8-6-15

Majorana Modes + Topo. SC's: Where we are + where we're headed

Outline

I. Big Picture (non-Abelian anyons / TQC)

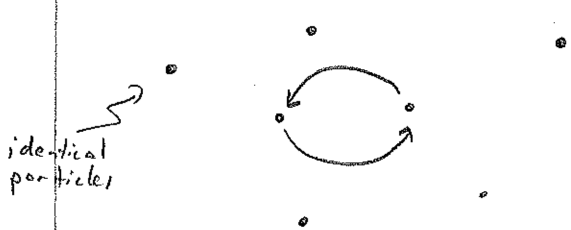
II. Toy Models
  A. Kitaev chain
  B. 2D Topo. SC

III. Experimental Blueprints

IV. Majorana Detection

V. Status + Future Milestones

Big Picture



Possible exchange statistics

$\psi \rightarrow \pm \psi$ (bosons/fermions; all elementary particles)

$\psi \rightarrow e^{i\alpha} \psi$ (Abelian anyons)

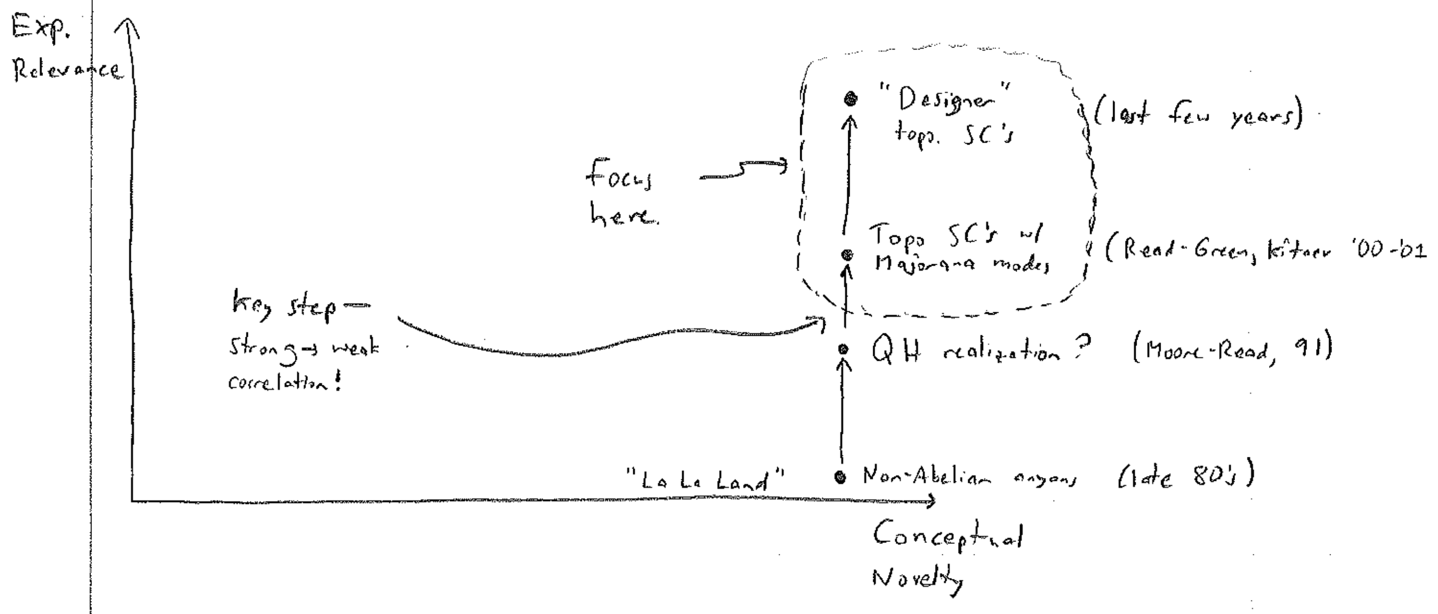
holy grail! $\rightarrow$ $\boxed{\psi_i \rightarrow U_{ij} \psi_j \quad \text{(non-Abelian anyons)}}$

$\{\psi_i\}$ = degenerate gd. sts. generated by anyons; locally indistinguishable!

U_{ij} = rotation w/in gd. st. manifold

Underlies "Topological Quantum Computing" (Kitaev)

"Fisher plot"



Toy Models I: Kitaev chain

Spinless fermions on N-site chain w/ P.B.C.'s (for now).

