Bose-Einstein condensation and superfluidity of magnons in Yttrium Iron Garnet

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Congratulations!

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ICTP, Trieste

Brief outline

1Introduction: magnons in YIG.

2. Bose-Einstein condensation of magnons in Yttrium-Iron-Garnet (YIG). Experiment.

3. BECM in YIG. Theory.

4. Superfluidity of magnons. Theory.

5. Conclusions.

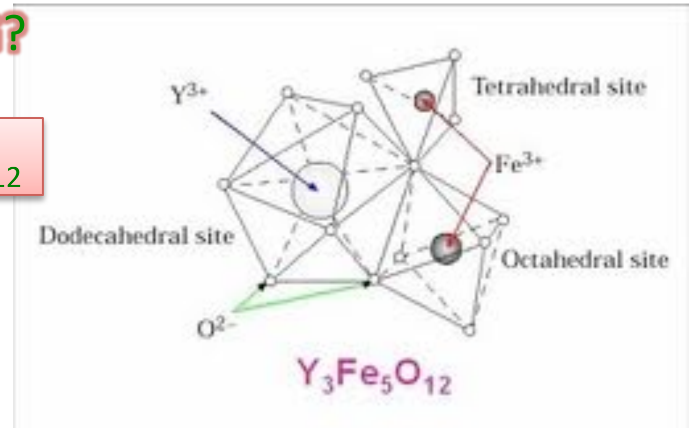
1. Magnons in YIG?

YIG crystal

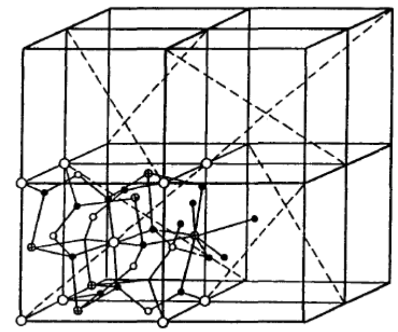
Chemical formula: $Y_3Fe_5O_{12}$

Crystal structure

a=1.2 nm



80 atoms in elementary cell



Magnetic properties: Ferrite T_c=560K

S_{cell}=14.5

Electric properties: Insulator

Magnons almost do not attenuate

20 spin-wave branches in the bulk

Low-energy spin waves in films

Spin of elementary cell rotates as a whole

Dipolar interaction is important

Hamiltonian:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + H_D - \gamma H \sum_i S_i^z ,$$

Holstein-Primakoff transformation works well: S=14.5

$$S_+ = S_x + iS_y = a^\dagger \sqrt{2S - a^\dagger a}; \quad S_- = S_x - iS_y = a\sqrt{2S - a^\dagger a}; \quad S_z = S - a^\dagger a$$

Quadratic Hamiltonian:

$$\mathcal{H}_0 = \sum_{\mathbf{k}} \left[\mathcal{A}_k a_k^\dagger a_k + \frac{1}{2}\mathcal{B}_k a_k a_{-k} + \frac{1}{2}\mathcal{B}_k^* a_k^\dagger a_{-k}^\dagger \right]$$

$$\mathcal{A}_k = \gamma H_0 + Dk^2 + \gamma 2\pi M (1 - F_k)\sin^2\theta + \gamma 2\pi M F_k$$

$$\mathcal{B}_k = \gamma 2\pi M (1 - F_k)\sin^2\theta - \gamma 2\pi M F_k$$

$D = 2JSa^2 = 0.24 \text{ eV\AA}^2$

$4\pi M = 1.76 \text{ kG}$

$F_k \equiv (1 - e^{-kd})/kd$

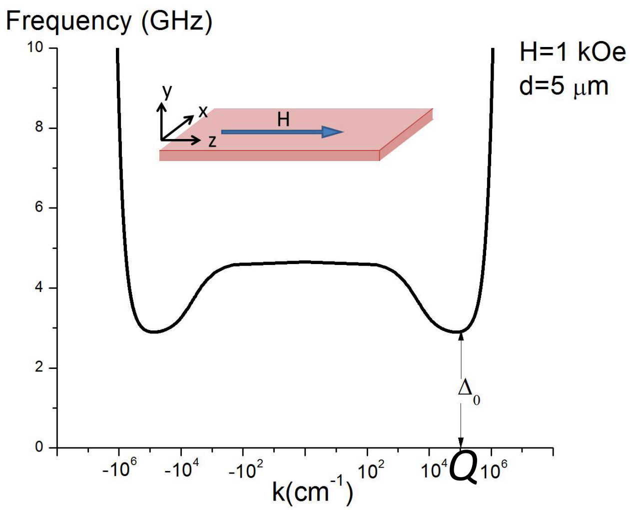
$$\hbar\omega_k = (\mathcal{A}_k^2 - |\mathcal{B}_k|^2)^{1/2}$$

$\gamma = 1.2\times10^{-5}\, eV / kOe$ - gyromagnetic ratio

θ - the angle between **k** and **H**

a - lattice constant

d - film thickness



Spectrum has two symmetric minima at k=+Q and k=-Q, Q about 10^5 cm^{-1}.

