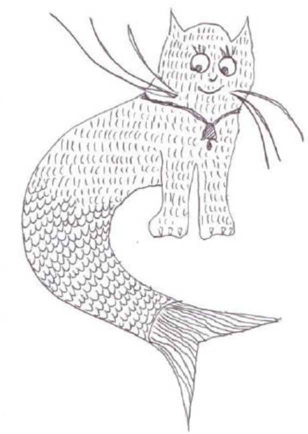
Light-Matter Correlations in Polariton Condensates



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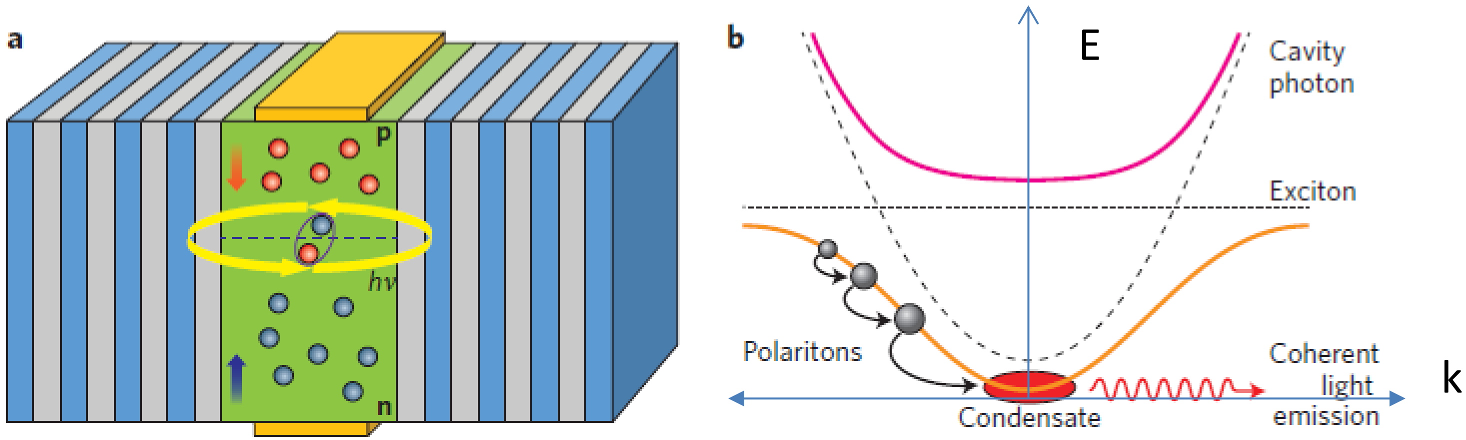
- Motivation: experimental work on polariton lasing and BEC of polaritons
- Exciton-polaritons as superposition quantum states of light and matter
- Exciton-Photon (XC) correlators
- Time evolution of the correlators
- Stochastic exciton-photon conversion: interpretation
- Proposed experiments

Exciton-polariton laser: the concept

A condensate of exciton-polaritons emits light spontaneously



No need of the inversion of population!

PHYSICAL REVIEW A VOLUME 53, NUMBER 6 JUNE 1996

Nonequilibrium condensates and lasers without inversion: Exciton-polariton lasers

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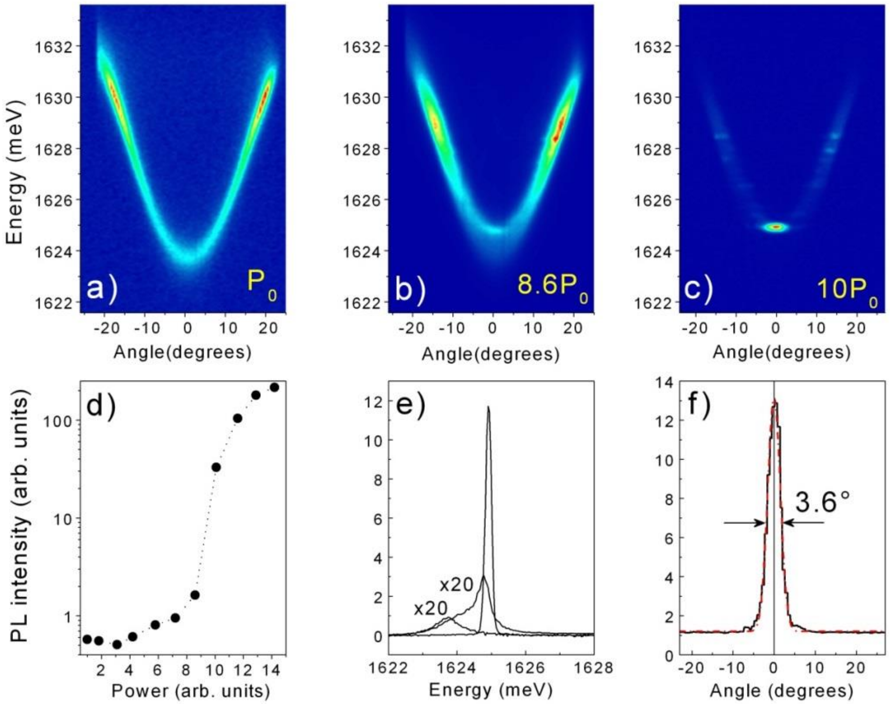
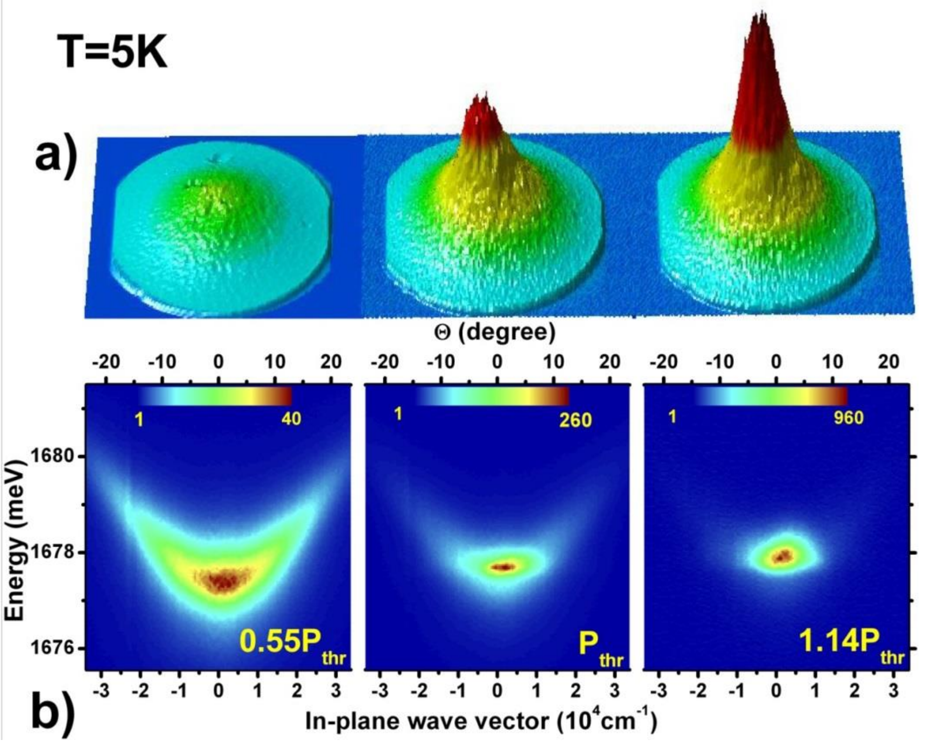
(Received 19 December 1995)

