Frontiers of Nanoscience
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Topological Cooper-pairing based on spin-orbit interactions

The Abdus Salam
International Centre
for Theoretical Physics

mpipks

Topological Superconductors or Superfluids

Why are they of interest? → possible candidates for quantum computing

reason : excitations (particles), topolog. protected against decoherence

→ Majorana fermions , non-Abelian statistics

Different types of topological superfluids : solids or cold atoms

(a) superconductivity induced by proximity effect in

- topolog. semiconductors
- convent. semimetals or metals with large SO interaction

(b) intrinsic topolog. superfluids , e.g., due to spin-orbit interactions

→ ^{3}He B-phase, $Cu_xBi_2Se_3$

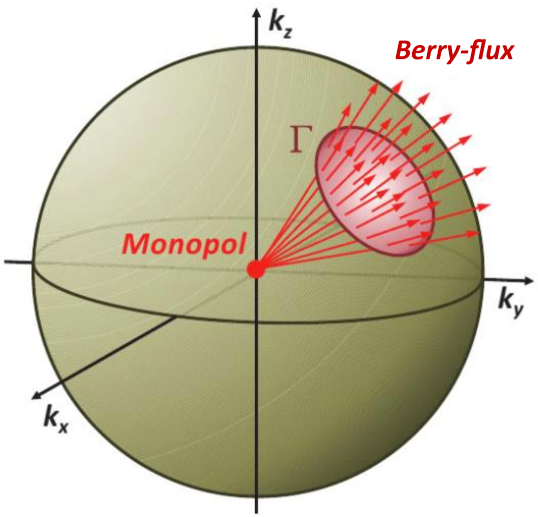
prerequisite → topolog. insulator or superconductor :

→ integer Chern number $\neq 0$ → topol. order parameter

class of Hamiltonians with the same Chern number , effective field theory analogue G–L

Berry phase: Bloch state $\psi_{m\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{kr}} u_{m\mathbf{k}}(\mathbf{r})$ m = band index

closed path C



$$|u_m(\text{end})\rangle = |u_m(\text{start})\rangle e^{i\gamma_m[C]}$$

$$\gamma_m[C] = i\int_C \langle u_m(\mathbf{k})|\nabla_k u_m(\mathbf{k})\rangle d\mathbf{k} \quad = i\int_C \mathbf{A}_m(\mathbf{k})d\mathbf{k}$$

$$= i\int \text{curl } A_m(\mathbf{k})dS$$

$$n_m = \frac{1}{2\pi}\int_{BZ} F_m(\mathbf{k})dS$$; „magnet. flux“ $F_m(\mathbf{k}) = \text{curl}\,\mathbf{A}_m(\mathbf{k})$

$$N = \sum_m^{occ} n_m$$ **Chern number**

Edge or surface modes of topological insulators and superconductors

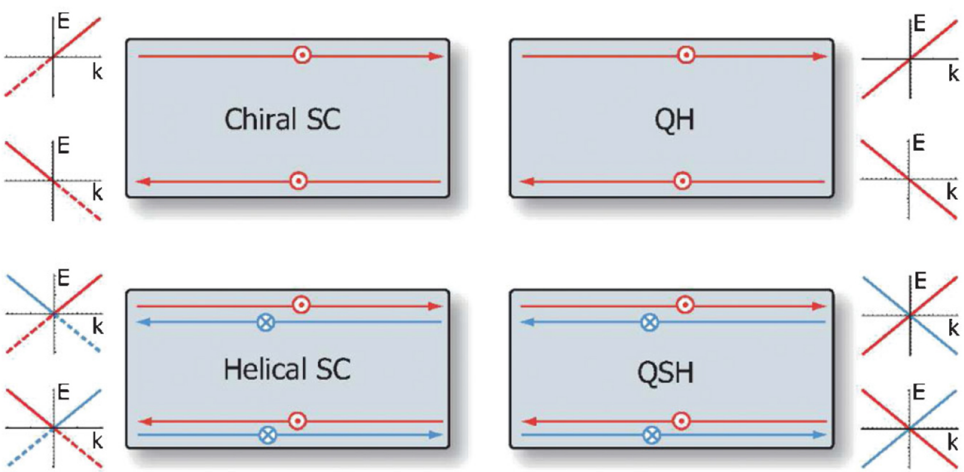
when $C \neq 0$ → edge modes ; well known example : QH effect

