NONLOCAL TRANSPORT IN GRAPHENE: VALLEY CURRENTS, HYDRODYNAMICS AND ELECTRON VISCOSITY

Leonid Levitov (MIT)

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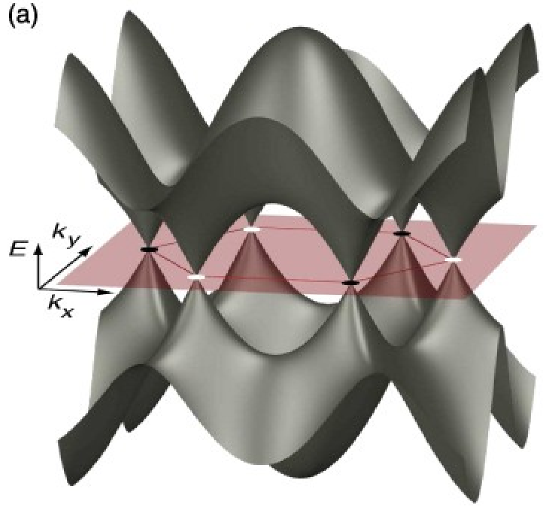
Boris @ 60

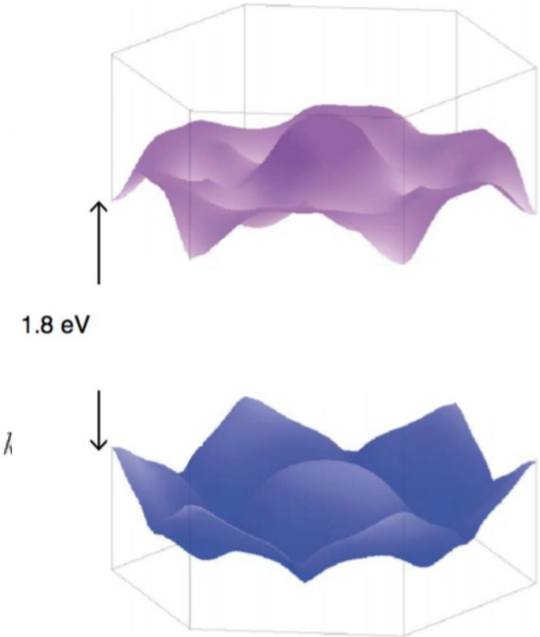
Boris Blinks the Antiquarian Bookseller

Valley currents

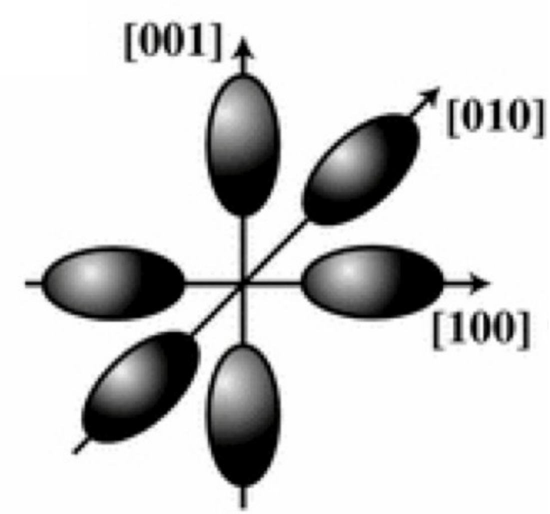
Valleys in Graphene



Valleys in MoS2



Valleys in Bulk Si



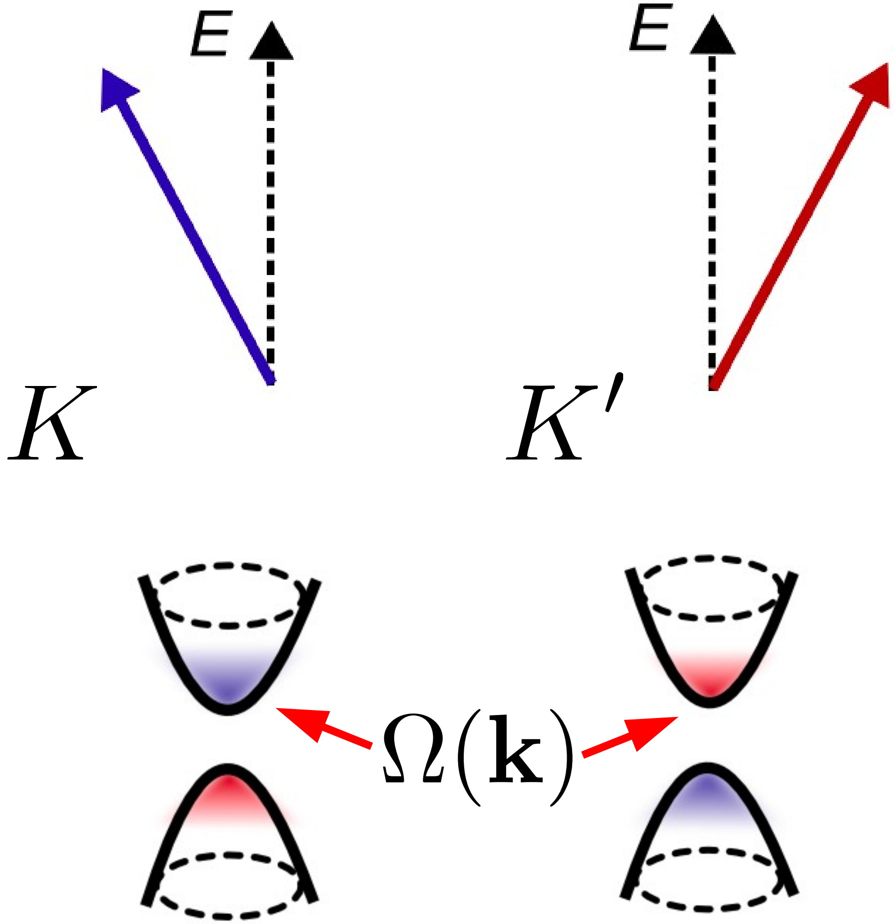
Berry curvature

$$\sigma_{xy}^{v} \neq 0$$

No Berry curvature

$$\sigma_{xy}^{v} = 0$$

Use Berry curvature to electrically manipulate valleys



$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar}\frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} \boxed{+ \dot{\mathbf{k}} \times \Omega(\mathbf{k})}$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$

Valley Hall effect:

Transverse charge-neutral currents

$$\vec{J}_v = \vec{J}_K - \vec{J}_{K'}$$

$$\vec{J}_v = \sigma_{xy}^{v}\, \vec{z} \times \vec{E}$$

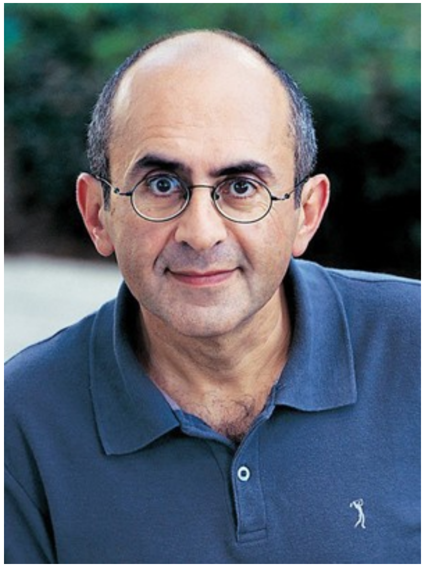
Collaboration

Justin Song

Polnop Samutpraphoot

Yuri Lensky

Andrey Shytov



Gregory Falkovich

Song, Shytov, LL PRL 111, 266801 (2013)
Song, Samutpraphoot, LL, PNAS (2015)
LL & Falkovich arXiv: 1508.00836 (2015)

Topological Bloch bands in graphene superlattices

- **Designer topological materials**: stacks of 2D materials which by themselves are not topological, e.g. graphene.
- **Previously**, topological bands in graphene were presumed either **impossible or impractical**
- Turn graphene into a robust platform with which **topological behavior** can be realized and explored.

Goal: develop graphene-based topological materials

Quantized transport, Topological bands, Anomalous Hall effects

Chern invariant $$C = \frac{1}{2\pi} \sum_k \Omega(k)$$

$$\Omega(k) = \nabla_k \times A_k, \quad A_k = i \langle \psi(k) | \nabla_k | \psi(k) \rangle$$

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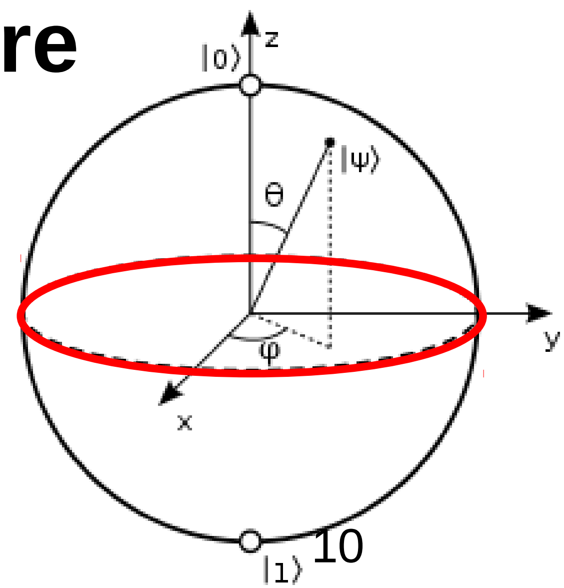
Pristine graphene: massless Dirac fermions, Berry phase **yet no Berry curvature**

$$\psi_{\pm,\mathbf{K}}(\mathbf{k})=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\theta_{\mathbf{k}}/2}\\ \pm e^{i\theta_{\mathbf{k}}/2}\end{pmatrix}$$



Goal: develop graphene-based topological materials

Quantized transport, Topological bands, Anomalous Hall effects

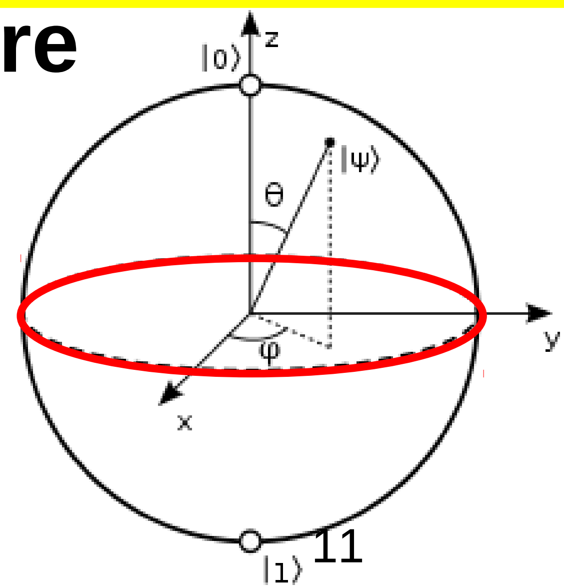
Chern invariant $C = \frac{1}{2\pi} \sum_k \Omega(k)$