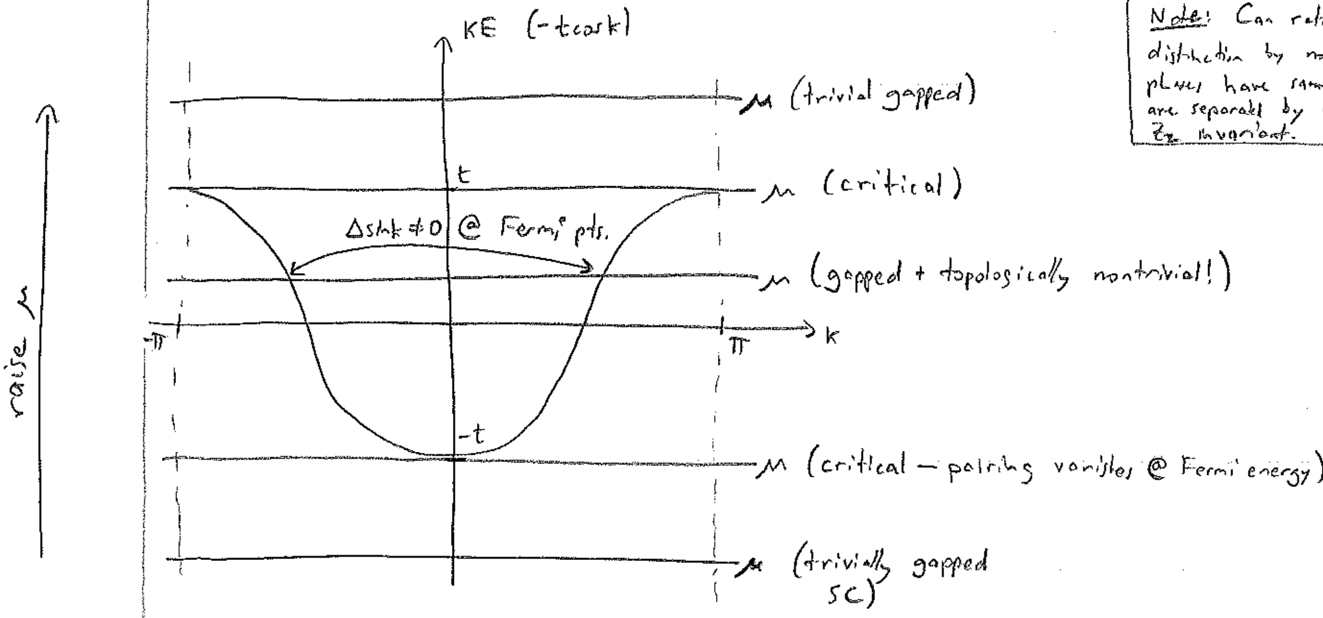
$$\boxed{H = \sum_x \left[-\mu c_x^\dagger c_x - \frac{1}{2}\left(t c_x^\dagger c_{x+1} + \Delta c_x c_{x+1} + h.c. \right) \right]}$$

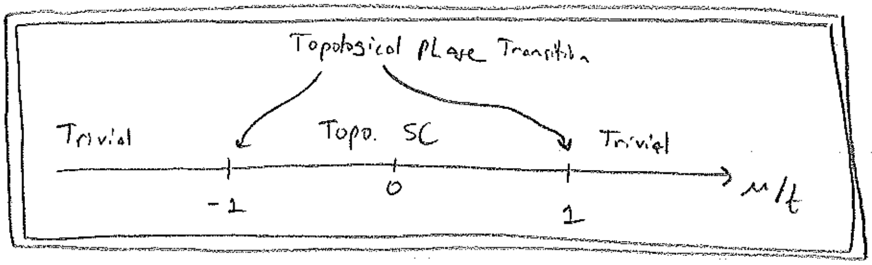
P.B.C. ⇒ go to k-space,

$$\boxed{H = \sum_{k \in BZ} \left[\left(-\mu - t\cos k \right) c_k^\dagger c_k + \left(\frac{i\Delta}{2} \underbrace{\sin k}_{\substack{\text{odd parity} \\ \text{(required by} \\ \text{spinlessness)}}} c_k c_{-k} + h.c. \right) \right]}$$

Phase Diagram as f^n of μ?

> **Note:** Can rationalize topo distinction by noting that gapped phases have same symmetry, yet are separated by critical pt. Mention $\mathbb{Z}_2$ invariant.





