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Non-analytic Vortex Core and Nonlinear Vortex Flow in Bosonic Superfluids

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Introduction

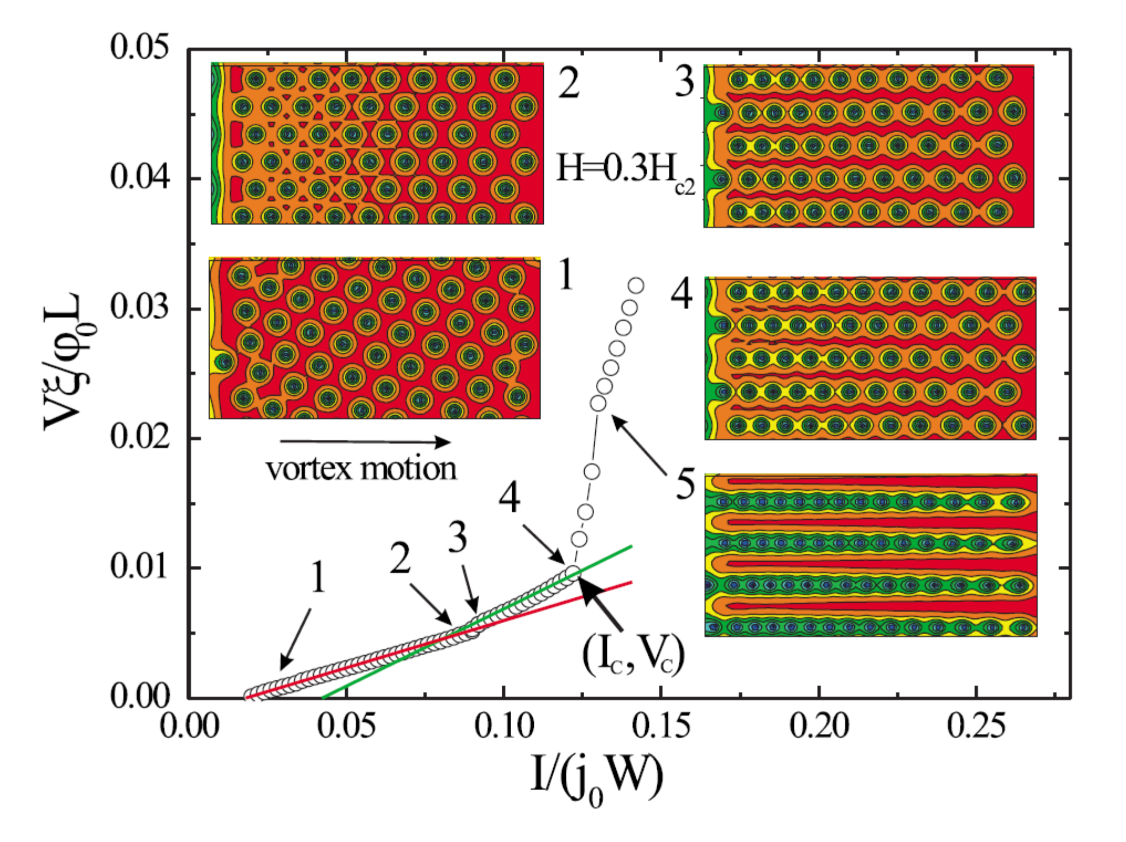
Entropy *production and* ***dissipation*** *in superfluids and superconductors are associated with* ***dynamics of vortices.***

The ***cores*** *of these vortices play a central role in these processes.*

E.g. Bardeen-Stephen friction force acting on a vortex comes, essentially, from the normal current flowing through the core.

Beyond linear response, ***Larkin*** *and* ***Ovchinnikov*** *showed that* ***heating*** *effects within vortex core (due to long relaxation time) lead to* ***nonlinear friction*** *force.*

Introduction *(cont.)*



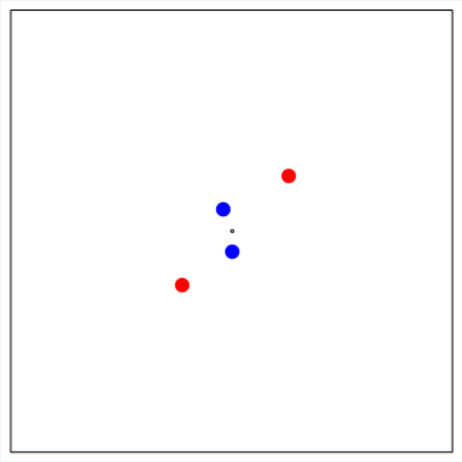
Vodolazov & Peeters PRB (2007)

Introduction (cont.)

