# Gapless Topological Phases and Unusual Magnetic Order in Iridates

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# Two Routes to Quantum Spin Liquid

1. Geometric Frustration (spin-rotation invariant)

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = \frac{J}{2} \sum_{\text{simplex}} \left( \sum_{i \in \text{simplex}} \mathbf{S}_i \right)^2 + \text{ constant}$$

#### $\sum \mathbf{S}_i = 0$ macroscopic degeneracy

 $i \epsilon$  simplex







## 2. Frustrated Interactions (spin-orbit)

Spin interactions depend on spatial directions

#### Kitaev Model on Honeycomb Lattice: Exact Solution



$$\mathcal{H}_{\mathrm{K}} = \frac{i}{2} \sum_{\alpha - \mathrm{links}} u_{ij}^{\alpha} c_i c_j \qquad (\text{where } u_{ij}^{\alpha} = i b_i^{\alpha} b_j^{\alpha}),$$

 $\mathcal{W}_P = \prod_{\text{loop}} u_{ij}^{\alpha}$  commute with the Hamiltonian  $\Longrightarrow \mathcal{W}_P = \pm 1$ 

Ground state is in the zero-flux sector  $u_{ij}^{\alpha} = +1 (\forall \langle ij \rangle)$ 



Majorana Fermions with Dirac Dispersion

#### Kitaev Spin Liquid as a Z2 Spin Liquid

Making connection to Slave-fermion approach (more conventional)

$$f_{i\uparrow} = \frac{1}{\sqrt{2}} (c_i + ib_i^z) \qquad S_i^a = \frac{1}{2} f_{i\alpha}^{\dagger} \sigma_{\alpha\beta}^a f_{i\beta}$$
$$f_{i\downarrow} = \frac{i}{\sqrt{2}} (b_i^x + ib_i^y) \qquad \text{with} \quad \sum_{\alpha} f_{i\alpha}^{\dagger} f_{i\alpha} = 1$$

 $H = \sum_{\langle ij \rangle \in a} \{ f_{i\alpha}^{\dagger} [T^{a}]_{\alpha\beta}^{ij} f_{j\beta} + f_{i\alpha} [\Delta^{a}]_{\alpha\beta}^{ij} f_{j\beta} \}$  exactly one particle per site (insulator)

If 
$$\Delta^{a} = 0$$
  $f_{i\alpha} \longrightarrow f_{i\alpha} e^{i\theta_{i}}$   $[T^{a}]_{ij} = |[T^{a}]_{ij}|e^{ia_{ij}}$  U(1)  
 $a_{ij} \longrightarrow a_{ij} + \theta_{i} - \theta_{j}$ 

 $\begin{array}{ll} \text{In general} & f_{i\alpha} \to f_{i\alpha} s_i & s_i = \pm 1 \\ & [T^a]_{ij}, & [\Delta^a]_{ij} \to s_i [T^a]_{ij} s_j, & s_i [\Delta^a]_{ij} s_j \end{array} \quad \textbf{Z}_2 \end{array}$ 

## a-A<sub>2</sub>IrO<sub>3</sub> Kitaev Interaction?

$$H = \sum_{\langle ij \rangle \in \gamma} -KS_i^{\gamma}S_j^{\gamma} + J\vec{S}_i \cdot \vec{S}_j$$
$$\gamma = x, y, z$$

G. Jackeli and G. Khaliullin, PRL 102, 256403 (2009)

J. Chaloupka, G. Jackeli and G. Khaliullin, PRL 105, 027204 (2010)

$$J_{eff}=1/2$$



$$|\uparrow_{j}\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_{s}\rangle + |yz,\downarrow_{s}\rangle + |xy,\uparrow_{s}\rangle)$$
$$|\downarrow_{j}\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_{s}\rangle - |yz,\uparrow_{s}\rangle + |xy,\downarrow_{s}\rangle)$$

Strong Spin-Orbit Coupling leads to Spin-Orbit entangled pseudo-spin basis (Kramers Doublet) Realization of Kitaev Quantum Spin Liquid?

