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A Metal-Insulator transition in the Kondo Chain

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- 1D systems – transport carried by collective modes (Charge Density Waves).
- In ideal systems CDWs are supposed to move ballistically.
- In reality even a weak disorder pins CDW (Giamarchi, Shulz, 1988).
- Exceptions? Transport in chiral edges (quantum Hall, spin Hall).
- Any other cases of symmetry protected ballistic transport?

The Model and the Results

$$H = \sum_n \left[-t\left(C^+_{n+1,\alpha} C_{n,\alpha} + H.c.\right) + J_a C^+_{n,\alpha}\left(\sigma^a_{\alpha\beta} S^a_n\right) C_{n\beta} \right]$$

We study the U(1) invariant case and have found two phases.

1. $J_x = J_y > J_z$ (easy plane anisotropy). The ground state has a broken Z_2 symmetry, quasiparticles with a one sign of helicity are gapless, with the other have a spectral gap.

The transport is ballistic, protected by the U(1) symmetry. **Perfect metal. The order parameter** $i < S^+_n S^-_{n+1} - S^-_n S^+_{n+1} >$

2. $J_x = J_y < J_z$ (easy axis anisotropy). The quasiparticles are gapped, the low lying excitations are Charge and Spin density waves. The CDW is pinned by the static potential disorder. **Insulator**.

TOWARDS THE CONTINUUM LIMIT

$$H = \sum_n \left[-t\left(C^+_{n+1,\alpha} C_{n,\alpha} + H.c.\right) + J_a C^+_{n,\alpha}\left(\sigma^a_{\alpha\beta} S^a_n\right) C_{n\beta} \right]$$

The continuum limit: $C_n = e^{-ik_F n a_0} R(x) + e^{ik_F n a_0} L(x),$

The band Lagrangian becomes $L = \Psi^+\left[(I \otimes I)\partial_\tau - i\left(I \otimes \tau^z\right)\partial_x \right]\Psi, \qquad \Psi = \begin{pmatrix} R \\ L \end{pmatrix},$

The interaction is $V = \int dx J_a \left[R^+\left(\sigma^a S_a\right) L e^{2ik_F n a_0} + H.c.\right]$

Now we have absorb the exponent into the spin configuration!

Slow variables for Spins.

$$\vec{S}_j/S = \sqrt{1-m^2}\left[\vec{e}_1(x)\cos[2k_F ja_0 + \alpha(x)] + \vec{e}_2(x)\sin[2k_F ja_0 + \alpha(x)]\right] + m_j\vec{e}_3(x), \qquad x = na_0,$$

$$\vec{e}_j^{\,2} = 1, \qquad \left(\vec{e}_i\vec{e}_j\right) = \delta_{ij}.$$

m_j is a fast variable, to be integrated out.

$$V = \int dx \left[J_\perp \sum_{a=x,y} R^+\sigma^a(e_1^a + ie_2^a)L + J_z R^+\sigma^z(e_1^z + ie_2^z)L + H.c.\right]\sqrt{1-m^2}$$

Parametrization:

$$\vec{e}_1 = \left(\sin\theta, -\cos\theta\cos\psi, -\cos\theta\sin\psi\right),$$

$$\vec{e}_2 = \left(0, \sin\psi, -\cos\psi\right),$$

$$\vec{e}_3 = \left(\cos\theta, \sin\theta\cos\psi, \sin\theta\sin\psi\right).$$

The backscattering generates gaps at least in a part of the fermionic spectrum

$$V = \int dx \Big[R^{+} e^{i\alpha} \Big(J_{\perp} \big[e^{-i\psi} \cos^2(\theta/2) \sigma^{-} - e^{i\psi} \sin^2(\theta/2) \sigma^{+} \big] + J_z \sin\theta \hat{I} \Big) L + H.c. \Big] (1 - m^2)^{1/2}.$$

$$\Delta_{\pm}^2 = S^2 \left(\sqrt{J_{\perp}^2 \cos^2\theta + J_z^2 \sin^2\theta} \pm J \right)^2 (1 - m^2).$$

Integrating over the fermions we obtain the following contribution to the ground state energy:

$$E = -\frac{1}{2\pi v_F} \sum_{a=\pm} \Delta_a^2 \ln(t/\Delta_a) = E(m=0,\theta) + Am^2/2.$$

$$A > 0.$$

Depending on whether the anisotropy is *easy axis* or *easy plane*, $E(\theta)$ has minimum at $\theta=\pi/2$ or $\theta=0$.

