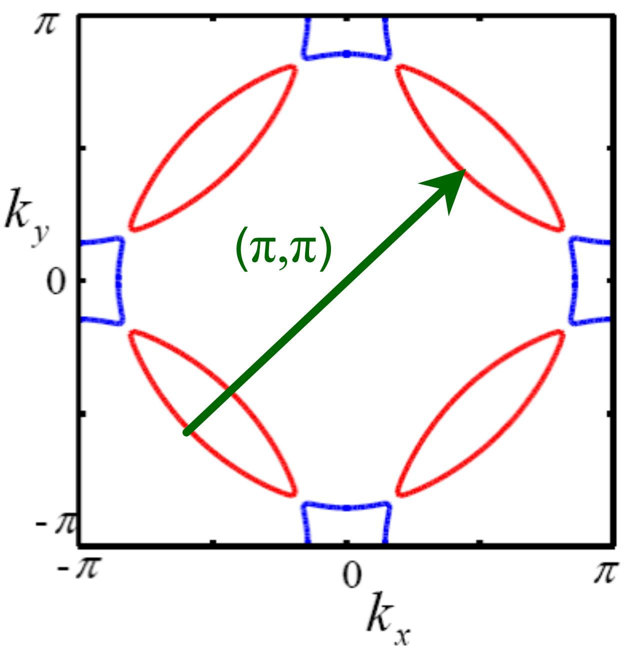


$d_{xy}+id_{x^2-y^2}$ density wave: modulation of plaquettes

The $d_{xy}+id_{x^2-y^2}$ density wave model

- breaks macroscopic time-reversal symmetry
- has non-zero Berry curvature
- exhibits anomalous (sponaneous) Hall effect with $\sigma_{xy}\neq 0$
- shows polar Kerr effect with $\theta_K\neq 0$

SdH magnetic oscillations, pockets, and the i$d_{x^2-y^2}$ density wave



Observation of the Shubnikov-de Haas quantum magnetic oscillations in the underdoped cuprates can be most naturally explained by formation of small pockets on the Fermi surface.

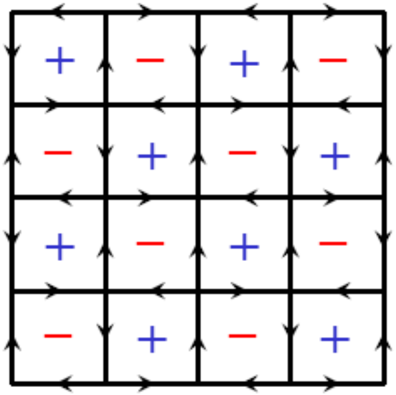
The pockets can be produced by a superstructure with the wave vector ***Q***=(π,π), which doubles the unit cell of the crystal.

The superstructure may originate from fluctuating antiferromagnetism or SDW.

However, there is another possibility: formation of the *d*-density wave.

$$i\left\langle \psi^{+}(x+a,y)\psi(x,y)-\psi^{+}(x,y)\psi(x+a,y)\right\rangle \neq 0$$
$$=-i\left\langle \psi^{+}(x,y+b)\psi(x,y)-\psi^{+}(x,y)\psi(x,y+b)\right\rangle \neq 0$$

$id_{x^2-y^2}$ density wave: staggering currents



In real space, the $id_{x^2-y^2}$ density wave represents staggering currents along the bonds of the lattice.

Strictly speaking, it is not a density-wave, because the electron density is not modulated, only the current is. All sites of the lattice remain equivalent. Thus, it may be difficult to detect directly.

The $id_{x^2-y^2}$ density wave correspond to the following electron-hole pairing:

$$\left\langle \psi^\dagger_{k+Q}\psi_k \right\rangle \propto iW_k, \quad W_k = \frac{\Delta_2}{2}\left(\cos k_x - \cos k_y\right) \qquad \mathbf{Q} = (\pi,\pi)$$

The *d*-density wave was first proposed by Halperin and Rice in 1960s.

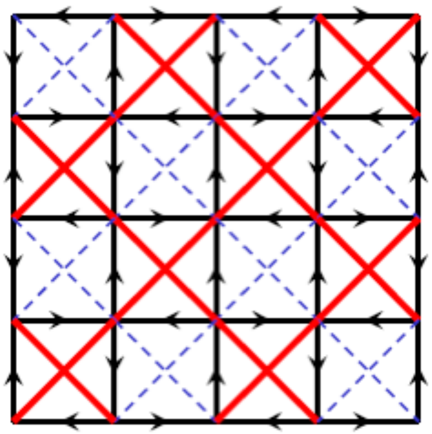
For cuprates, it was discussed in 1980s and 1990s by many people. In 2000s, it was advocated for cuprates by Chakravarty *et al*.

The staggering flux breaks the micro time-reversal symmetry, but not macro, because the combined symmetry with translation is preserved. Thus, $id_{x^2-y^2}$ density wave alone is not enough to explain the Kerr effect.

The chiral $d_{xy}+id_{x^2-y^2}$ density wave

Let us consider an additional d_{xy} density wave with a small amplitude Δ_1:

$$\langle \psi^\dagger_{\boldsymbol{k}+\boldsymbol{Q}} \psi_{\boldsymbol{k}} \rangle \propto V_{\boldsymbol{k}} + iW_{\boldsymbol{k}}, \quad V_{\boldsymbol{k}} = \Delta_1 \sin k_x \sin k_y, \quad W_{\boldsymbol{k}} = \frac{\Delta_2}{2}(\cos k_x - \cos k_y)$$



The d_{xy} density wave represents staggering modulation of the diagonal tunneling amplitudes between the next-nearest neighboring sites. It breaks the symmetry between the plaquettes. The d_{xy} density wave may originate from an electronic instability or a structural transition.

The combined $d_{xy}+id_{x^2-y^2}$ density wave breaks the time-reversal symmetry, because it breaks the symmetry between the staggered fluxes.

Yakovenko, *PRL* **65**, 251 (1990) calculated the intrinsic (spontaneous) quantum Hall effect for the $d_{xy}+id_{x^2-y^2}$ density wave.

The d_{xy} and the $id_{x^2-y^2}$ transitions generally occur at different temperatures.

Acoustic anomaly around 300 K (Boebinger, Migliori, et al. March 2011) − d_{xy}?

