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# $1/f$ noise: Implications for solid-state quantum information

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I will talk about a relationship between thermodynamics of a simple driven quantum system and its decoherence.

# Dephasing and dissipation in qubit thermodynamics

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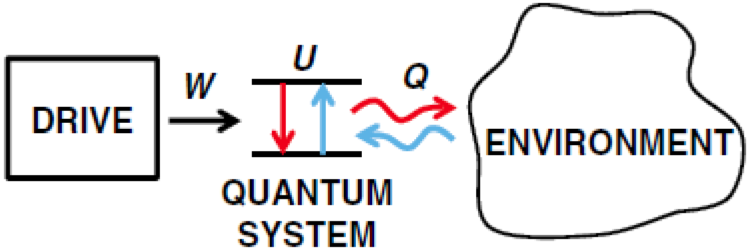
Questions:

- Is thermodynamics of a driven quantum system related to its decoherence?
- Which protocol can be used to determine this relationship?

We address these issue using the quantum jump (QJ) approach applied to a driven quantum two-level system (TLS)




Support: EU through FP7 Project INFERNOS

# Are the question proper?

**Can one tell something general about thermodynamics of a nonequilibrium system?**

In conventional thermodynamics, the free energy difference between two equilibrium states, *A* and *B*, is related to the work *W* done on the system through the *inequality*,

$$\Delta F \leq W,$$

with equality holding only in the case of a *quasistatic* process.

**Out of equilibrium**:

According to the conventional thermodynamics, one has to treat each system individually

> Happy families are all alike; every unhappy family is unhappy in its own way.
>
> From "Anna Karenina" by Leo Tolstoy

# Jarzynski equality

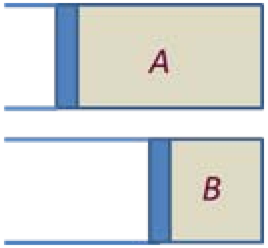
The **Jarzynski equality** (JE) is an equation that relates free energy differences between two *equilibrium* states and *non-equilibrium* processes. It is named after Christopher Jarzynski (then at Los Alamos National Laboratory) who derived it in 1997.

In contrast to conventional thermodynamics, the JE remains valid no matter how fast the process happens. The JE states:

$$e^{-\beta\Delta F} = \langle e^{-\beta W} \rangle$$

The brackets indicate an average over all possible realizations of an external process that takes the system from the equilibrium state *A* to a new, generally nonequilibrium state *under the same external conditions as that of the equilibrium state B*.

Example - a gas compressed by a piston: the gas is equilibrated at piston position *A* and compressed to piston position *B*; in the Jarzynski equality, the final state of the gas does not need to be equilibrated at this new piston position, but there exist the equilibrium state, B, which corresponds to new external conditions.



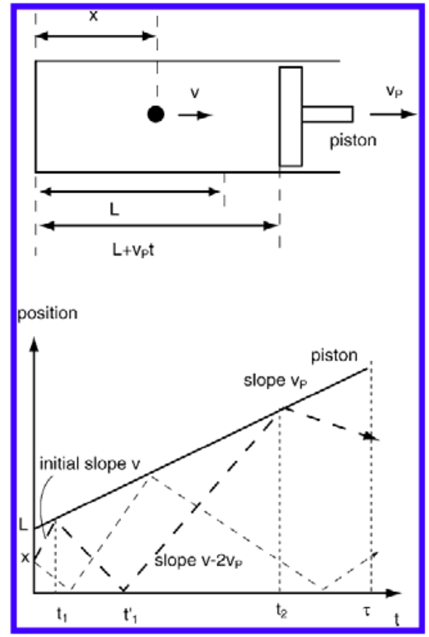
There are already quite a number of works exploring various aspects of the Jarzynski formula addressing mathematical foundations of the JE in the context of fundamental statistical mechanics.

Some authors describe the Jarzynski identity in terms of “transient violations of the second law of thermodynamics”, “temporary violations of the second law”.

“… the Jarzynski identity is based on proper exploration of a representative set of fluctuations.”

Lua & Grosberg, *J. Phys. Chem. B* **2005,** *109,* 6805

They analyzed the case of a gas and showed that JE is valid due to the very rapid molecules belonging to the tail of the Maxwell distribution.




Not so simple:

If (i) the system volume is large and (ii) the piston is moving with great speed (compared to thermal velocity) for a very short time, the necessary number of independent experimental runs to obtain a reasonable approximation for the free energy from averaging the nonequilibrium work grows exponentially with the system size.

# Quantum TLS

Total work: $W = U + Q$

$U = E_f - E_i$ - "useful" work

$Q$ - heat dissipated into environment

According to the Jarzynski equality (JE) for a cycle procedure, $\langle \exp(-\beta W) \rangle = 1$

A proposal to study fluctuation theorems was put forward as the so-called two-measurement protocol (TMP), where the state of the system is measured first before the work is applied, and second after the application of this work. This yields the difference *U* in the internal energy.

J. Kurchan, e-print cond-mat/0007360.
P. Talkner, E. Lutz, and P. Hänggi, Phys. Rev. E **75**, 050102(R) (2007).

For a closed system, *Q=0* and *W=U*. Therefore, $\langle \exp(-\beta U) \rangle = \langle \exp(-\beta W) \rangle = 1$
This is not the case for an open system where a TMP does not yield the total work *W*.

Q: Can we extract a useful information from the average $\langle \exp(-\beta U) \rangle$ for an open system?

We will analyze this issue using the so-called quantum jumps (QJ) method and show that the difference $\langle \exp(-\beta U) \rangle - \langle \exp(-\beta W) \rangle$ depends on the measurement protocol and on the amount and mechanism of decoherence in the system.

## QJ approach in quantum optics:

VOLUME 68, NUMBER 5 PHYSICAL REVIEW LETTERS 3 FEBRUARY 1992

### Wave-Function Approach to Dissipative Processes in Quantum Optics

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(Received 15 October 1991)

A novel treatment of dissipation of energy from a "small" quantum system to a reservoir is presented. We replace the usual master equation for the small-system density matrix by a wave function evolution including a stochastic element. This wave-function approach provides new insight and it allows calculations on problems which would otherwise be exceedingly complicated. The approach is applied here to a two- or three-level atom coupled to a laser field and to the vacuum modes of the quantized electromagnetic field.

## Application to statistics of work in a driven TLS:

PRL 111, 093602 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

### Quantum Jump Approach for Work and Dissipation in a Two-Level System

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(Received 22 May 2013; published 27 August 2013)

QJ method:

It is assumed that we are able to register the acts of photon (or phonon) emission or absorption. After that we know for sure in which state, ground |*g*> or excited |*e*> the system is.