Ω

Benefit from graphene's superior electronic properties

Pris ... Berry phase **yet no Berry curvature**

$$\psi_{\pm,\mathbf{K}}(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_{\mathbf{k}}/2} \\ \pm e^{i\theta_{\mathbf{k}}/2} \end{pmatrix}$$



Massive (gapped) Dirac particles

A/B sublattice asymmetry a gap-opening perturbation
Berry curvature hot spots above and below the gap

T-reversal symmetry: $\Omega(-k) = -\Omega(k) \qquad \Omega(k) \neq 0$

Valley Chern invariant
(for closed bands)

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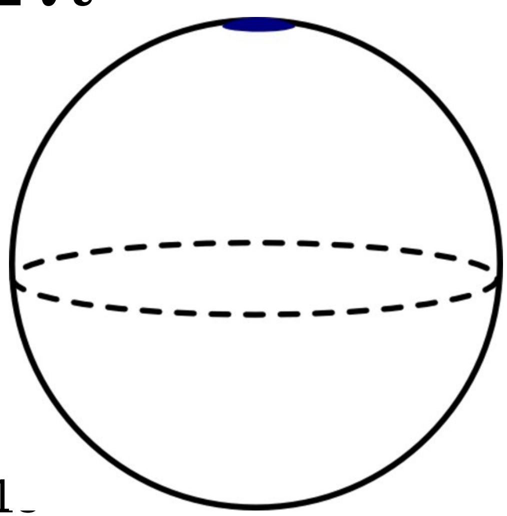
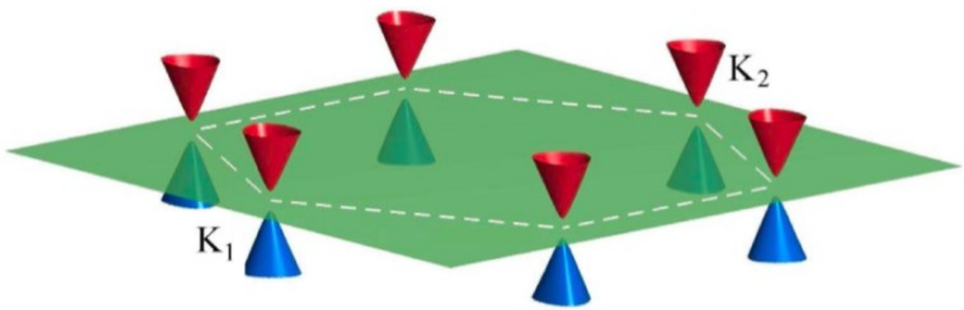
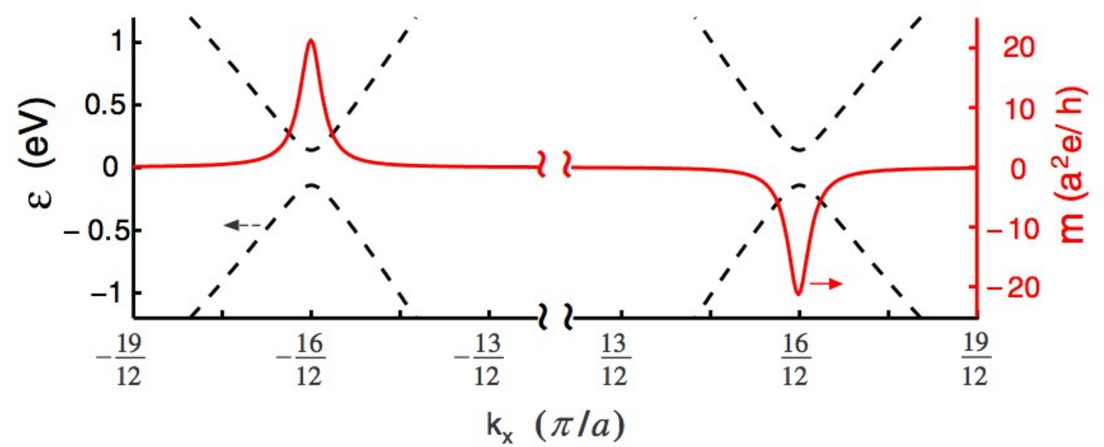






D. Xiao, W. Yao, and Q. Niu, PRL 99, 236809 (2007)

The variety of van der Waals heterostructures: G/hBN superlattices

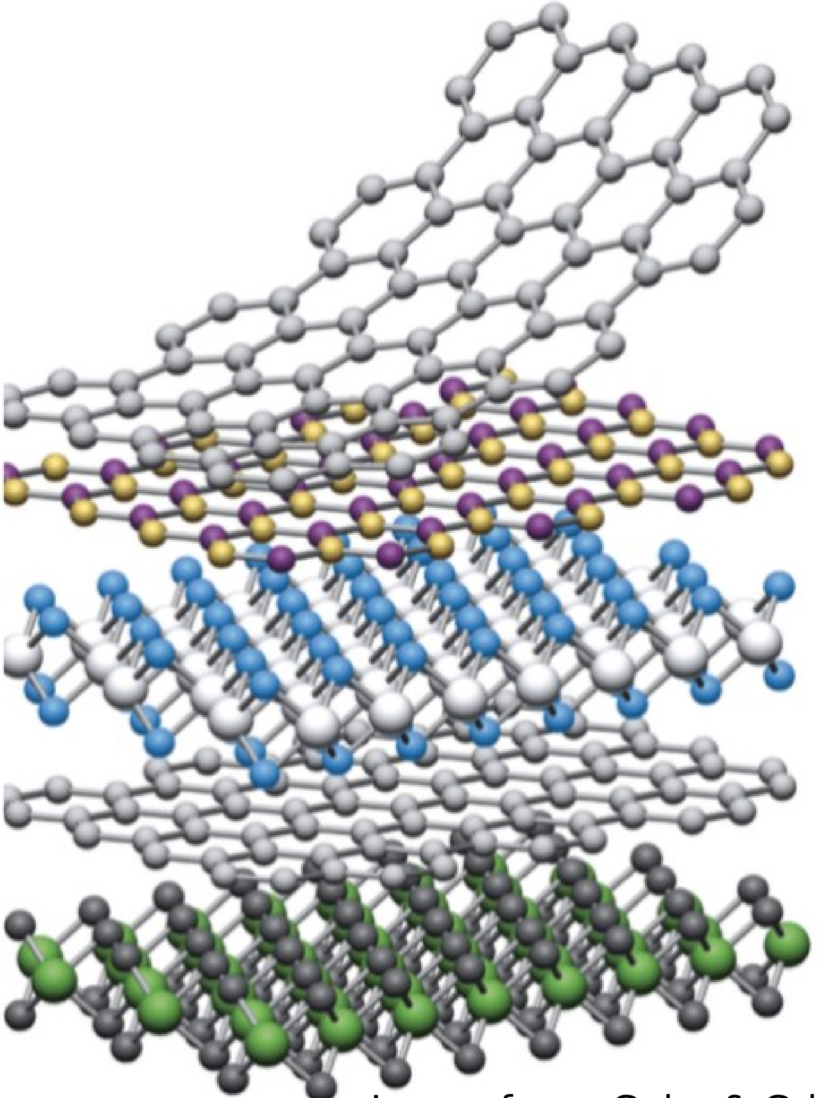
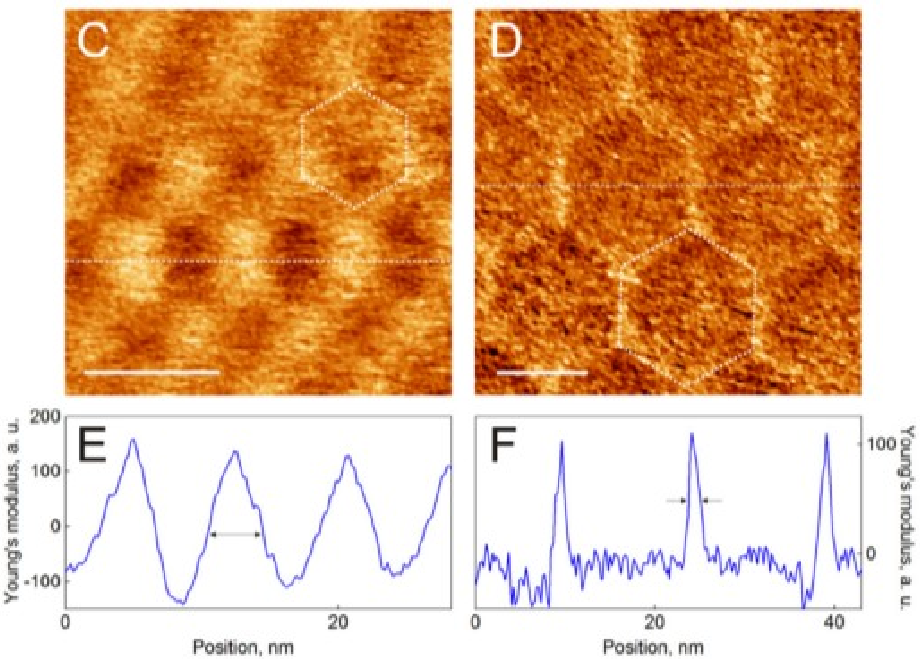


Image from: Geim & Grigorieva, Nature 499, 419 (2013)

Stacked vdW materials exhibit spatial structure

AFM Spatial Map



CR Woods, et.al. Nat. Phys (2014)

Bloch bands in G/hBN superlattices

Song, Shytov, LL, *PRL* ***111***, *266801 (2013)*

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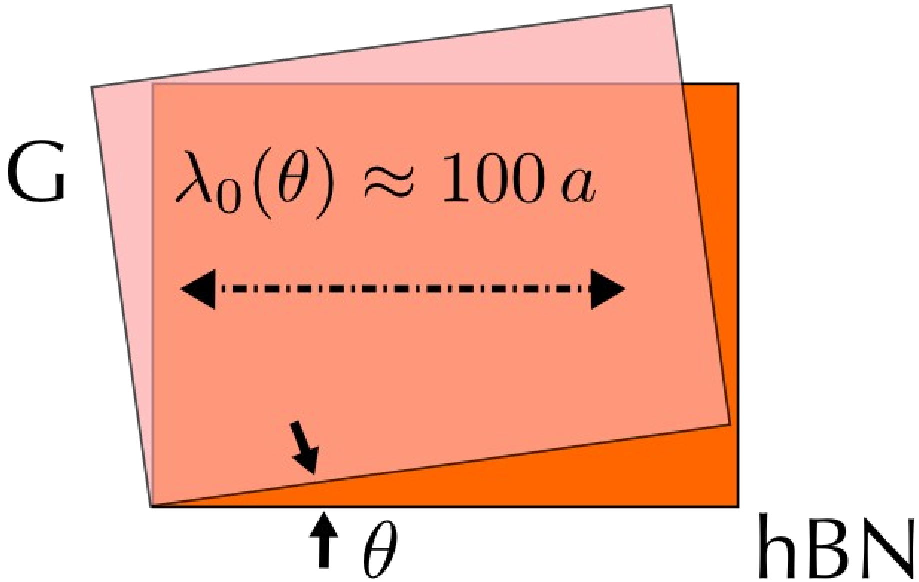