time-reversal symm. plays important role

no TR symm : chiral edge states , e.g. QH effect , classified by integer

when SC → edge states give raise to Majorana fermions

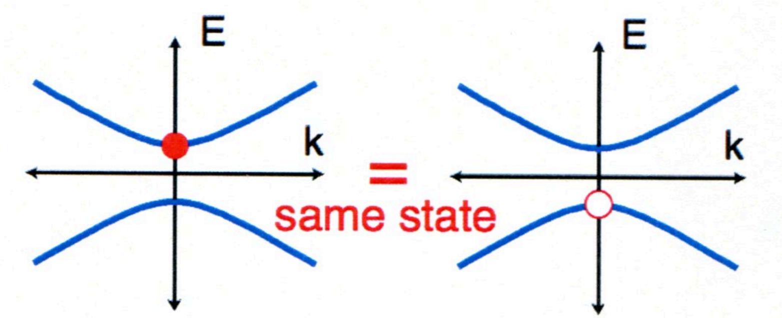
with TR symm : helical edge states , e.g. SQH effect , classified by $\mathbb{Z}_2$



Special features of topol. superconductors

Bogoliubov – de Gennes Hamiltonian

$$H = \frac{1}{2}\sum_{\mathbf{k}} \left(c^{+} c\right) \begin{pmatrix} H_0 & \Delta \\ \Delta^{*} & -H_0 \end{pmatrix} \begin{pmatrix} c \\ c^{+} \end{pmatrix}$$

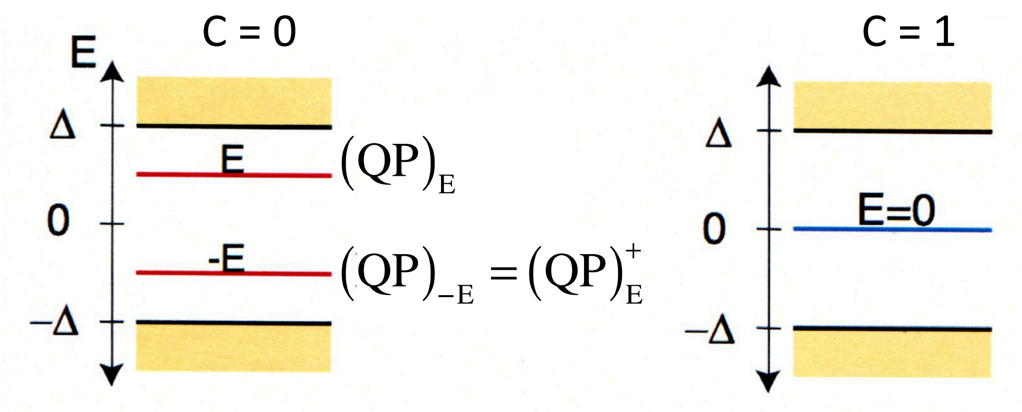


for $\Delta = 0$ ⟶ two copies of H_0

particle-hole symm.

$$\Xi = \tau_x K \;\;\Rightarrow\;\; \Xi H(\mathbf{k}) \Xi^{-1} = -H(-\mathbf{k})$$

bound states :



Zero energy mode

$$(QP)_{E=0}^{+} = (QP)_{E=0} \equiv \gamma$$

single state
associated with $(QP)_E^{+}$ and $(QP)_{-E}$!

Majorana bound states stored nonlocally at the end of a chain or in a vortex

Majorana Fermions

particle is own antiparticle → 1 electron = 2 majoranas

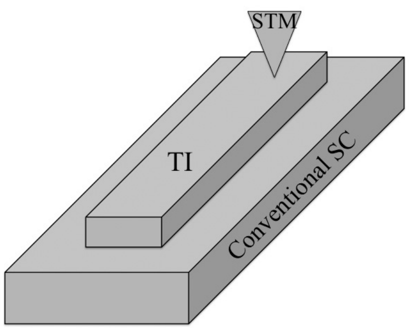
→ fractionalization

$$c^{+} = \frac{1}{\sqrt{2}}(\gamma_A - i\gamma_B) \qquad c^{+} = \frac{1}{\sqrt{2}}(\gamma_A + i\gamma_B) \qquad ; \quad \gamma_A = \gamma_A^{+} \quad , \quad \gamma_B = \gamma_B^{+}$$

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

prerequisite : Chern number $C \neq 0$, e.g. ^{3}He-B phase , $Cu_xBi_2Se_3$

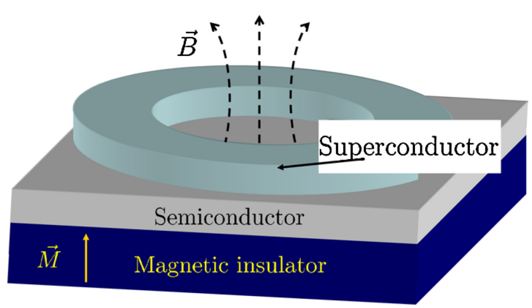
how to generate topological superconductor? → proximity effect