Even if there is no jump during some time interval, a possibility for such a jump must be taken into account. This is done constructing an effective non-Hermitian Hamiltonian, *H.*

It has been shown that during the small non-jump time interval Δt the wave function evolves according to the Schrödinger-like equation

Non-Hermitian

$$|\psi(t+\Delta t)\rangle = (1-p)^{-1/2}\left(1 - \frac{i}{\hbar} H \Delta t\right)|\psi(t)\rangle$$

where $|\psi(t)\rangle = a(t)|g\rangle + b(t)|e\rangle,\ p = (\Gamma_{\uparrow}|a|^2 + \Gamma_{\downarrow}|b|^2)\Delta t,\ \Gamma_{\downarrow,\uparrow}$ are the relaxation (excitation) rates, $\Gamma_{\uparrow} = e^{-\beta\hbar\omega_0}\Gamma_{\downarrow}$, $\hbar\omega_0$ is the level spacing of the qubit.

These rates determine the probabilities of interrupting the evolution, $p_{\uparrow} = \Gamma_{\uparrow}|a|^2\Delta t$ and $p_{\downarrow} = \Gamma_{\downarrow}|b|^2\Delta t$, during the time $\Delta t$.

Hamiltonian:

$$H = H_0 + \delta H + V + N$$

$\delta\omega(t)$ - classical stochastic process.

$H_0 = (\hbar\omega_0/2)(|e\rangle\langle e| - |g\rangle\langle g|)$ - TLS

$\delta H = (\hbar\delta\omega/2)(|e\rangle\langle e| - |g\rangle\langle g|)$ - Noise

$V = \lambda(t)(|g\rangle\langle e| + |e\rangle\langle g|)$ - Drive

$N = -(i\hbar/2)(\Gamma_\downarrow|e\rangle\langle e| + \Gamma_\uparrow|g\rangle\langle g|)$ - Non-Hermitian

In the interaction picture, representing the wave function as $|\psi_I(t)\rangle = a|g\rangle + b|e\rangle$ we express the dissipative "Schrödinger equation" as a set of equations for the amplitudes

$$\dot{a} = -\frac{i}{\hbar}\lambda(t)e^{-i\omega_0 t}b + i\frac{\delta\omega}{2}a + \frac{\Delta\Gamma}{2}a|b|^2,$$
$$\dot{b} = -\frac{i}{\hbar}\lambda(t)e^{i\omega_0 t}a - i\frac{\delta\omega}{2}b - \frac{\Delta\Gamma}{2}|a|^2 b.$$

$$\Delta\Gamma \equiv \Gamma_\downarrow - \Gamma_\uparrow$$

A jump resets the TLS to its ground or excited state.

It has been shown that the above scheme (for $\Gamma\Delta t \ll 1$ ) is fully equivalent to the Bloch-Redfield equations for the density matrix of the system.

# Thermodynamics

Long interval between measurements, $\Gamma_{\downarrow\uparrow}\tau \gg 1$ :

$$\langle e^{-\beta U}\rangle = p_g^i\, p_{g|g}^f\, e^0 + p_g^i\, p_{e|g}^f\, e^{-\beta\hbar\omega_0} + p_e^i\, p_{g|e}^f\, e^{\beta\hbar\omega_0} + p_e^i\, p_{e|e}^f\, e^0 .$$

Populations are thermally distributed:

$$p_g^i = 1 - p_e^i = p_{g|g,e}^f = 1 - p_{e|g,e}^f = (1 + e^{-\beta\hbar\omega_0})^{-1}$$

$$\downarrow$$

$$\langle e^{-\beta U}\rangle - 1 = \tanh^2(\beta\hbar\omega_0/2)$$

$$\boxed{\langle e^{-\beta U}\rangle - \langle e^{-\beta W}\rangle = \tanh^2(\beta\hbar\omega_0/2)}$$

- The difference does not depend on the decoherence and characteristics of the TLS.
- At low temperatures, dissipation into environment is important. This result holds for any driving protocol between the two measurements.

Our aim is to evaluate and compare the averages $\langle e^{-\beta W}\rangle$ and $\langle e^{-\beta U}\rangle$ for the case of weak dissipation, $\Gamma_{\downarrow\uparrow}\tau \ll 1$, where τ is the time between the measurements.

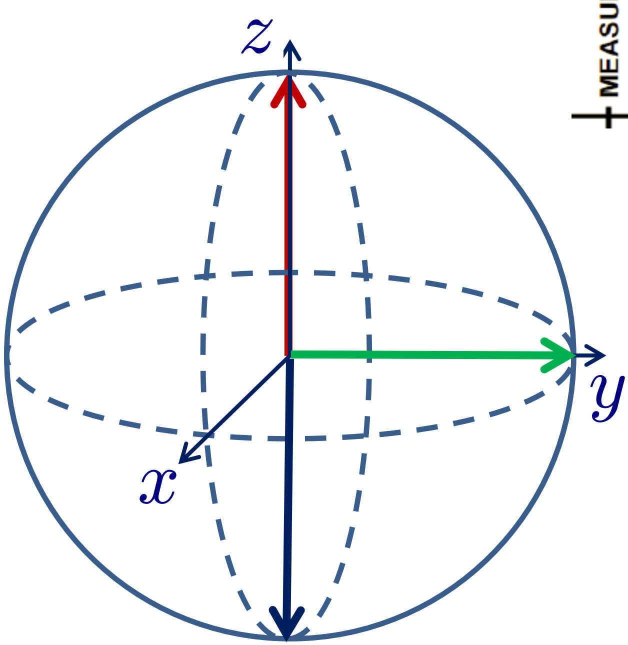
<u>Small dissipation limit</u> – at most one quantum jump (excitation or relaxation event) between measurements.

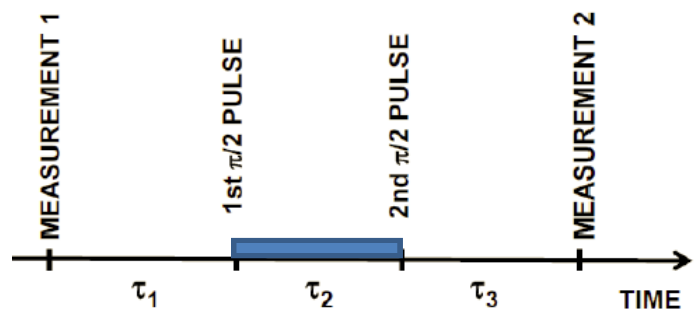
$$\langle e^{-\beta W}\rangle = P_0\langle e^{-\beta W}\rangle_0 + P_1\langle e^{-\beta W}\rangle_1$$

$P_0$, $P_1$ are the probabilities of zero- and one-photon processes, respectively

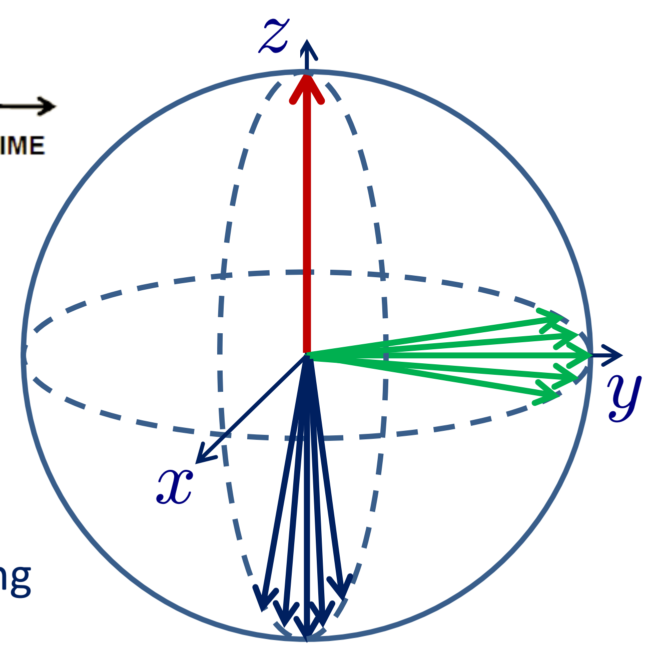
In a TMP we measure $\langle e^{-\beta U}\rangle = P_0\langle e^{-\beta U}\rangle_0 + P_1\langle e^{-\beta U}\rangle_1$ and for the zero-photon process *W*=*U*.