Moiré wavelength λ_0 can be as large as 14nm ≈100 C-C spacings

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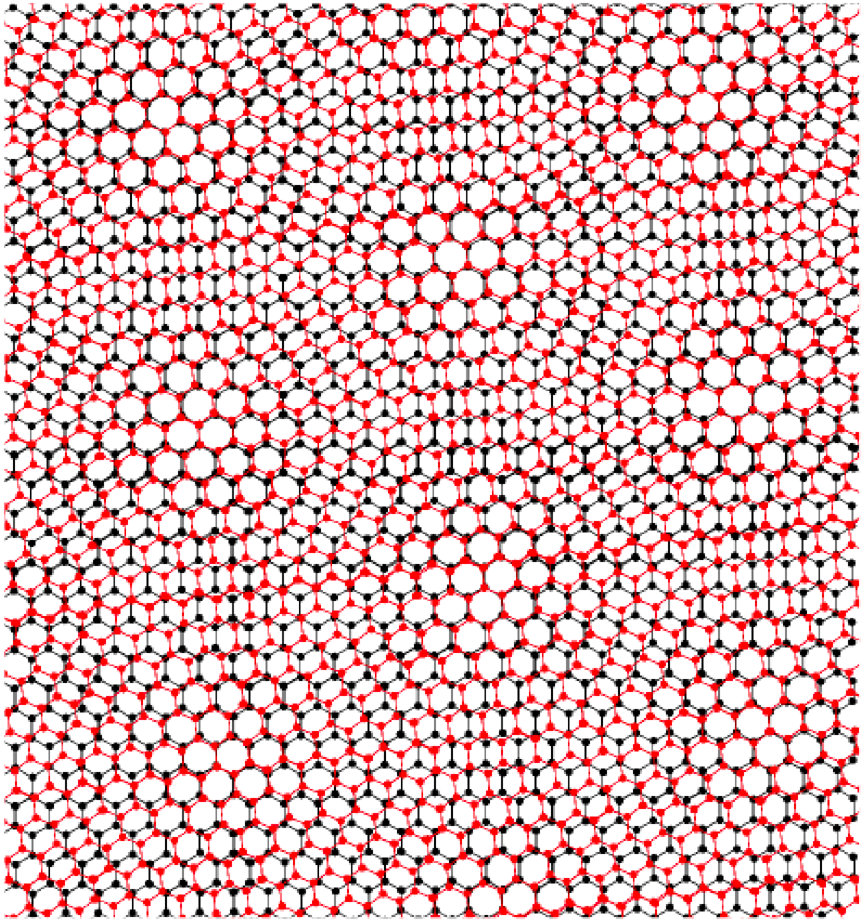

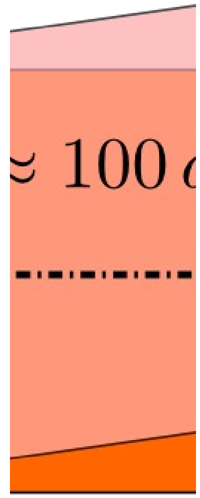

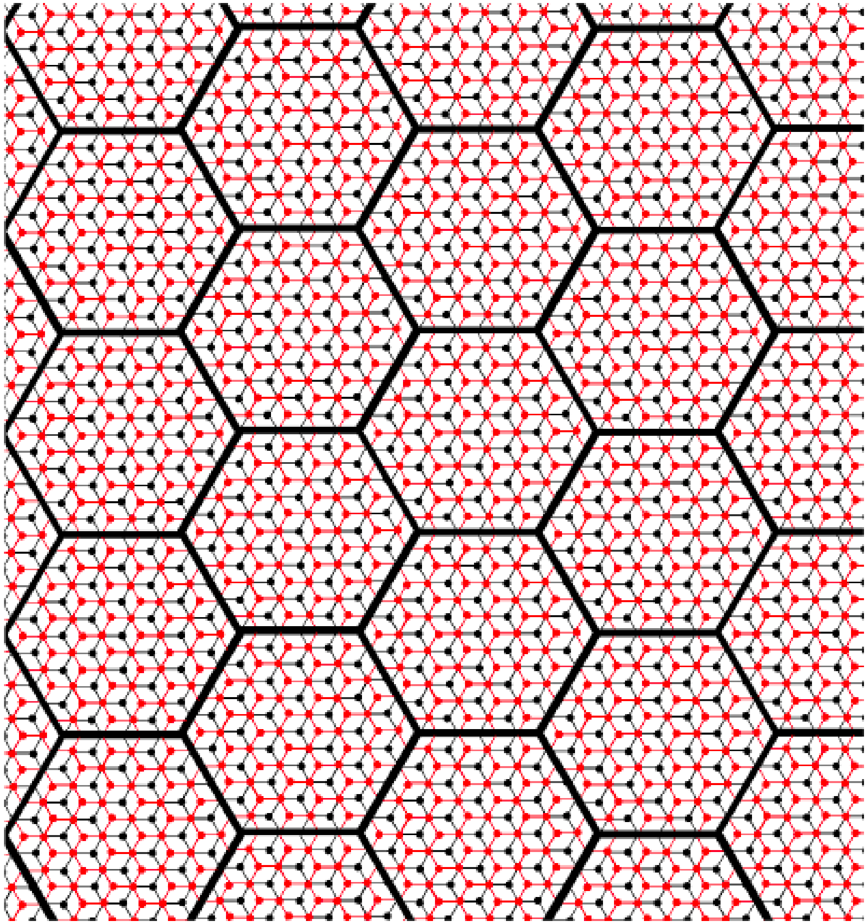

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Focus on one valley, K or K'
Bloch Chern minibands w/ Valley Chern invariants

Moiré wavelength λ_0 can as large as 14nm ≈100 times C-C spacing

Low-energy Hamiltonian

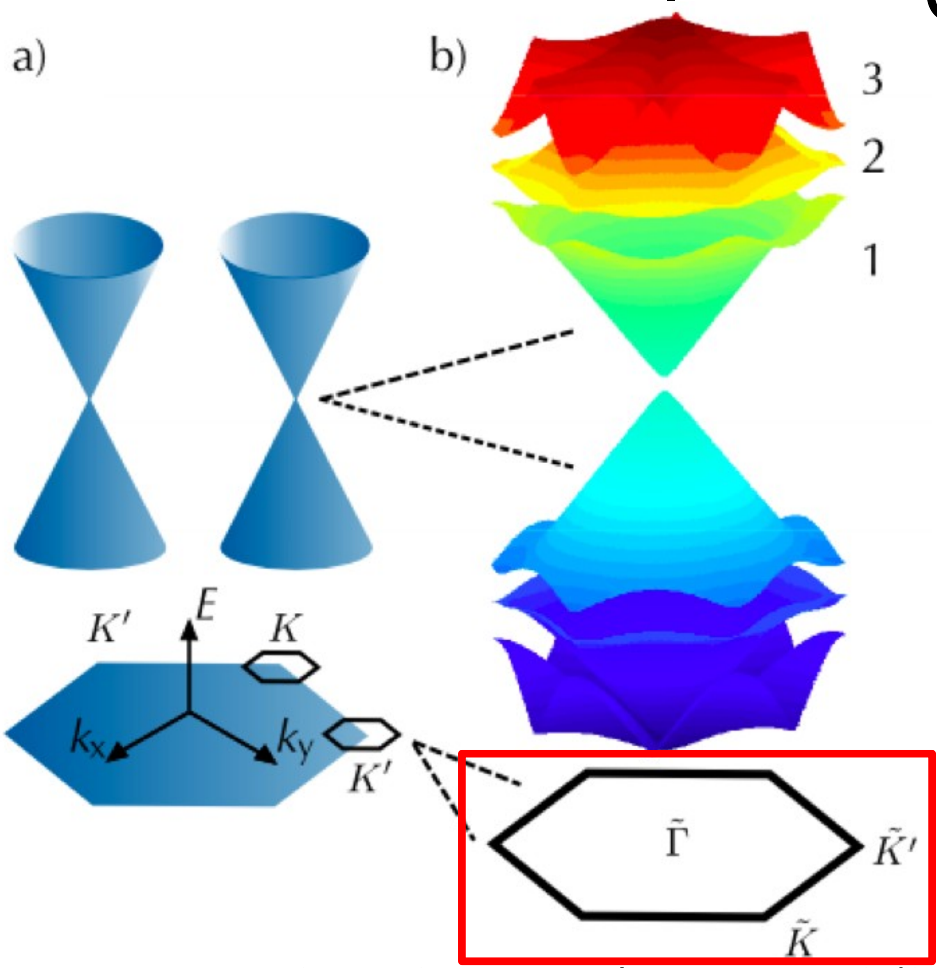
San-Jose et al. (2014), Jung et al (2014), Song, Shytov LL PRL (2013)
Wallbank et al PRB (2014), Kindermann PRB (2012) Sachs, et. al. PRB (2011)

$$\mathcal{H} = \int d^2x \sum_{i=1}^{N} \psi_i^\dagger(\mathbf{x})[v\sigma\mathbf{p} + m(\mathbf{x})\sigma_3]\psi_i(\mathbf{x})$$

Constant global gap at DP

$$m(\mathbf{x}) = \Delta + m \sum_{j=1}^{6} e^{i\mathbf{b}_j\cdot\mathbf{x}}$$

Spatially varying gap,
Bragg scattering



Song, Samutpraphoot, LL PNAS (2015)