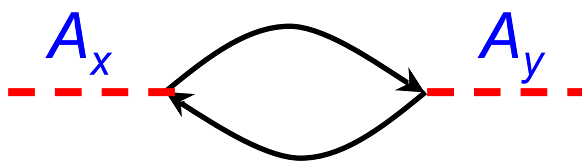
Calculation of the Kerr angle

A textbook formula relates the Kerr angle θ_K with the ac Hall conductivity $\sigma_{xy}(\omega)$

$$\theta_K = \mathrm{Im}\frac{4\pi\sigma_{xy}(\omega)}{n(n^2-1)\omega d}$$

where n is the refraction coefficient,
d is the interlayer distance.

The ac Hall conductivity $\sigma_{xy}(\omega)$ is given by the one-loop response function



The Hamiltonian of the problem is a 2×2 matrix

$$\hat{H} = \begin{pmatrix} \varepsilon_{\boldsymbol{k}} & V_{\boldsymbol{k}} + iW_{\boldsymbol{k}} \\ V_{\boldsymbol{k}} - iW_{\boldsymbol{k}} & \varepsilon_{\boldsymbol{k}+\boldsymbol{Q}} \end{pmatrix} = \hat{\tau}\cdot\boldsymbol{w}(\boldsymbol{k}) + \hat{\mathrm{I}}\,w_0(\boldsymbol{k})$$

$$\varepsilon_{\boldsymbol{k}} = -2t(\cos k_x + \cos k_y) + 4t'\cos k_x \cos k_y$$

The Hamiltonian has a non-zero Berry curvature when Δ_1 and Δ_2 are present

$$\Omega(\boldsymbol{k}) = -2\boldsymbol{w}\cdot\left[\frac{\partial \boldsymbol{w}}{\partial k_x}\times\frac{\partial \boldsymbol{w}}{\partial k_y}\right] = 4t\Delta_1\Delta_2\left(\sin^2 k_x + \sin^2 k_y - \sin^2 k_x \sin^2 k_y\right)$$

The anomalous Hall conductivity

The Berry curvature results in a non-zero Hall conductivity $\sigma_{xy}(\omega)$ in the absence of an external magnetic field (the intrinsic or spontaneous Hall effect)

$$\sigma_{xy}(\omega) = \int \frac{d^2k}{(2\pi)^2} \frac{\Omega(\boldsymbol{k})\left\{n_F\left[E_+(\boldsymbol{k})\right] - n_F\left[E_-(\boldsymbol{k})\right]\right\}}{w(\boldsymbol{k})\left[\omega - 2w(\boldsymbol{k})\right]\left[\omega + 2w(\boldsymbol{k})\right]}$$

where $n_F(E)$ is the Fermi occupation function of the upper and lower bands.

1) For $\omega=0$, we get the dc Hall conductivity

$$\sigma_{xy}(0) = \frac{e^2}{h} \int_{\text{BZ-pockets}} \frac{d^2k}{(2\pi)^2} \frac{\Omega(\boldsymbol{k})}{4w^3(\boldsymbol{k})}$$

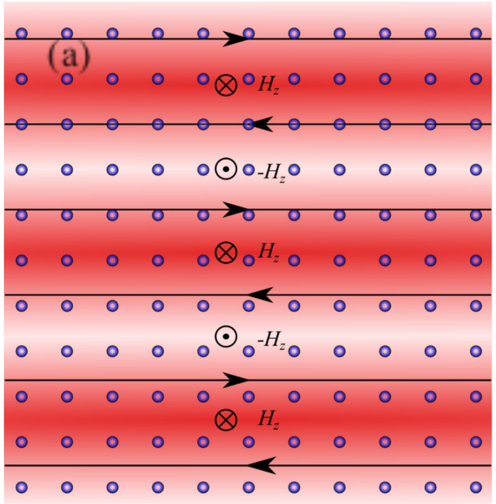
2) For optical $\omega \gg T$, we get

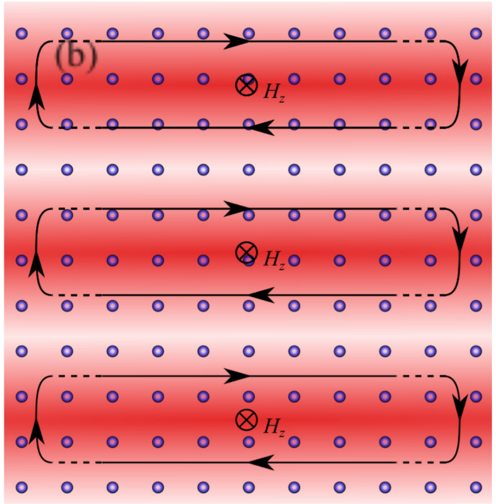
$$\sigma''_{xy}(\omega) = \frac{1}{\omega^2} \int_{\text{BZ-pockets}} \frac{d^2k}{4\pi} \Omega(\boldsymbol{k}) \delta\left[\omega - 2w(\boldsymbol{k})\right]$$

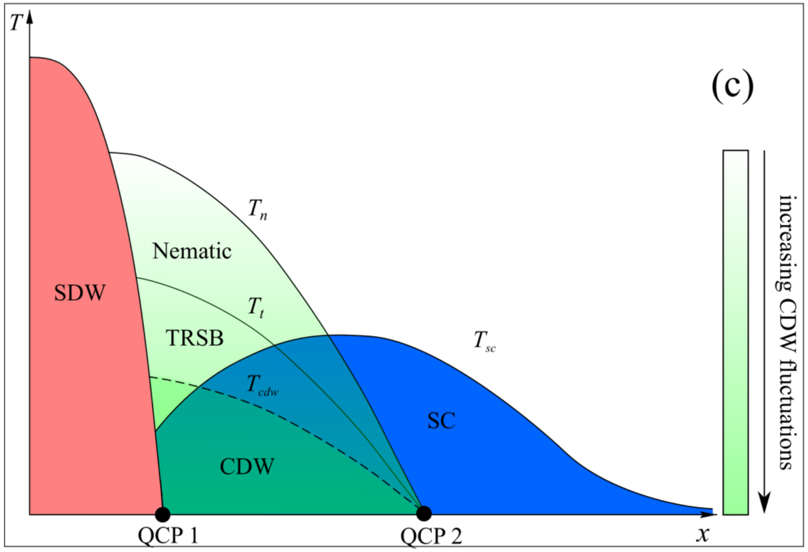
The imaginary part $\sigma''_{xy}(\omega)$ represents vertical transitions across the gap $2w$.