We choose the following protocol:



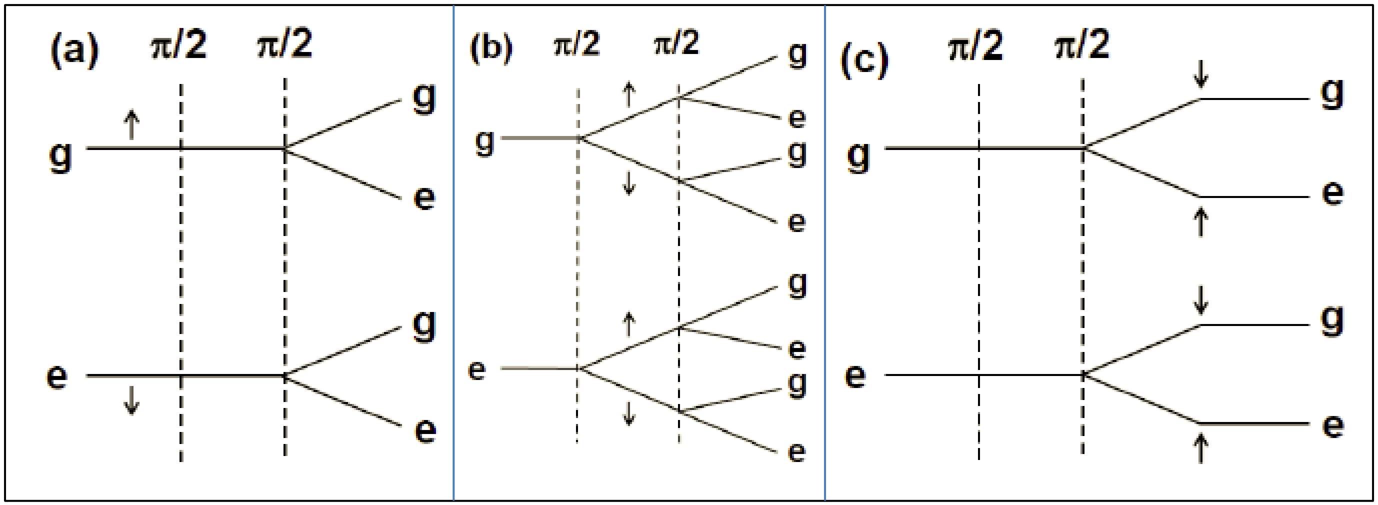

Dephasing




With dephasing

## Sketch of derivation for low dissipation

For the selected protocol, we calculate evolution of amplitudes taking into account the trajectories involving 0 and 1 jumps - first order contributions in Γ's) to various $P_1 \langle \cdots \rangle_1$




$0 < t_j < \tau_1$ $\quad \tau_1 < t_j < \tau_1 + \tau_2$ $\quad \tau_1 + \tau_2 < t < \tau_1 + \tau_2 + \tau_3$

Knowing the amplitudes we evaluate the probabilities of different trajectories and then calculate the thermal averages up to the first order in Γ's.

(Algebra is simple, but rather tedious)

Results:

(1) $$\langle e^{-\beta W}\rangle_{\delta\omega} \equiv P_0\langle e^{-\beta W}\rangle_0 + \sum_{i=a,b,c} P_{1,i}\langle e^{-\beta W}\rangle_{1,i} = 1$$

Sequences of pulses and jumps

for a given realization of δω . That proves the JE is valid for any distribution of δω.

(2) $$\langle e^{-\beta U}\rangle_{\delta\omega} = 1 + [\tau_3 - \tau_1 \cos(\delta\varphi_2)]\,\Gamma_\Sigma \tanh^2(\beta\hbar\omega_0/2).$$

$$\boxed{\delta\varphi_2 \equiv \int_{\tau_1}^{\tau_1+\tau_2} \delta\omega(t)\, dt}$$

$$\boxed{\Gamma_\Sigma = \Gamma_\uparrow + \Gamma_\downarrow}$$

Averaging over the stochastic processes $\delta\omega(t)$ we get:

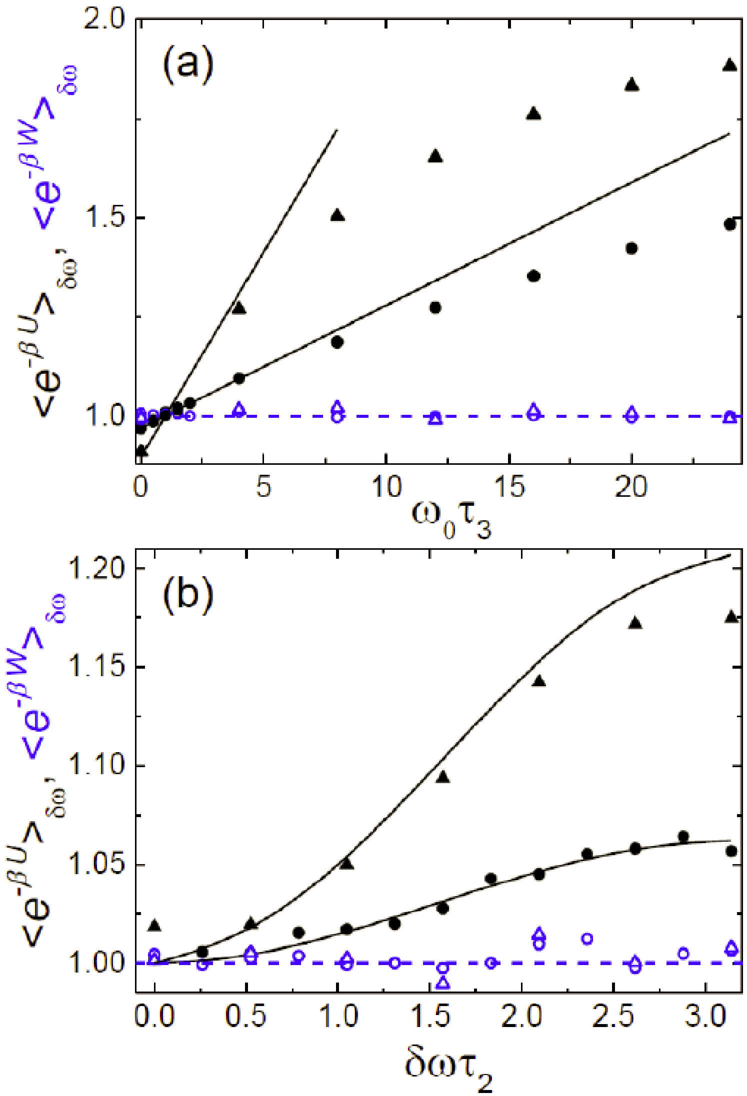
$$\boxed{\langle e^{-\beta U}\rangle = 1 + [\tau_3 - \tau_1 \langle\cos(\delta\varphi_2)\rangle]\,\Gamma_\Sigma \tanh^2(\beta\hbar\omega_0/2).}$$