THE SPIN BERRY PHASE

- The standard formulation for spin path integral:

$$\int D\cos\theta D\psi \exp\left[-iS\int d\tau \cos\theta \partial_\tau \psi\right]$$

where the spin is represented as $\vec{S} = S\vec{n}, \qquad \vec{n} = (\cos\theta, \sin\theta\cos\psi, \sin\theta\sin\psi)$

This formulation is connected with a particular basis. We need to have one which is independent on the basis. We know such formulation exists. Let us try the following one

$$L = \frac{iS}{2}\left[(\vec{e}_1\partial_\tau\vec{e}_2) - (\vec{e}_2\partial_\tau\vec{e}_1)\right], \qquad \vec{e}_1 \perp \vec{e}_2 \perp \vec{n}.$$

In the basis where **n** is as above we can choose

$\vec{e}_1 = (\sin\theta, -\cos\theta\cos\psi, -\cos\theta\sin\psi), \qquad \vec{e}_2 = (0, \sin\psi, -\cos\psi),$ and reproduce the known expression.

or take any

$$\vec{e}_1{}' = \vec{e}_1\cos A + \vec{e}_2\sin A,$$
$$\vec{e}_2{}' = -\vec{e}_1\sin A + \vec{e}_2\cos A,$$

with time-independent A.

Change of the basis:

$$\vec{e}_3' = \sqrt{1-m^2}\Big[\vec{e}_1(x)\cos[2k_F ja_0 + \alpha(x)] + \vec{e}_2(x)\sin[2k_F ja_0 + \alpha(x)]\Big] + m_j\vec{e}_3(x),$$
$$\vec{e}_2' = -\vec{e}_1(x)\sin[2k_F ja_0 + \alpha(x)] + \vec{e}_2\cos[2k_F ja_0 + \alpha(x)],$$
$$\vec{e}_1' = -m\Big[\vec{e}_1(x)\cos[2k_F ja_0 + \alpha(x)] + \vec{e}_2(x)\sin[2k_F ja_0 + \alpha(x)]\Big] + \sqrt{1-m^2}\vec{e}_3.$$

Substituting this into the Berry phase and omitting the oscillatory terms we get

$$L_{WZ} = iS\int d\tau[m(\partial_\tau\alpha + \cos\theta\partial_\tau\psi)].$$

Now we can integrate over m and after that look for the saddle point in θ.

Easy Axis Anisotropy.

- The fermionic spectrum is gapped. The low energy spectrum consists of gapless collective excitations: the fluctuations of angles α,ψ:

$$L_{eff} = \frac{1}{4\pi}\left[v^{-1}\left(\partial_\tau\psi\right)^2 + v\left(\partial_x\psi\right)^2\right] +$$
$$\frac{1}{4\pi}\left[v^{-1}K^{-2}\left(\partial_\tau\alpha\right)^2 + v\left(\partial_x\alpha\right)^2\right].$$
$$K = \left[1 + \frac{(2\pi v/b_0)^2}{2\left(J_{II}^2 + J_\perp^2\right)\ln(t/JS)}\right]^{-1/2}.$$

α is a slow charge density, ψ is a fast spin density mode. The cut-off is the smallest gap $\Delta_- = cS(J_{II} - J_\perp) << \varepsilon_F$.

Disorder.

- The electron density couples to the $2k_F-$ component of the potential disorder only quadratically:

$$V \sim (J_{II}^2 - J_{\perp}^2)\int dx[g^2(x)e^{2i\alpha(x)} + H.c.]$$

The scaling dimension of this operator is 2K and since K <<1, the disorder is relevant.

Correlation functions

- The smooth part of the density is

$$\rho_{smooth} = \sqrt{2}\partial_x \alpha,$$

Hence we have

$$<< \rho(-\omega,-q)\rho(\omega,q) >>= \frac{1}{\pi}\frac{vq}{(\omega + i0)^2 - (vq)^2}.$$

Easy plane anisotropy

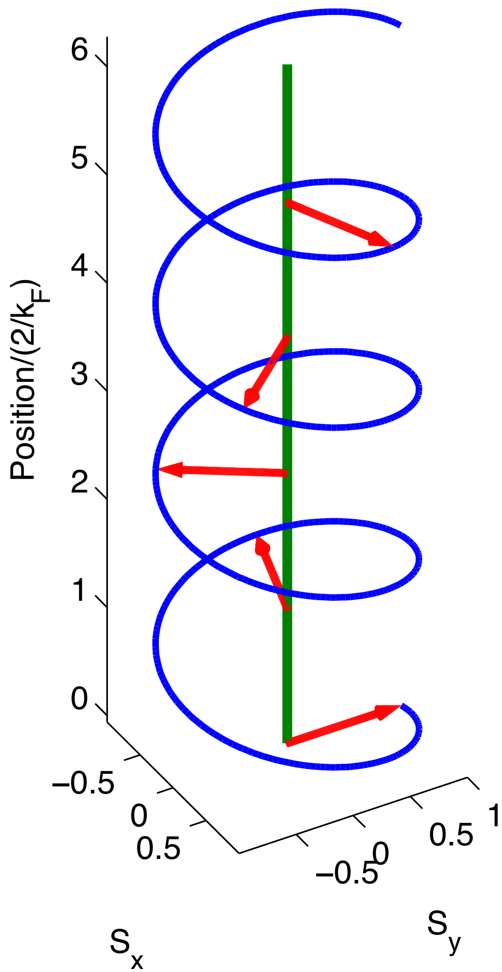
- The helicity symmetry is spontaneously broken at T=0.