Charge-density wave with staggered currents

Wang and Chubukov, *PRB* **90,** 035149 (2014)







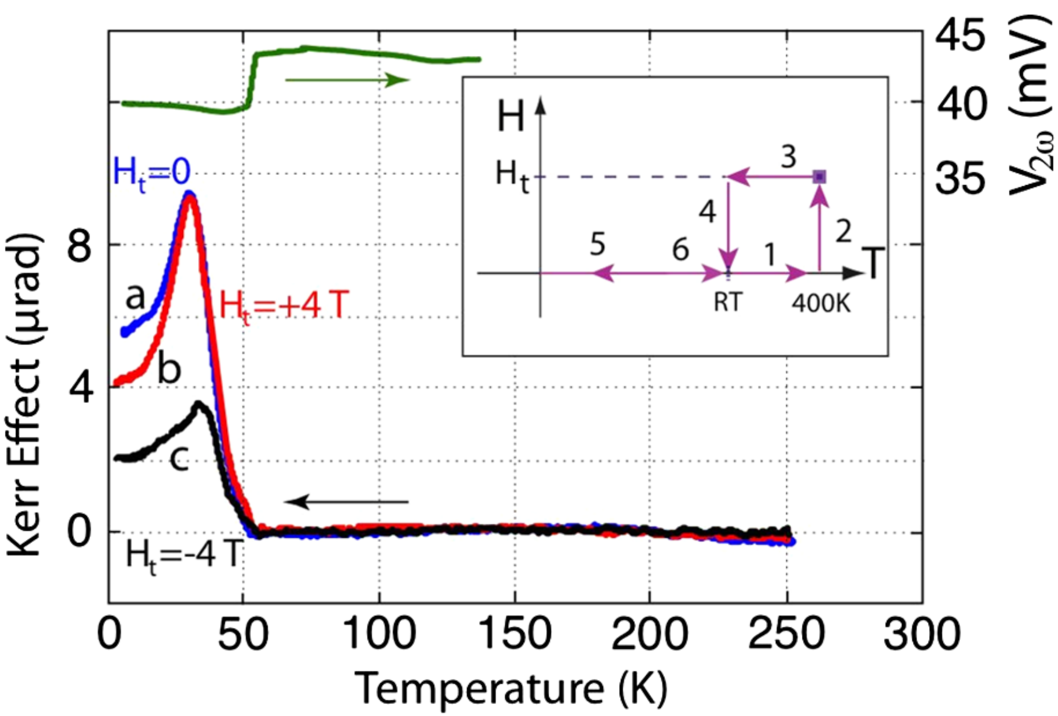
Anomalous Hall conductivity and Kerr effect due to

- Impurities: Wang, Chubukov, Nandkishore, *PRB* **90**, 205130 (2014)
- Berry curvature: Gradhand, Eremin, Knolle, *PRB* **91**, 060512 (2015)

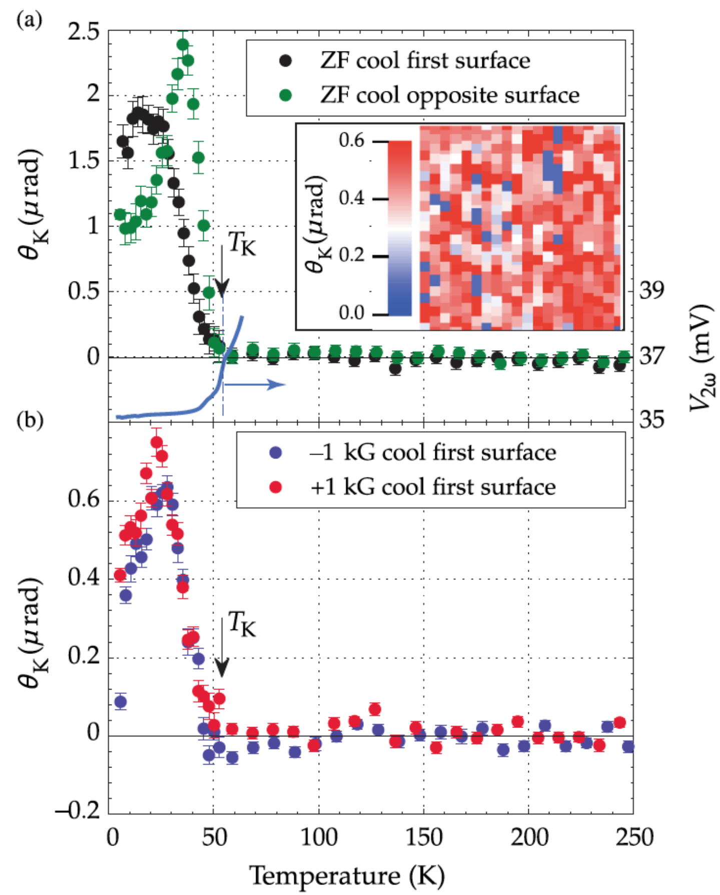
New developments: Karapetyan *et al.* *PRL* **109**, 147001 (2012), *PRL* **112**, 047003 (2014)

- Sign of θ_K cannot be trained by a magnetic field
- Sign of θ_K is the same on the opposite surfaces
- θ_K changes linearly with applied uniaxial strain

Conclusion: not TRSB!



$La_{1.875}Ba_{0.125}CuO_4$



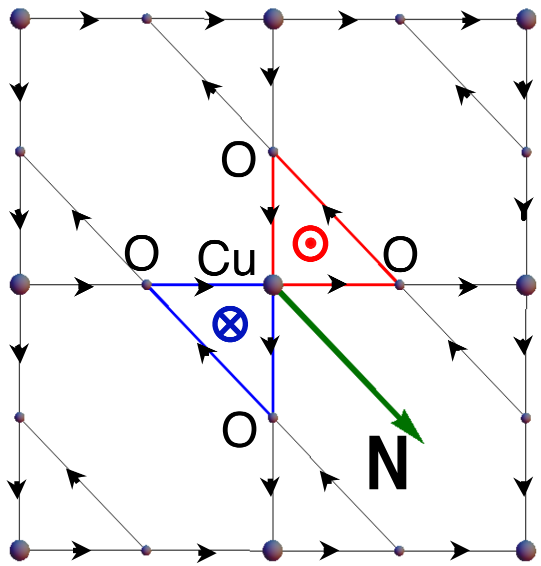
Circular dichroism due to spatial dispersion if inversion and mirror symmetries are broken (chirality or natural optical activity)

$$\varepsilon_{\alpha\beta}(\omega,\mathbf{k})=\varepsilon_{\alpha\beta}(\omega,0)+\gamma(\omega)\varepsilon_{\alpha\beta\delta}k_\delta+O(k^2),\qquad \gamma(\omega)\text{ is a pseudoscalar}$$