1D

Majorana end states

(L. Fu + C. Kane)

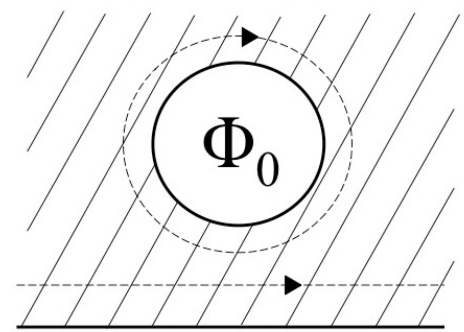


2D

Majorana vertex

(J. Sau et al.)

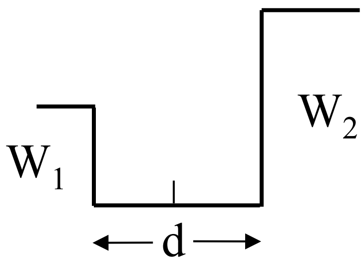
2D T - SC



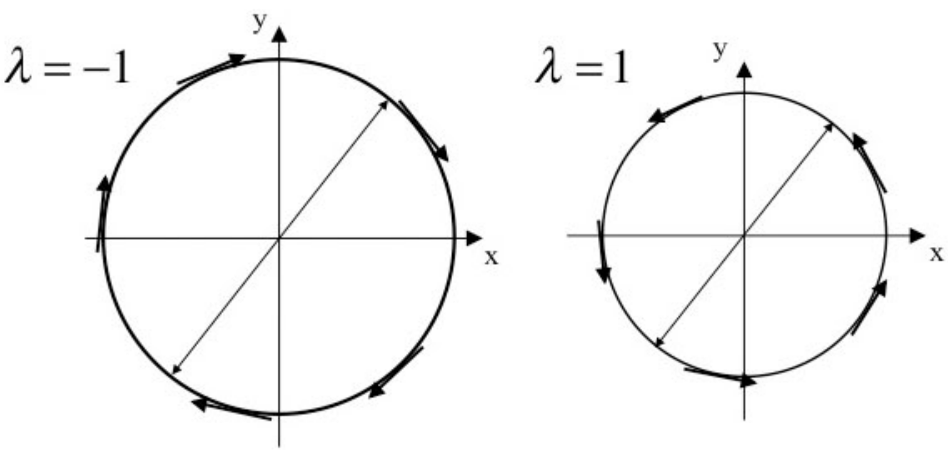
Superconductors with Rashba-type spin-orbit interaction

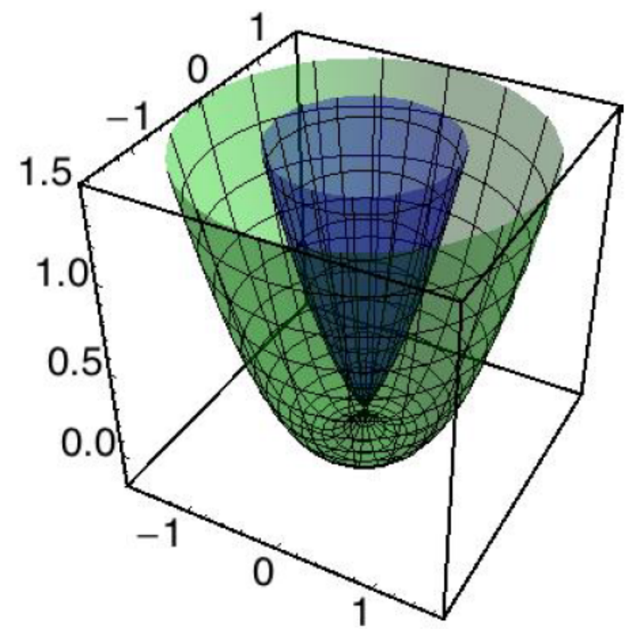
e.g., monolayer of Pb or ultracold atoms or via proximity effect

Rashba $H_{s0} = \alpha v_{F0} \sum_{\mathbf{k}} (\mathbf{e}_z \times \mathbf{k}) \boldsymbol{\sigma}$



$$\varepsilon(\mathbf{k}\lambda; h=0) = \varepsilon_0(\mathbf{k}) + \lambda \alpha v_{F0} k$$