# Honeycomb Iridates Family

2D Honeycomb  $a - Na_2 IrO_3 a - Li_2 IrO_3 (2D)$ 

3D Hyper-Honeycomb  $\beta$ -Li<sub>2</sub>IrO<sub>3</sub> (3D)

3D Harmonic-Honeycomb Y-Li<sub>2</sub>IrO<sub>3</sub> (3D)



Quantum Spin Liquid robust to small Heisenberg interaction

Monoclinic Distortions Strong Trigonal Distortions

#### Discovery of Three dimensional "Honeycomb" lattice

B- Li<sub>2</sub>IrO<sub>3</sub>
Dresden, July 2013
arXiv:1403.3296
H. Takagi
Hyper-Honeycomb



y- Li<sub>2</sub>IrO<sub>3</sub> arXiv:1402.3254 James Analytis Radu Coldea Stripy-Honeycomb



#### Hyper-Honeycomb β-Li<sub>2</sub>IrO<sub>3</sub> Hide Takagi arXiv:1403.3296



Close to ideal structure AF ordering But with positive (ferro-like) Curie-Weiss temperature



#### Stripy-Honeycomb y-Li<sub>2</sub>IrO<sub>3</sub> arXiv:1402.3254



Radu Coldea James G. Analytis

AF ordering  $T_c = 38K$ 

strong magnetic anisotropy

## Is J<sub>eff</sub>=1/2 picture val



E. K.-H. Lee, S. Bhattacharjee, K. Hwang, H.-S. Kim, H. Jin, Y. B. Kim, arXiv:1402.2654 (2014)

H.-S.Kim, E. K.-H. Lee, Y. B. Kim, arXiv:1502.00006 (2015)

# Localized Pseudo-Spin Model in the Strong Coupling Limit J<sub>eff</sub>=1/2

#### Edge-sharing Oxygen Octahedra Structure



## Strong Coupling Limit: Localized Pseudo-Spin Model



(a) Ir-Ir overlap for  $t_1$ 



(c) Ir-Ir overlap for  $t_2$ 



(b) Ir-O-Ir overlap for  $t_2$ 



(d) Ir-Ir overlap for  $t_3$ 

$$t_1 = \frac{t_{dd\pi} + t_{dd\delta}}{2}, \ t_2 = \frac{t_{pd\pi}^2}{\Delta_{pd}} + \frac{t_{dd\pi} - t_{dd\delta}}{2}, \ t_3 = \frac{3t_{dd\sigma} + t_{dd\delta}}{4},$$

Strong Coupling Limit: Localized Pseudo-Spin Model

$$H = \sum_{\langle ij \rangle \in \alpha \beta(\gamma)} \left[ J \vec{S}_i \cdot \vec{S}_j + K S_i^{\gamma} S_j^{\gamma} + \left[ \Gamma \left( S_i^{\alpha} S_j^{\beta} + S_i^{\beta} S_j^{\alpha} \right) \right] \right]$$

e.g. In the limit of  $U, J_H \gg \lambda \gg t$ 



$$t_1 = \frac{t_{dd\pi} + t_{dd\delta}}{2}, \quad t_2 = \frac{t_{pd\pi}^2}{\Delta_{pd}} + \frac{t_{dd\pi} - t_{dd\delta}}{2}, \quad t_3 = \frac{3t_{dd\sigma} + t_{dd\delta}}{4},$$

J. G. Rau, E. K.-H. Lee, H. Y. Kee, PRL 2014

#### Kitaev Model: Exact Solution

$$\mathcal{H}_{\mathrm{K}} = -\sum_{\alpha-links} S_{i}^{\alpha} S_{j}^{\alpha}$$

 $S_i^{lpha} = i b_i^{lpha} c$   $\{b_i^x, b_i^y, b_i^z, c\}$  Four Majorana Fermions

$$\mathcal{H}_{\mathrm{K}} = \frac{\imath}{2} \sum_{\alpha - \mathrm{links}} u_{ij}^{\alpha} c_i c_j \qquad (\text{where } u_{ij}^{\alpha} = i b_i^{\alpha} b_j^{\alpha}),$$

 $\mathcal{W}_P = \prod_{\text{loop}} u_{ij}^{\alpha} \text{ commute with the Hamiltonian} \Longrightarrow \mathcal{W}_P = \pm 1$ 

**Hyper-Honeycomb** 5. Mandal and N. Surendran (2009) Ground state is in the zero-flux sector  $u_{ij}^{\alpha} = +1 (\forall \langle ij \rangle)$ 

**Stripy-Honeycomb** R. Schaffer, E. Lee, Y.-M. Lu, Y. B. Kim, PRL 2015 Ground state is in a  $\pi$ -flux sector some  $W_p = -1$ 



Nodal Ring in 3D