Polariton lasing in CdTe cavities





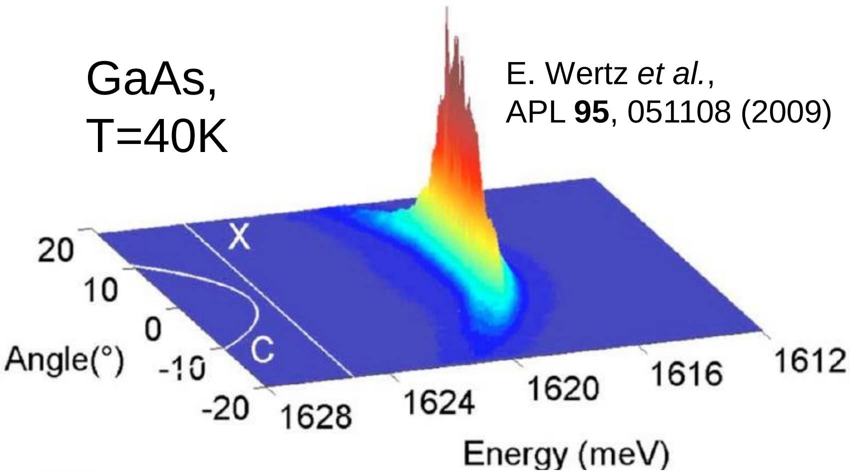
M. Richard, …, AK, Experimental evidence for nonequilibrium Bose condensation of exciton polaritons, Phys. Rev. B **72**, 201301 (2005).

J. Kasprzak et al., Bose-Einstein condensation of exciton polaritons, Nature, **443**, 409 (2006).

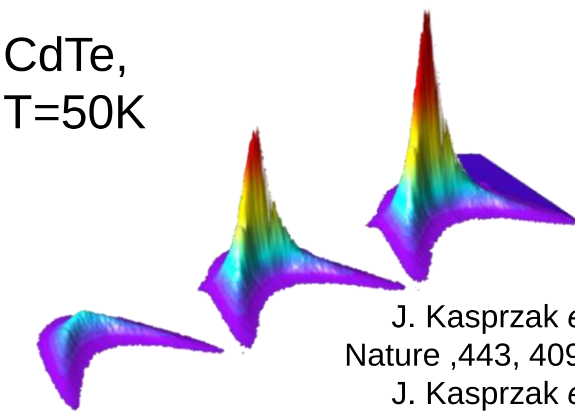
Lasers based on bosonic condensates of exciton-polaritons

GaAs,
T=40K

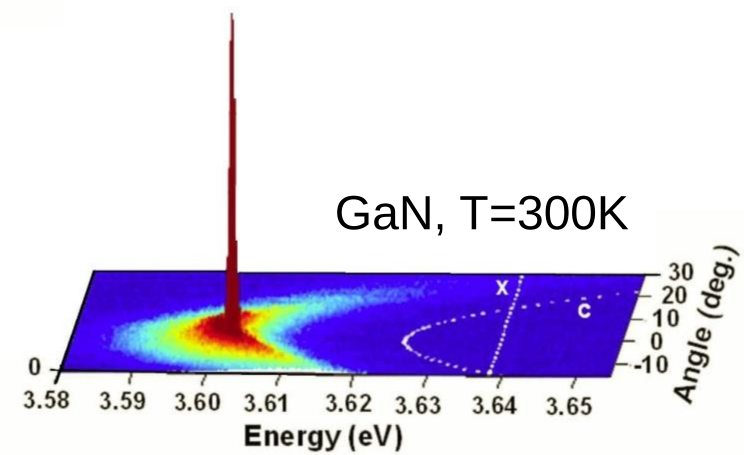
E. Wertz *et al.*,
APL **95**, 051108 (2009)



CdTe,
T=50K



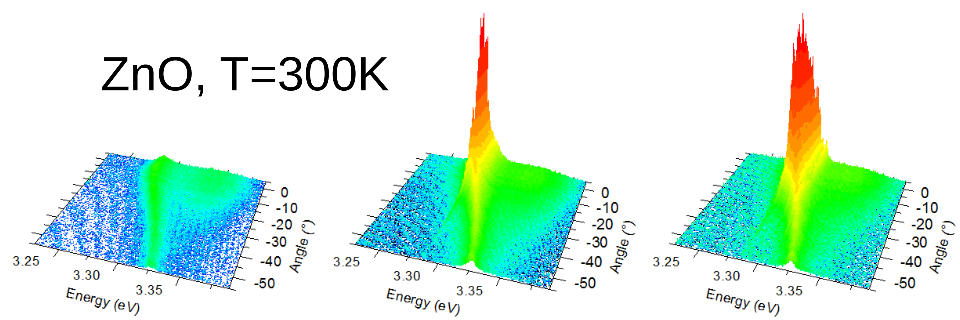
J. Kasprzak *et al.*
Nature ,443, 409 (2006)
J. Kasprzak *et al.*
PRL ,101, 146404 (2008)