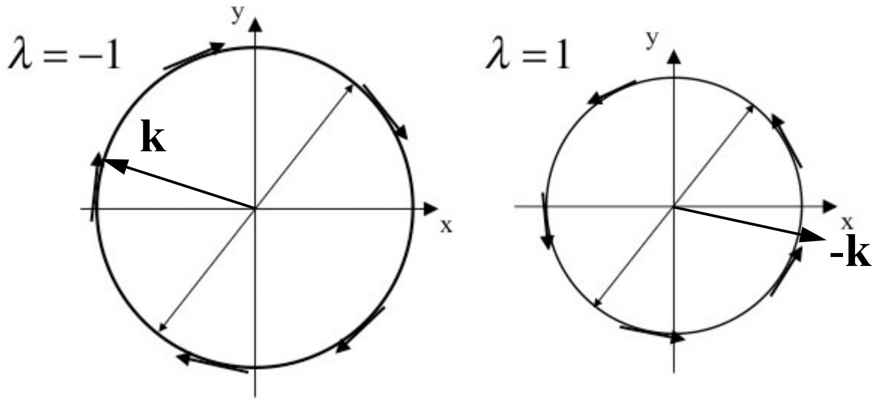


intraband vs inter-band pairing

➔ intraband pairing : spin singlet + triplet
inter-band pairing : spin triplet

G. Zwicknagl

inter-band pairing : finite pairing momentum $\left\langle c_{\mathbf{k}+\mathbf{q}/2,\lambda}\; c_{-\mathbf{k}+\mathbf{q}/2,\bar{\lambda}} \right\rangle$

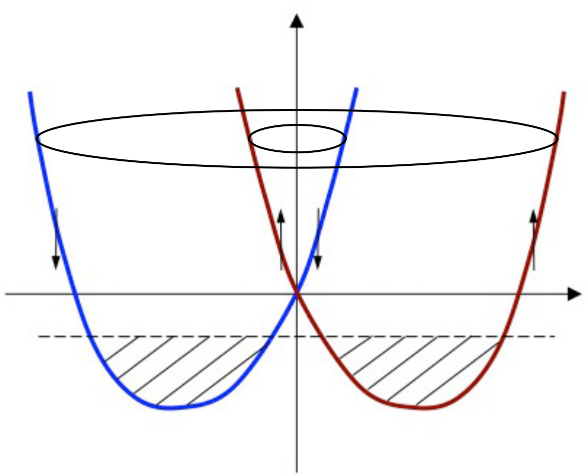


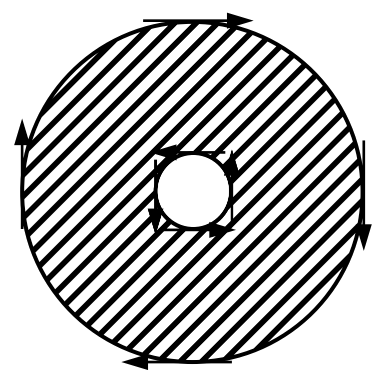
SO interaction is pair breaking

$|\mathbf{q}|$ degeneracy

$$\Delta = \sum_{\nu} \Delta_{\nu} e^{i\mathbf{q}_{\nu}\mathbf{r}}$$

intraband pairing : time-reversed pairing





topolog.
trivial

Large spin-orbit interaction → only intraband pairing left

Hamiltonian : $H = H_0 + H_{int}$ $\qquad H_0(\mathbf{k}) = \varepsilon_0(\mathbf{k}) - \alpha\left(k_x\sigma_y - k_y\sigma_x\right)$

electron-phonon interaction :

$$H_{int} = \sum_{\substack{\mathbf{k}\lambda \\ k_x\geq 0}} \sum_{\substack{\mathbf{k}'\lambda' \\ k'_x\geq 0}} v_0 \lambda\lambda' c^+_{\mathbf{k}\lambda} c^+_{-\mathbf{k}\lambda} c_{-\mathbf{k}'\lambda'} c_{\mathbf{k}'\lambda'}$$

mean field : $\Delta_\lambda = \sum_{\substack{\mathbf{k}'\lambda' \\ k_x\geq 0}} v_0 \lambda\lambda' \left\langle c_{-\mathbf{k}'\lambda'} c_{\mathbf{k}'\lambda'} \right\rangle$