Proposals for Kerr effect due to chiral order without TRSB:

- Arfi & Gor'kov, *PRB* **46**, 9163 (1992): Broken inversion symmetry
- Hosur, Kapitulnik, Kivelson, Orenstein, Raghu, *PRB* **87**, 115116 (2013): Various density-wave chiral structures
- Orenstein & Moore, *PRB* **87**, 165110 (2013): Berry curvature
- Mineev, *PRB* **88**, 134514 (2013): Noncentrosymmetric media with spin-orbit
- Pershoguba, Kechedzhi, & Yakovenko, *PRL* **111**, 047005 (2013): Helical texture of loop currents

Varma's loop currents



Anapole moment

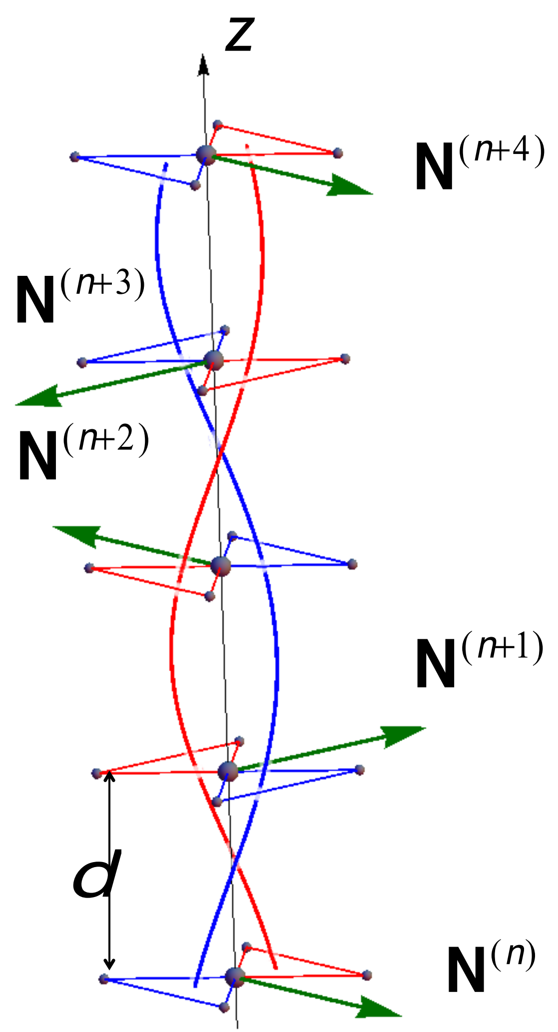
$$\mathbf{N} = \int [\mathbf{r} \times \mathbf{m}(\mathbf{r})] d^2 r$$

Symmetries:

Time-reversal odd

Inversion odd

Helical structure

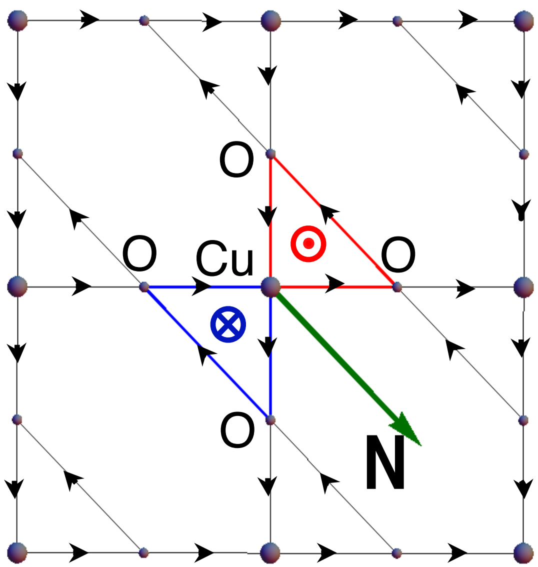


Neutron scattering (Bourges et al.) finds tilted magnetic fields with an in-plane component.

Twisted model gives the tilt due to the double-helix spiral pattern of magnetic field lines.

Chirality: $\Xi = \left\langle \hat{\mathbf{z}}[\mathbf{N}^{(n)} \times \mathbf{N}^{(n+1)}] \right\rangle$

Magneto-electric effect in Varma's model



$$S[\mathbf{E},\mathbf{B}] = \int \beta(\omega)\,\mathbf{N}[\mathbf{E}\times\mathbf{B}]$$

Electric polarization

$$\mathbf{P} = \frac{\delta S}{\delta \mathbf{E}} = \beta\,[\mathbf{B}\times\mathbf{N}]$$

Magnetization

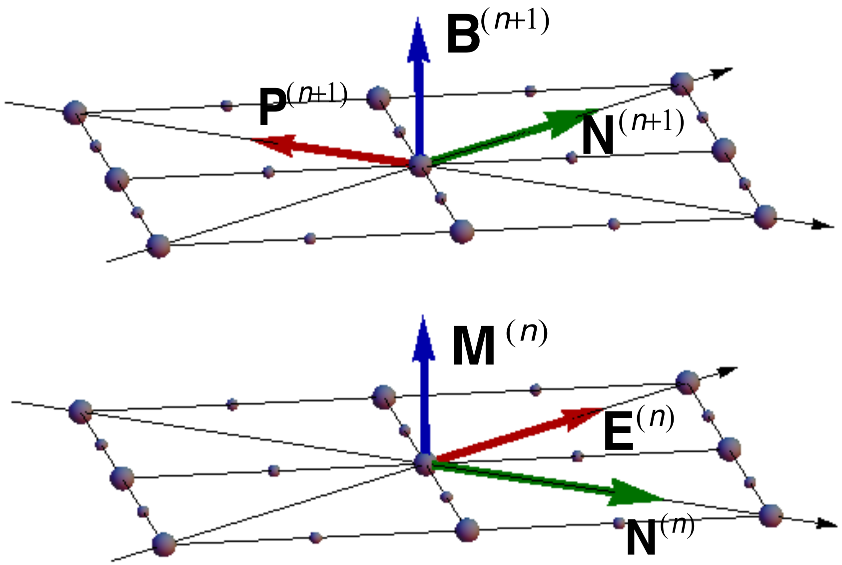
$$\mathbf{M} = \frac{\delta S}{\delta \mathbf{B}} = \beta\,[\mathbf{N}\times\mathbf{E}]$$

Physical Intuition:
External field breaks symmetry between red and blue currents.