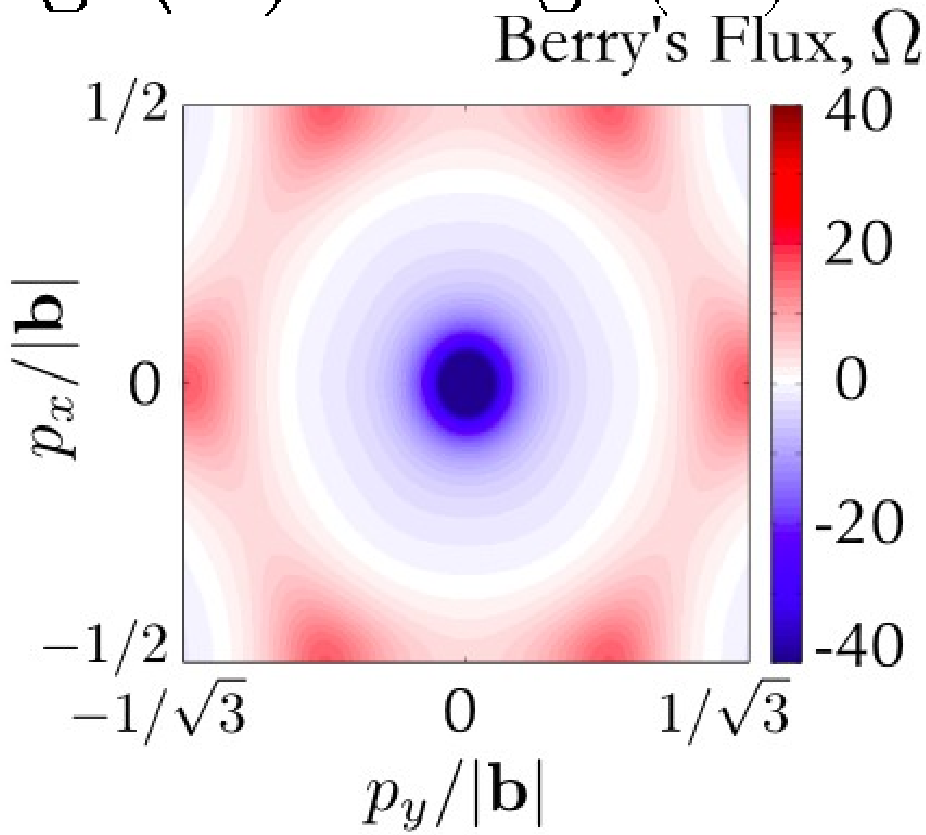
Focus on one valley

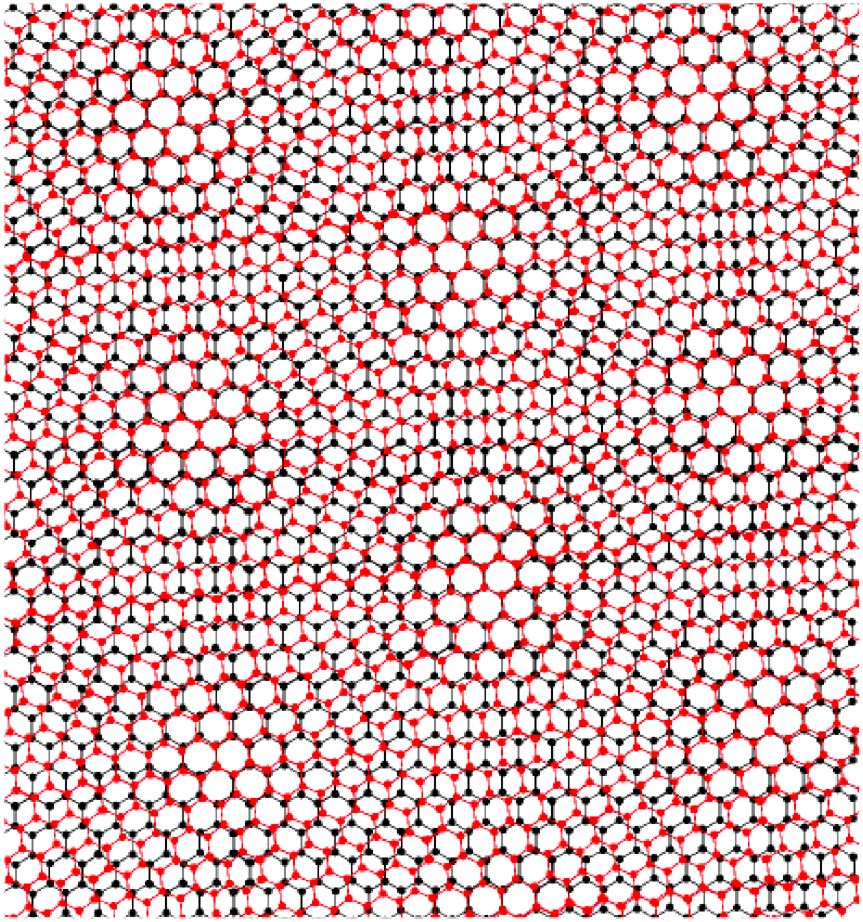
Incommensurate/Moire case

$$\mathcal{H} = \int d^2x \sum_{i=1}^{N} \psi_i^\dagger(\mathbf{x})[v\sigma\mathbf{p} + m(\mathbf{x})\sigma_3]\psi_i(\mathbf{x})$$

$$m(\mathbf{x}) = \Delta + m\sum_{j=1}^{6} e^{i\mathbf{b}_j\cdot\mathbf{x}}$$

$$\mathrm{sgn}(\Delta) = -\mathrm{sgn}(m)$$





Commensurate case

$$\mathcal{H} = \int d^2x \sum_{i=1}^{N} \psi_i^\dagger(\mathbf{x})[v\sigma\mathbf{p} + m(\mathbf{x})\sigma_3]\psi_i(\mathbf{x})$$

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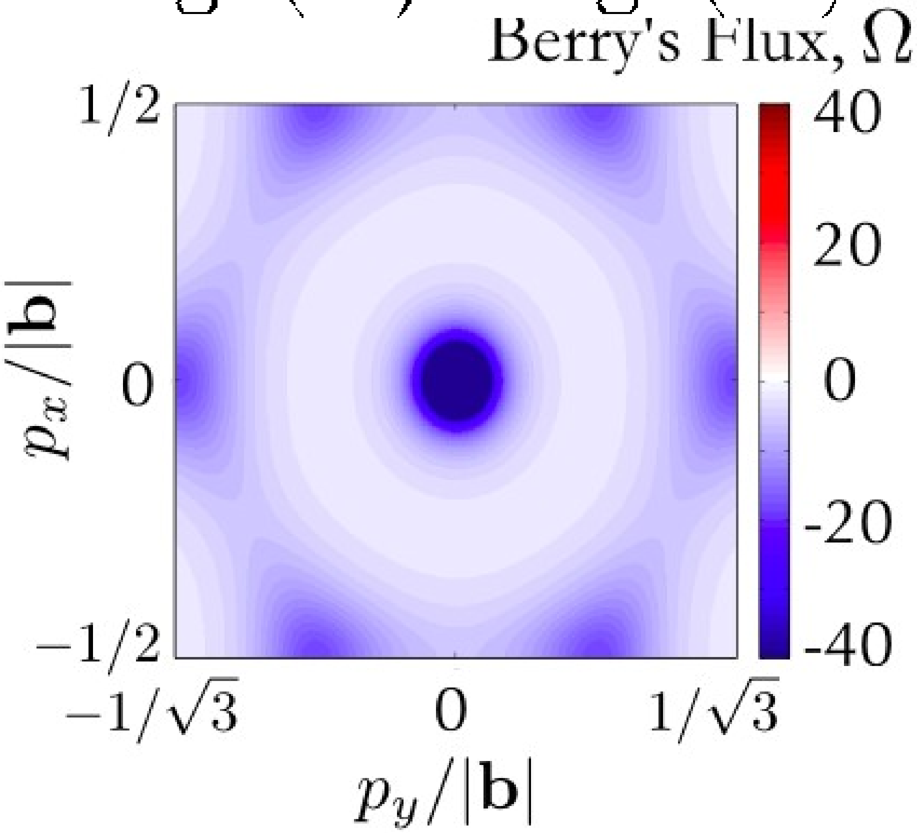
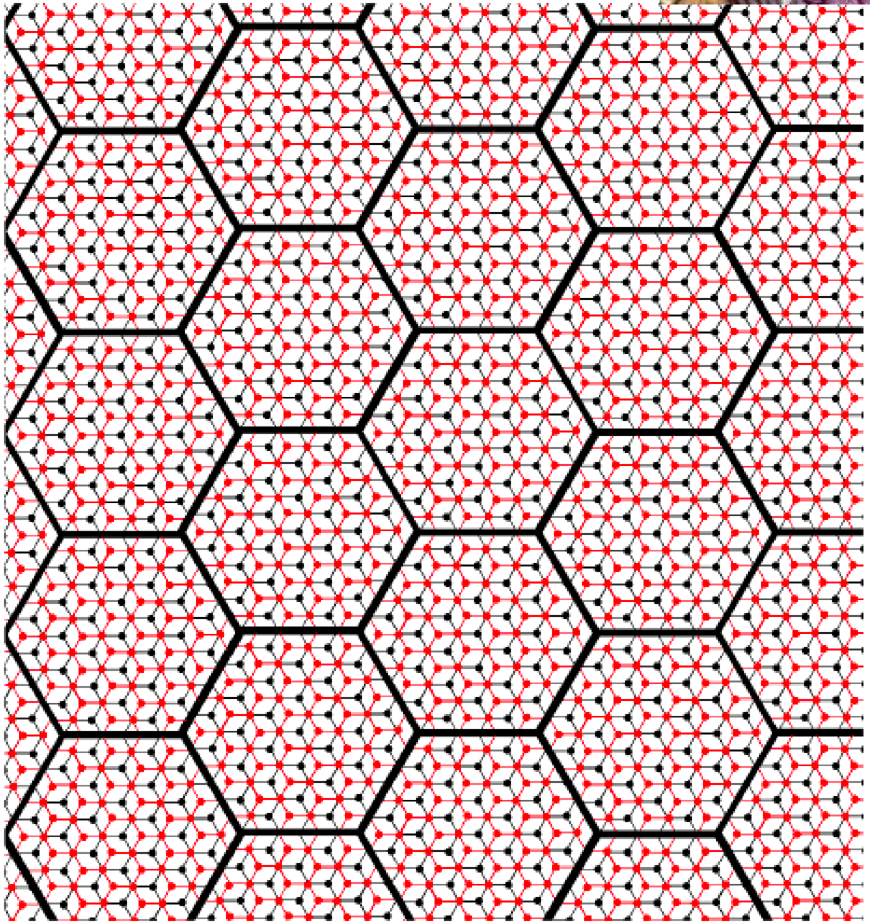
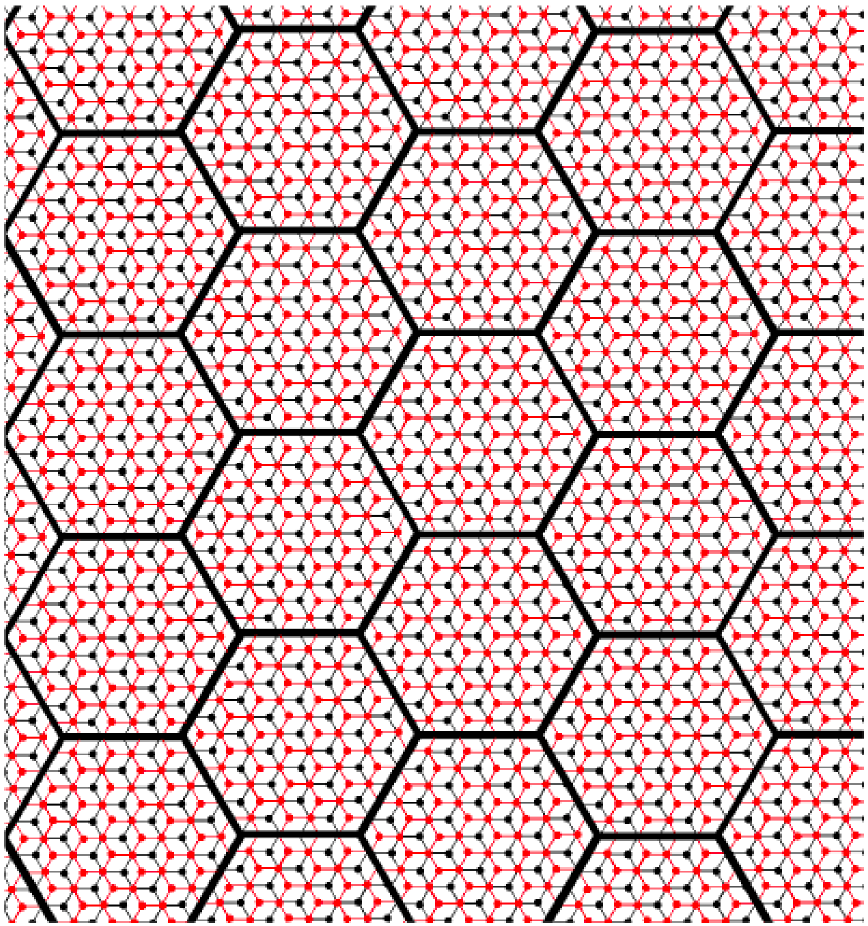