GaN, T=300K

G. Christmann *et al.*,
APL **93**, 051102 (2008)

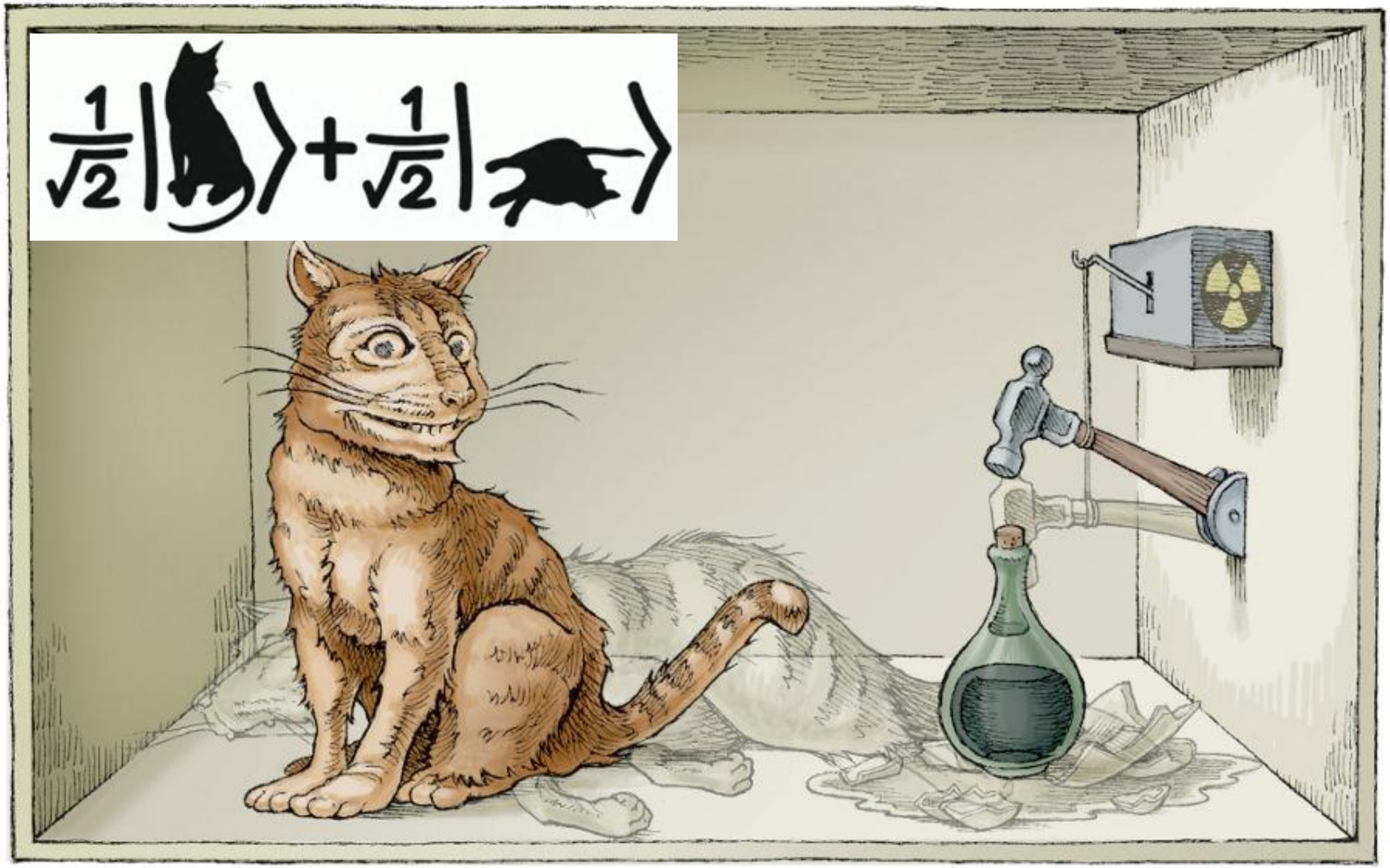
ZnO, T=300K



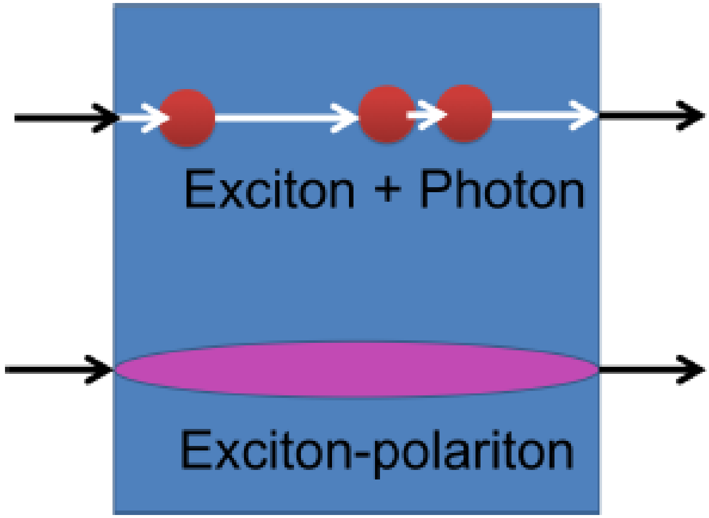
T. Guillet et al, APL 2012

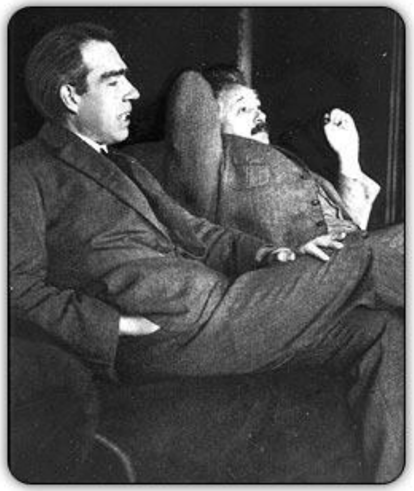
What is an exciton-polariton?

It is a superposition of a matter quasiparticle (exciton) and a quantum of light (photon)



Exciton-Polaritons: superposition light-matter quasiparticles

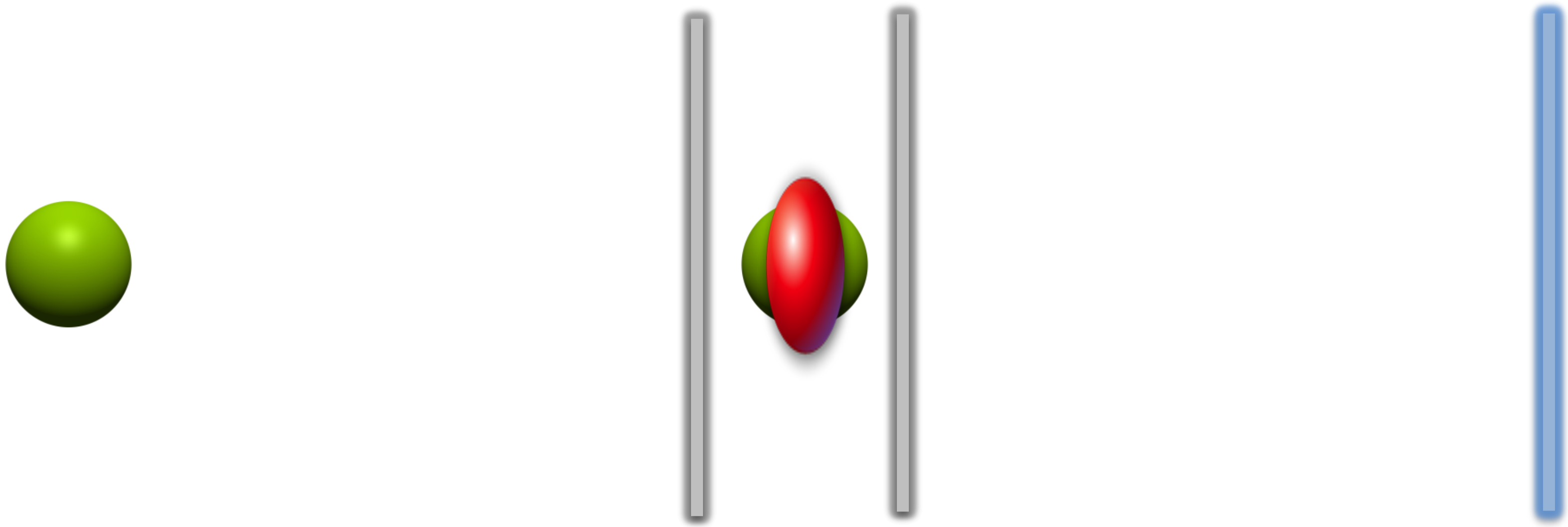




What is exciton-polariton?

1) Bohr-Heisenberg: a superposition quantum state. It is neither exciton nor photon until you do the measurement.
2) Einstein-Schroedinger: a chain of emission-absorption acts, it leaves part time as exciton, part time as photon

Do exciton-photon conversions really take place?
Difference between weak and strong coupling?