In-plane electric field --------- out-of-plane magnetization

Out-of-plane magnetic field --------- in-plane polarization

Dielectric response in a multilayer helical structure with magnetic interlayer coupling



Consider a transverse electromagnetic wave propagating along z axis.

$$\mathbf{P}^{n+1} \propto \hat{\mathbf{z}} \times \mathbf{E}^{n} - \hat{\mathbf{z}} \times \mathbf{E}^{n+2} \propto \hat{\mathbf{z}} \times \left(\mathbf{E}^{n+2} - \mathbf{E}^{n} \right) \propto \left[\nabla_z \times \mathbf{E} \right]$$

$$\varepsilon_{\alpha\beta}(\omega, \mathbf{k}) = \varepsilon_{\alpha\beta}(\omega, 0) + \gamma(\omega)\varepsilon_{\alpha\beta z} k_z$$

However, polarization on the surface layer is $\boldsymbol{P}^0 = \boldsymbol{N}^1(\boldsymbol{N}^0 \boldsymbol{E}^1)$.

In calculation of the Kerr effect upon reflection, the bulk and the surface contributions cancel out, so the Kerr effect vanishes!

By Onsager's reciprocity principle, reflection matrix is symmetric for a time-reversal system, so the Kerr effect must vanish and cannot be obtained due to chiral order

- Bert Halperin, *High-T_c Proceedings* (1992)
- Peter Armitage, *PRB* **90**, 035135 (2014)
- Alex Fried, *PRB* **90**, 121112 (2014)

Retractions of claims for Kerr effect due to chiral order

- Mineev and Yoshioka, *PRB* **89**, 139902 (2014)
- Hosur, Kapitulnik, Kivelson, Orenstein, Raghu, Cho, Fried, *PRB* **91**, 039908 (2015)
- Pershoguba, Kechedzhi, and Yakovenko, *PRL* **113**, 129901 (2014)

Different forms of constituent relations with surface terms:

$4\pi \boldsymbol{P} = \gamma \boldsymbol{\nabla} \times \boldsymbol{E}$ - incorrect

$4\pi \boldsymbol{P} = \boldsymbol{\nabla} \times (\gamma \boldsymbol{E}) = \gamma \boldsymbol{\nabla} \times \boldsymbol{E} + (\boldsymbol{\nabla}_z \gamma) \times \boldsymbol{E}$ - incorrect

$4\pi \boldsymbol{P} = \gamma \boldsymbol{\nabla} \times \boldsymbol{E} + (1/2)(\boldsymbol{\nabla}_z \gamma) \times \boldsymbol{E}$ - correct, zero Kerr effect

“Multipole Theory in Electromagnetism”

by R. E. Raab and O. L. de Lange (Oxford University Press, 2005)

Electric dipole density

$$P_\alpha = (\kappa_{\alpha\beta} - i\cancel{\kappa'_{\alpha\beta}})E_\beta + \frac{1}{2}(\cancel{a_{\alpha\beta\gamma}} - ia'_{\alpha\beta\gamma})\nabla_\gamma E_\beta + (G_{\alpha\beta} - i\cancel{G'_{\alpha\beta}})B_\beta$$

Magnetic dipole density

$$M_\alpha = (G_{\beta\alpha} + i\cancel{G'_{\beta\alpha}})E_\beta$$

Electric quadrupole density

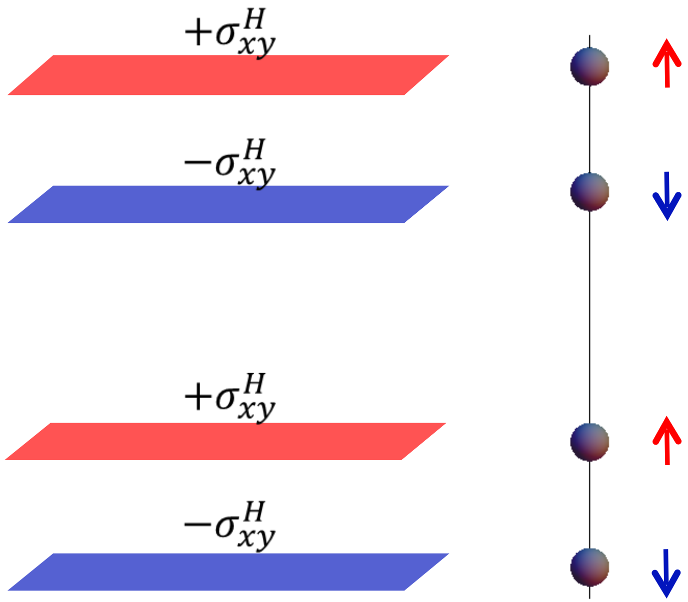
$$Q_{\alpha\beta} = (\cancel{a_{\gamma\alpha\beta}} + ia'_{\gamma\alpha\beta})E_\gamma$$

The primed functions are odd with respect to time reversal

Electric current density

$$J_\alpha = \dot{P}_\alpha - \frac{1}{2}\nabla_\beta \dot{Q}_{\alpha\beta} + \epsilon_{\alpha\beta\gamma}\nabla_\beta M_\gamma$$

Microscopic TRSB & Kerr effect in antiferromagnets



Dzyaloshinskii *Phys Lett A* **155**, 62 (1991)

Cr_2O_3

Magnetoelectric effect in Cr_2O_3

$$F = c_1 E_z B_z + c_2 (E_x B_x + E_y B_y)$$

predicted by Dzyaloshinskii *Sov. Phys. JETP* **10**, 628 (1960)

observed by Astrov *Sov. Phys. JETP* **11**, 708 (1960)

Kerr effect in Cr_2O_3

predicted by Hornreich and Shtrikman, *PR* **171**, 1065 (1968)

observed by Krichevtsov et al. *J Phys Cond Mat* **5**, 8233 (1993)