Wave vectors to minima are parallel to magnetization.

The gap in the spectrum is due to Zeeman energy

$$Q \sim \left(\frac{\pi M}{Dd}\right)^{1/3} \approx 10^5\, cm^{-1} \qquad D = Ja^3$$

Creation and annihilation operators of spin waves (Bogoliubov transformation):

$$a_k = u_k c_k + v_k c^{\dagger}_{-k}$$

$$u_k = \left(\frac{A_k + \hbar\omega_k}{2\hbar\omega_k}\right)^{1/2}$$

$$v_k = sgn(B_k)\left(\frac{A_k - \hbar\omega_k}{2\hbar\omega_k}\right)^{1/2}$$

References:

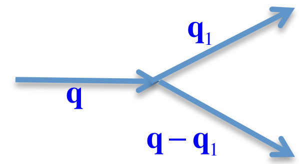
Kalinikos, B. A. Excitation of Propagating Spin Waves in Ferromagnetic Films. *IEE Proc., Part H: Microwaves, Opt. Antennas* , **127**, 4-10(1980).

Rezende, S. M. Theory of Coherence in Bose-Einstein Condensation Phenomena in a Microwave Driven Interacting Magnon Gas. *Phys. Rev. B* **79**, 174411 (2009).

Tupitsyn, I. S., Stamp, P. C. E. & Burin, A. L. Stability of Bose-Einstein Condensates of Hot Magnons in Yttrium Iron Garnet Films. *Phys. Rev. Lett.* **100**, 257202(2008).

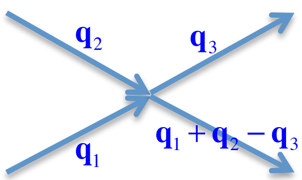
Spin wave interaction

3-d and 4-th order terms in spin-wave amplitudes



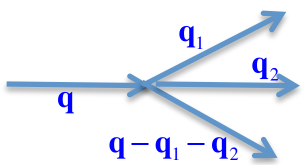
$c_{\mathbf{q}} c^{\dagger}_{\mathbf{q}_1} c^{\dagger}_{\mathbf{q}-\mathbf{q}_1}$

Decay 1->2 exists in the bulk, vanishes at $\omega(q)<2\Delta_0$



$c_{\mathbf{q}_1} c_{\mathbf{q}_2} c^{\dagger}_{\mathbf{q}_3} c^{\dagger}_{\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3}$

Scattering 2->2 conserves number of spin waves



$c_{\mathbf{q}} c^{\dagger}_{\mathbf{q}_1} c^{\dagger}_{\mathbf{q}_2} c^{\dagger}_{\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2}$

Decay 1->3 vanishes at $\omega(q)<3\Delta_0$

Nevertheless it is very important!

1. Bose-Einstein condensation of spin waves in YIG-Experiment

nature

Vol 443|28 September 2006|doi:10.1038/nature05117

LETTERS

Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

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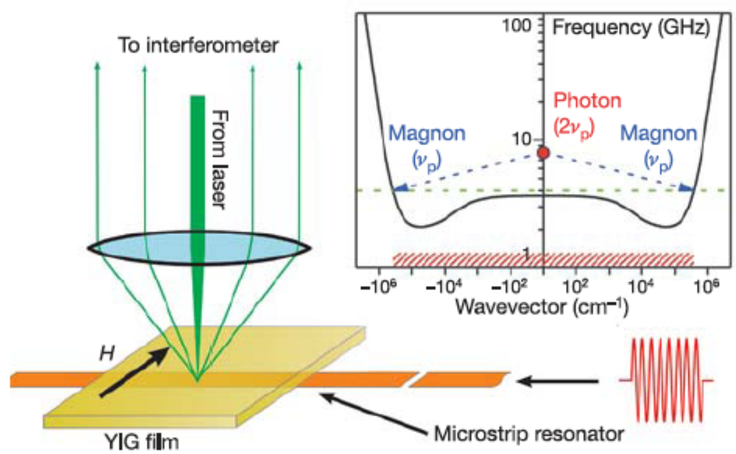


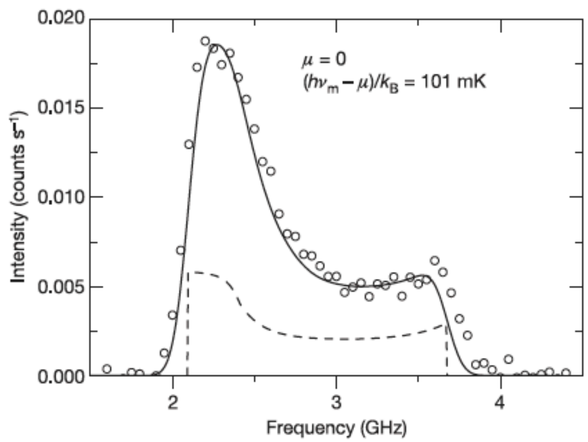


Figure 2 | BLS spectrum of thermal magnons recorded without pumping. The reduced density of states, $\tilde{D}(\nu)$, obtained from the fit of the experimental data (solid line) using equation (1) with the zero chemical potential, μ, is shown by the dashed line. ν_m is the minimum frequency of magnons, h is Planck's constant, and k_B is the Boltzmann constant.