Want to explore <u>universal</u> properties of phases/transitions. Convenient to take $\boxed{\Delta = t}$ hereafter.

hopping

$$\Rightarrow \boxed{H = \sum_x \left[-\mu c_x^\dagger c_x - \tfrac{1}{2} t \left(c_x^\dagger + c_x\right)\left(c_{x+1} - c_{x+1}^\dagger\right) \right]}$$

pairing

Gapped phases

Take open B.C.'s now + use Majorana rep.:

$$\boxed{c_x = \frac{1}{2}\left(\gamma_{B,x} + i\gamma_{A,x}\right)}$$

Majorana ops.
(only pairs have well-defined occupation #'s)

$$\boxed{\begin{gathered}\gamma_\alpha = \gamma_\alpha^+ , \quad \gamma_\alpha^2 = I \\ \{\gamma_\alpha, \gamma_{\alpha' \neq \alpha}\} = 0\end{gathered}}$$

(Majorana op. algebra — reproduces $c_x^2 = (c_x^+)^2 = 0$, $\{c_x, c_{x'}^+\} = \delta_{x,x'}$)

[Note: this is always a legitimate rep. of any ordinary fermion op, like c_x. Does not however guarantee that a system supports Majorana-like excitations as we'll see!]

Rewrite H:

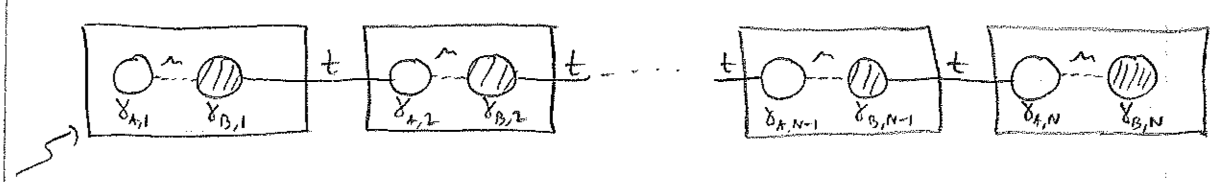
$$c_x^+ c_x = \frac{1}{4}\left(\gamma_{B,x} - i\gamma_{A,x}\right)\left(\gamma_{B,x} + i\gamma_{A,x}\right)$$

$$= \frac{1}{4}\left(1 + 1 + i\gamma_{B,x}\gamma_{A,x} - i\gamma_{A,x}\gamma_{B,x}\right)$$

$$= \frac{1}{2}\left(1 + i\gamma_{B,x}\gamma_{A,x}\right)$$

$$\boxed{(c_x^+ + c_x)(c_{x+1} - c_{x+1}^+) = i\gamma_{B,x}\gamma_{A,x+1}}$$

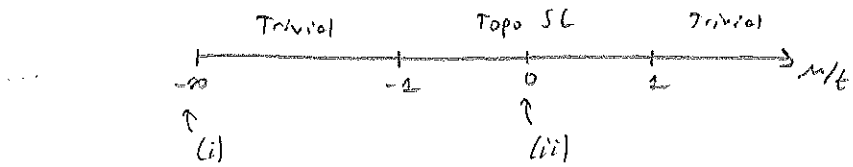
$$\Rightarrow \boxed{H = \frac{-i}{2}\sum_x \left(\mu\gamma_{B,x}\gamma_{A,x} + t\gamma_{B,x}\gamma_{A,x+1}\right)}$$



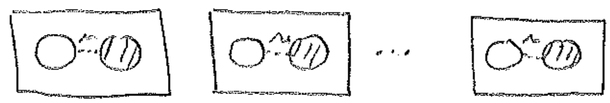
c_x fermions

Majorana chain w/ competing dimerizations μ, t!

For revealing snapshots of gapped phases, examine 2 limits:

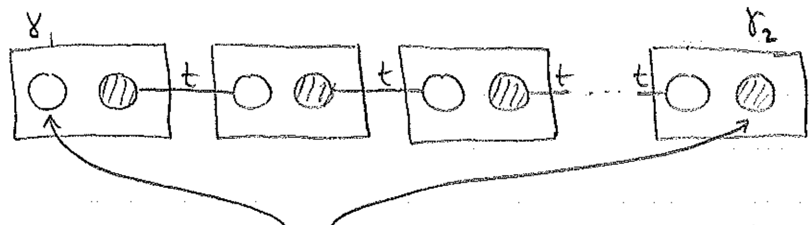


(i) $\mu < 0,\ t = 0$



Trivial product state w/ no entanglement between sites. Unique gd. state. (Vacuum of c_x fermions.)