- Same sign of the Kerr effect on the opposite surfaces
- Magnetoelectric training by applying ***E*** and ***B***

Measurements of the Kerr effect in Cr_2O_3

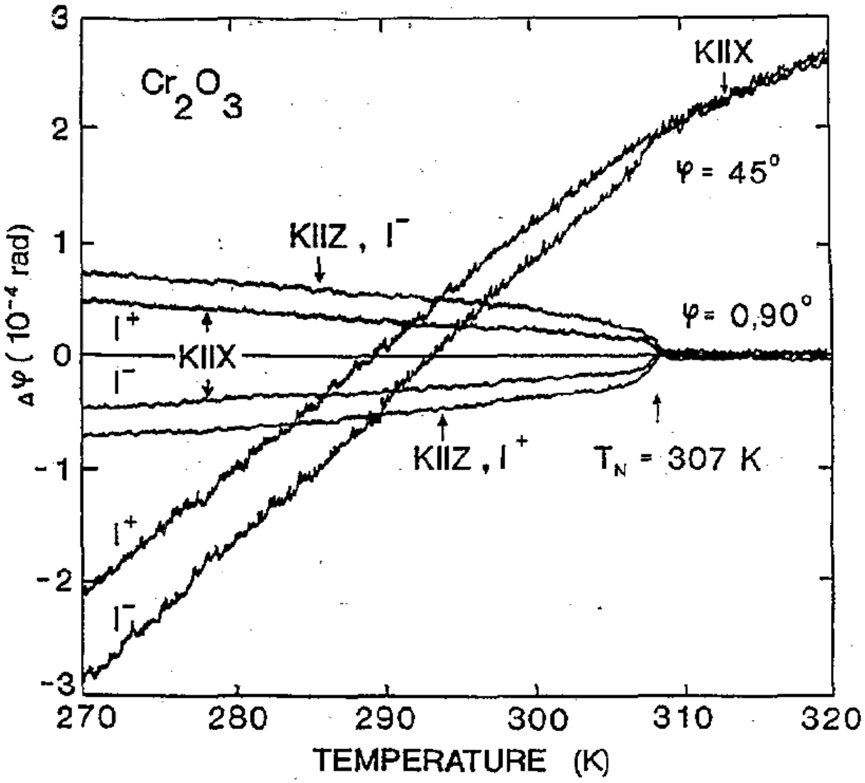
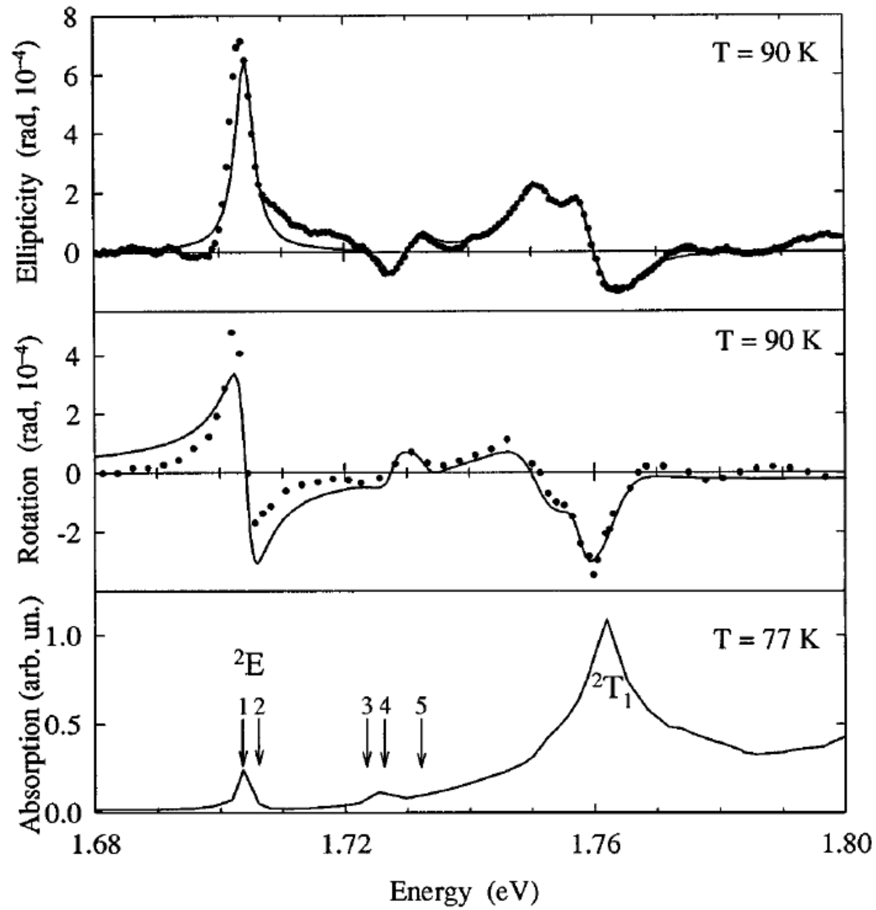


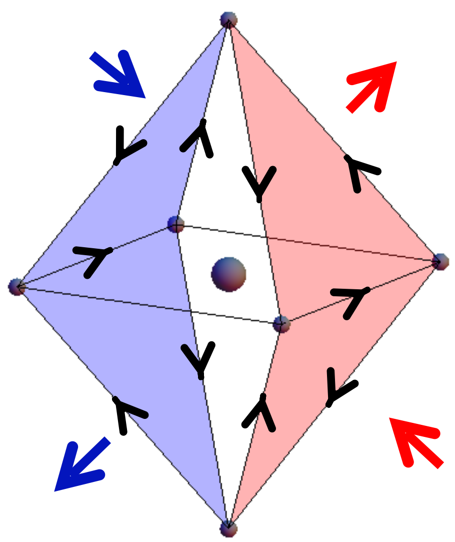
Figure 1. The NR rotation $\Delta\varphi$ versus temperature in two antiferromagnetic states l^+ and l^- and for two principal orientations of the wavevector k of the incident–reflected light wave.