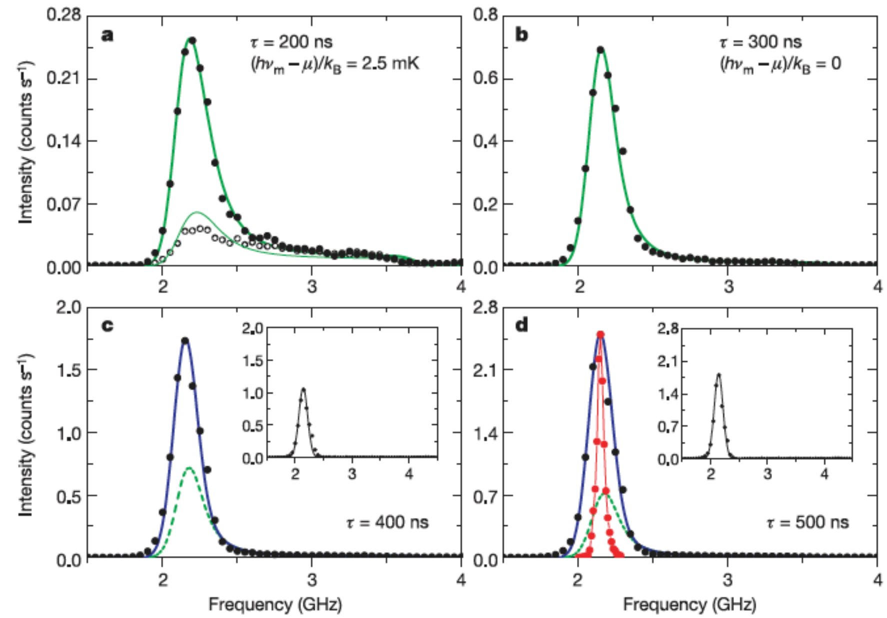


Figure 3 | BLS spectra from pumped magnons at different delay times, τ. **a**, τ = 200 ns; **b**, 300 ns; **c**, 400 ns; and **d**, 500 ns. Black and red filled circles (all panels) show data points recorded at pumping power P = 5.9 W, whereas open circles (panel **a**) represent the data recorded at P = 4 W. Green solid lines in **a** and **b** show the results of the fit of the spectra based on equation (1) with the chemical potential being a fitting parameter. The fit of the spectra in **c** and **d** (blue solid lines) are the sums of the magnon density calculated using equation (1) (green dashed line) with $\mu = h\nu_m$ and the magnon density due to the singularity at $\nu = \nu_m$. Red circles in **d** indicate data obtained with a resolution of 50 MHz; red line is a guide for the eye, connecting the red circles. Insets in **c** and **d** illustrate the difference between the corresponding raw spectra and that at τ = 300 ns; axes as main panels.

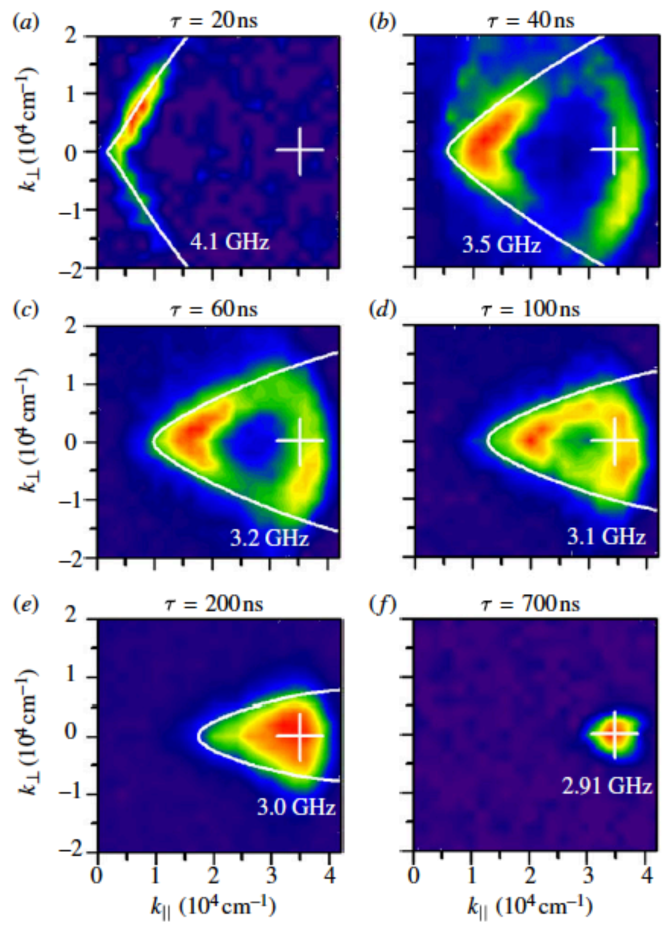
Room temperature!