(ii) $\mu = 0,\ t > 0$



unpaired Majorana ops — zero modes!

Several comments in order:

- $\boxed{\gamma_1 = \frac{c_1 - c_1^\dagger}{i}, \quad \gamma_2 = c_N + c_N^\dagger}$. $\boxed{[H, \gamma_{1,2}] = 0}$

- Non-local fermion $\boxed{d = \frac{\gamma_1 + i\gamma_2}{2}}$ can be filled, empty w/ no energy cost.

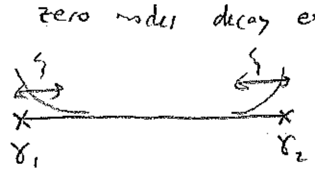
  $\Rightarrow$ 2-fold topological gd. state deg.!

  $\boxed{|0\rangle, \quad d^\dagger|0\rangle \equiv |1\rangle}$

  carry opposite fermion parity

  [Very unusual — most SC's prefer even parity so that all e^-'s can pair. Excitation energy for unpaired e^- vanishes for topo. reasons here.]

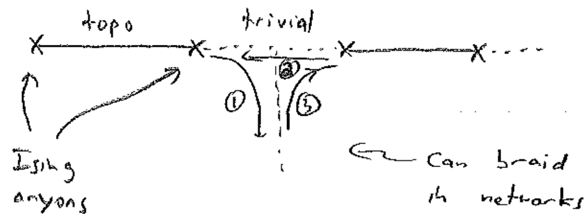
- For $\mu \neq 0$ zero modes decay exponentially into bulk



- Deg. stable to local perturbations; no local measurement can distinguish gd. states. [contrast to deg. $\uparrow, \downarrow$ spin states]

- $\gamma_{1,2}$ are <u>not</u> particles (or quasiparticles)!

- Ends of topo SC $\simeq$ "Ising non-Abelian anyons". Zero-modes are "internal" deg. of freedom that encode gd. state deg.

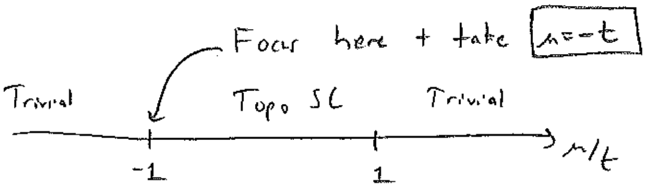
- 

Each anyon pair gives 2 gd. states

$$\boxed{\psi_i \to U_{ij}\,\psi_j}$$

[Braids effectively exchange "half" of one fermion w/ "half" of another — hence nontriviality.]

Topo. Phase Transition



Low-energy physics @ criticality?

$$\Rightarrow \quad H \to \boxed{H_{crit}} = \frac{-it}{2}\sum_x \left(-\gamma_{B,x}\gamma_{A,x} + \gamma_{B,x}\gamma_{A,x+1}\right) = \frac{-it}{2}\sum_x \gamma_{B,x}\left(\gamma_{A,x+1} - \gamma_{A,x}\right)$$

$$\underset{\text{continuum limit}}{\sim} \frac{-it}{2}\int_x \gamma_B\,\partial_x\gamma_A$$

Write $\boxed{\gamma_{A/B} = \gamma_R \pm \gamma_L}$ $\Rightarrow$ $H_{crit} = \frac{-it}{2}\int_x (\gamma_R - \gamma_L)\partial_x(\gamma_R + \gamma_L)$

$$= \frac{-it}{2}\int_x \Big(\gamma_R\partial_x\gamma_R - \gamma_L\partial_x\gamma_L + \underbrace{\gamma_R\partial_x\gamma_L - \gamma_L\partial_x\gamma_R}_{\text{cancel}}\Big)$$

So we get $\boxed{\boxed{H_{crit} = \int_x \left(-iv\gamma_R\partial_x\gamma_R + iv\gamma_L\partial_x\gamma_L\right)}}$ $(v \propto t)$