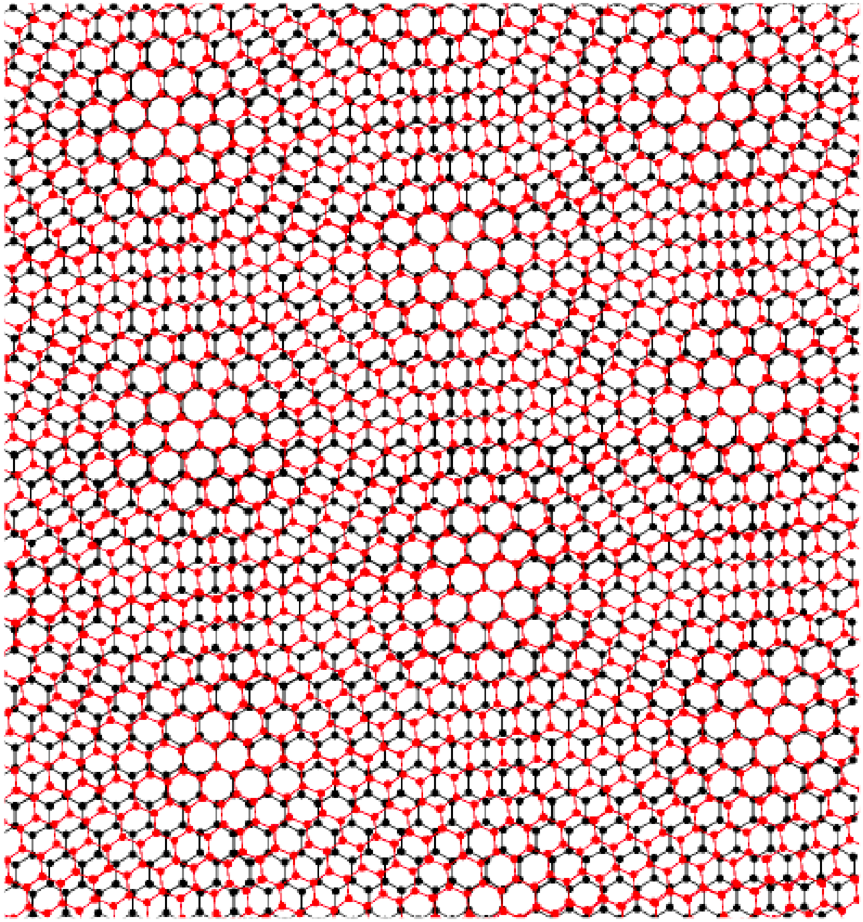


Band topology tunable by crystal axes alignment

Topological bands C=1

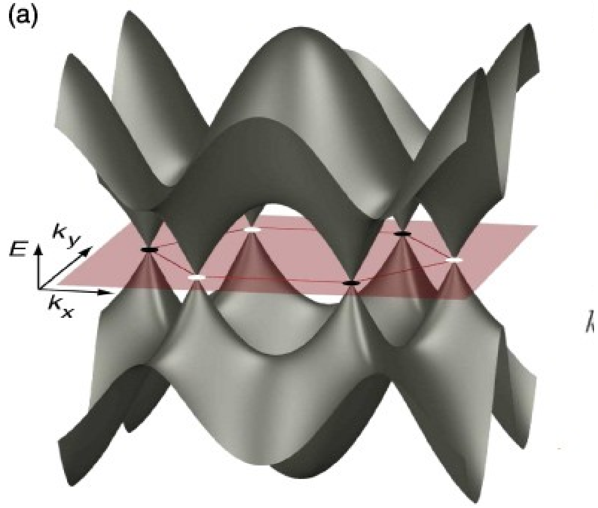


Trivial bands C=0



Valley currents

Valleys in Graphene



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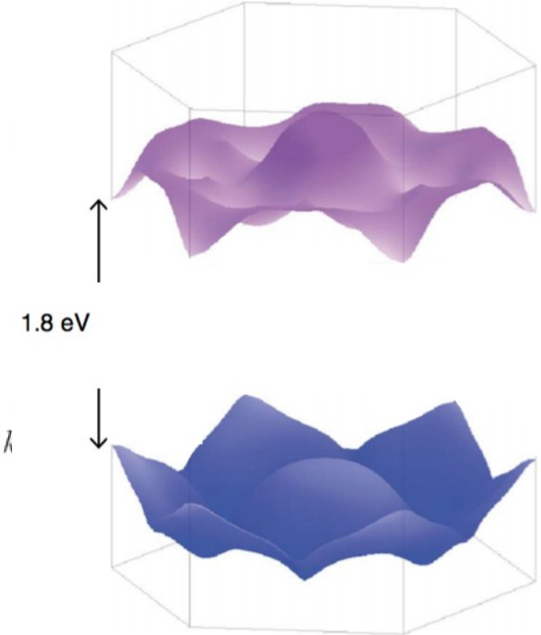

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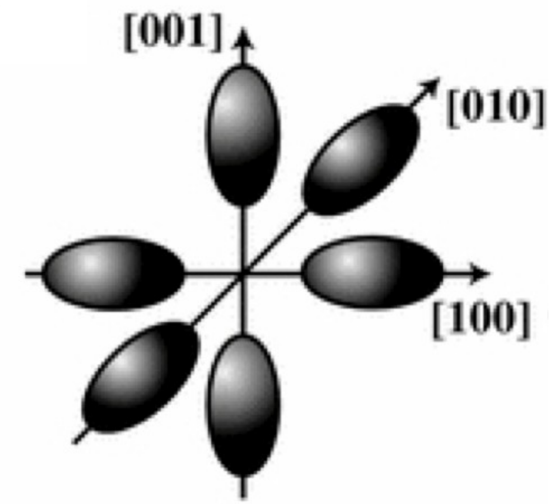

Berry curvature

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No Berry curvature

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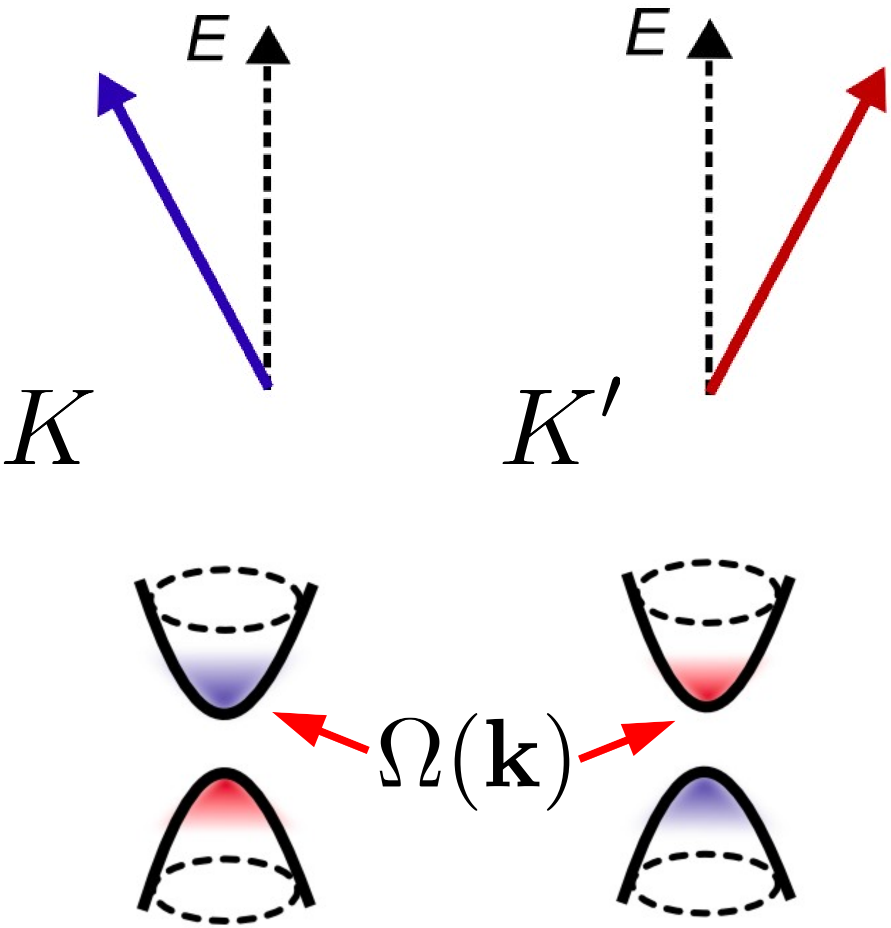
Use Berry curvature to electrically manipulate valleys