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

Phil. Trans. R. Soc. A (2011) **369**, 3575–3587
doi:10.1098/rsta.2011.0128

Bose–Einstein condensation of spin wave quanta at room temperature

By O. Dzyapko[1], V. E. Demidov[1], G. A. Melkov[2] and S. O. Demokritov[1,*]



What is going on? Simple theoretical ideas

Frequency of pumped spin waves is less than doubled gap frequency
➔ the number of spin waves is conserved

Bun'kov and Volovik

Pumping establishes a stationary number N_p of spin waves $N_p = W_p \tau / (\hbar\omega_p)$

Pumping power — W_p; Lifetime — τ

Low energy spin waves relax to a metastable thermal equilibrium with non-zero chemical potential: $f_{\mathbf{k}} = \left[\exp\left(\frac{\hbar\omega_{\mathbf{k}} - \mu}{T}\right) - 1\right]^{-1} \approx \frac{T}{\hbar\omega_{\mathbf{k}} - \mu}$

$$\sum_{\mathbf{k}} \left[\exp\left(\frac{\hbar\omega_{\mathbf{k}} - \mu}{T}\right) - 1\right]^{-1} \equiv N(T,\mu)$$

$$N_p = N(T,\mu) - N(T,0) \approx T\mu \sum_{\mathbf{k}} \left[\hbar\omega_{\mathbf{k}}(\hbar\omega_{\mathbf{k}} - \mu)\right]^{-1}$$

μ is monotonically growing function of N_p limited by the value Δ.

$N_{pc} = N(T,\Delta) - N(T,0)$ -- equation of condensation line.

$N_c = N_p - N_{pc} = N_p - N(T,\Delta) + N(T,0)$ -- number of spin waves in condensate

Condensation is possible at room temperature since only low energy spin waves condense

SCIENTIFIC REPORTS

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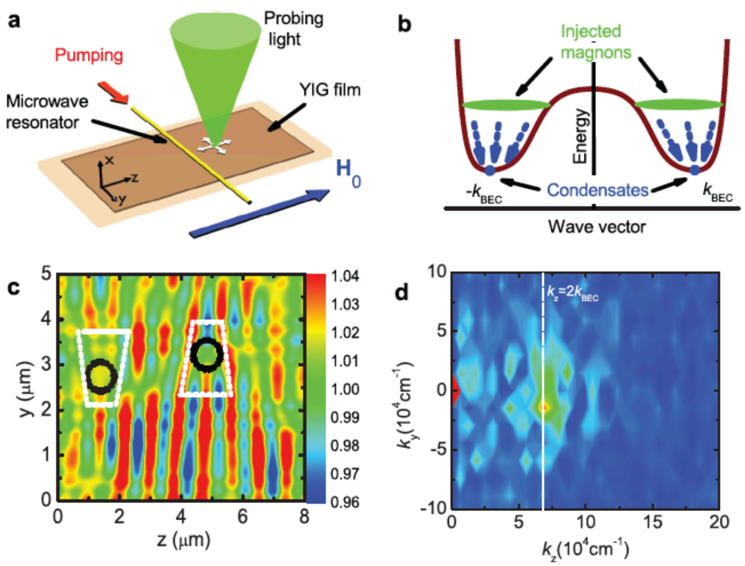
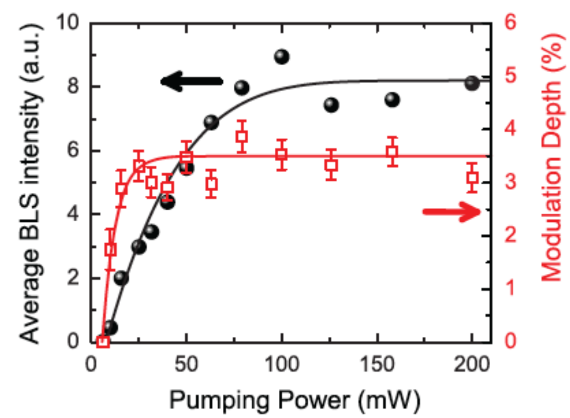
OPEN

SUBJECT AREAS:
MAGNETIC MATERIALS AND DEVICES
QUANTUM PHYSICS

Spatially non-uniform ground state and quantized vortices in a two-component Bose-Einstein condensate of magnons

P. Nowik-Boltyk[1], O. Dzyapko[1], V. E. Demidov[1], N. G. Berloff[2] & S. O. Demokritov[1]





The BLS signal is proportional to $\delta M_z = |c_Q e^{i\mathbf{Qr}} + c_{-Q} e^{-i\mathbf{Qr}}|^2$

$$\delta M_z = N_Q + N_{-Q} \pm 2\sqrt{N_Q N_{-Q}} \cos(2\mathbf{Qr})$$

If $N_Q = N_{-Q}$, the contrast is 100%

In their experiment it was 3%

Question: Why the symmetry Q ⟺ -Q is violated?