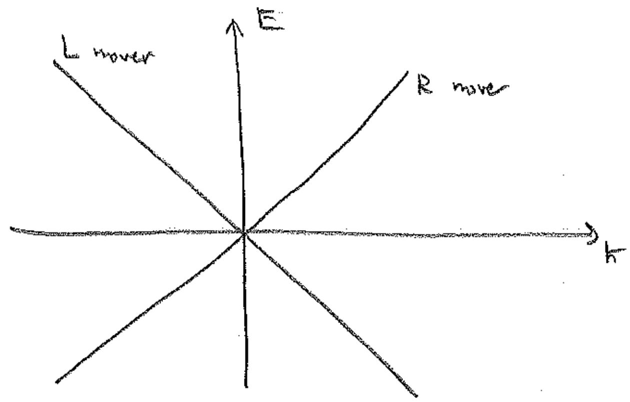
chiral, gapless Majorana fermions!

Go to k-space: $\boxed{\gamma_{R/L}(x) = \int_k e^{ikx}\gamma_{R/L}(k)}$ [Implicitly thinking about P.B.C.'s again.]

Hermicity $\Rightarrow$ $\boxed{\gamma_{R/L}(k) = \gamma^\dagger_{R/L}(-k)}$ $(*)$

$$\Rightarrow \boxed{H_{crit} = \int_k \left[vk\,\gamma_R^\dagger(k)\gamma_R(k) - vk\,\gamma_L^\dagger(k)\gamma_L(k)\right]}$$

not distinct from E>0 levels.



By $(*)$, $E>0$, $E<0$ states not distinct!

$\Rightarrow$ $\boxed{\text{"half" of usual single-channel wire.}}$

[can be made very precise; e.g., thermal transport exactly half that in a usual wire ($c = \frac{1}{2}$ vs. 1).]

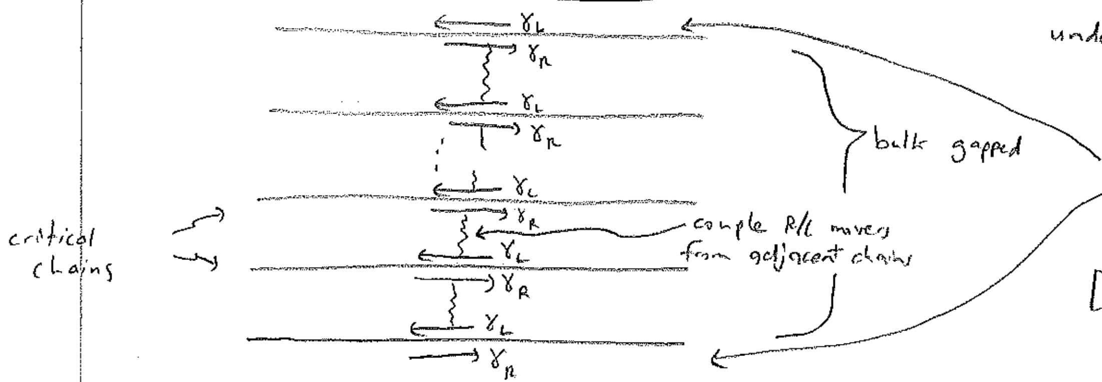
So Majorana physics in Kitaev chain appears in 2 ways:

(i) Localized zero modes in topo. phase

(ii) Gapless propagating deg. of freedom at criticality.

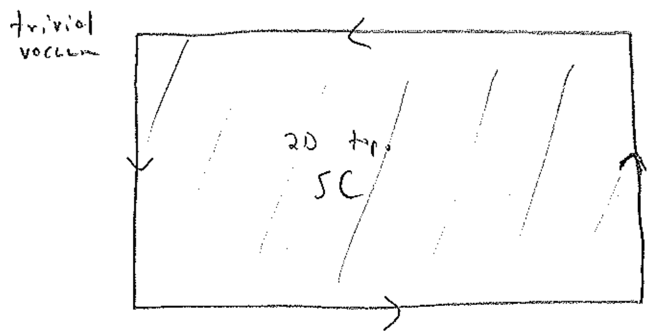
Toy Models II: 2D Topo. SC's

Build from array of critical Kitaev chains [efficiently leverages our understanding of 1D case.]



[hallmark of 2D topo SC. Note similarity to nontrivial dimerization in Kitaev chain.]