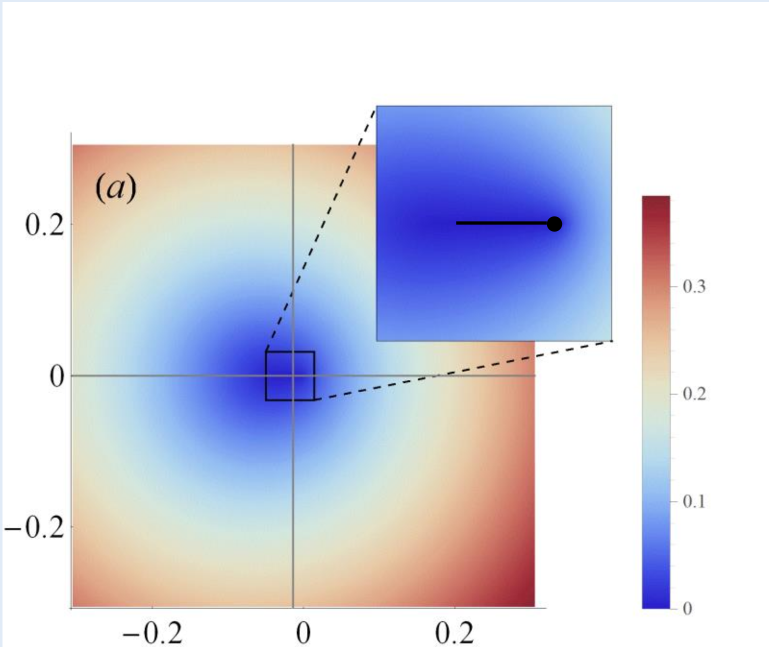
What about superfluids?

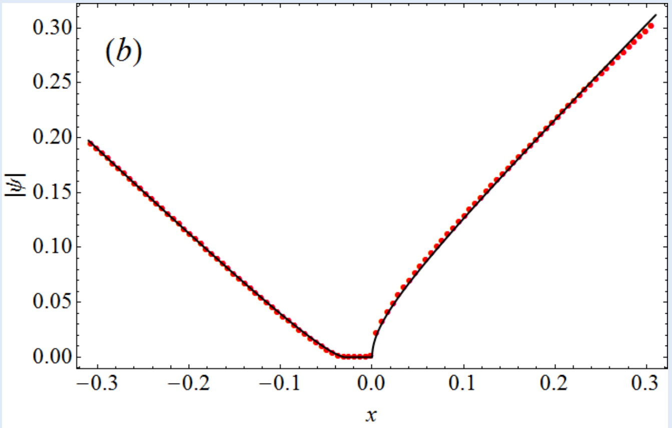
No analogous scenario because vortices in bosonic superfluids are considered to have Featureless cores.

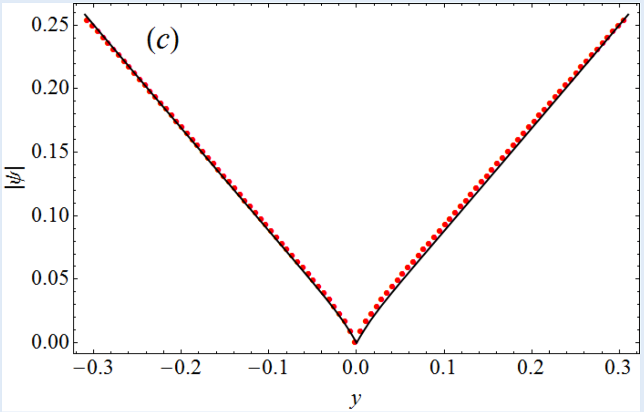
Vortex mass is essentially zero implying Kelvin's circulation theorem, i.e. a vortex moves with the flow.



Introduction (cont.)







Weak solutions of GP equation in the weak coupling limit.

Introduction *(cont.)*

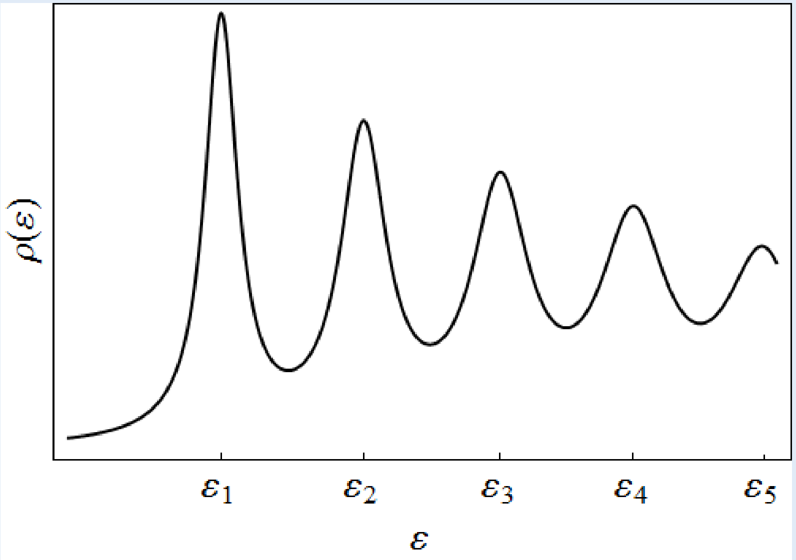
Implications:

- ***Large vortex mass.***
- ***Excited state well within the phonon spectrum.***
- ***Slow relaxation time.***

$$\varepsilon_j \simeq \hbar\Omega_c^j \left(j + \frac{1}{2}\right)$$

$$\Omega_c^j \sim \frac{1}{\ln\left(\frac{1}{\lambda j}\right)} \ll 1$$

$$\frac{1}{\tau_{in}} \sim \Omega_c^4 \sim \frac{1}{\ln^4 \, {}^{1}\!/_{\lambda}} \ll \varepsilon_1 \sim \frac{1}{\ln \, {}^{1}\!/_{\lambda}}$$



$\lambda \ll 1$ *(weak coupling limit)*

Introduction (cont.)

Excited many excited states + *Long relaxation time*

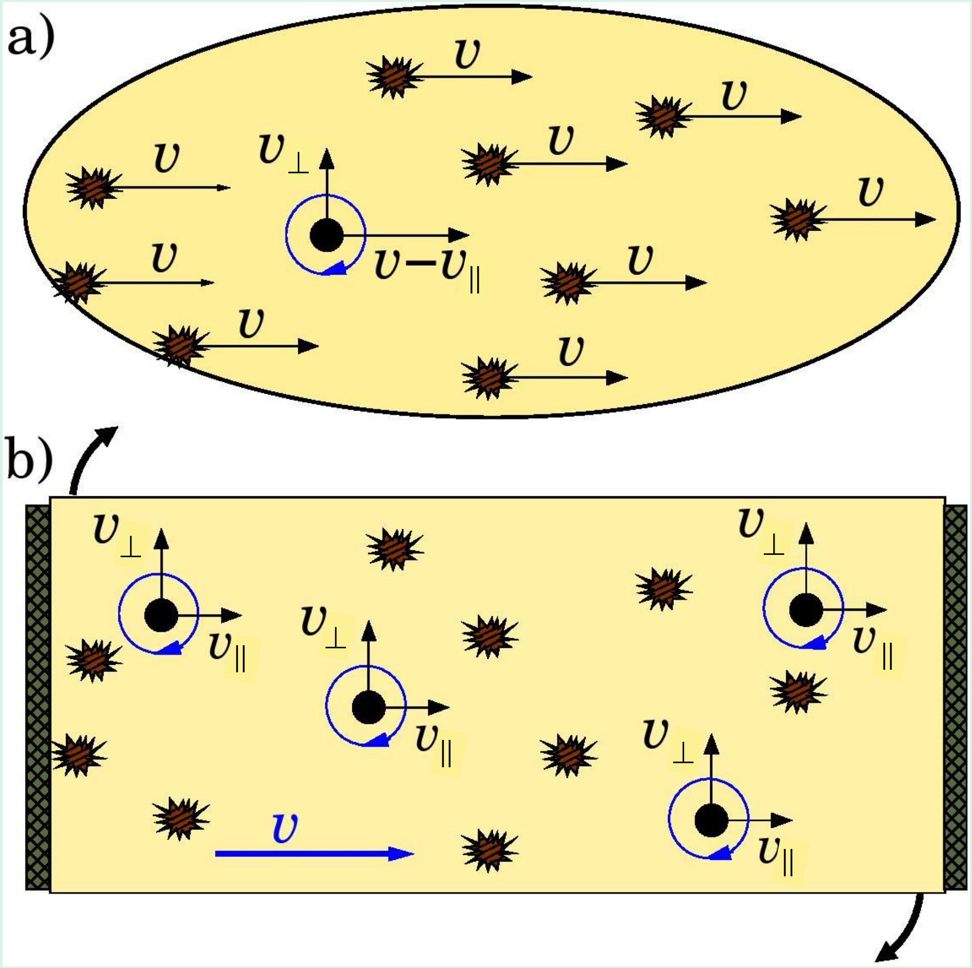
Heating

Nonlinear vortex flow behavior

Outline

- *The experimental setup*
- *Results*
- *The effective Hamiltonian*
- *Qualitative discussion*
- *Kinetic equations*
- *Solution*
- *Summary*