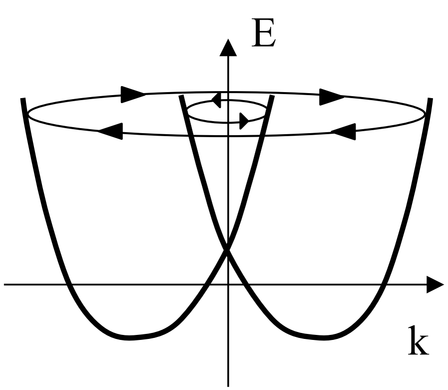
$$\Delta_+ = -\Delta_- = \Delta$$

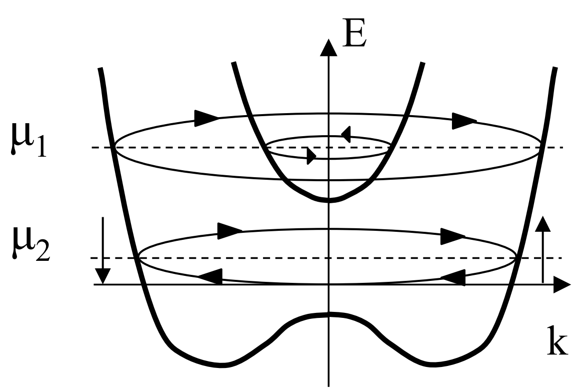
with G. Zwicknagl

topological superfluids

(C. Zhang et al. (2008), Sato et al. (2009))

2D : out of plane magnetic field $\qquad H_{ze} = \sigma_z h$





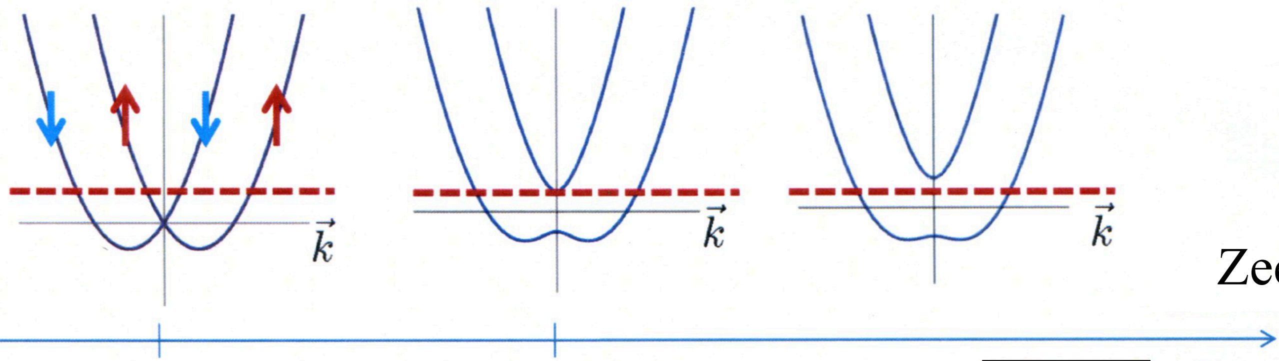
$$H(\mathbf{k}) = \begin{pmatrix} H_0\left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) & -i\Delta\sigma_y \\ i\Delta^*\sigma_y & -H_0^*\left(-\mathbf{k} + \frac{\mathbf{q}}{2}\right) \end{pmatrix}$$

$\mathbf{q}$ = pairing momentum

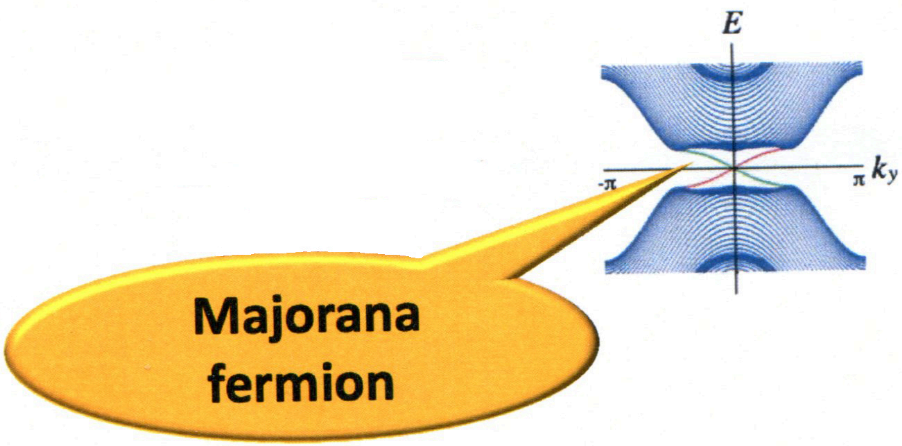
Topological edge states

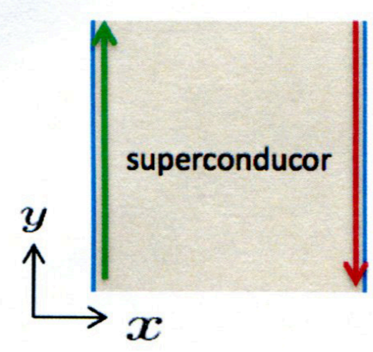
external magnetic field perpendicular to plane