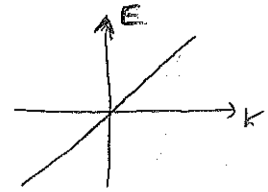
$\Downarrow$



edge coord. chiral Majorana field

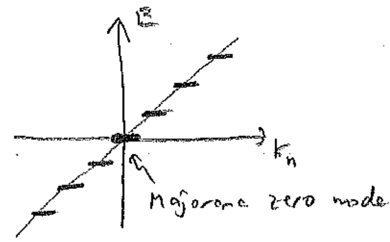
$$\boxed{H_{edge} = \int dx \left(-iv\,\gamma \partial_x \gamma\right)}$$

$$\boxed{E = vk}$$



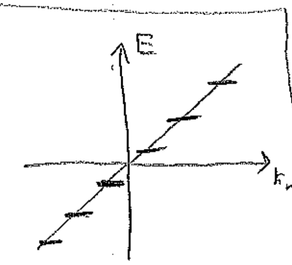
Important Q: Spectrum for finite perimeter L?

i.e. is k quantized to (i) $k_n = \frac{2\pi}{L} n$ (PBC)



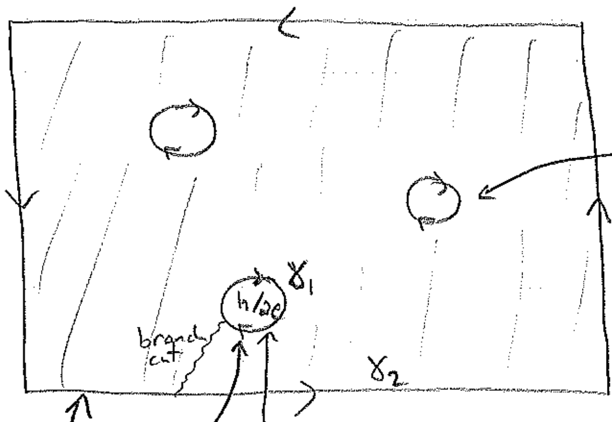
or

$$\boxed{\text{(ii)}\quad k_n = \frac{2\pi}{L}\left(n+\tfrac{1}{2}\right) \quad \text{(anti-PBC)}}$$



correct answer; PBC ruled out because you can't have just one Majorana zero mode! [Hilbert space wouldn't make sense.]

Drill holes:



any boundary binds chiral Majorana

thread $\frac{h}{2e}$ flux quantum;
unpaired fermions get − sign when encircling flux (Aharanov-Bohm)

Majorana B.C.'s shift from anti-periodic $\rightarrow$ periodic!
$\Rightarrow$ zero modes γ_1, γ_2!

Lessons: (i) boundaries of 2D topo SC host chiral Majorana fermions [cousins of Majorana end-states in 1D]

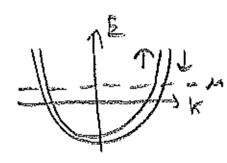
(ii) $\frac{h}{2e}$ flux localizes a Majorana zero mode [deeply related to (i)!]
+ ∴ forms Ising non-Abelian anyon

<u>Experimental Blueprints</u>

Wanted: "Spinless" 1D, 2D SC's [both harbor Ising anyons, albeit in different ways]

Challenges: (1) We live in 3D

(2) e^-'s carry spin



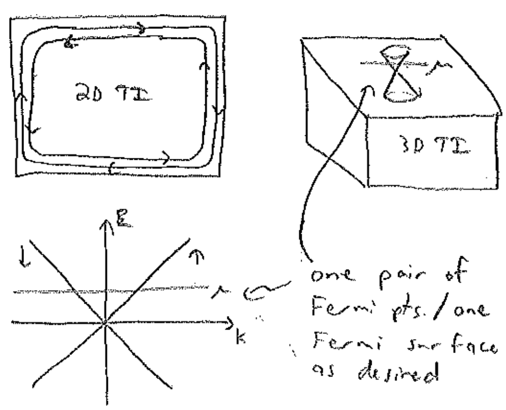
Need to remove half of the carriers.

(3) Nearly all SC's arise from spin singlet Cooper pairs.

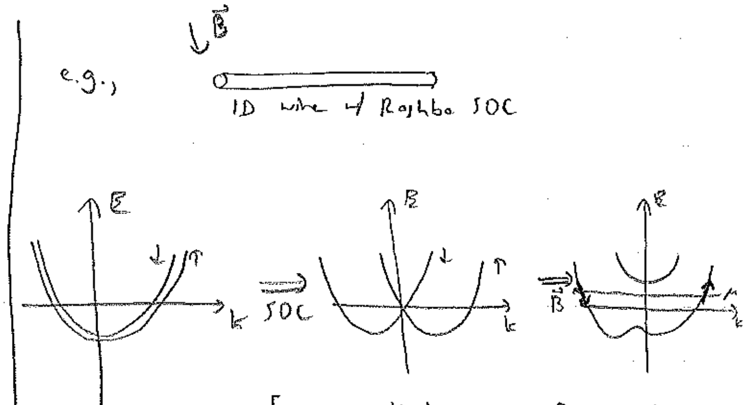
Likely no "intrinsic" realizations in solid state − despite 1000's of known SC's!