The experimental setup



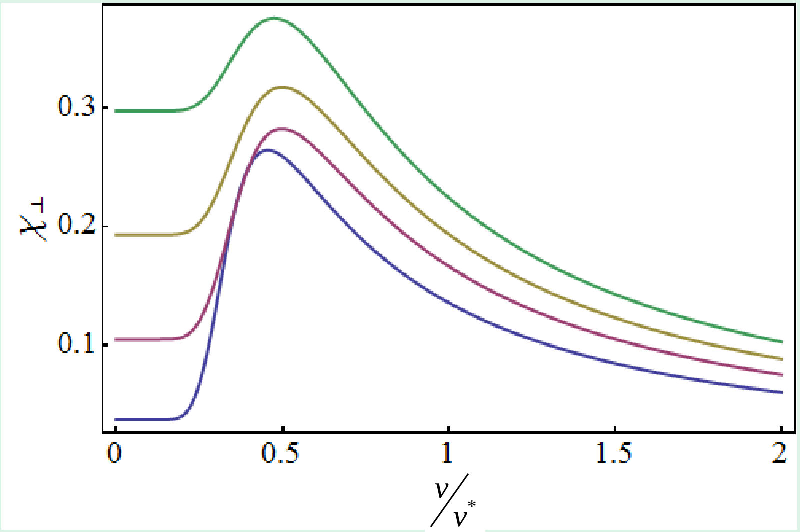
Vortices in BECs with moving disordered potential.

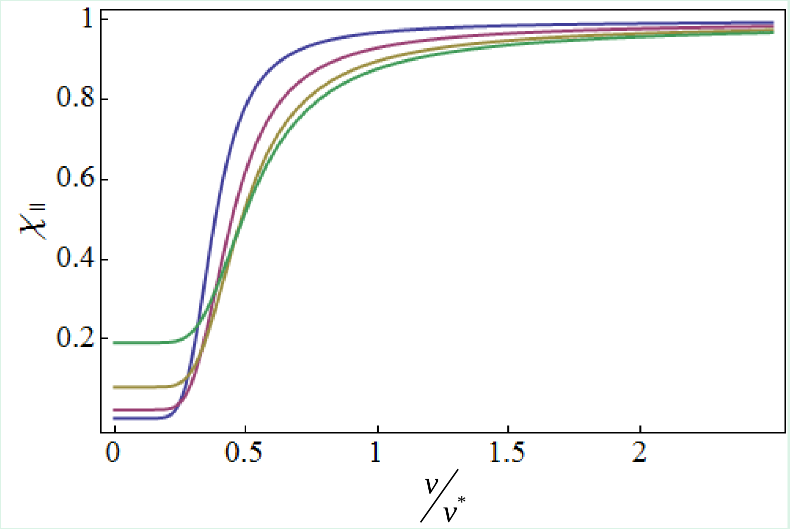
Vortices in rotating samples of thin helium films.

Results

Drag coefficients

$$v_\perp = \chi_\perp v \qquad\qquad v_{||} = \chi_{||} v$$

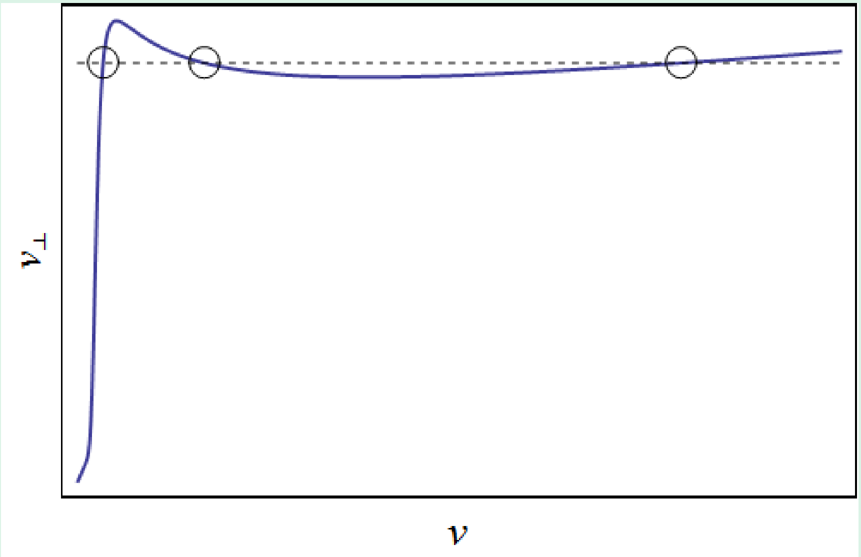




v^* is the "threshold" velocity which depends on the disorder strength and the temperature

Results

$v_\perp$ *has an N-shape behavior as function of* v *, i.e. the same vortex current can be realized for 3 possible values of* v *:*



Filament instability

The effective Hamiltonian *(Popov's notation)*

Single vortex Hamiltonian (sparse vortices)

$$H = \beta \frac{\vec{p}^{\,2}}{2\log \frac{1}{p}} + \vec{p}\hat{\varepsilon}\vec{E} + 2\pi\sigma\varphi(\vec{r}) + \pi B(\vec{r})$$

$\vec{p} = \vec{\mathcal{P}} - 2\pi\sigma\vec{A}$, $\vec{r}$ *the kinetic momentum and the position of the vortex*