two Fermi surfaces one Fermi surface



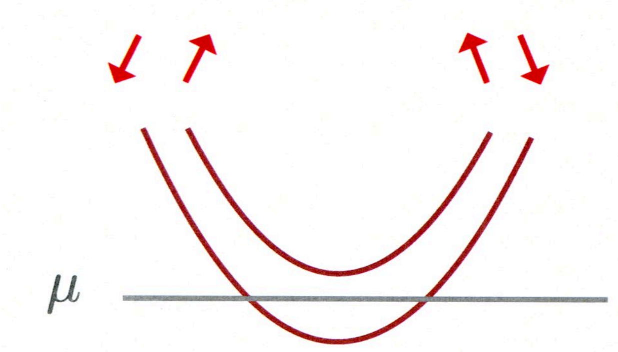
$\mu_B H = 0$ $\mu_B H = \sqrt{\Delta_0^2 + \mu^2}$ $\mu_B H > \sqrt{\Delta_0^2 + \mu^2}$

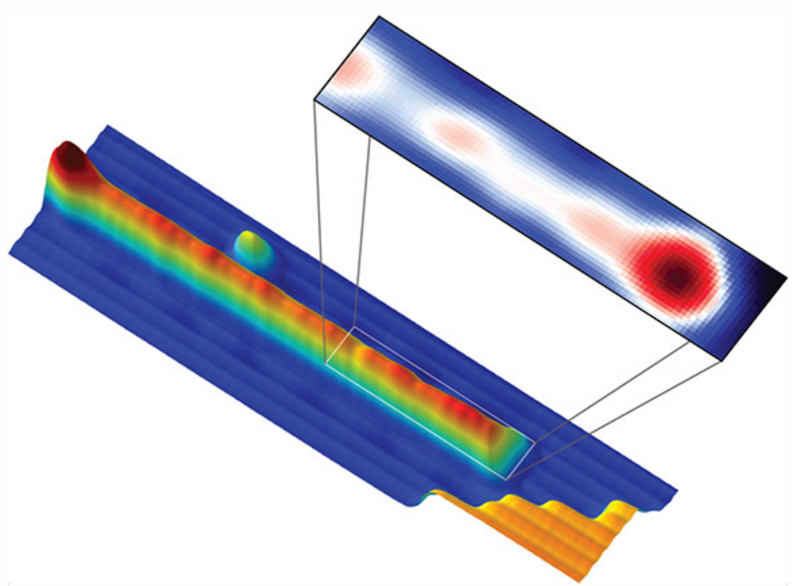




Sato, Takahashi, Fujimoto

A. Yazdani, A. Bernevig et al. (2014)





chain of Fe atoms on Pb (in a trench)

→ strong SO interaction

→ induction of SC on Fe chain (proximity effect)

STM along chain

modified version (J. Alicea) : splitting of SO bands by in-plane field

prerequisite : semiconduct. grown along (110) direction, e.g., InSb

Rashba + Dresselhaus SO interaction

Dresselhaus : favors spin alignment normal to plane

Rashba : spins within plane are favored

$$H_0 + H_0 = \int d^2\mathbf{r}\,\psi^+\left[-\left(\frac{1}{2m_x}\partial_x^2 + \frac{1}{2m_y}\partial_y^2\right) - \mu - i\alpha^D\boldsymbol{\sigma}_z\partial_x\right]\psi$$

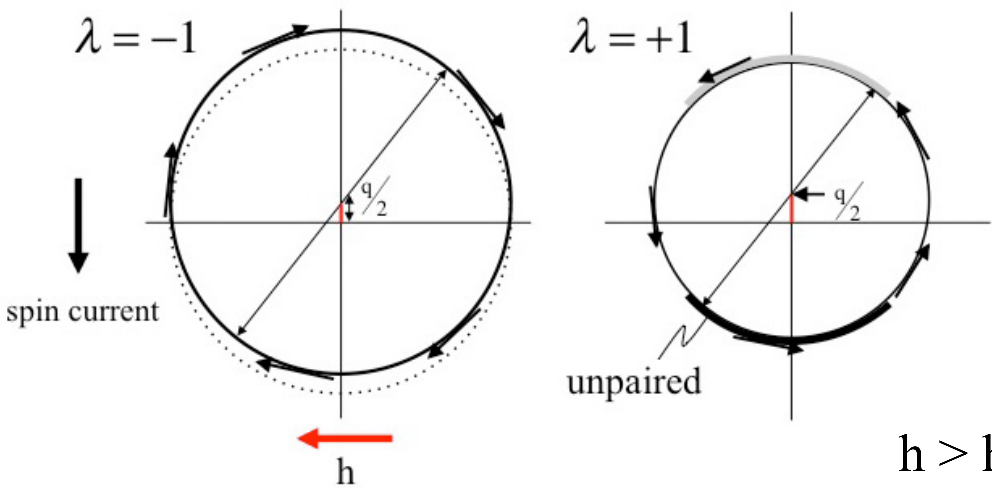
$$H_R = -i\int d^2\mathbf{r}\,\psi^+\left(\alpha_x\boldsymbol{\sigma}^x\,\partial_y - \alpha_y\boldsymbol{\sigma}^y\,\partial_x\right)\psi \qquad H_{z_e} = g\frac{\mu_B}{2}h_y\int d^2\mathbf{r}\,\psi^+\boldsymbol{\sigma}_y\psi$$