Can instead "engineer" topo. SC's! $O(100)$ papers revealing various strategies. Most follow a common recipe:

Step I Use TI boundary **or** Break $\mathcal{T}$ in low-D systems w/ SOC



e.g.,

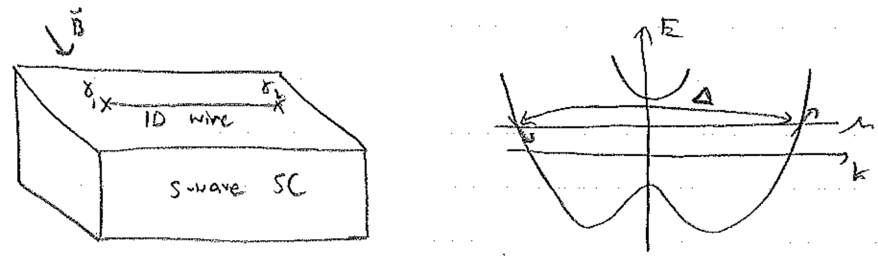


[crucial that carriers @ μ not spin polarized; c.f. 2D TI]

Solves challenges 1, 2! [in a way that makes challenge 3 "easy"]

Step II Couple systems above to s-wave SC.

e.g.,



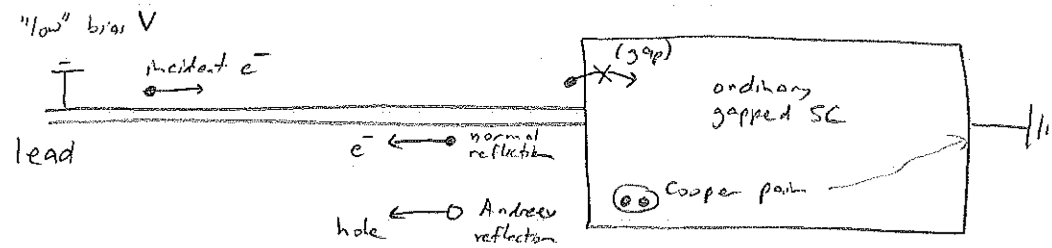
"proximity effect" drives 1D, 2D topo SC!!

Lots of expts followed, but first...

Majorana Detection

Focus on tunneling methods – most common so far.

Primer – SC tunneling as scattering problem



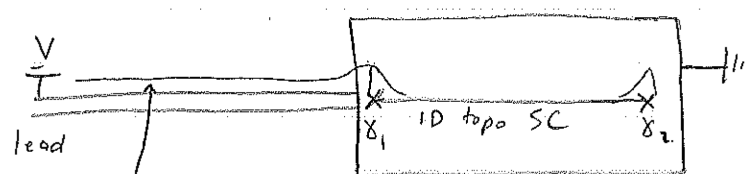
Conductance $\boxed{G = \frac{dI}{dV} = 2 \times \frac{e^2}{h} \times (\text{Andreev ref. prob.})}$

reflects pair injection

conductance quantum

Solve just like potential barrier scattering in intro QM
– get wavefn, extract normal/Andreev ref. coefficients.

Result for topo SC



γ_1 delocalizes into lead

[generic property of localized modes coupled to gapless deg. of freedom]

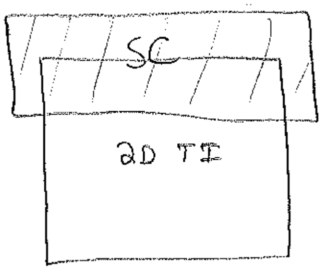
$\Rightarrow$ low-energy wavefn acquire equal e^-/hole character

$\Rightarrow$ Majorana-mediated "perfect Andreev reflection"!