Krichevtsov *et al.*
J Phys Cond Mat
5, 8233 (1993)

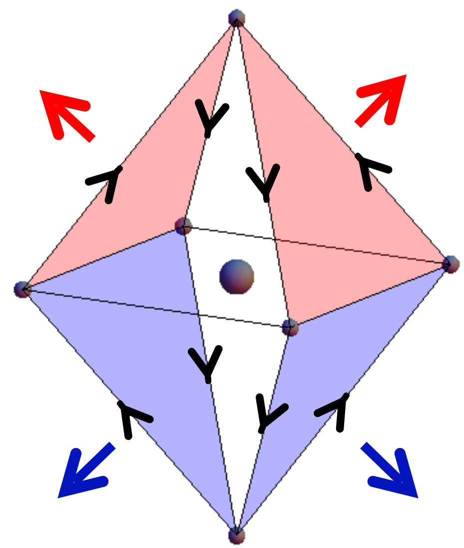


Krichevtsov *et al.*
PRL **76**, 4628 (1996)

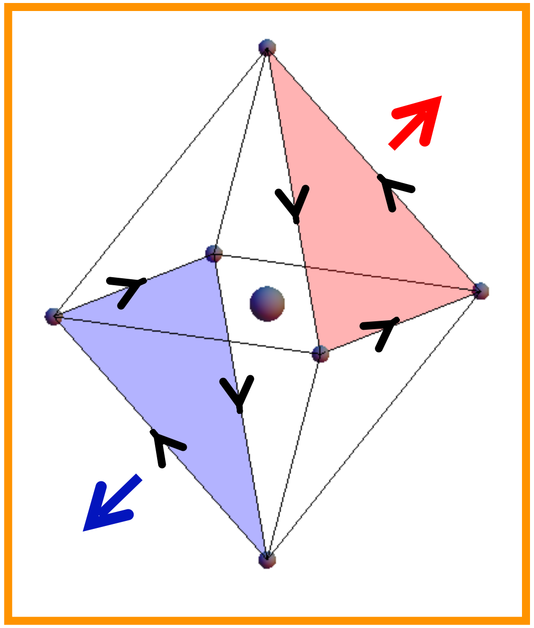
Tilted loop current models for cuprates



Weber et al. *PRL* **102,** 017005 (2009) gives $\theta_K=0$, no Kerr effect



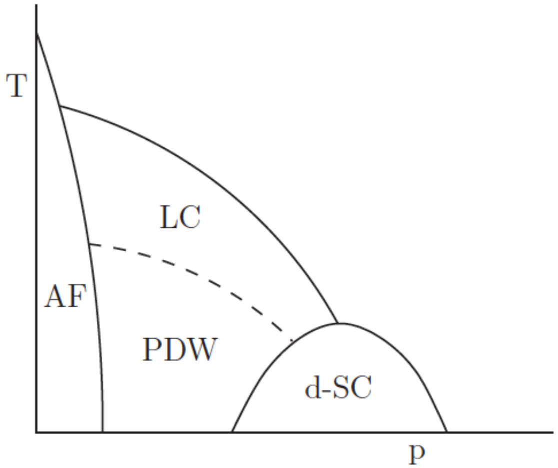
Orenstein, *PRL* **107**, 067002 (2011) $\theta_K\neq 0$ but disagrees with neutrons

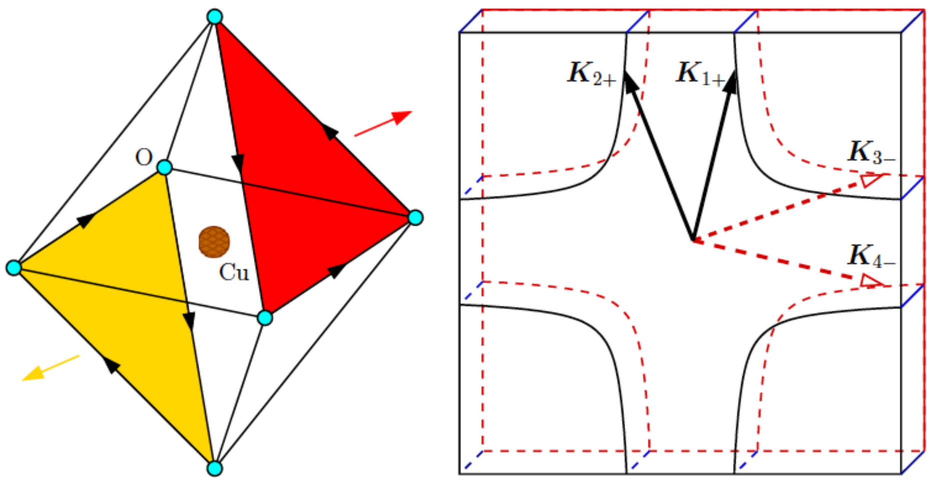


Yakovenko *Physica B* (2015) Yuan Li, *PhD thesis* 2010 $\theta_K\neq 0$, agrees with neutrons

Emergent loop current (LC) order from pair density wave (PDW) superconductivity

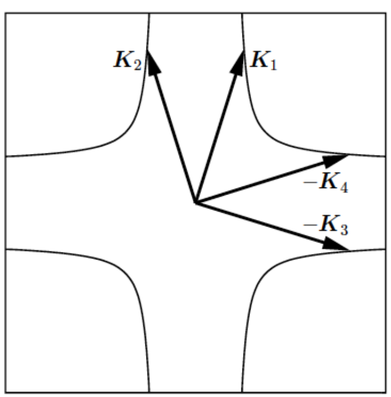
Agterberg *et al.* *PRB* **91**, 054502 (2015)

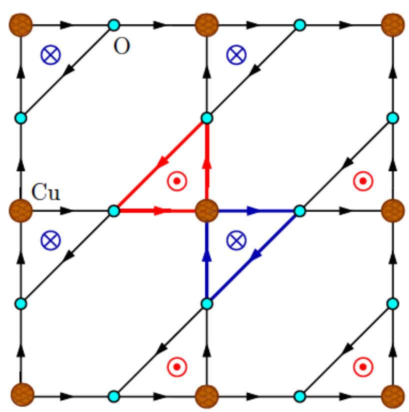




(a) (b)

FIG. 5. (Color online) (a) Tilted loop current state proposed by Yakovenko [26]. The arrows on the bonds depict the direction of the current, the longer arrows depict the associated magnetic moments. (b) PDW state with the same symmetry properties as the tilted loop current state. The arrows $\boldsymbol{K}_i$ depict the nonzero components of the PDW order parameter. Wave vectors labeled "+" are above the x-y plane and those labeled "−" are below the x-y plane.