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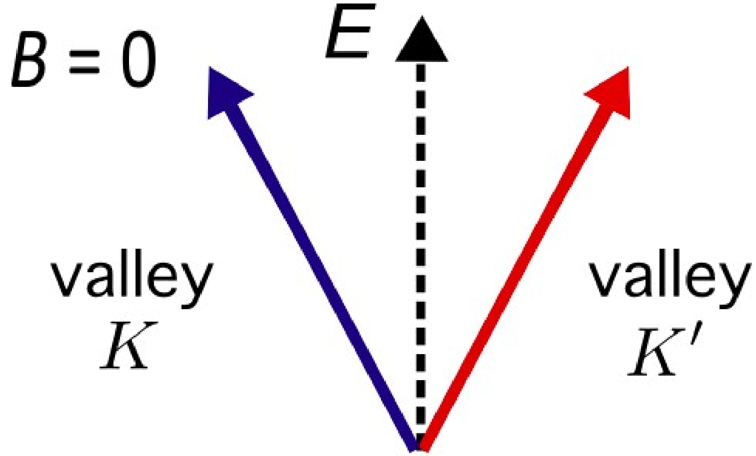
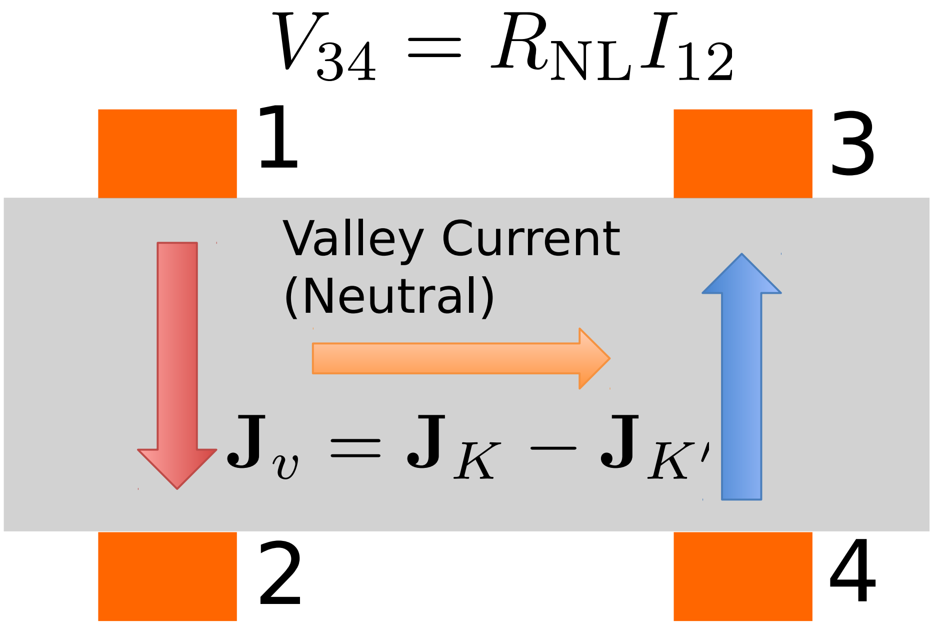
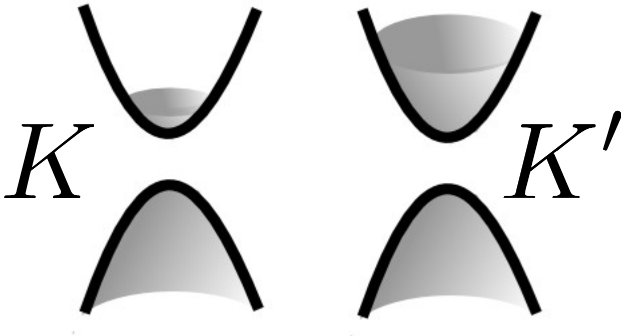
Detecting valley currents



$$\sigma_{xy}^{v} = N\frac{e^2}{h}\int d^2k\Omega(k)f(k)$$



Pump valley imbalance



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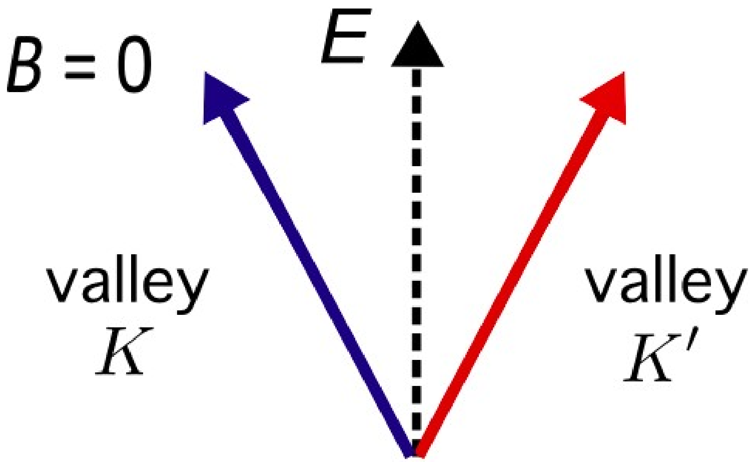
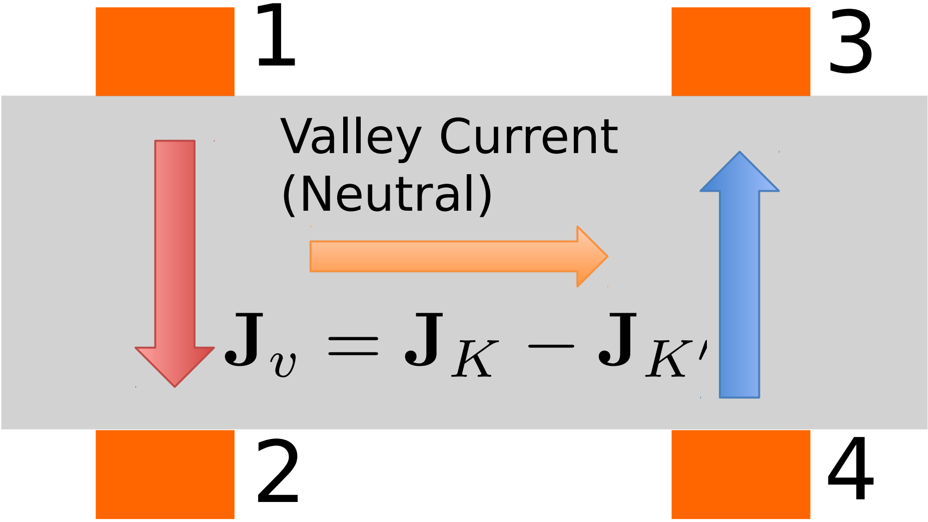


$$\sigma^{v}_{xy} = N\frac{e^2}{h}\int d^2k\,\Omega(k)\,f(k)$$

$$V_{34} = R_{\mathrm{NL}} I_{12}$$



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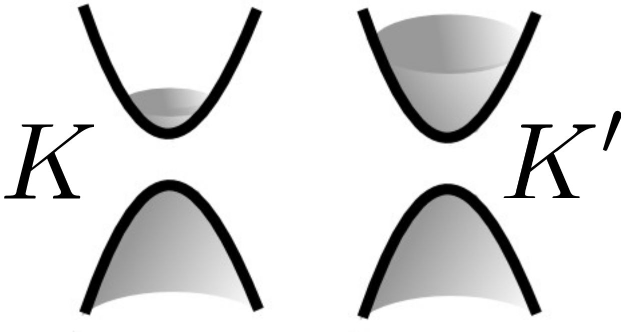

Valley Hall Effect (VHE):

$$\mathbf{J}_v = \frac{\sigma^{v}_{xy}}{\sigma}\mathbf{j}\times\hat{\mathbf{z}}$$

Reverse Valley Hall Effect (RVHE):

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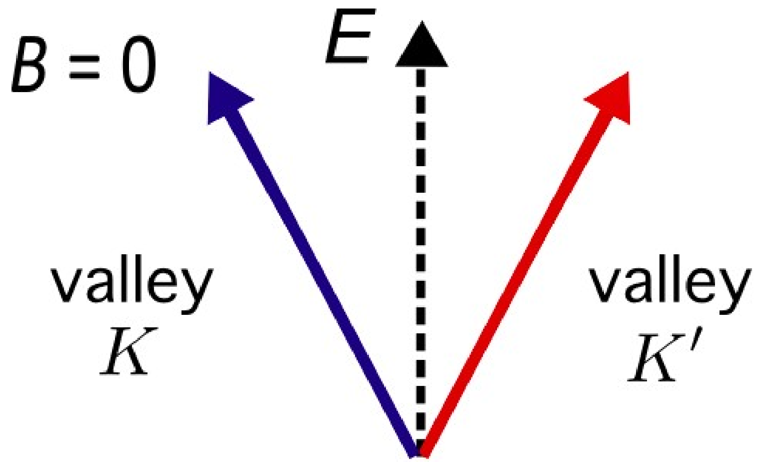
Detecting valley currents



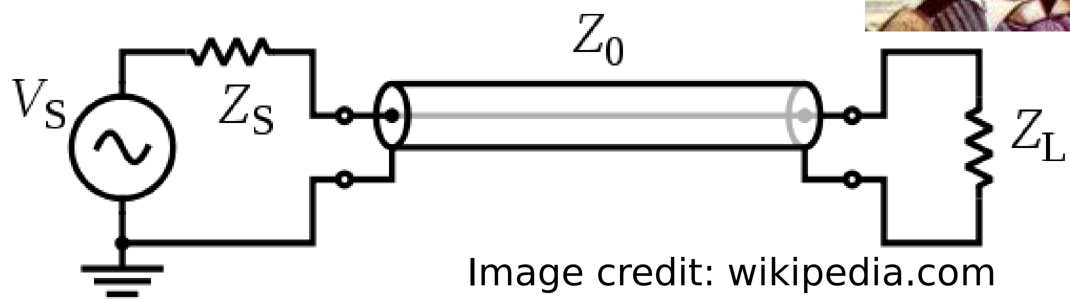


Image credit: wikipedia.com

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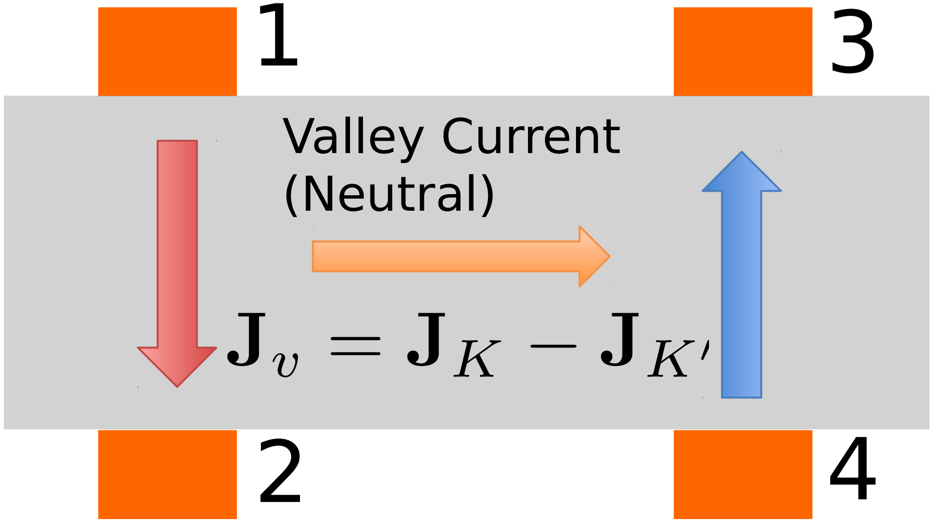