2. BECM - Theory

SCIENTIFIC REPORTS

Published
4 March 2013

OPEN

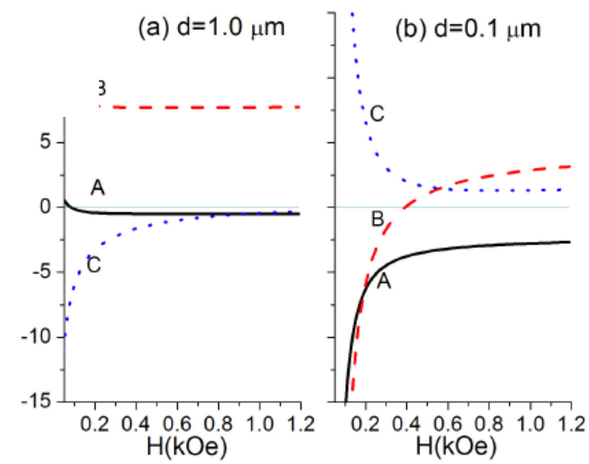
SUBJECT AREAS:
BOSE-EINSTEIN CONDENSATES

Phase Diagram for Magnon Condensate in Yttrium Iron Garnet Film

Fuxiang Li[1], Wayne M. Saslow[1] & Valery L. Pokrovsky[1,2]

Magnons with the same momentum attracts each other; Magnons from different minima repulse each other.

$A < 0; B > 0$



$A < 0$; B changes sign

Then $N_Q=N_c$, $N_{-Q}=0$ ⟶ No oscillations

Question: Why the oscillations appear?

The coupling of the two condensates is determined by the 4-th order Hamiltonian

$$\hat{V}_4 = A[c_Q^\dagger c_Q^\dagger c_Q c_Q + c_{-Q}^\dagger c_{-Q}^\dagger c_{-Q} c_{-Q}] + 2B c_Q^\dagger c_{-Q}^\dagger c_{-Q} c_Q + C[c_Q^\dagger c_Q c_Q c_{-Q} + c_{-Q}^\dagger c_{-Q} c_{-Q} c_Q + h.c.].$$

Condensate amplitudes $c_{\pm Q} = \sqrt{N_{\pm Q}} e^{i\phi_{\pm Q}}$

$\Phi = \phi_Q + \phi_{-Q}$

$$V_4 = A(N_Q^2 + N_{-Q}^2) + 2BN_Q N_{-Q} + 2C\cos\Phi (N_Q^{\frac{3}{2}} N_{-Q}^{\frac{1}{2}} + N_Q^{\frac{1}{2}} N_{-Q}^{\frac{3}{2}}),$$

Minimization over phase:

$\cos\Phi = -\mathrm{sign} C$ ⟶ $\Phi = \pi$ if $C > 0$ and $\Phi = 0$ if $C < 0$

Phase coherence!

Two states: pi-state and zero-state

Minimization over condensate numbers:

$$V_4 = A\left(N_Q^2 + N_{-Q}^2\right) + 2BN_Q N_{-Q} - 2|C| N_Q^{1/2} N_{-Q}^{1/2}\left(N_Q + N_{-Q}\right)$$

Constraint: $N_Q + N_{-Q} = N_c = \text{const}$

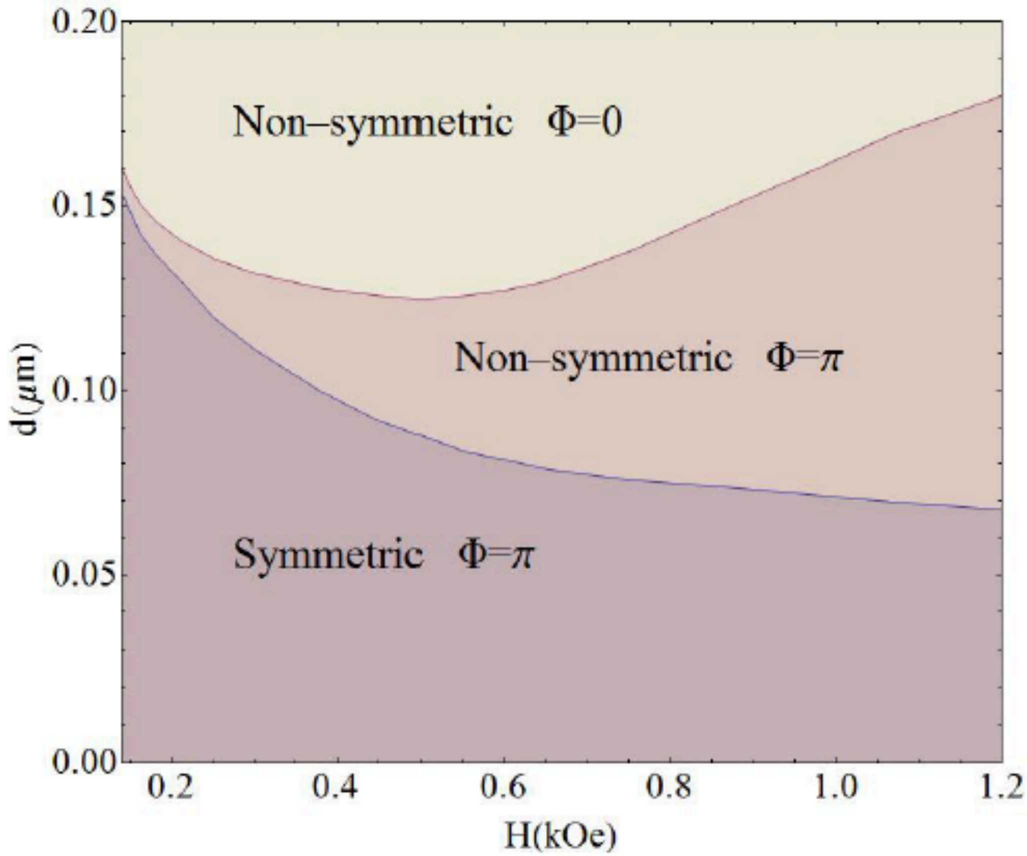
$$V_4 = \frac{A+B}{2} N_c^2 + \frac{A-B}{2}\delta^2 - |C| N_c \sqrt{N_c^2 - \delta^2}$$