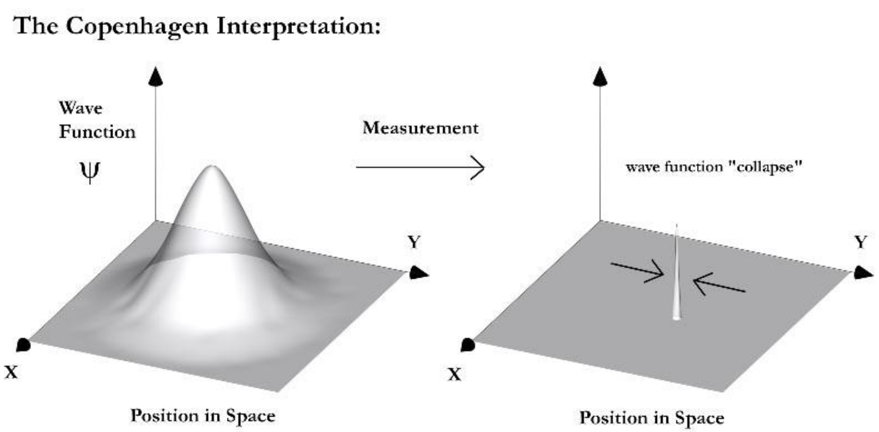
Two Interpretations of Quantum Mechanics

Copenhagen School

Werner Heisenberg

Niels Bohr

- Uncertainty principle
- Collapse of the wave-function

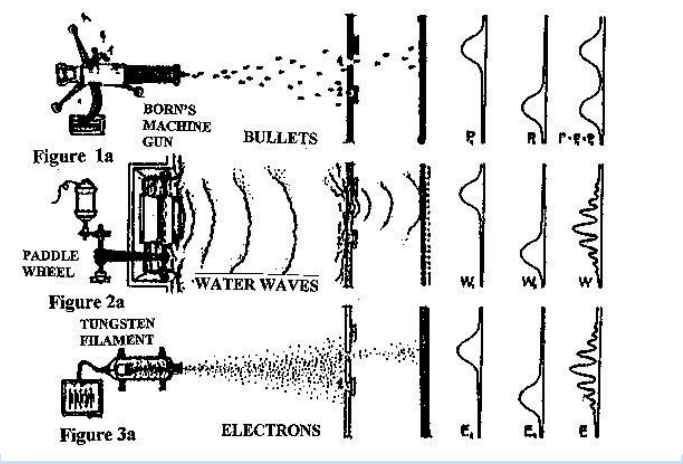


Statistical Interpretation

Erwin Schroedinger

Albert Einstein

- Matter is real, local and casual
- Wave-function describes real trajectories



Tracing Schroedinger Cats with Exciton-Polaritons

Bosonic condensates of Exciton-Polaritons:

a) Statistical interpretation:

$$N_{Pol}(t) = N_{Ex}(t) + N_{Ph}(t)$$

$|Ex\rangle \leftrightarrow |Ph\rangle$ convert to each other

b) Copenhagen-school view:

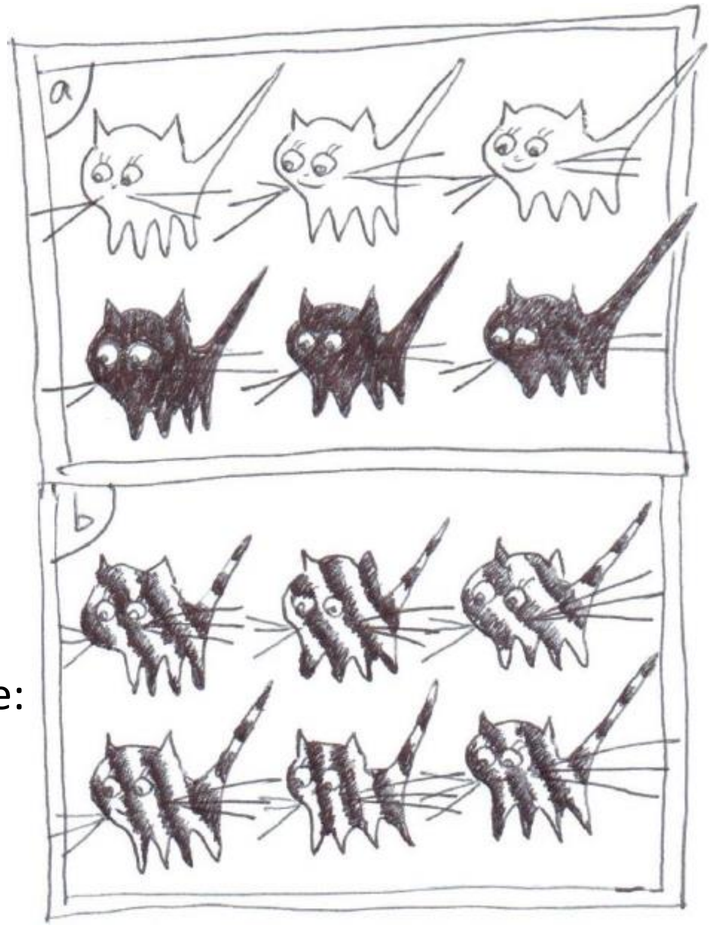
$$|Pol\rangle = \frac{1}{\sqrt{2}}(|Ex\rangle \pm |Phot\rangle)$$

A polariton condensate is a superposition. Its fractions are:

$$N_{Ex}(t) = |X|^2 N_{Pol}(t)$$

$$N_{Ph}(t) = |C|^2 N_{Pol}(t)$$

$$|X|^2 + |C|^2 = 1$$

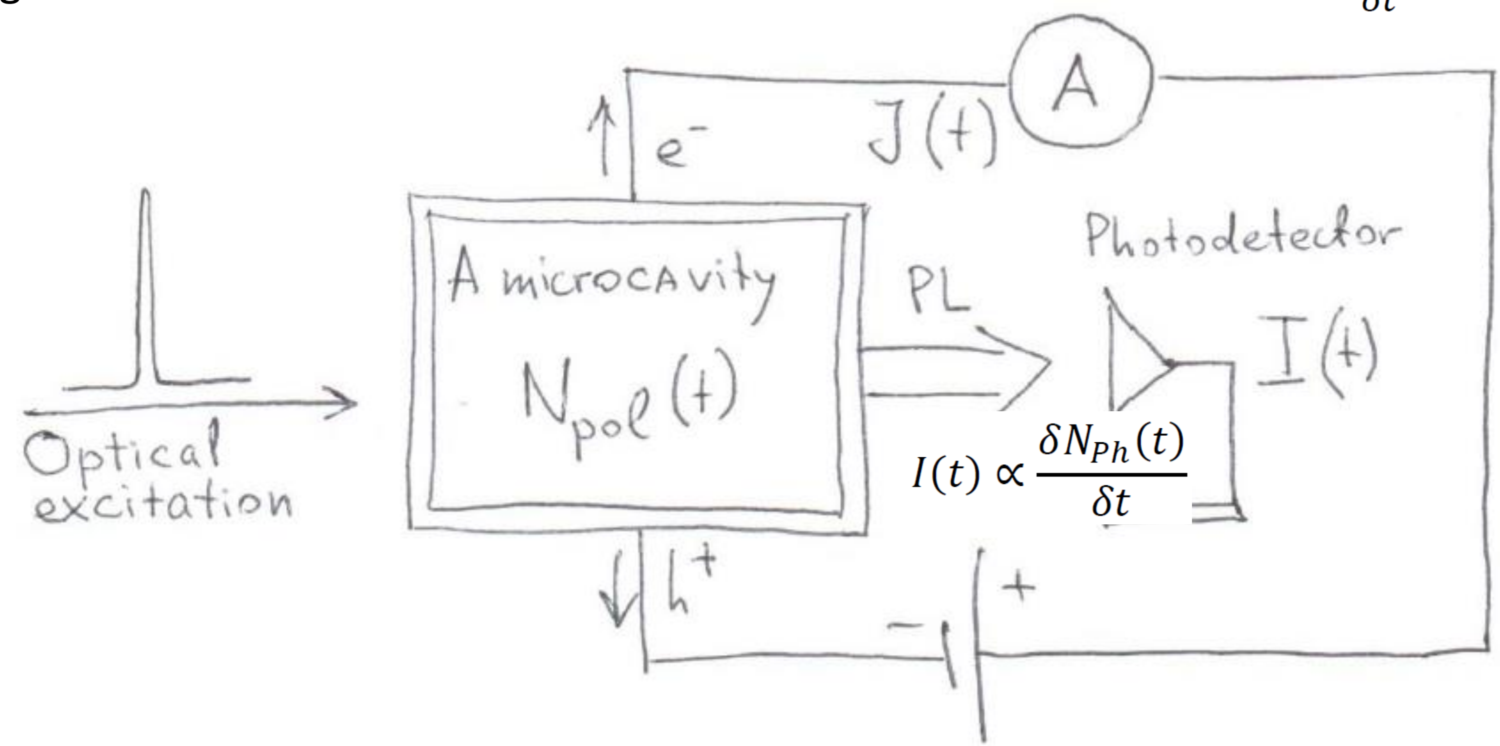


Can one experimentally distinguish between these two models??

1) Correlations between photocurrent and photoluminescence noise

Statistical: Yes!
Copenhagen: No!

$$J(t) \propto \frac{\delta N_{Ex}(t)}{\delta t}$$



$$I(t) \propto \frac{\delta N_{Ph}(t)}{\delta t}$$

Correlators of Interest:

“Small” exciton-photon correlator

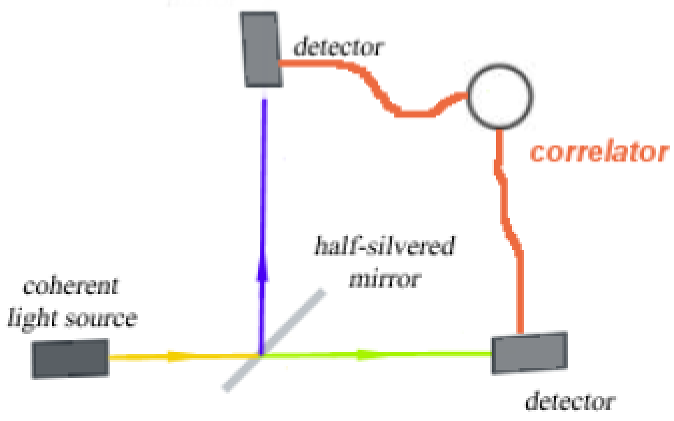
$$g_{XC}(t) = \frac{\langle n_a(t) n_b(t) \rangle}{\langle n_a(t) \rangle \langle n_b(t) \rangle}$$

Photon-photon coherence:

$$g_{2ph}(t) = \frac{\langle n_a(t)^2 - n_a(t) \rangle}{\langle n_a(t) \rangle^2}$$

Exciton-exciton coherence:

$$g_{2ex}(t) = \frac{\langle n_b(t)^2 - n_b(t) \rangle}{\langle n_b(t) \rangle^2}$$



“Big” exciton-photon correlator

$$G_{XC} = \frac{(g_{XC})^2}{g_{2ex} g_{2ph}} = \frac{\langle a^+ b^+ ab \rangle^2}{\langle a^{+2} a^2 \rangle \langle b^{+2} b^2 \rangle} = \frac{\langle I_x I_c \rangle^2}{\langle I_x^2 \rangle \langle I_c^2 \rangle} = \frac{\langle n_a(t) n_b(t) \rangle^2}{\langle n_a(t)^2 - n_a(t) \rangle \langle n_b(t)^2 - n_b(t) \rangle}$$

Exciton-photon correlators in different models

1) Copenhagen interpretation: we have polaritons (no excitons, no photons)

$$a = \frac{1}{\sqrt{2}}(c_L + c_U) \qquad b = \frac{1}{\sqrt{2}}(c_U - c_L) \qquad |c_U\rangle = |0\rangle$$ (empty upper brunch)

$g_{XC}(t)$ is formally equivalent to

$$g_{XC}(t) = \frac{\langle c_L^+ c_L^+ c_L c_L \rangle}{\langle c_L^+ c_L \rangle \langle c_L^+ c_L \rangle} = \frac{\left\langle N(t)^2 \right\rangle - \langle N(t) \rangle}{\langle N(t) \rangle^2}$$

Text book answers:

Coherent state: $g_{XC}(t) = 1$ Thermal state: $g_{XC}(t) = 2$

Number state: $g_{XC}(t) = 1 - \frac{1}{N}$

The "big" correlator:

$$G_{XC} = \frac{\langle a^+ b^+ a b \rangle^2}{\langle a^{+2} a^2 \rangle \langle b^{+2} b^2 \rangle} = \frac{\left\langle c_L^+ c_L^+ c_L c_L \right\rangle^2}{\langle c_L^{+2} c_L^2 \rangle \left\langle c_L^{+^2} c_L^2 \right\rangle} = 1$$ for any polariton statistics!

Exciton-photon correlations: XC correlator in the statistical model

2) Statistical interpretation: no polaritons, excitons convert to photons and backward

Stochastic conversions of excitons to photons and backward are characterised by a time τ_{XC}

The probability to find *na* photons and *nb* excitons is described by the Boltzmann-master equation:

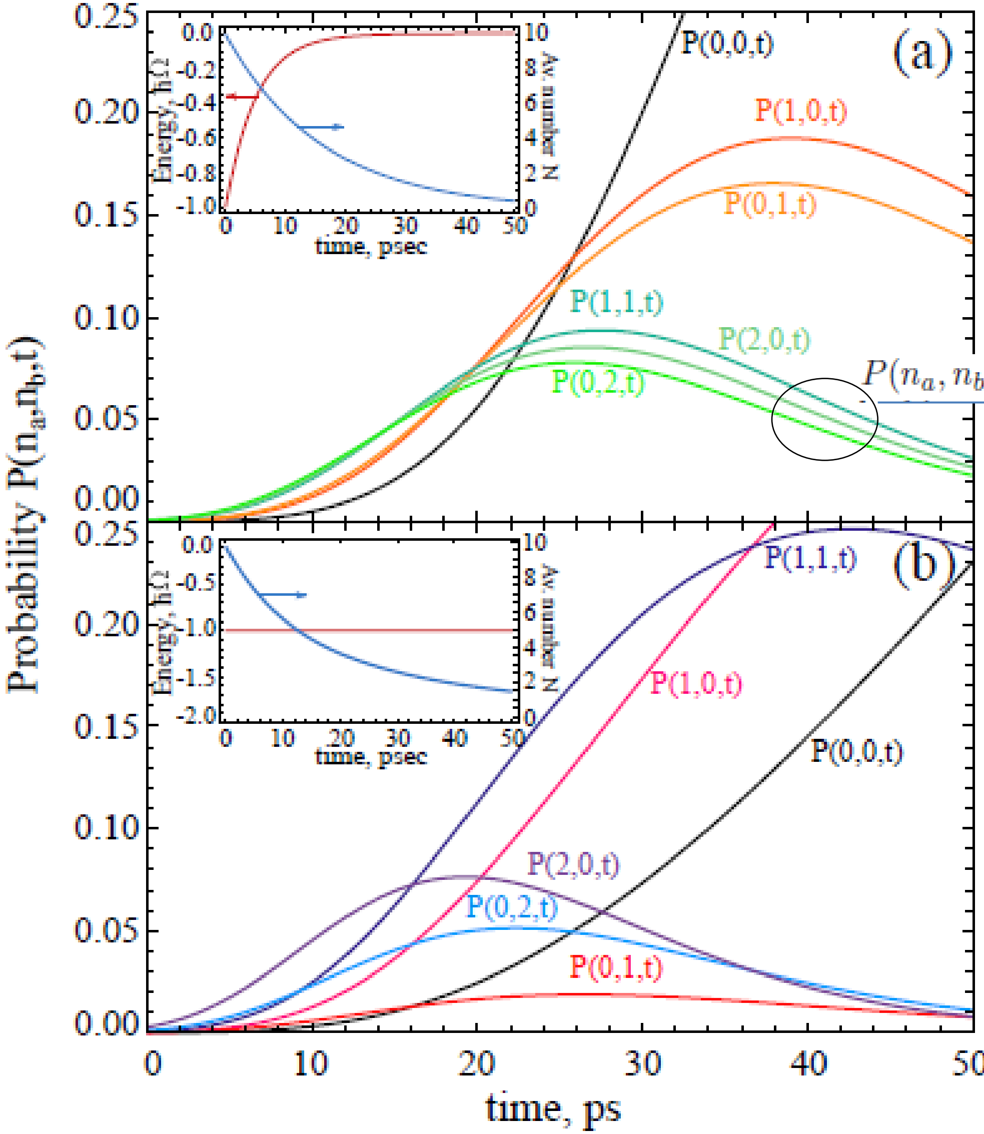
$$\begin{aligned}
\frac{P(n_a, n_b, t)}{dt} &= \frac{1}{\tau_{ph}}\left[(n_a+1)P(n_a+1, n_b, t) - n_a P(n_a, n_b, t)\right] \\
&+ \frac{1}{\tau_{ex}}\left[(n_b+1)P(n_a, n_b+1, t) - n_b P(n_a, n_b, t)\right] \\
&+ \frac{1}{\tau_{XC}}\left[n_a(n_b+1)P(n_a-1, n_b+1, t) + (n_a+1)n_b P(n_a+1, n_b-1, t)\right. \\
&- \left. n_a(n_b+1)P(n_a, n_b, t) - (n_a+1)n_b P(n_a, n_b, t)\right]
\end{aligned}$$

This can be solved assuming some initial condition

E.g. a coherent distribution $P(n_a, n_b, 0) = \frac{e^{-N}}{n_a!\, n_b!}\left(\frac{N}{2}\right)^{n_a+n_b}$

or the number state of *N* polaritons: $P(n_a, n_b, 0) = \frac{1}{2^{n_a(0)+n_b(0)}}\frac{(n_a(0)+n_b(0))!}{n_a(0)!n_b(0)!} = \frac{1}{2^N}\frac{N(0)!}{n_a(0)!(N(0)-n_a(0))!}$

Finite life-time effect:



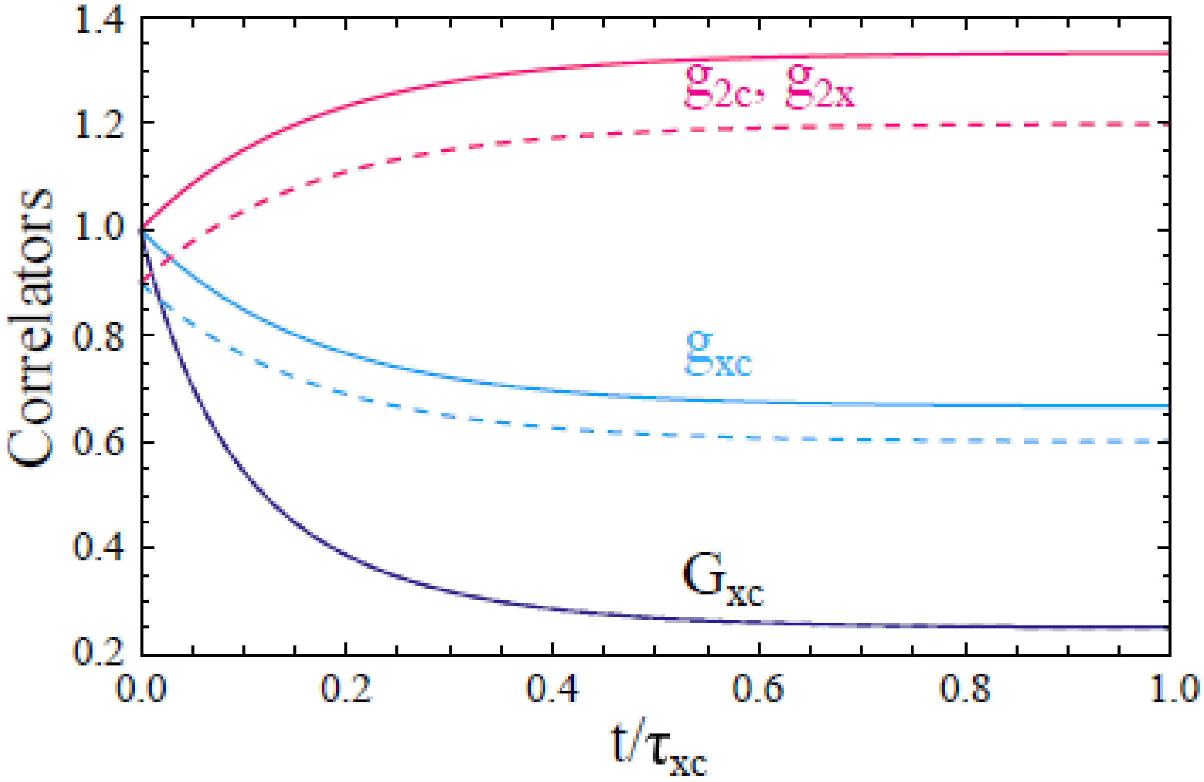
In the presence of stochastic exciton-photon conversions

$P(n_a, n_b, t) \approx P(n_b, n_a, t)$ if $t > \tau_{xc}$

In the absence of stochastic exciton-photon conversions

The initial condition: a coherent state with 10 polaritons in average

Results of the statistical model:



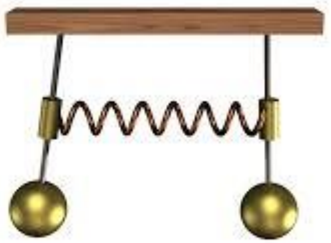
Solid lines: coherent initial state

Dashed lines: number initial state (10 polaritons)

For the “big” correlator dashed and solid coincide: it is independent on the statistics!