Depends on the dephasing rate

$$\boxed{\langle e^{-\beta U}\rangle - \langle e^{-\beta W}\rangle = [\tau_3 - \tau_1 \langle\cos(\delta\varphi_2)\rangle]\,\Gamma_\Sigma \tanh^2(\beta\hbar\omega_0/2)}$$

Our central result

# Analytic expressions compared to numerical results obtained by stochastic simulations




$$\beta\hbar\omega_0 = 5,\ \frac{\Gamma_\downarrow}{\hbar\omega_0} = \begin{cases} 0.03 \text{ (circles)} \\ 0.1 \text{ (triangles)} \end{cases}$$

(a) - $\omega_0\tau_1 = \omega_0\tau_2 = 1,\ \delta\omega = 0$

Open symbols refer to $\langle e^{-\beta W}\rangle_{\delta\omega}$, while the filled ones – to $\langle e^{-\beta U}\rangle_{\delta\omega}$. The solid lines represent the analytic predictions.

(b) - $\langle e^{-\beta U}\rangle_{\delta\omega}$ is plotted for

$$\omega_0\tau_1 = \omega_0\tau_2 = \omega_0\tau_3 = 1$$

versus $\delta\varphi_2 \equiv \delta\omega\tau_2$ (solid lines and filled symbols).

The results for $\langle e^{-\beta W}\rangle_{\delta\omega}$, indicated by the corresponding open symbols, are again concentrated around unity.

In all cases we employed $10^7$ repetitions of the protocol for each data point.

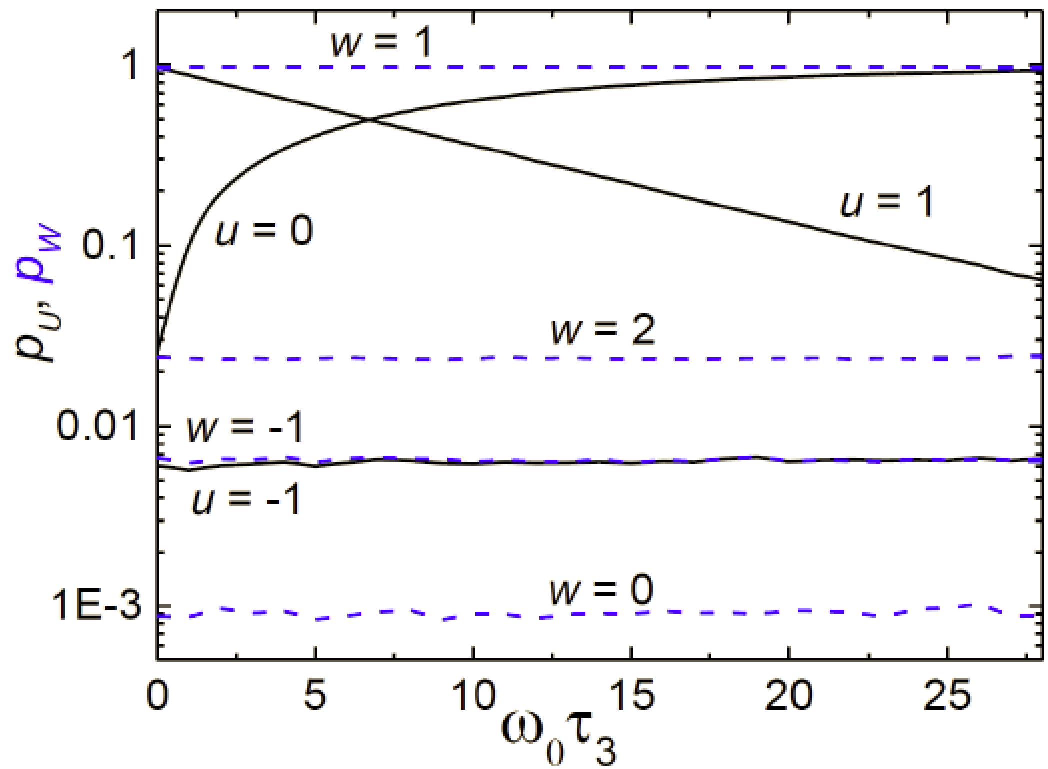
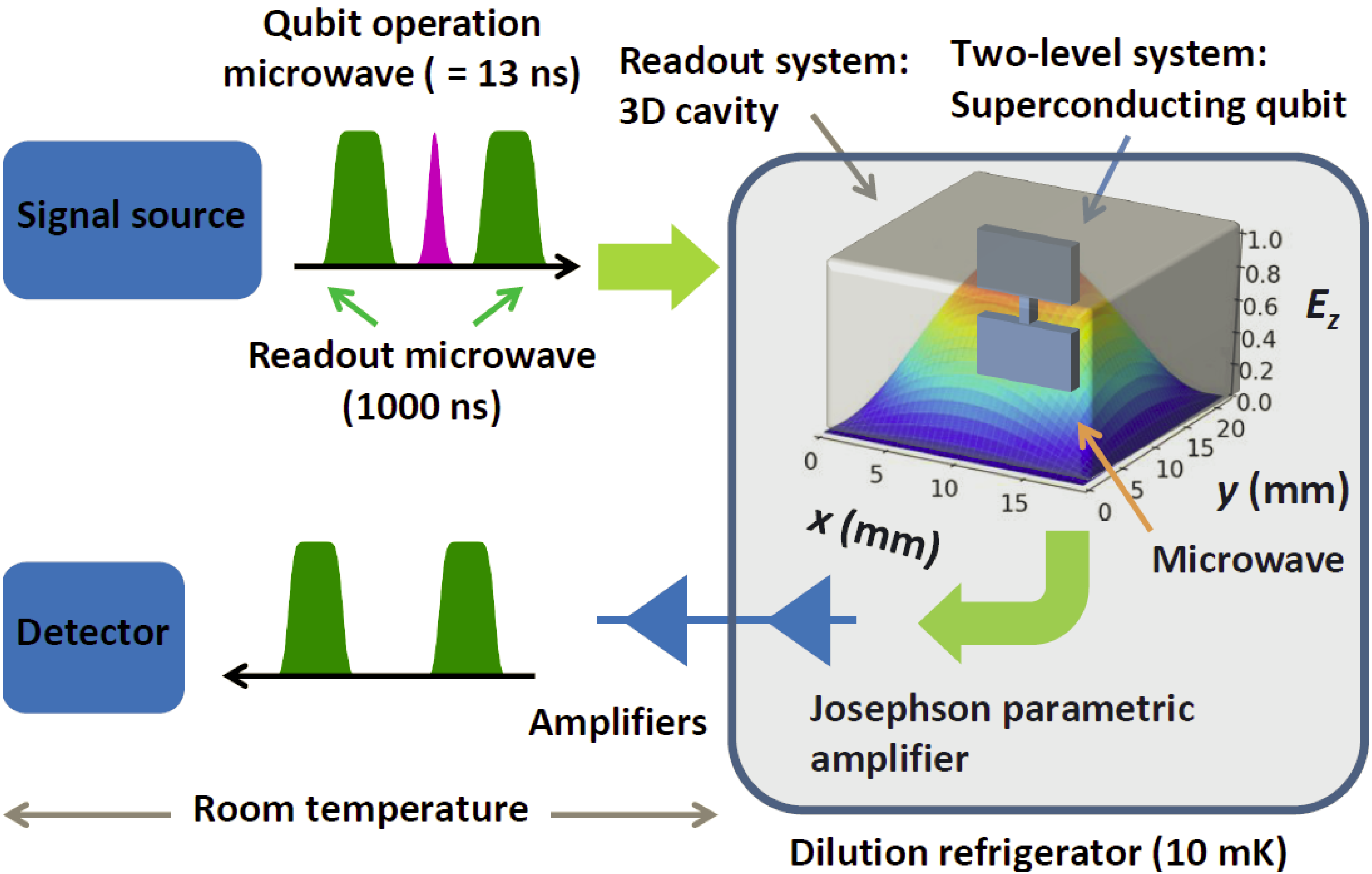
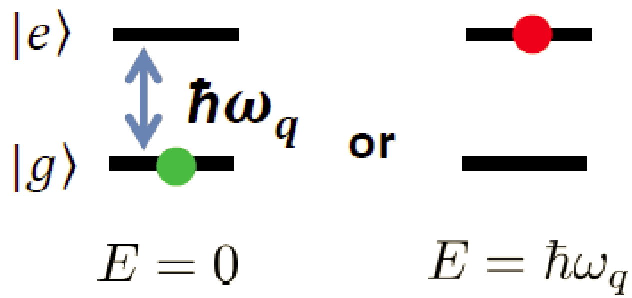