SC via proximity effect ➡ in-plane field opens gap when

$$|\mu| < g\frac{\mu_B}{2}h_y$$

2D case : magnet. field in plane ; $h = \mu_B H / \Delta_0$

$\alpha = 0.1$ for $h > 0.85$ ⟶ $\Delta_\lambda(q) = \sum_{\substack{\mathbf{k}'\lambda' \\ k_x > 0}} v_0 \lambda \lambda' \left\langle c_{-\mathbf{k}' + \frac{q}{2}\lambda'} \, c_{\mathbf{k}' + \frac{q}{2}\lambda'} \right\rangle$

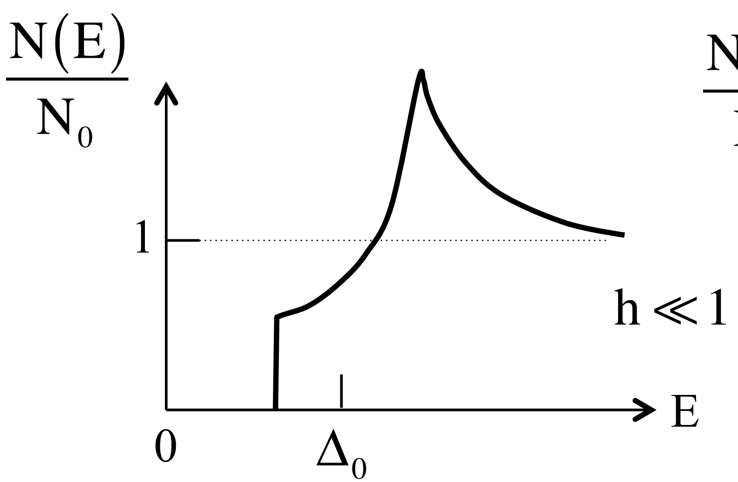


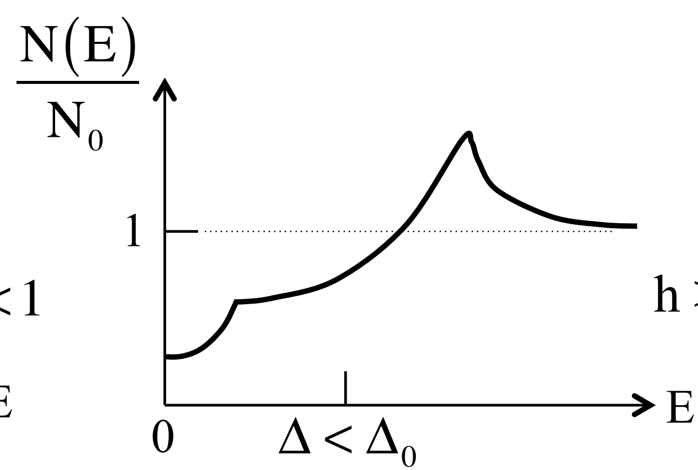
inhomogeneous sc. state !