$$\delta = N_Q - N_{-Q}$$

The ground state depends on a criterion $\Gamma = A - B + |C|$

$\Gamma > 0$ or $A > B$ → $N_Q = N_{-Q}$ — Symmetric phase

$\Gamma < 0$ and $A < B$ → $\delta = N_c \sqrt{1 - \frac{|C|^2}{(B-A)^2}}$ — Non-symmetric phase



Phase diagram in the plane *d* - *H*

Interference and contrast

The BLS signal is proportional to $\delta M_z = \left| \sqrt{N_Q} e^{i(Qz+\phi_Q)} + \sqrt{N_{-Q}} e^{-i(Qz-\phi_{-Q})} \right|^2$

$$\delta M_z = N_Q + N_{-Q} \pm 2\sqrt{N_Q N_{-Q}} \cos(2\mathbf{Qr} + \varphi); \qquad \varphi = \phi_Q - \phi_{-Q}$$

Displacement of the interference pattern – Goldstone mode

Contrast

$$\beta \equiv \frac{\delta M_{z\max} - \delta M_{z\min}}{\delta M_{z\max}} = \sqrt{1 - \frac{\delta^2}{N_c^2}} = \begin{cases} 1 & \text{in symmetric phase} \\ \dfrac{|C|}{B - A} & \text{in non-symmetric phase} \end{cases}$$

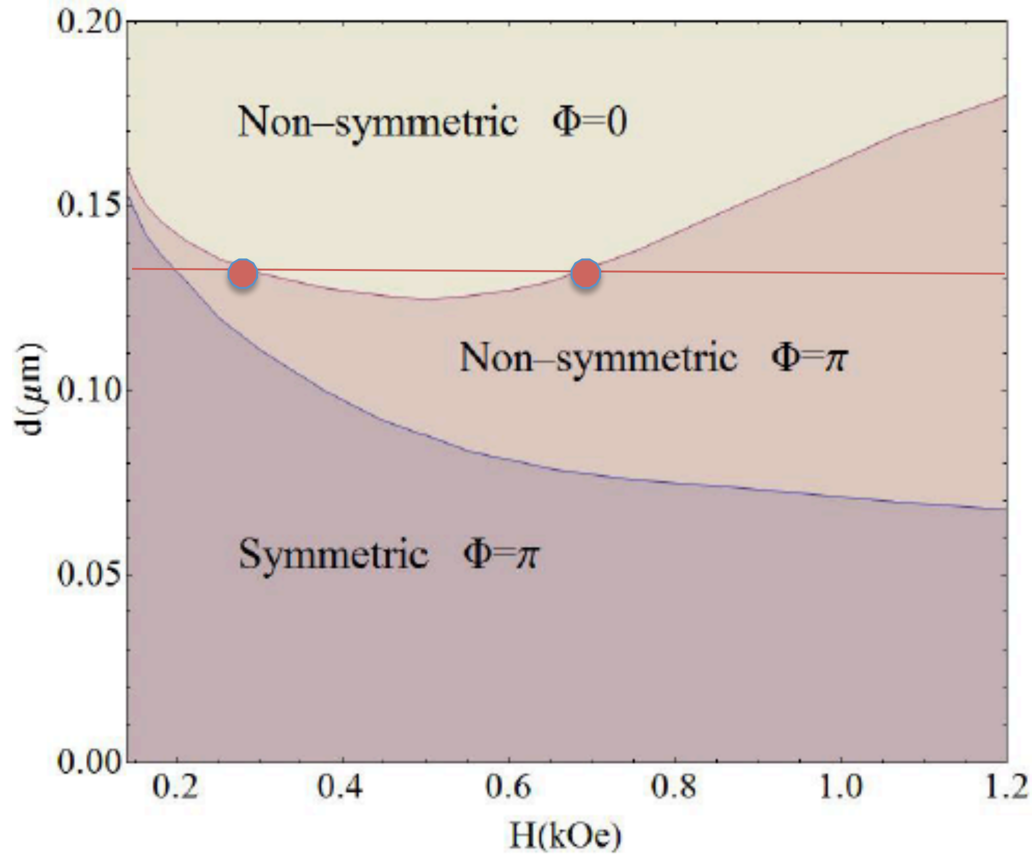
In the experiments *A* = - 0.168 mK, *B* = 8.218 mK, C = - 0.203 mK