FIG. 3. Probabilities of different possible outcomes of $u \equiv U/\hbar\omega_0$ and $w \equiv W/\hbar\omega_0$. We assume the same protocol as before with $\beta\hbar\omega_0 = 5$, $\Gamma_\downarrow/\hbar\omega_0 = 0.1$, $\delta\omega = 0$, and $\omega_0\tau_1 = \omega_0\tau_2 = 1$. We vary the delay time of the second measurement, $\tau_3$. The solid lines refer to $U$ and the dashed ones to $W$. We employed $3{\cdot}10^5$ repetitions of the protocol for each data point.

# Experiment (Y. Nakamura and Y. Masuyama, in progress)



## Two level system: superconducting qubit



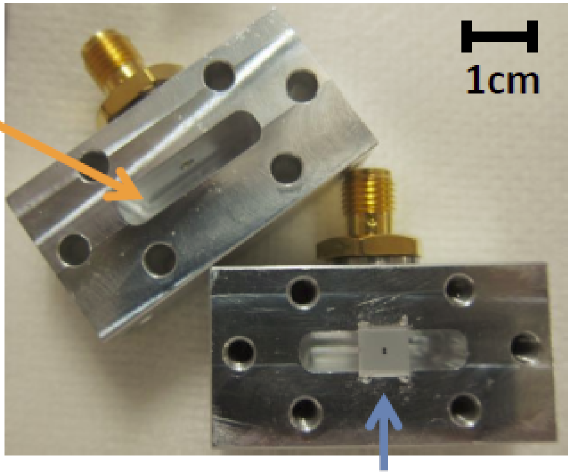

The state of superconducting qubit is measured via 3D cavity.

Free energy difference: $\Delta F = 0$

Population of 1st meas.
→Temp. of system
( ≈ 100 mK)

## Parameters

| | |
|---|---|
| Readout frequency | 10.9213 GHz |
| Qubit frequency | 6.7159 GHz |
| T1 | 24.0 μs |
| T2* | 4.3 μs |
| T2 echo | 5.3 μs |



# Simple Gaussian model

Assuming that $\delta\omega$ is time-independent and $\langle\delta\omega\rangle = 0$ after averaging over the Gaussian distribution with $\langle\delta\omega^2\rangle \equiv 2\Gamma_\varphi^2$ we get:

$$\langle\cos(\delta\varphi_2)\rangle = e^{-(2\Gamma_\varphi\tau_2)^2}$$

$\Gamma_\varphi \equiv \langle\delta\omega^2\rangle/2$ is the dephasing rate for the Gaussian model

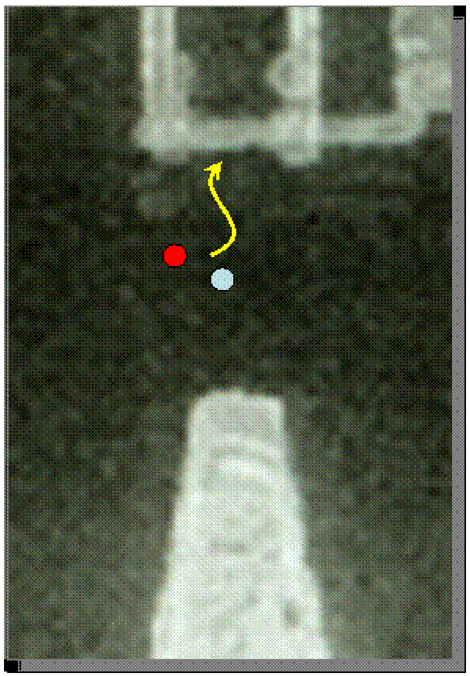
Finally, we obtain:

$$\langle e^{-\beta U}\rangle - 1 = \left(\tau_3 - e^{-(2\Gamma_\varphi\tau_2)^2}\tau_1]\right) \Gamma_\Sigma \tanh^2(\beta\hbar\omega_0/2)$$

Analyzing $\tau_2$ -dependence of the thermal average $\langle e^{-\beta U}\rangle$ one can relate thermodynamics with the the dephasing rate, $\Gamma_\varphi$ .

# Decoherence due to *1/f*-noise


**Fluctuators:** structural defects, charge traps, which can exist in dielectric parts of the device



The fluctuators randomly switch between their states due to interaction with extended modes of environment – phonons or electronic modes.