$\vec{E} = -\hat{\varepsilon}\,\vec{j} = -\vec{\nabla}\varphi(\vec{r}) - \partial_t\vec{A}$ *the (rotated) superfluid current*

$B(\vec{r}) = \rho(\vec{r}) = \vec{\nabla}\times\vec{A}$ *the superfluid density in the vicinity of the vortex*

disorder enters here

Equations of motion

$$\frac{\partial \vec{p}}{\partial t} = 2\pi\sigma\left(\vec{E} + B\hat{\varepsilon}\frac{\partial \vec{r}}{\partial t}\right) - \pi\vec{\nabla}B(\vec{r})$$

$$\frac{\partial \vec{r}}{\partial t} = \omega_c(p)\,\vec{p} + \hat{\varepsilon}\frac{\vec{E}}{B}$$

$$\omega_c(p) \simeq \frac{\beta}{\log\left(1/p\right)}$$

Magnus (Lorentz) force

$$\frac{\partial \vec{p}}{\partial t} = \omega_c\hat{\varepsilon}\,\vec{p} - \pi\vec{\nabla}B(\vec{r})$$

Excitations of the vortex are induced by scattering on inhomogeneities of the superfluid density.

Qualitative discussion

(time scales associated with disorder)

$$B = n = n_0 + \delta n$$

some dimensionless function whose precise form is not important

$$\left\langle \delta n(\vec{r}_1)\,\delta n(\vec{r}_2) \right\rangle_q = \gamma n_0 G(q r_c)$$

disorder averaging & Fourier transform

disorder strength

correlation length

$$\gamma \ll 1$$

$$r_c \geq \xi, \qquad p^2 \gg 1/r_c^2 \quad \Longrightarrow$$

small angle scattering

healing length

$$\frac{1}{\tau_{tr}} = \omega_c \left(\frac{\varepsilon_d}{\varepsilon} \right)^{3/2}$$

disorder energy scale

kinetic energy of the vortex

Fermi's golden rule gives the relaxation time for the momentum direction

Qualitative discussion

(time scales associated with disorder)

If $\varepsilon \gg \varepsilon_d$ *the vortex precesses many times before changing its position. In the other limit,* $\varepsilon \ll \varepsilon_d$, *the vortex change its position before completing one circle.*

The characteristic time for scattering in all angles:

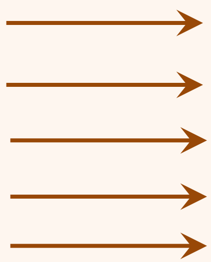
$$\frac{1}{\tau_q} = \omega_c \left(\frac{\varepsilon_q}{\varepsilon} \right)^{1/2} \qquad\qquad \varepsilon_q \gg \varepsilon_d$$

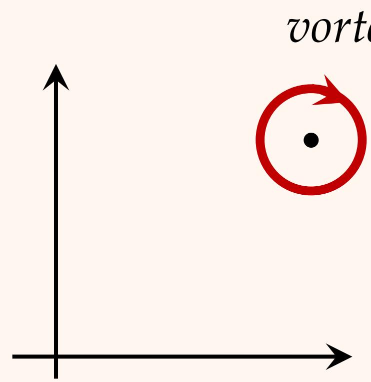
We assume $\omega_c \tau_q > 1$ *so that one can neglect interference effects (Shubnikov-de-Haas effect) .*

Qualitative discussion

(vortex dynamics in disordered landscape: Power production)

$$\vec{v} = \hat{\varepsilon}\frac{\vec{E}}{B} = \frac{\vec{j}}{n}$$





Coordinate frame moving at velocity $\vec{v}$

If there is no disorder, $\tau_{tr} \to \infty$, *the vortex is essentially moving with the flow , namely it is at rest in the moving frame of reference.*

Finite disorder generates scattering by small angles of order $\omega_c \tau_{tr} \ll 1$ *as a result the vortex acquires component along* $\vec{E}$ *direction:*

$$v_{\perp} \simeq \sigma \omega_c \tau_{tr} \frac{\vec{E}}{B}$$

Qualitative discussion

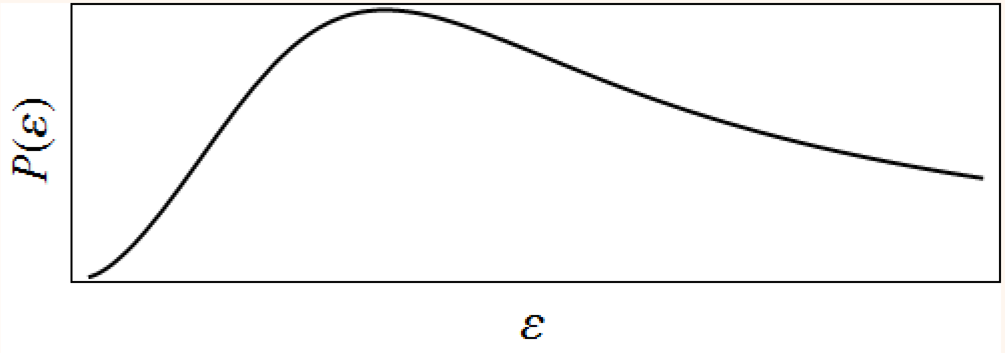
(vortex dynamics in disordered landscape: Power production)