β = 2.5-5%

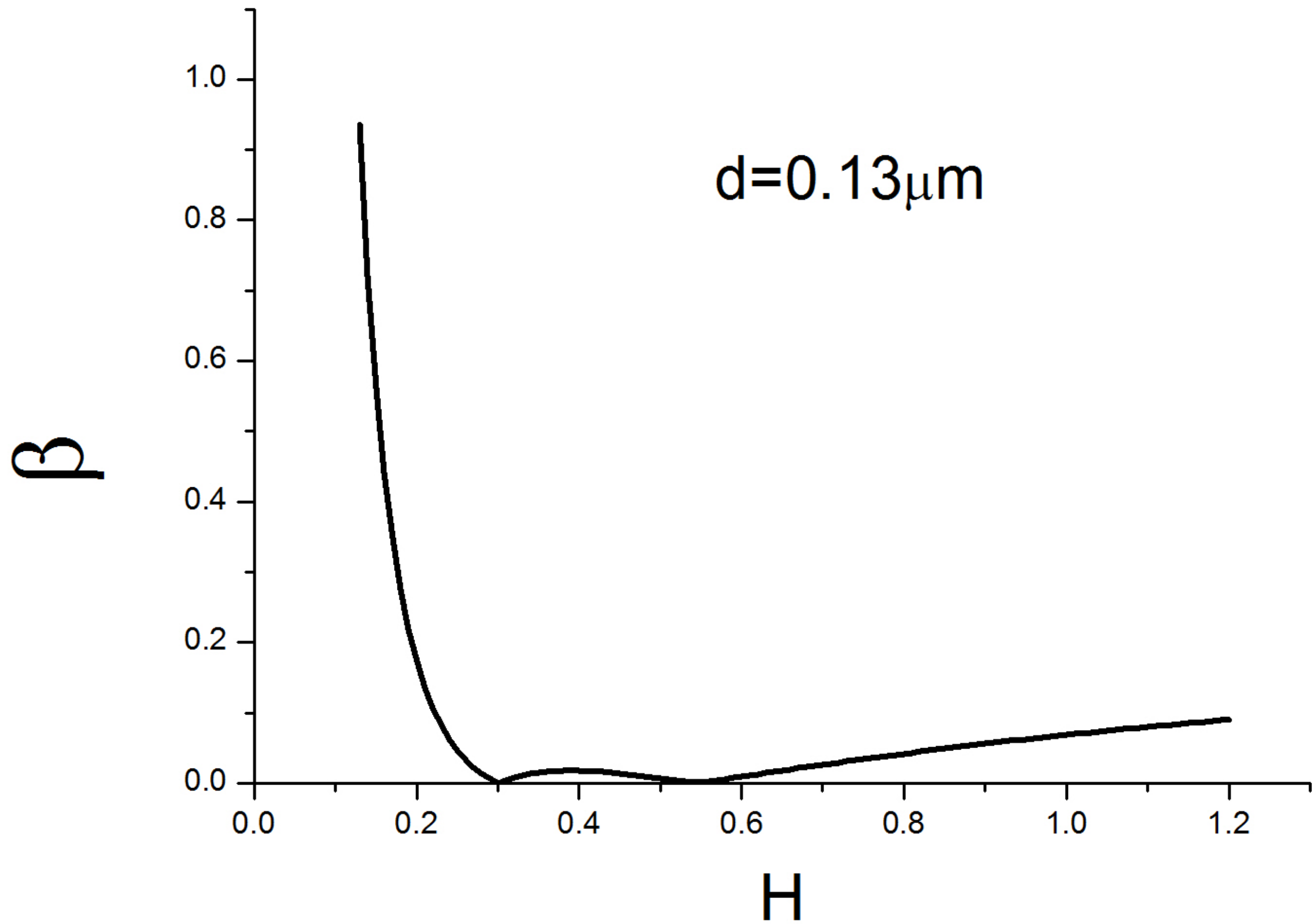
In the experiment β = 3-10%

Why β is small?

$$\frac{|C|}{B} \sim \frac{1}{Qd} \approx \frac{1}{30}$$



Phase diagram in the plane *d* - *H*



Cusps are manifestations of the 0-π transition

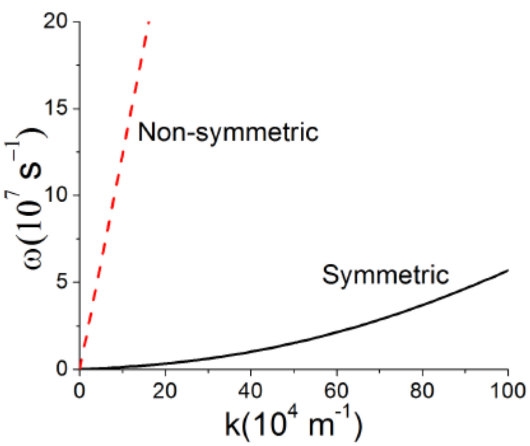
Zero sound

Goldstone mode $\phi=\phi_Q-\phi_{-Q..}$ It induces oscillations of $\delta = N_Q - N_{-Q}$

Local displacements of a “crystal” interference pattern.