Switching $\Rightarrow$ random fields $\Rightarrow$ decoherence

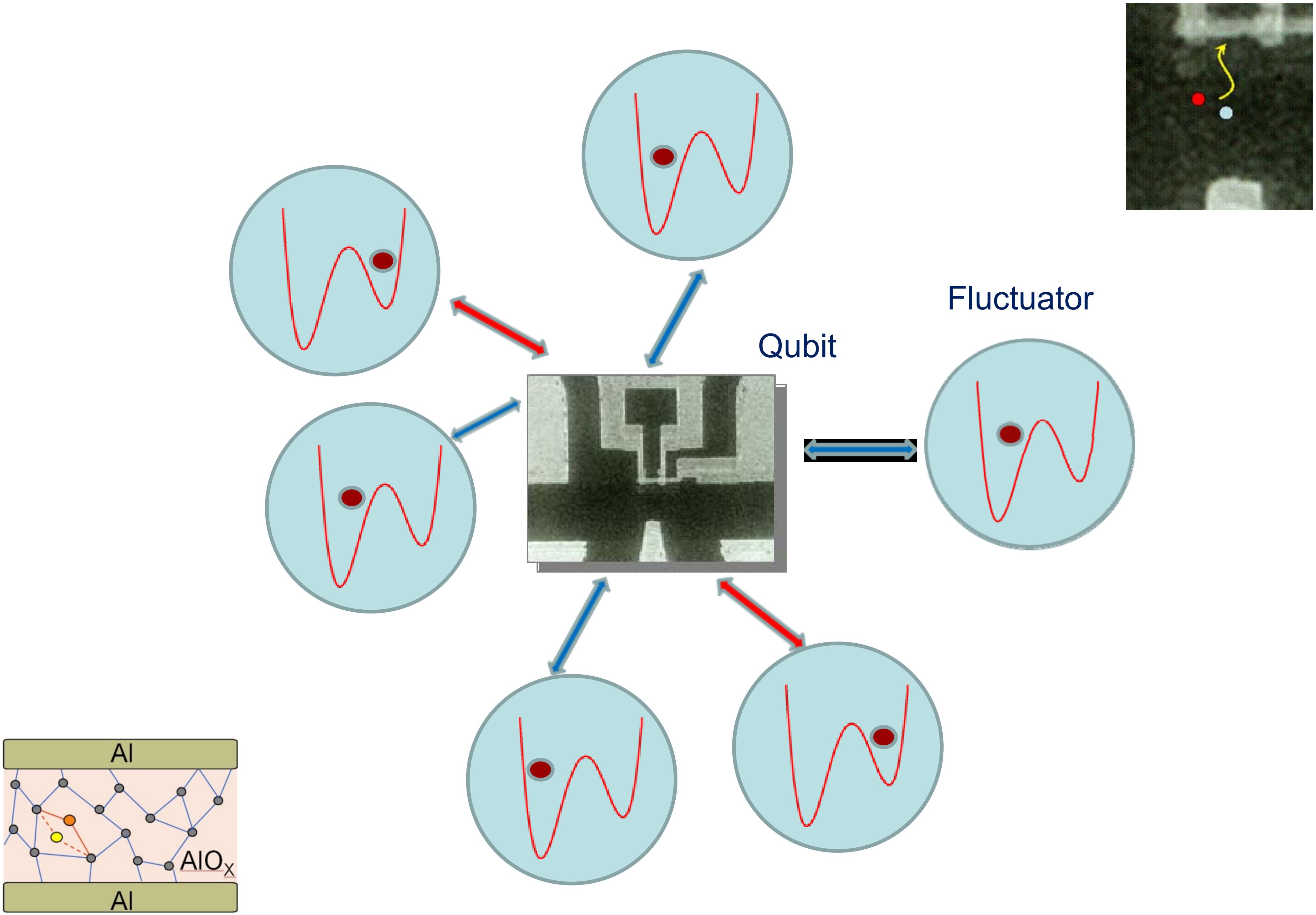
## Spin-fluctuator model

VOLUME 88, NUMBER 22 PHYSICAL REVIEW LETTERS 3 JUNE 2002

**Decoherence and 1/$f$ Noise in Josephson Qubits**

E. Paladino,[1] L. Faoro,[2] G. Falci,[1] and Rosario Fazio[3]

# Spin-fluctuator mode: Cartoon

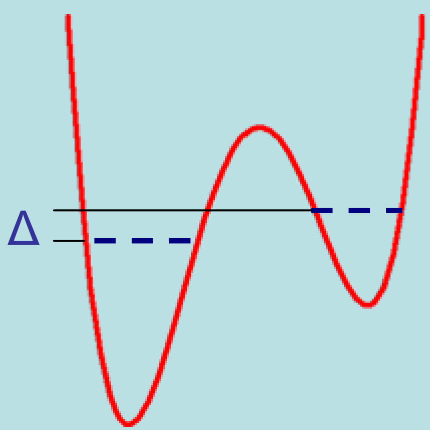


# Hamiltonian:

$$\mathcal{H} = -(B/2)\,\sigma_z + (1/2)F(t)\sigma_x$$ qubit

fluctuator $$+(1/2)\sum \boxed{E_i}\tau_z^{(i)} + \boxed{\mathcal{H}_{\text{env}} + \mathcal{H}_{F-\text{env}}}$$

interaction $$+\sum_i \left( v_i\,\sigma_z^{(i)}\tau_z^{(i)} + \ldots \right)$$




$$E_i = \sqrt{\Delta_i^2 + \Lambda_i^2}$$

$$v_i = g(r_i)A(\mathbf{n}_i)(B_z/B)(\Delta_i/E_i)$$

$$\mathcal{H}_{\text{env}} = \sum_\mu \omega_\mu \left( \hat{b}^\dagger \hat{b} + \frac{1}{2} \right)$$

$$\mathcal{H}_{F-\text{env}} = \sum_{i\mu} C_{i\mu}\tau_x^{(i)}\,(\hat{b}_\mu + \hat{b}_\mu^\dagger)$$

Spin-fluctuator model - continued

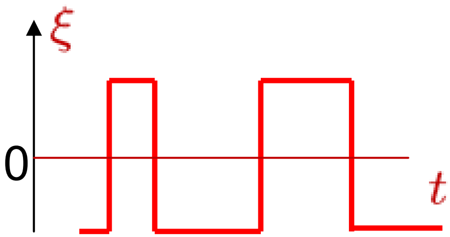
## Simple classical model:

Validity is considered by Wold *et al.*, PRB **86**, 205404 (2012)

Classical low-frequency fluctuations $\xi_i(t)$ acting upon the qubit:

$$\mathcal{H}_{qF} = \mathcal{X}_1(t)\,\sigma_z\,, \quad \mathcal{X}_1(t) = \sum_i v_i \xi_i(t)$$