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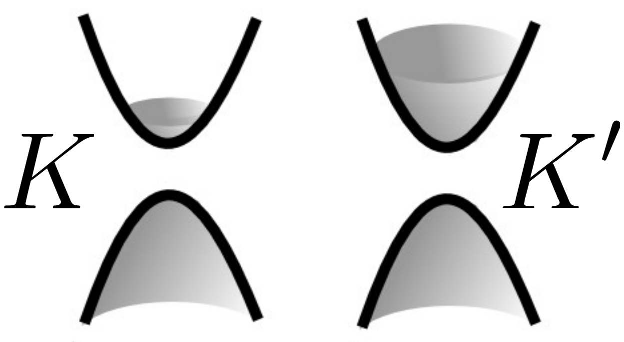

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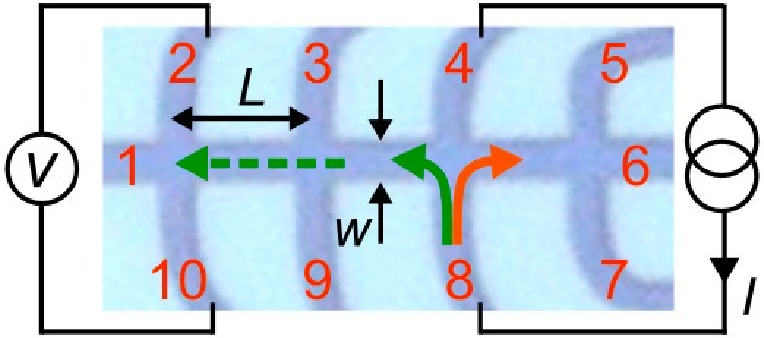
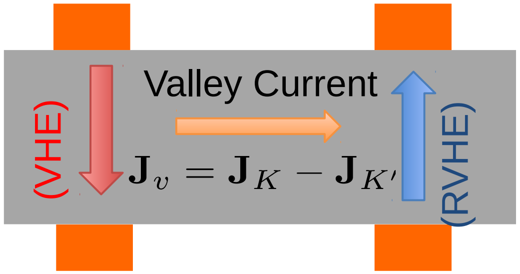
Nonlocal response in aligned G/hBN



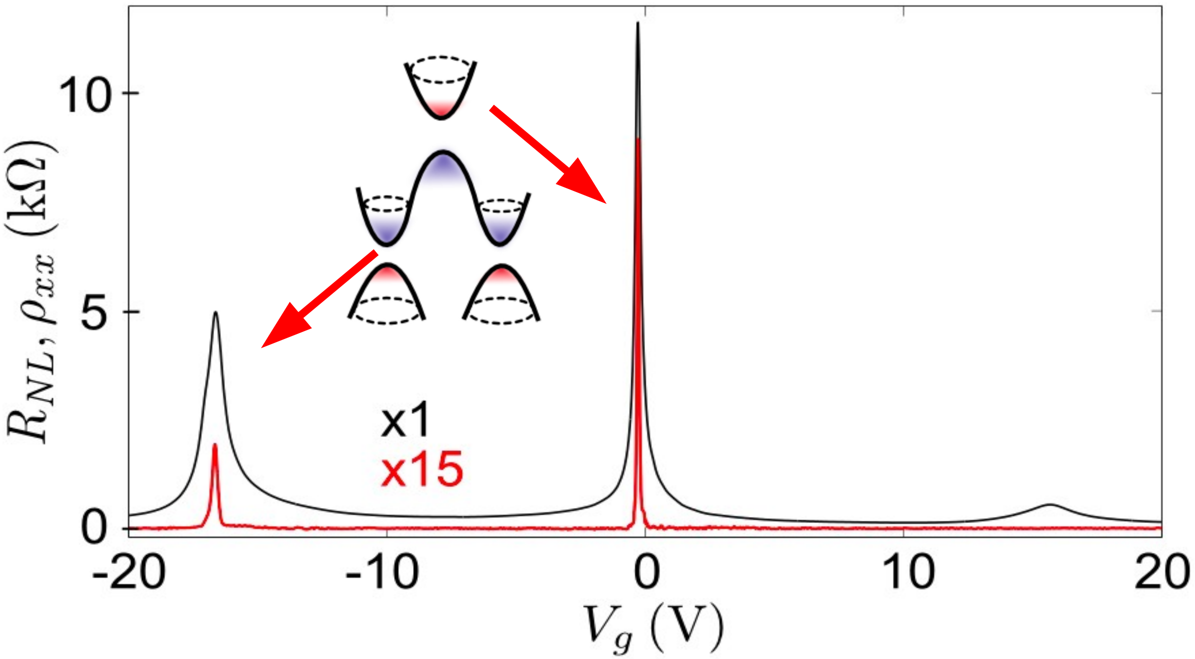
Gorbachev, Song et al
Science 346, 448 (2014)



$$V_{2,10} = R_{\rm NL} I_{4,8}$$

Van der Pauw bound: $R_{\rm NL}^{VdP} \approx \rho_{xx} e^{-\pi L/w}$

Berry hot spots



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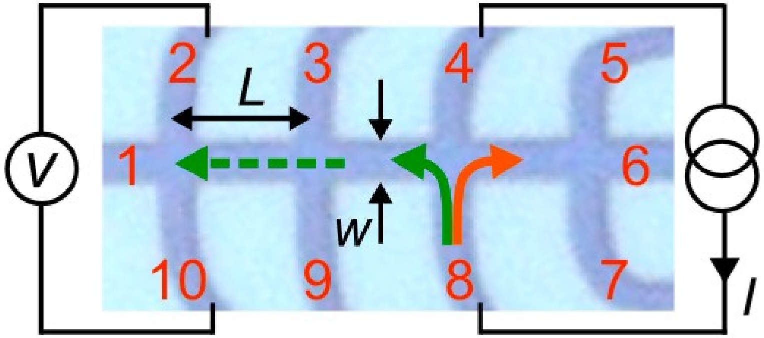

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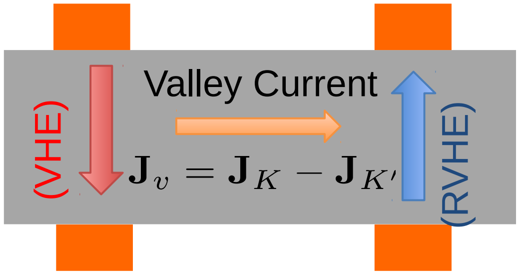

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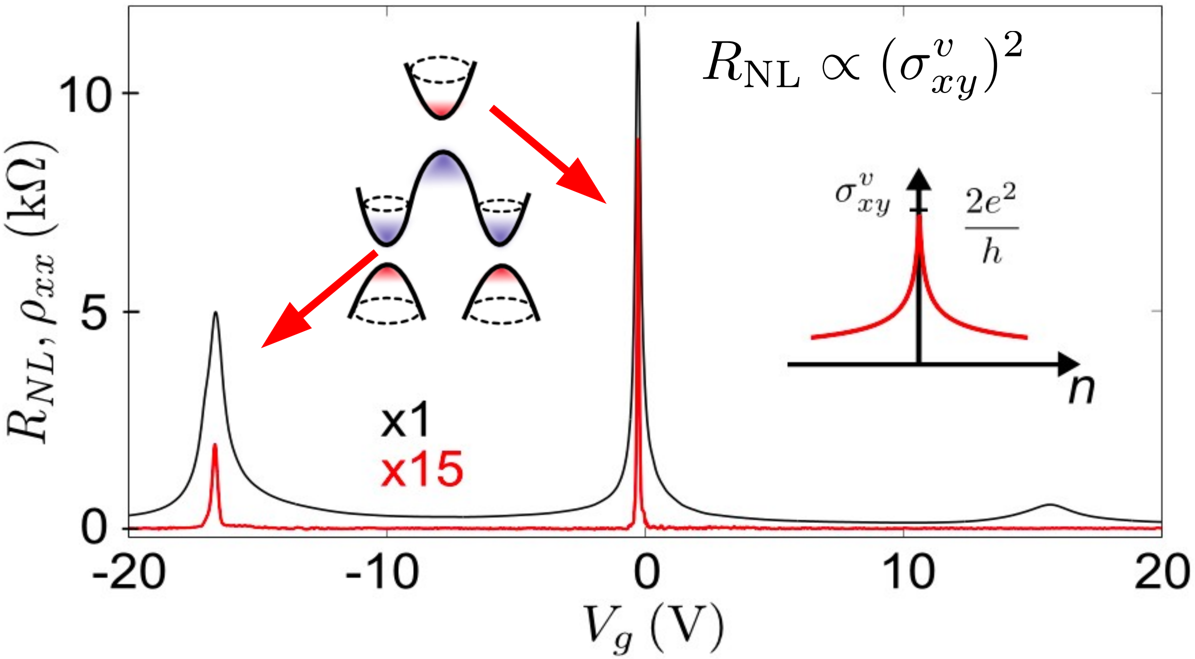

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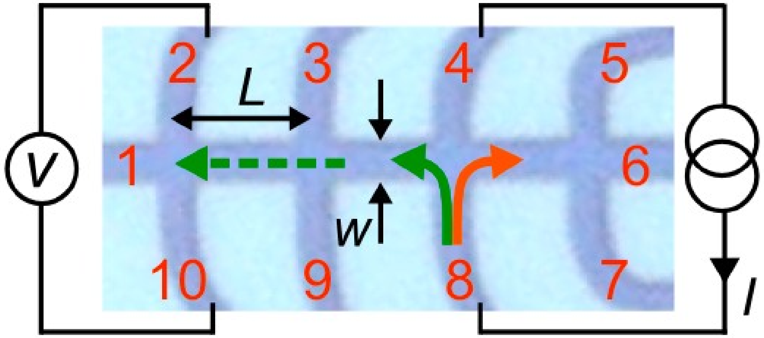

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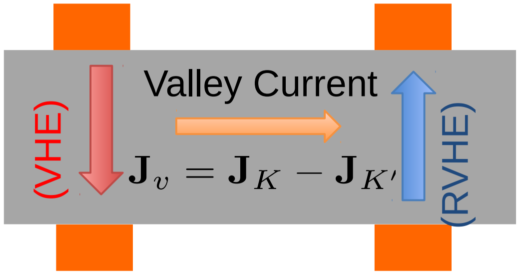