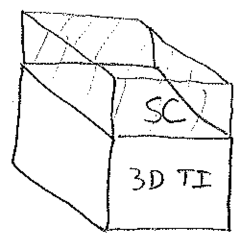
$$\boxed{G = \frac{2e^2}{h} \quad (V \to 0)}$$

Experimental Status

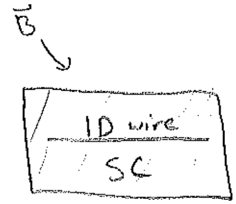
Devices built:



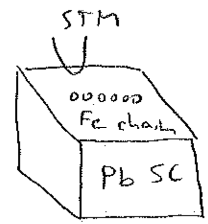
(Yacoby, Kouwenhoven, Du,...)



(van Harlingen, Hasan,...)



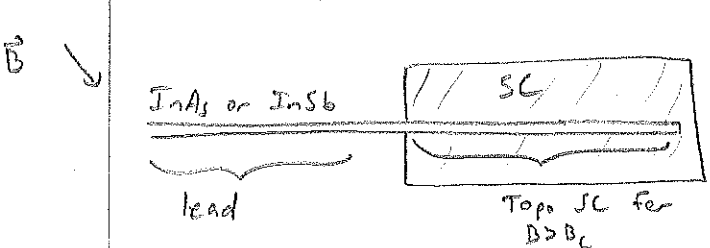
(Kouwenhoven, Heiblum, Marcus,.)

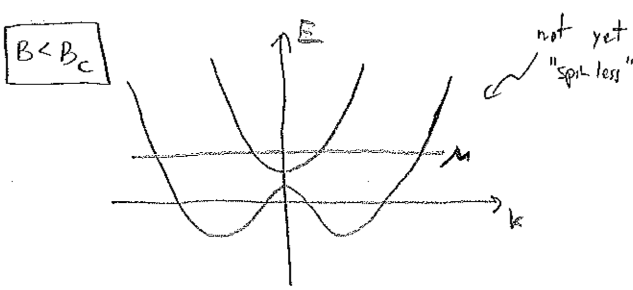


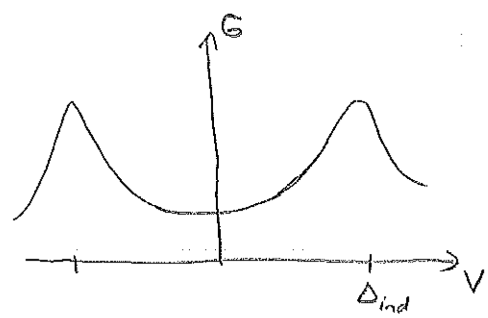
(Yazdani, Berlin group)

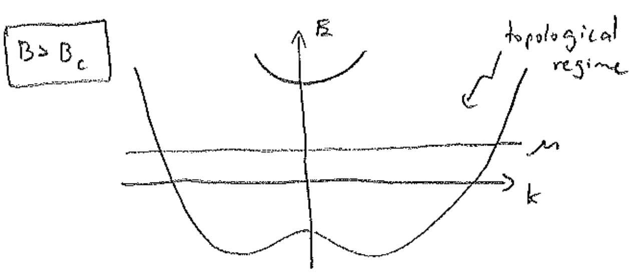
↑ Majorana evidence via tunneling here.

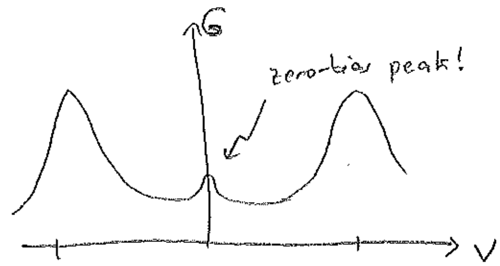
1D wire expts









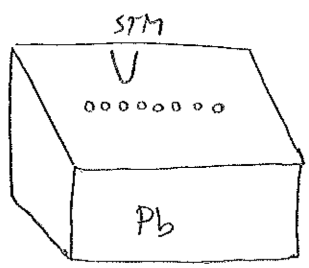


Consistent w/ Majorana zero-mode formation, but

- peak height far below $\frac{2e^2}{h}$
- no sign of phase transition
- other explanations proposed [but harder sell as device quality improves]

My take: analogous features in latest high quality devices likely of Majorana origin. Reasonable alternatives not obvious.

Fe chain expts



Huge virtue: spatial resolution of tunneling conductance.

Zero-bias peaks localized to edges.

Tantalizing Majorana evidence! Future expt./theory will be exciting.

Future Milestones