Uncorrelated random telegraph processes: $\xi_i(t) = -1 \text{ or } +1$




Switching times are distributed according to Poisson distribution

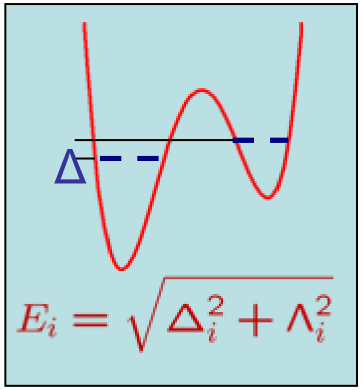
$$\langle \xi_i(t)\xi_k(t')\rangle = \delta_{ik} e^{-2\gamma_i|t-t'|}$$

The switching rates, $\gamma$, are calculated in the 2nd order in the interaction between the fluctuator and the thermal bath:

$$\boxed{E_i \lesssim kT}$$

$$\gamma_i = (1/2)\gamma_0(T)\,(\Lambda_i/E_i)^2 \propto e^{-2\lambda r}$$

Tunneling splitting of fluctuator



## Q: Does ensemble of fluctuators produce 1/f noise?

$$
\begin{aligned}
S_{\mathcal{X}}(\omega) &= 2\int_0^\infty dt\, e^{i\omega t}\langle \mathcal{X}(t)\mathcal{X}(0)\rangle \\
&= 2\sum_i v_i^2 \int_0^\infty dt\, e^{i\omega t}\langle \xi_i(t)\xi_i(0)\rangle \\
&\propto \sum_i v_i^2 \frac{\gamma_i}{\omega^2+\gamma_i^2} \to \langle v^2\rangle \int \mathcal{P}(\gamma)\, d\gamma \frac{\gamma}{\omega^2+\gamma^2}
\end{aligned}
$$

$\gamma \propto \Lambda^2 \propto e^{-2\lambda r} \longrightarrow$ For $\mathcal{P}(r) = \text{const},\ \mathcal{P}(\gamma) \propto 1/\gamma$ and $S_{\mathcal{X}}(\omega) \propto 1/\omega$

Exponentially-broad distribution of relaxation rates.

Interplay between decoherence and noise spectrum can depend on actual distribution of fluctuators in the device

# Single fluctuator: Essentially non-Gaussian situation

$$\langle\cos(\delta\varphi_2)\rangle = \frac{e^{-\gamma\tau_2}}{2\mu}\left[(\mu+1)e^{\gamma\mu\tau_2} + (\mu-1)e^{-\gamma\mu\tau_2}\right] \equiv \psi(2v,\gamma|\tau_2)$$

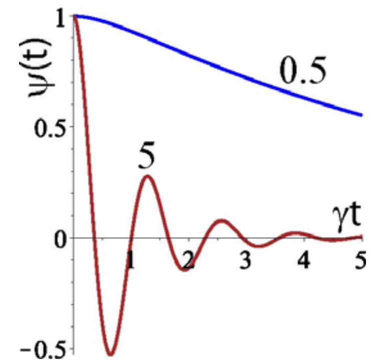
$$\boxed{\mu \equiv \sqrt{1-(v/\gamma)^2}}$$

Dimensionless parameter: $v/\gamma$

$$\langle\cos(2\delta\varphi_2)\rangle = \begin{cases} e^{-(v^2/2\gamma)\tau_2}, & v \ll \gamma \\ e^{-\gamma\tau_2}\cos(v\tau_2), & v \gg \gamma \end{cases}$$

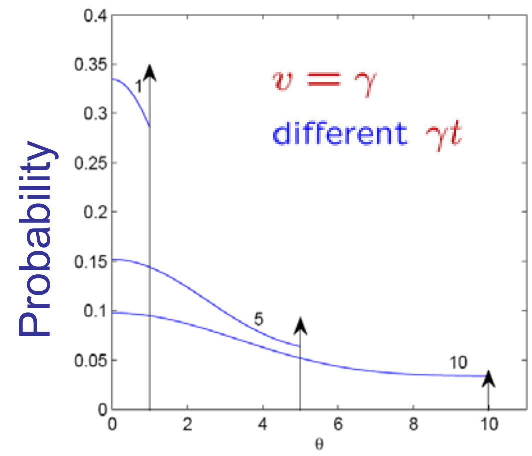
Motional narrowing

Beatings




Distribution of phase shifts: Essentially non-Gaussian

$$p(\theta,t) = \frac{1}{2}e^{-\gamma t}\left[\delta(\theta+vt)+\delta(\theta-vt)\right] + \frac{\gamma}{2v}e^{-\gamma t}\left[\Theta(\theta+vt)-\Theta(\theta-vt)\right]$$
$$\times\left[\frac{I_1\left(\gamma t\sqrt{1-\frac{\theta^2}{v^2t^2}}\right)}{\sqrt{1-\frac{\theta^2}{v^2t^2}}} + I_0\left(\gamma t\sqrt{1-\frac{\theta^2}{v^2t^2}}\right)\right] \qquad (23)$$



# Many fluctuators; 1/f noise

We assume that the random processes of different fluctuators are not correlated and their total number $N >> 1$.

Then $\langle\cos(2\delta\varphi_2)\rangle$ can be written as $e^{-\mathcal{K}(\tau_2)}$ where

$$\mathcal{K}(\tau_2) = \int dv\, d\gamma\, \mathcal{P}(v,\gamma)\, [1 - \psi(v,\gamma|\tau_2)]$$

The distribution function, $\mathcal{P}(v,\gamma)$ , depends on the model for fluctuators and their arrangement in space.

J. Bergli, Y. M. Galperin, and B. L. Altshuler, New. Journ. Phys. **11**, 025002 (2009).
E. Paladino, Y. M. Galperin, G. Falci, and B. L. Altshuler, Rev. Mod. Phys.**86**, 361 (2014).

To formulate the results let us introduce a typical coupling strength η as the interaction strength at the average distance between fluctuators with the interlevel spacing of *T*.

Let us consider the case when fluctuators are uniformly distributed in the space around qubit and the interaction between the qubit and a fluctuator decays as $1/r^3$.

The result reads:

$$\mathcal{K}(\tau_2) \approx \eta \cdot \begin{cases} \gamma_0 \tau_2^2 & \text{for} \quad \gamma_0\tau_2 \ll 1\,; & \text{quasi-Gaussian,} & \Gamma_\varphi \sim \sqrt{\eta\gamma_0} \propto T^2 \\ \tau_2 \ln \gamma_0\tau_2 & \text{for} \quad \gamma_0\tau_2 \gg 1\,. & \text{quasi-exponential} & \Gamma_\varphi \sim \eta \propto T \end{cases}$$

The result for the quasi-Gaussian regime has a clear physical meaning: the decoherence occurs only provided that at least one of the fluctuators flips.

Each flip provides a contribution $\sim\eta\tau_2$ to the phase, while $\gamma_0\tau_2$ is a probability for a flip during the observation time.

The result for $\gamma_0\tau_2 >> 1$ is less intuitive since in this region the dephasing is non-Markovian.

# Summary

- TMP turns out to be an independent tool for studying thermodynamic relations in driven systems.