Power production when $\omega_c \tau_{tr} \ll 1$

$$P \sim \sigma v_\perp E \simeq \omega_c \tau_{tr} \frac{E^2}{B} \simeq \frac{E^2}{B}\left(\frac{\varepsilon}{\varepsilon_d}\right)^{3/2}$$

On the other hand if $\omega_c \tau_{tr} \gg 1$ *(i.e.* $\varepsilon \gg \varepsilon_d$ *) the circular motion averages out the dissipative current*

$$P \sim \frac{1}{\omega_c \tau_{tr}} \frac{E^2}{B} \simeq \frac{E^2}{B}\left(\frac{\varepsilon_d}{\varepsilon}\right)^{3/2}$$

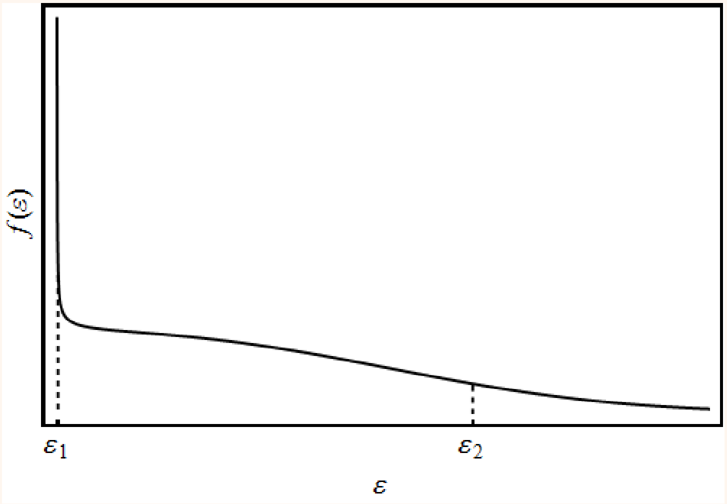


Qualitative discussion

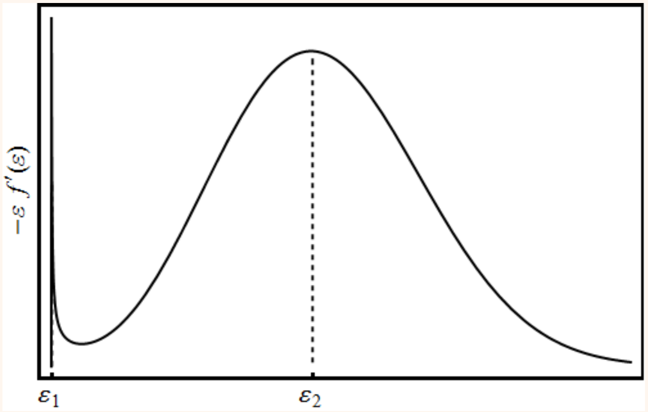
Phonon emissions remove energy from the vortex core and the energy accumulated with reference to starting energy ε *can be estimated to be:*

$$\Delta(\varepsilon) \sim \tau_{in} \frac{E^2}{B} \frac{\left(\frac{\varepsilon}{\varepsilon_d}\right)^{3/2}}{1+\left(\frac{\varepsilon}{\varepsilon_d}\right)^3}$$

For large enough E *, there is a region where* $\Delta(\varepsilon) \geq \varepsilon$ *. The vortex distribution function in this region is almost constant:*



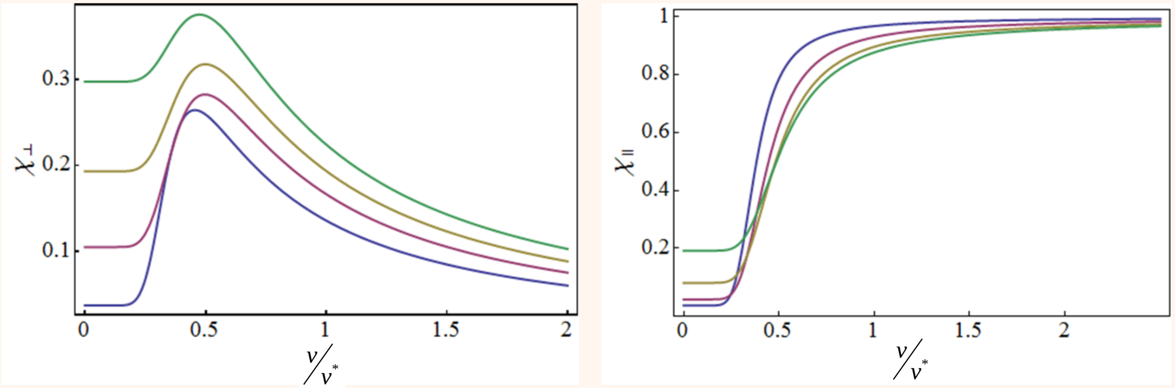
Qualitative discussion



Nonequilibrium currents are determined by $-\varepsilon \dfrac{\partial f}{\partial \varepsilon}$:

Thus currents are determined by regions of small and large energies where dissipative currents are suppressed. This explains the nonlinear behavior as function of E/B *and the drop of the dissipative current at large* E/B.

$$v_\perp = \chi_\perp v \qquad v_{||} = \chi_{||} v$$



Kinetic Equations:

Suppressing spatial dependence of the distribution function:

$$\frac{\partial f}{\partial t} + \sigma\omega_c(\varepsilon)\frac{\partial f}{\partial \phi} = St_{el}[f] + St_{in}[f]$$

where small angle scattering by disorder is described by:

$$St_{el}[f] = \left[\frac{\partial}{\partial \phi} - \frac{\partial}{\partial \varepsilon}\frac{\vec{E}\cdot\vec{p}}{B}\right]\frac{1}{\tau_{tr}}\left[\frac{\partial}{\partial \phi} - \frac{\partial}{\partial \varepsilon}\frac{\vec{E}\cdot\vec{p}}{B}\right] f$$

And neglecting effects of the field $\vec{E}$ and the disorder on the inelastic collision:

$$St_{in}[f] = \frac{\partial}{\partial \varepsilon}\left[\frac{\varepsilon}{\tau_{in}(\varepsilon)}\left(1 + T\frac{\partial}{\partial \varepsilon}\right)\right] f$$

Phonon temperature

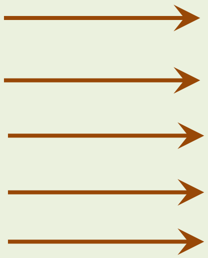
$$\frac{1}{\tau_{in}(\varepsilon)} \sim \omega_c^4$$

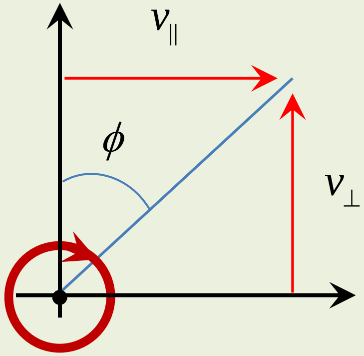
Vortex velocities:

$$v_\perp = \int d\varepsilon d\phi f \frac{\partial \varepsilon}{\partial p} \cos\phi$$

$$v_\parallel = \frac{E}{B} + \int d\varepsilon d\phi f \frac{\partial \varepsilon}{\partial p} \sin\phi$$

$$\frac{\vec{j}}{n} = \hat{\varepsilon} \frac{\vec{E}}{B}$$





vortex

Solution:

$$f(\varepsilon,\phi) = f_0(\varepsilon) + f_1(\varepsilon,\phi) \qquad \textit{with} \qquad \int d\phi f_1(\varepsilon,\phi) = 0$$

$f_1(\varepsilon,\phi)$ *is found by perturbation theory in* E.

In the same approximation inelastic collision effects on can be neglected and one obtains

$$f_1(\varepsilon,\phi) = \frac{Ep(\varepsilon)}{B}\frac{\sigma\omega_c(\varepsilon)\tau_{tr}(\varepsilon)\cos\phi - \sin\phi}{1+\left[\omega_c(\varepsilon)\tau_{tr}(\varepsilon)\right]^2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)$$

Substituting back to the kinetic equation gives $\dfrac{\partial f_0}{\partial \varepsilon} = -\dfrac{f_0}{T_{eff}(\varepsilon)}$

with the effective temperature: $T_{eff}(\varepsilon) = T + \dfrac{2\pi\tau_{in}E^2}{B}\dfrac{(\varepsilon_d\varepsilon)^{\frac{3}{2}}}{\varepsilon_d^3+\varepsilon^3}$

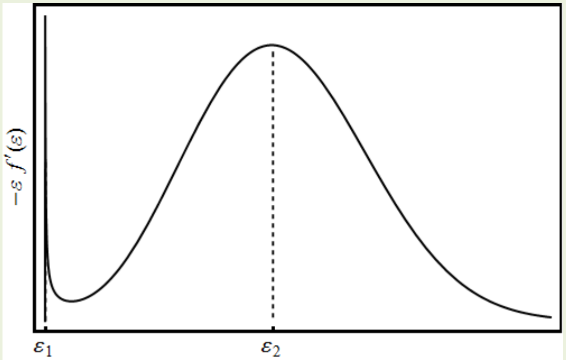
$\Delta(\varepsilon)$

Solution:

Substituting the above result to the formulae for the velocities:

$$v_\perp = \frac{E}{B}\int d\varepsilon \frac{\pm(\varepsilon_d \varepsilon)^{3/2}}{\varepsilon_d^3+\varepsilon^3}\left(-\varepsilon\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$v_\parallel = \frac{E}{B}\int d\varepsilon \frac{\varepsilon^3}{\varepsilon_d^3+\varepsilon^3}\left(-\varepsilon\frac{\partial f_0}{\partial \varepsilon}\right)$$



Matching the large and the small E asymptotics of the integrals one obtains

$$v_\perp = \chi_\perp v \qquad v_\parallel = \chi_\parallel v$$

$$\chi_\perp = \frac{0.58\left(\frac{2T}{\varepsilon_d}\right)^{\frac{5}{2}} + 0.583\left(\frac{v}{v^*}\sqrt[4]{\frac{\varepsilon_d}{T}}\right)^{-\frac{2}{5}}\exp\left[-\left(\frac{v^*}{v}\right)^{\frac{4}{3}}\right]}{\frac{T}{\varepsilon_d}+3.53\left(\frac{v}{v^*}\sqrt[4]{\frac{\varepsilon_d}{T}}\right)^{-\frac{2}{5}}\exp\left[-\left(\frac{v^*}{v}\right)^{\frac{4}{3}}\right]} \qquad \chi_\parallel = \frac{\frac{3}{2}\left(\frac{2T}{\varepsilon_d}\right)^{4} + 3.53\left(\frac{v}{v^*}\sqrt[4]{\frac{\varepsilon_d}{T}}\right)^{\frac{4}{5}}\exp\left[-\left(\frac{v^*}{v}\right)^{\frac{4}{3}}\right]}{\frac{T}{\varepsilon_d}+3.53\left(\frac{v}{v^*}\sqrt[4]{\frac{\varepsilon_d}{T}}\right)^{-\frac{2}{5}}\exp\left[-\left(\frac{v^*}{v}\right)^{\frac{4}{3}}\right]}$$

Solution:

The "threshold" velocity is

boson mass

$$v^* = \left[\frac{2.25}{n_0\xi^2}\frac{m}{M_V}\frac{\hbar\omega_c}{T}\right]^{\frac{1}{4}}\left[\frac{m\xi^2\varepsilon_d}{\hbar^2}\right]^{\frac{3}{4}}\left[\frac{1}{\omega_c\tau_{in}}\right]^{\frac{1}{2}}c$$

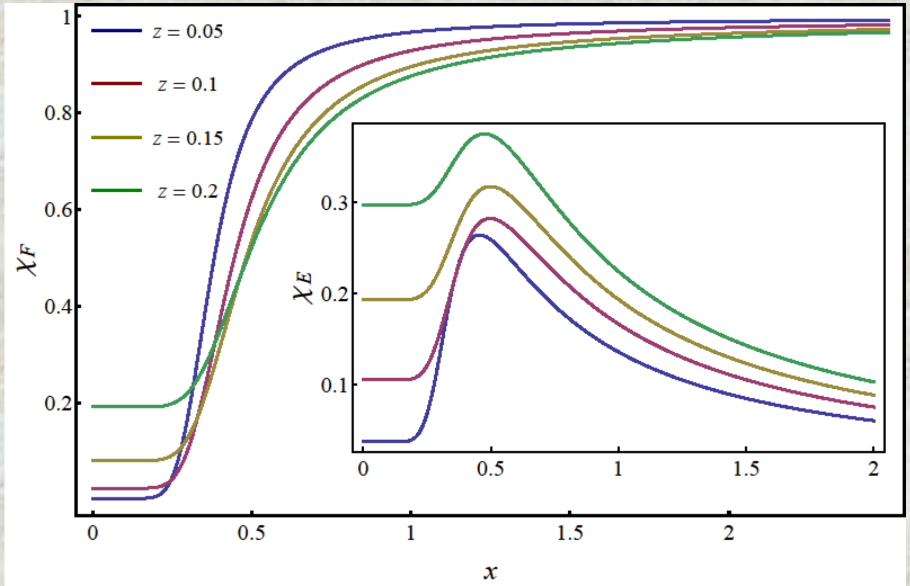
superfluid density

healing length *vortex mass* *temperature* *speed of sound*

Each factor is smaller than one, thus nonlinearity occurs at superfluid velocities much smaller than critical velocity

Summary

- *Low energy excitations associated with nonanalytic core reconstruction lead to nonlinear transport phenomena similar to those in superconductors.*
- *Confirmation of the peak effect in the dissipation will provide evidence for the existence of core reconstruction .*



$$z = \frac{T}{\varepsilon_d}$$