Interpretation: the classical limit

Consider two coupled oscillators with amplitudes A and B



We impose the energy conservation condition

$$A^2 + B^2 = N = \mathrm{const}$$

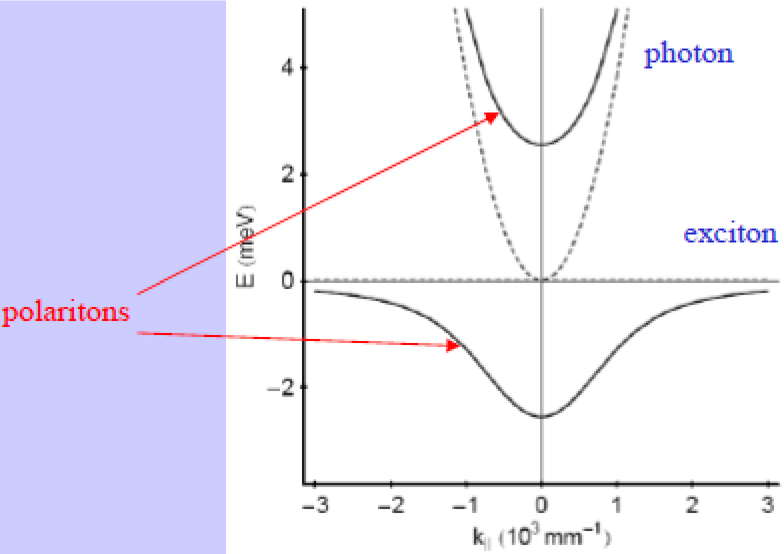
If the initial phases of oscillators are random, they are distributed with a function:

$$\mathcal{P}(A,B) = \frac{1}{\pi^2 N}\delta(A^2 + B^2 - N)$$

With this distribution $\langle A^2 \rangle = \langle B^2 \rangle = N/2$ $\quad$ $\langle A^2 B^2 \rangle = N^2/6$

Consequently $g_{XC} = \dfrac{\langle A^2 B^2 \rangle}{\langle A^2 \rangle \langle B^2 \rangle}$ = 2/3

Interpretation: the mixing of lower and upper polariton branches



At t=0 we have all particles at the lower polariton branch

$$\langle E(0)\rangle = -N\Omega$$

The evolution of the energy of the system is given by:

$$E = Tr\left[H_0\rho\right] = \Omega Tr\left[A^\dagger\rho + A\rho\right]$$

$$H_0 = \Omega\left(A^\dagger + A\right)$$

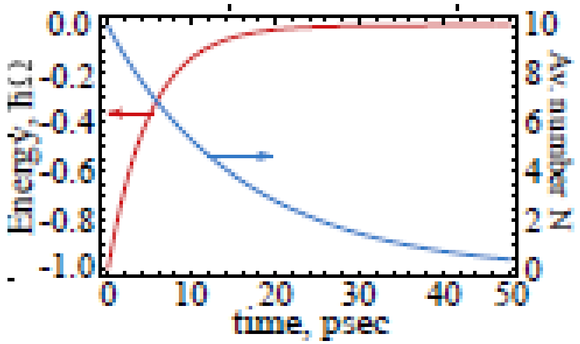
$$A^\dagger = \hat{a}^\dagger\hat{b} \qquad A = \hat{b}^\dagger\hat{a}$$

At $t \to \infty$

$$\langle E(\infty)\rangle = 0$$

The energy variance per particle is:

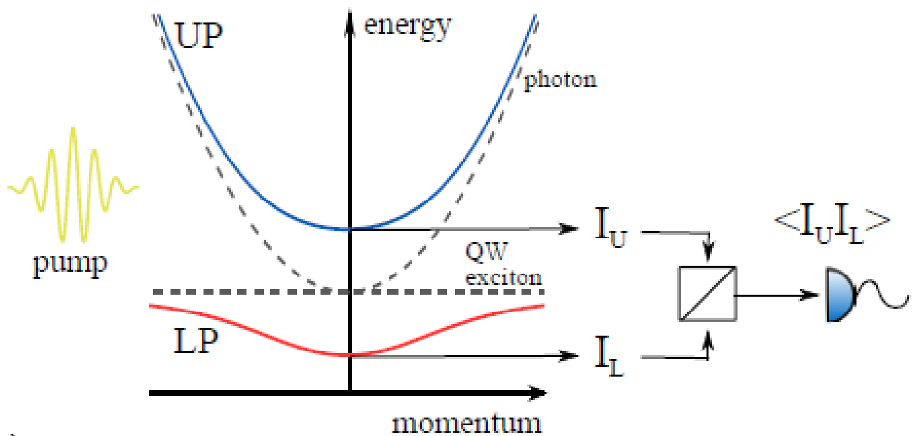
$$\delta E = \frac{1}{N}\sqrt{\langle E^2\rangle - \langle E\rangle^2} = \frac{\sqrt{\langle E^2\rangle}}{N} = \Omega\sqrt{\frac{N+2}{3N}}.$$

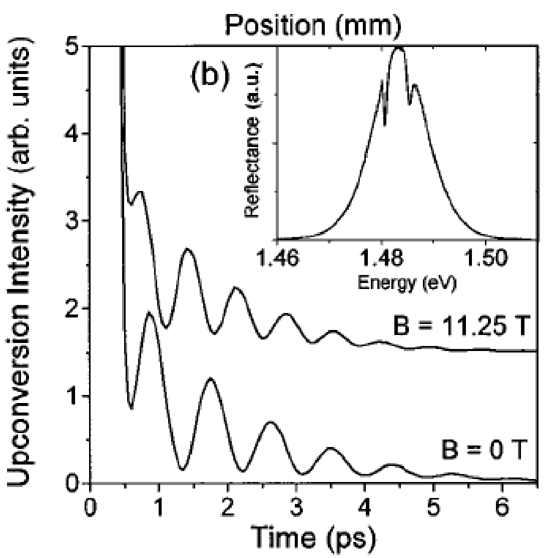


Stochastic exciton-photon conversion mixes two polariton branches with a characteristic time τ_{XC} Eventually, we achieve the weak coupling regime!

Gedankenexperiment 2 (easier to realise)

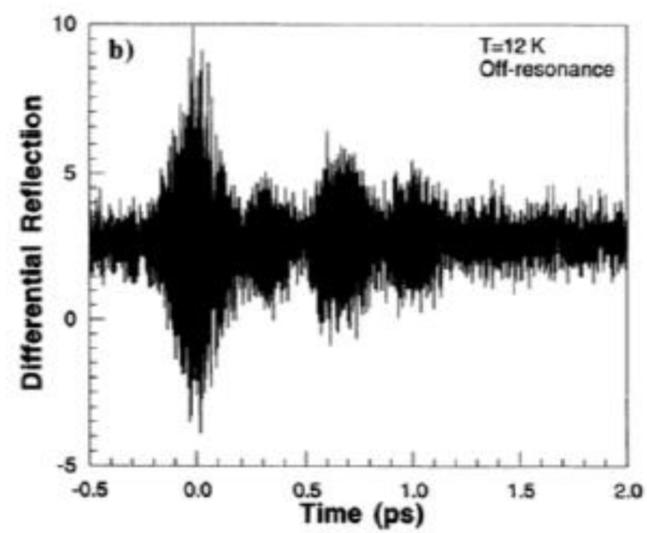
Correlations between upper and lower polariton branches in the Rabi oscillation regime