Spectrum: $$\omega = \sqrt{\frac{\hbar^2 k^4}{4m^2} + \Gamma n_c \frac{k^2}{m}}$$ in symmetric phase

$$\omega = \sqrt{\frac{\hbar^2 k^4}{4m^2}\kappa + (B-A) n_c (\kappa - 1)\frac{k^2}{m}}; \ \kappa = \left(\frac{B-A}{C}\right)^2$$ in non-symmetric phase



Dispersion of zero sound. For non-symmetric phase H=1kOe and d=5μm

4. Superfluidity of magnon gas. Theory.

Chen Sun, T. Nattermann and VP, in press

Is it observable? $\rho_n \approx 100\rho_s$ at $T = 300K$

But $v_s \approx 10^4 \div 10^5 v_n$ at $|\nabla H| \sim 10T/cm \Rightarrow j_s \gg j_n$

Main obstacle: The motion of the condensate as a whole violates the phase trapping.

$$\frac{\partial n}{\partial t} + \nabla \mathbf{j} = \eta \sin\Phi; \ \Phi = \phi_{\mathbf{Q}} + \phi_{-\mathbf{Q}}$$

$$\eta = \frac{Cn^2}{\hbar} \quad \text{symmetric}$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \tilde{\mathbf{j}} = 0$$

$$\eta = \frac{C^2 n^2}{\hbar(B-A)} \quad \text{non-symmetric}$$

$$n = n_Q + n_{-Q}; \ \mathbf{j} = \frac{\hbar}{m}\left(n_Q \nabla\phi_Q + n_{-Q}\nabla\phi_{-Q}\right)$$

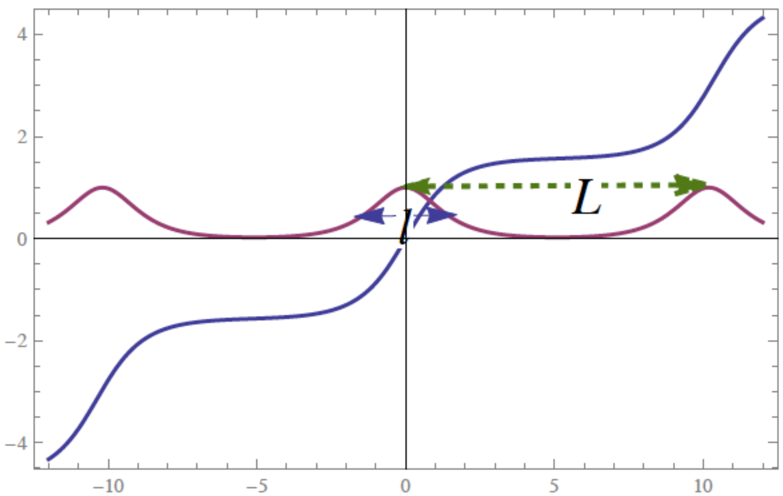
Condensate is conserved globally, but not locally

$$\tilde{n} = n_Q - n_{-Q}; \ \tilde{\mathbf{j}} = \frac{\hbar}{m}\left(n_Q \nabla\phi_Q - n_{-Q}\nabla\phi_{-Q}\right)$$

What is new in comparison to standard superfluidity?

Superfluid flow appears only after submission an energy E exceeding some threshold value E_{th} to the condensate

If $E-E_{th} \ll E_{th}$, the phase on long intervals L is almost trapped and then jumps by 2π on a short interval l



$$v_s = \frac{2\pi\hbar}{mL}; \quad E - E_{th} = \frac{\varepsilon_s}{L}$$

$$E_{th} \sim \begin{cases} Cn^2 & \text{in symmetric phase} \\ C^2 n^2 / (B - A) & \text{in non-symmetric phase} \end{cases}$$

$$l \sim \sqrt{Dn / E_{th}}; \quad \varepsilon_s \sim \sqrt{DE_{th} n}$$

Domain wall width

Domain wall energy

5. Conclusions

- SWBEC at rf pumping proceeds in low-frequency part of spin wave spectrum and therefore is possible at room temperature

- The number of spin waves is conserved since the decay processes are forbidden in low-energy part of spectrum.

- Coherence of two condensates is established due to the interaction of spin waves that violates the number of spin wave conservation at high frequency. There exist two coherent states with the sum of phases zero or pi.

- There should exist symmetric and non-symmetric phases of the condensates with equal or different numbers of spin waves at the two energy minima.

- Transition between phases is driven by thickness of the film and by magnetic field

- Dipolar interaction leads to the phase trapping. Superfluid motion is possible after submission of a finite amount of energy to the condensate

- Near the threshold energy the superfluid velocity remains zero on long intervals and is positive on short phase jump intervals.