- The central result is the relationship

$$\boxed{\Delta \equiv \langle e^{-\beta U}\rangle - 1 = [\tau_3 - \tau_1\langle\cos(\delta\varphi_2)\rangle]\,\Gamma_\Sigma \tanh^2\left(\frac{\beta\hbar\omega_0}{2}\right).}$$

✓ If the system is well characterized and the r.h.s. is known , then JE can be checked only by measuring the values of the internal energy.

✓ Contrary, determining the $\Delta$ versus times $\tau_i$ from experiment (say, via registering of absorber or emitted photons) one can judge on the underlying dephasing mechanism from the relation

$$\boxed{\langle\cos(\delta\varphi_2)\rangle = \frac{\tau_3}{\tau_1}\left[1 - \left(\frac{\partial \ln \Delta}{\partial \ln \tau_3}\right)^{-1}\right]}$$

- In case when the decoherence is induced by back action of the detector one can relate the dephasing to the so-called *accessible information* gained by the detector.

Averin, cond-mat/0004364; Pilgram *et al*., PRL **89**, 200401 (2002); Clerk *et al*., PRB **67**, 165324 (2003).

Then we can relate thermodynamics to the accessible information, the relationship being dependent on the detector type and measurement protocol.

- The same is true if the decoherence is induced by a strong fluctuator, which we can measure independently.

Wold *et al*., PRB **86**, 205404 (2012)

TMP allows relating thermodynamics of a quantum system to its decoherence.

If inelastic processes are independently registered, this relationship allows to characterize quantum dynamics from thermodynamic averages.

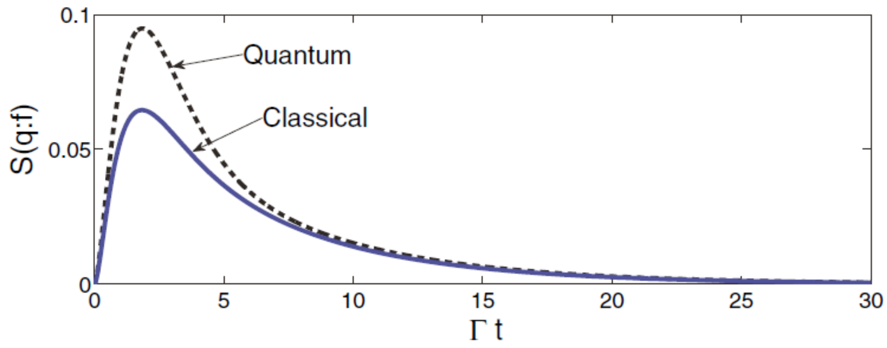
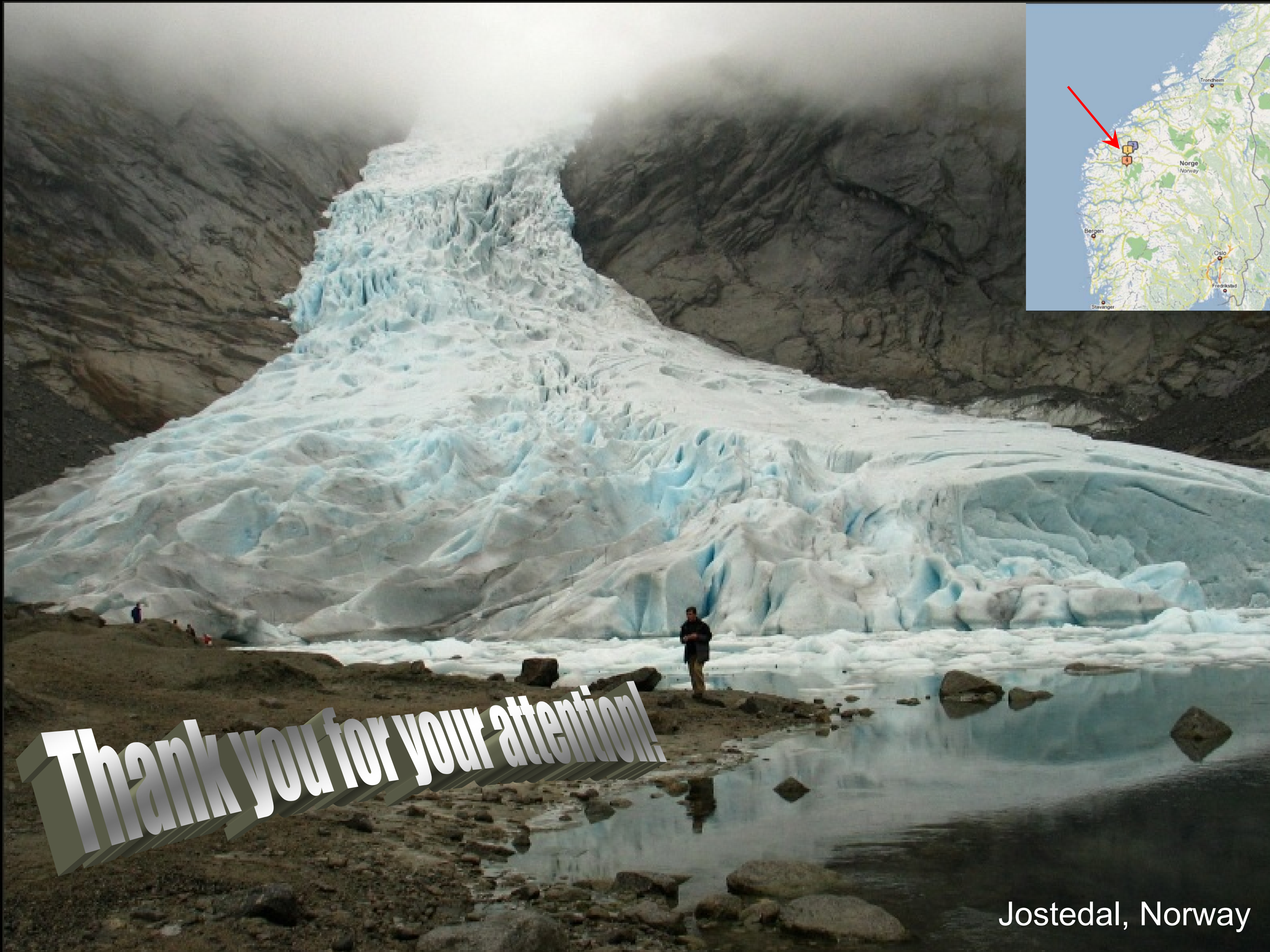



FIG. 3. (Color online) Mutual information $S(q:f)$ for the qubit coupled to the quantum fluctuator (black, dashed) and the qubit subject to the classical spin fluctuator (blue, solid). The mutual information is larger when both systems are treated as quantum objects, due to quantum entanglement between the two systems. In this simulation the parameters, in units of $E$, are $\xi = 0.1$, $\Delta = \Delta_0 = 1/\sqrt{2}$, $\gamma_1 = 1.0$, $\gamma_2 = 20$, and $E/T = 1.0$.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Thank you for your attention!
Jostedal, Norway