corresponds to type II superconductors

interband pair scattering is still taking place

$h > h_{cr}$ ⟶ Cooper pairs break up





$N_0 = N_- + N_+$

$h > 0.85$

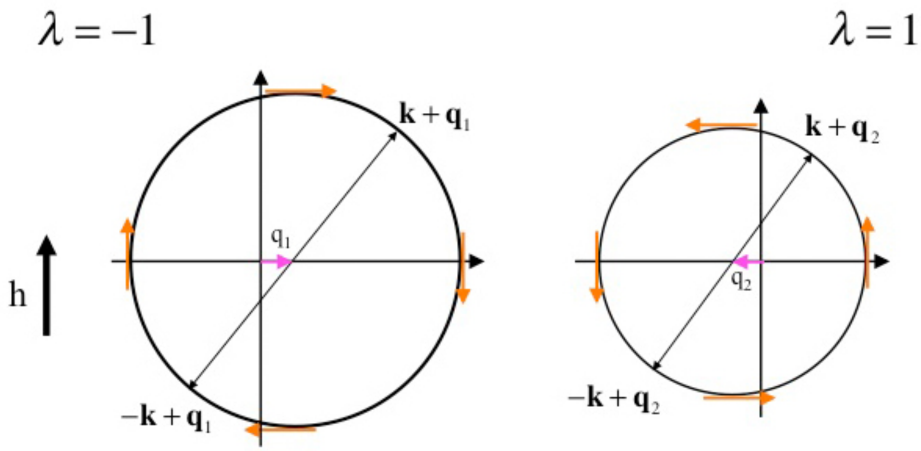
with G. Zwicknagl

as h increases more and more → 2. phase transition is expected

Cooper pairs with pairing momentum

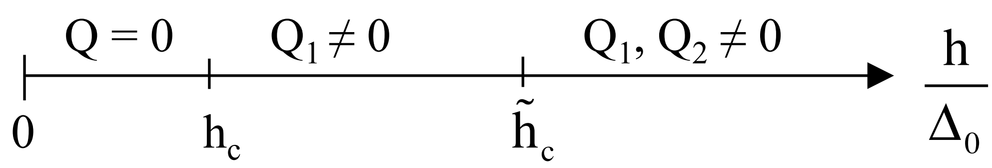
$$Q_1 = 2q_1$$
$$Q_2 = -2q_1$$

$$|q_1| = |q_2| = \frac{h}{v_F}$$



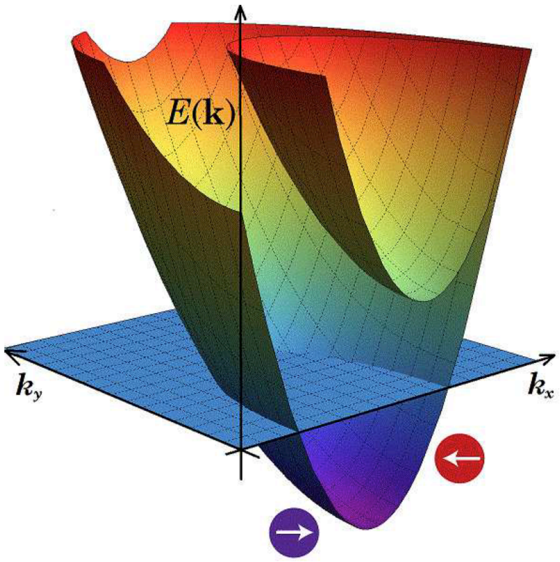
character of 2. phase transition is unknown

details rather different for thin films and for ultracold atoms due to different energy scales!



(W. Zhang + W. Yi (2013)
Y. Cao at al. (2014), C. Qu et al. (2014))

Two applied magnetic fields : $h_x + h_\perp$



finite pairing momentum

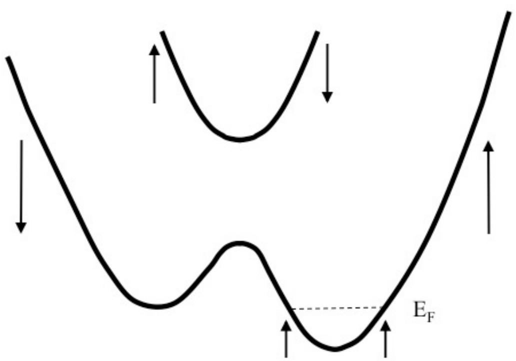
$$\Delta = \Delta_0 e^{iqy}; \quad \mathbf{q} = (0,q,0)$$

no charge current

t-FFLO state

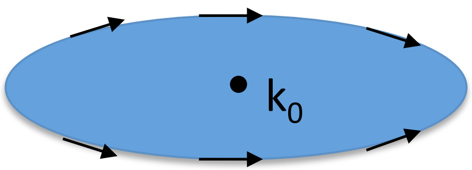
gapless t-sc?

special case



polarized fermionic system → like paired spinless

fermions similar to Fe chain



no spin current

Conclusion

- Topological superfluidity is feasible in systems with sufficiently large SO interaction in thin films (absence of inversion symmetry) or in optical lattice with ultracold atoms
- SC may be induced by proximity effects
- Magnetic fields perpendicular and parallel to the film play an important role in order to generate topological superconductivity
- Topological superconductors will have Majorana bound states in vortices