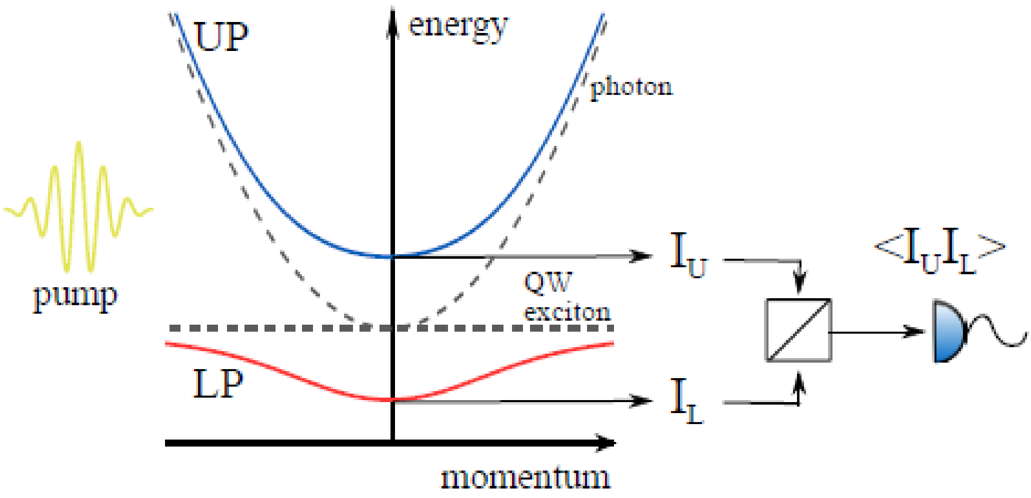
J. Berger et al, PRB 54, 1975 (1996)

Polariton Rabi Oscillations



N. Kopteva, unpublished, 2015

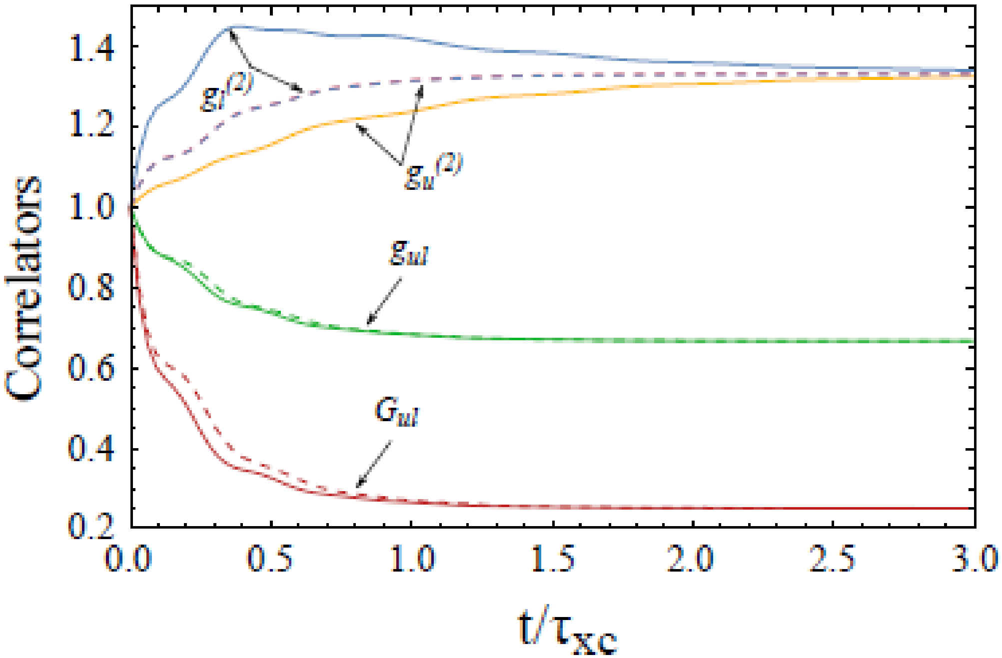
Upper-lower branch correlations: theory



$$\frac{d\hat{\rho}(t)}{dt} = \frac{\mathrm{i}}{\hbar}[\hat{\rho}, \hat{H}_0] - \sum_j \frac{g_j}{2}(\hat{A}_j^\dagger \hat{A}_j \hat{\rho} + \hat{\rho}\hat{A}_j^\dagger \hat{A}_j - 2\hat{A}_j \hat{\rho}\hat{A}_j^\dagger) \qquad (j = x, c, u)$$

$$\hat{H}_0 = \frac{\hbar}{2}\left\{\Delta(\hat{b}^\dagger \hat{b} - \hat{a}^\dagger \hat{a}) + \omega_R(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a})\right\} \qquad \hat{A}_x = \hat{a},\ \hat{A}_c = \hat{b}, \text{ and } \hat{A}_u = (\hat{a} + \hat{b})$$

Results for upper-upper, lower-lower and upper-lower correlators



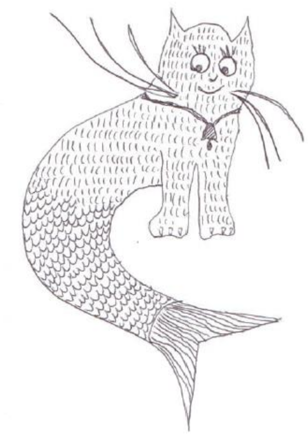
$$g_i^{(2)} = n_i^{-2} \langle \hat{c}_i^\dagger \hat{c}_i^\dagger \hat{c}_i \hat{c}_i \rangle$$

$$g_{ul} = \frac{\langle \hat{c}_u^\dagger \hat{c}_l^\dagger \hat{c}_u \hat{c}_l \rangle}{\langle \hat{c}_u^\dagger \hat{c}_u \rangle \langle \hat{c}_l^\dagger \hat{c}_l \rangle}$$

$$G_{ul} = \frac{g_{ul}^2}{g_u^{(2)} g_l^{(2)}} = \frac{\langle \hat{c}_u^\dagger \hat{c}_l^\dagger \hat{c}_u \hat{c}_l \rangle^2}{\langle \hat{c}_u^\dagger \hat{c}_u^\dagger \hat{c}_u \hat{c}_u \rangle \langle \hat{c}_l^\dagger \hat{c}_l^\dagger \hat{c}_l \hat{c}_l \rangle}$$

Upper-lower correlators strongly go below 1 due to exciton-photon conversions!

Exciton photon correlations: Conclusions



- Stochastic exciton-photon correlation processes are described by a “hidden variable” τ_{XC}
- If $\tau_{XC} = \infty$ the “Copenhagen” solutions are matched
- If $\frac{1}{\Omega} < \tau_{XC} < \tau_{ex}, \tau_{ph}$ the most interesting regime is hold,

strong deviations of the correlators from the “Copenhagen” prediction are expected

- If $\tau_{XC} < \frac{1}{\Omega}, \tau_{ex}, \tau_{ph}$ the weak coupling regime takes place

- Exciton-photon conversion mixes two polariton branches and changes the energy of the condensate.
- In the regime of Rabi oscillations, the Upper-Lower correlator is expected to go below 1 due to stochastic processes

AVK, A.S. Sheremet, I.A. Shelykh, P.G. Lagoudakis and Y.G. Rubo, *Exciton-photon correlations in bosonic condensates of exciton-polaritons,* Scientific Reports, 5:12020 (2015).