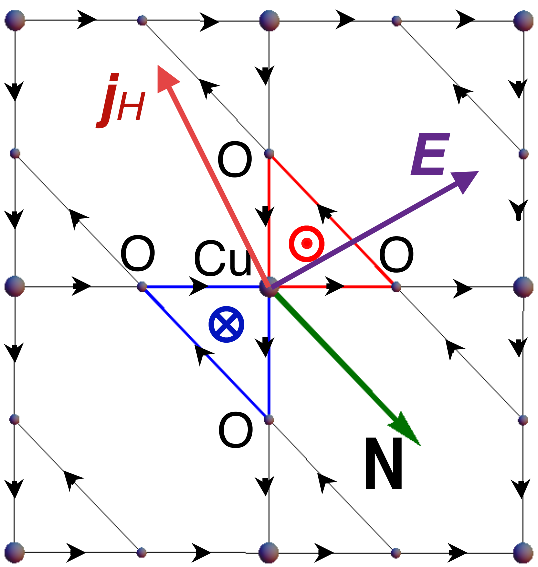




Experimental proposals

Magnetic-field-induced polarity to be observed by STM

Nonlinear Hall effect [Gao, Yang, Niu, *PRL* **112**, 166601 (2014)]



In Varma's model, an in-plane electric field $\boldsymbol{E}$ induces an out-of-plane magnetic field

$$\boldsymbol{B}_{\text{eff}} \propto \beta[\boldsymbol{E} \times \boldsymbol{N}]$$

E and B_{eff} produce an in-plane Hall current

$$\boldsymbol{j}_H \propto [\boldsymbol{E} \times \boldsymbol{B}_{\text{eff}}] \propto \beta \boldsymbol{E} \times [\boldsymbol{E} \times \boldsymbol{N}]$$

Possible experimental manifestations:

(1) dc Hall current proportional to the intensity of ac radiation:

$$\boldsymbol{j}_H(\text{dc}) \propto \boldsymbol{E}(\omega) \times [\boldsymbol{E}(-\omega) \times \boldsymbol{N}]$$

(2) Second harmonic generation: $\boldsymbol{j}_H(2\omega) \propto \boldsymbol{E}(\omega) \times [\boldsymbol{E}(\omega) \times \boldsymbol{N}]$

Permitted because Varma's model breaks inversion and time reversal

Second harmonic generation and visualization of AFM domains in Cr_2O_3

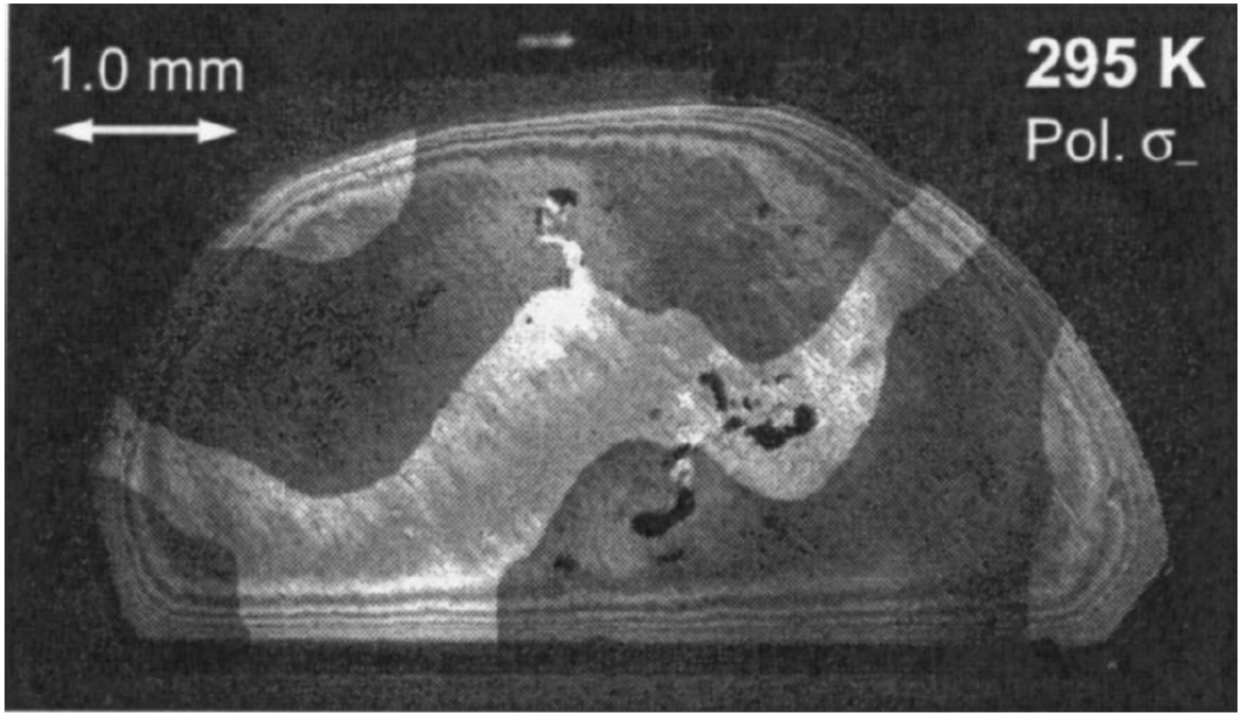


Fig. 6. Antiferromagnetic 180° domains in Cr_2O_3 exposed to circularly polarized light for SHG at 2.1 eV. Exposure time was 35 min but was reduced to 1–5 min in subsequent experiments.

Fiebig et al. *J Opt Soc Am. B* **22**, 96 (2005)

Summary: Yakovenko, *Physica B* **460**, 159 (2015)

The tilted loop current model for cuprates explains

- Kerr effect of the same sign on the opposite surfaces
- No magnetic-field training, but proposes magneto-electric one
- Tilted intra-unit-cell magnetic moments observed by neutrons
- Optical axes rotation away from ***a*** and ***b***,
  Lubashevsky et al. *PRL* **112**, 147001 (2014)

Proposed experiments for inversion and time reversal breaking

- Magnetic-field-induced polarity in STM
- Nonlinear Hall effect:
  - Second-harmonic generation
  - Photogalvanic effect, dc current proportional to ac intensity

Surprising connection with experiments in Sr_2IrO_4 by D. Hsieh

Not explained

- The absence of local magnetic field on apical oxygen in NMR