- A branch of the quasiparticles with a certain helicity (product of velocity and spin) remains gapless, the other is gapped. The situation is as on the edge of *topological insulator*, but here it is *generated by the interactions*.

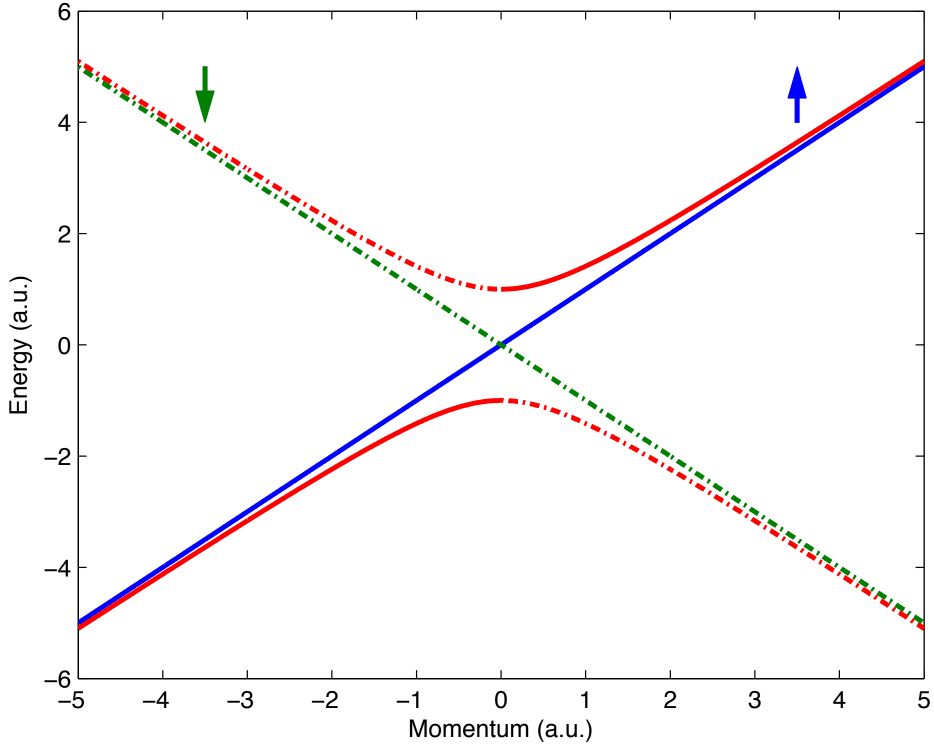
- The angle $\rho = \alpha + \psi$ $(\alpha - \psi)$ remains gapless:

$$L = \frac{1}{8\pi}\left[v^{-1}K^{-2}\left(\partial_\tau \rho\right)^2 + v\left(\partial_x \rho\right)^2\right].$$

$$K = \left[1 + \frac{(2\pi v / b_0)^2}{2\left(J_{II}^2 + J_\perp^2\right)\ln(t/JS)}\right]^{-1/2}.$$



Position/(2/k_F)
S_x
S_y



Energy (a.u.)
Momentum (a.u.)

Finite temperatures

- At finite T we have domains with different helicity separated by domain walls. Their density is

$$n \sim \exp\left(-E_{wall}/T\right), \qquad E_{wall} \sim \frac{S^2 J^2}{v} \ln(v/SJa_0).$$

- Now electrons may scatter from potential disorder (it does not require spin flip). There is a T-dependent metallic conductivity.

Conclusions

- A simple model of a Kondo chain with a potential disorder with an incommensurate electron density displays T=0 metal-insulator transition in anisotropy of the exchange interaction.
- At easy plane anisotropy the system spontaneously breaks the helical symmetry. As a result electrons with a particular helicity remain gapless and others acquire a gap.
- Since a low energy scattering process now requires a spin flip, the potential disorder becomes ineffective.