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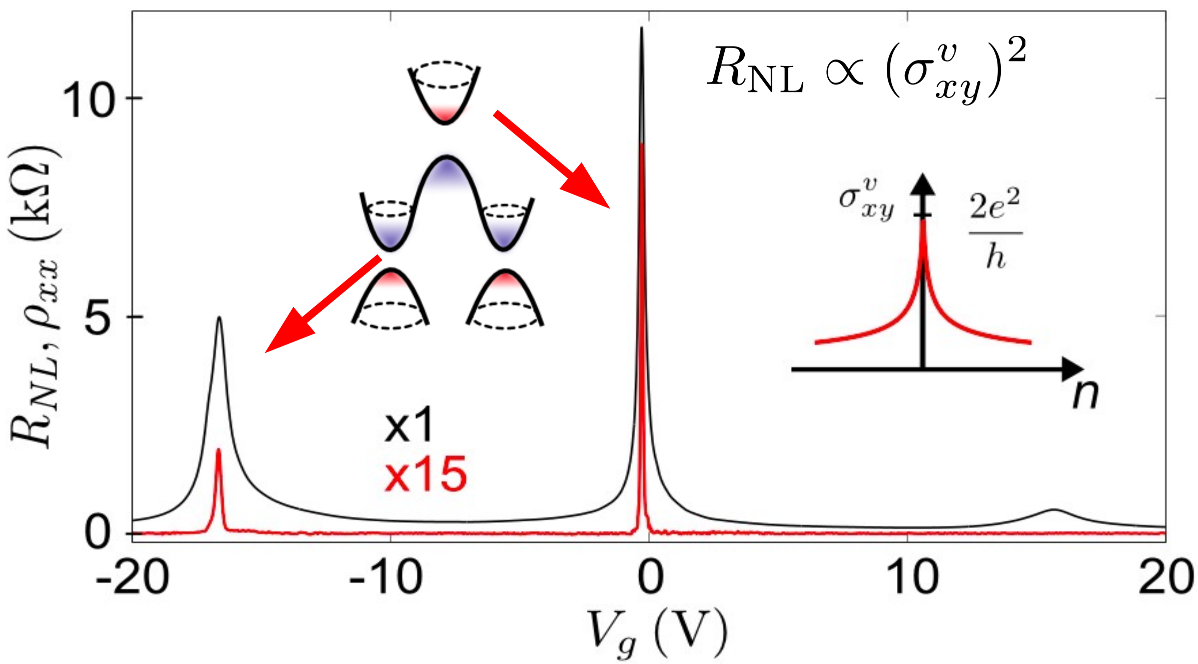
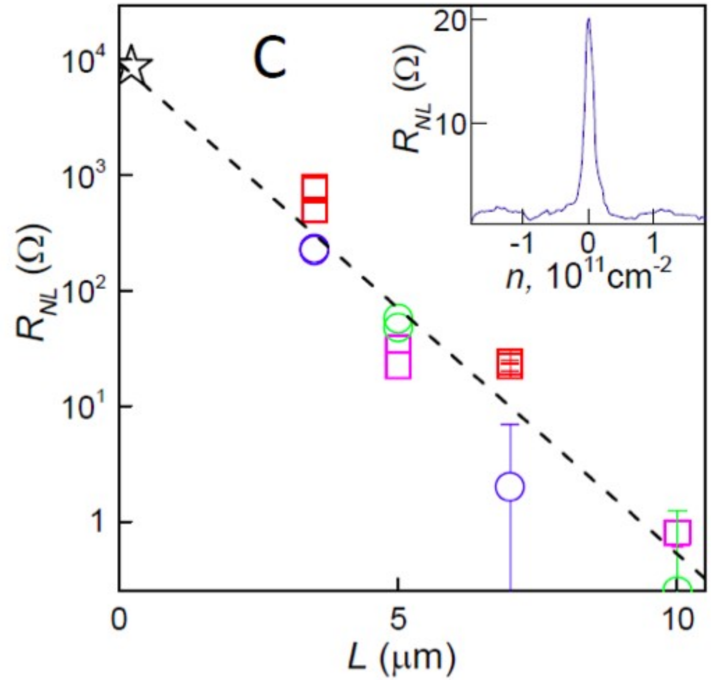


Distance dependence



Checklist

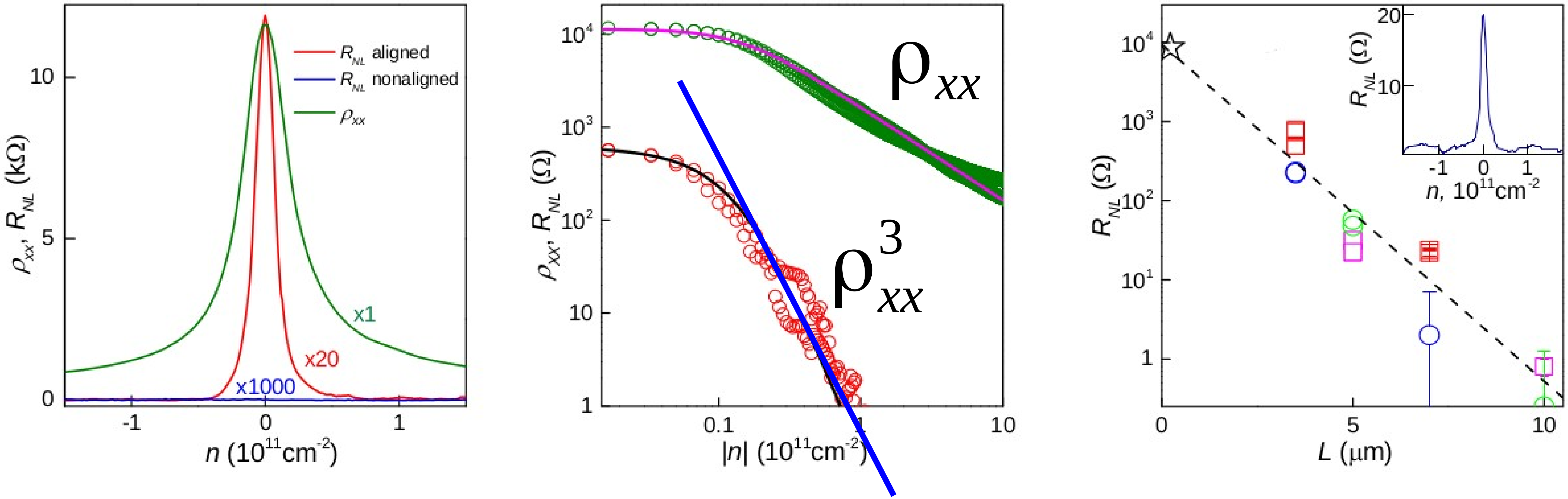
☑ 1) Stray ohmic currents too small, peaks in density dependence line up w Berry pockets; mediated by long-range neutral currents

2) Observed at B=0, excludes energy and spin (prev work)

3) Good quantitative agreement w/ topo valley currents for Berry curvature induced by gap opening

4) Seen in aligned G/hBN devices, never in nonaligned devices

5) Scales as cube of ρxx as expected for valley currents



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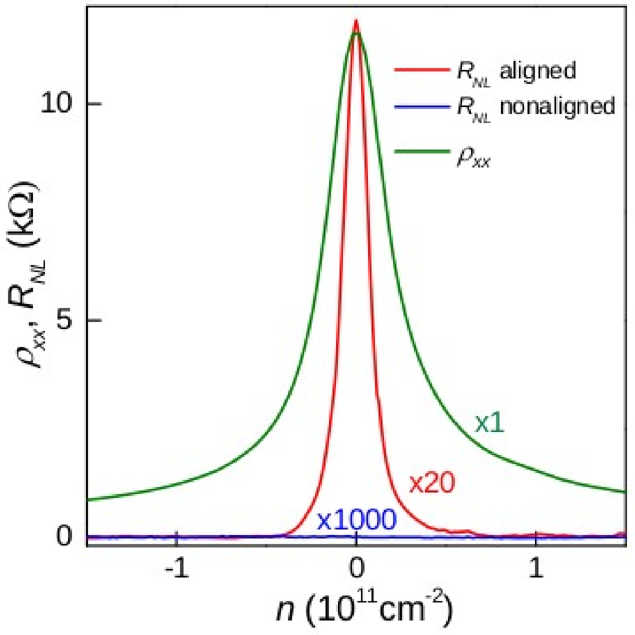
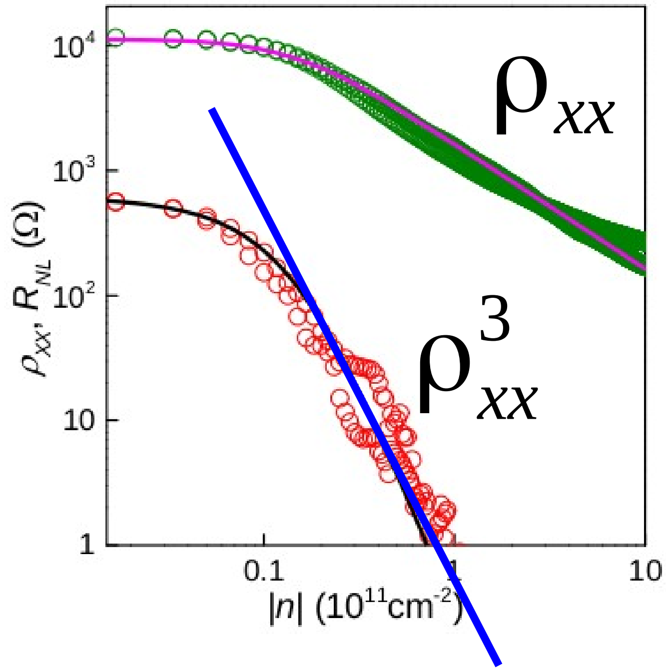
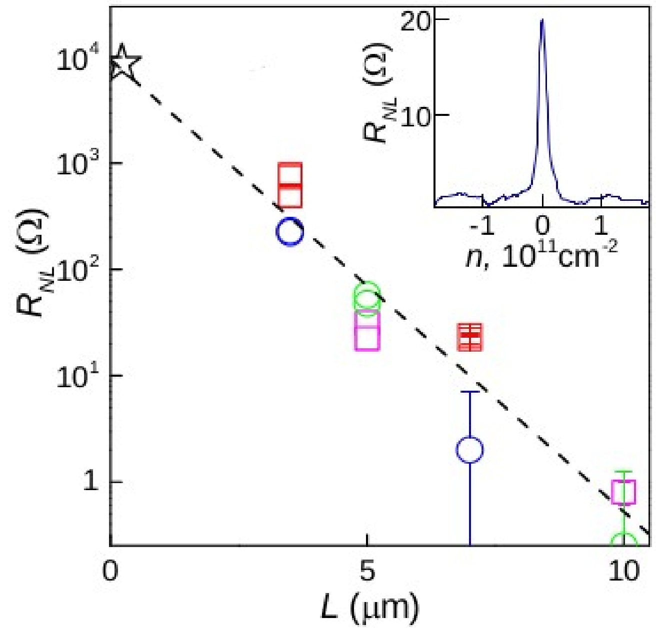
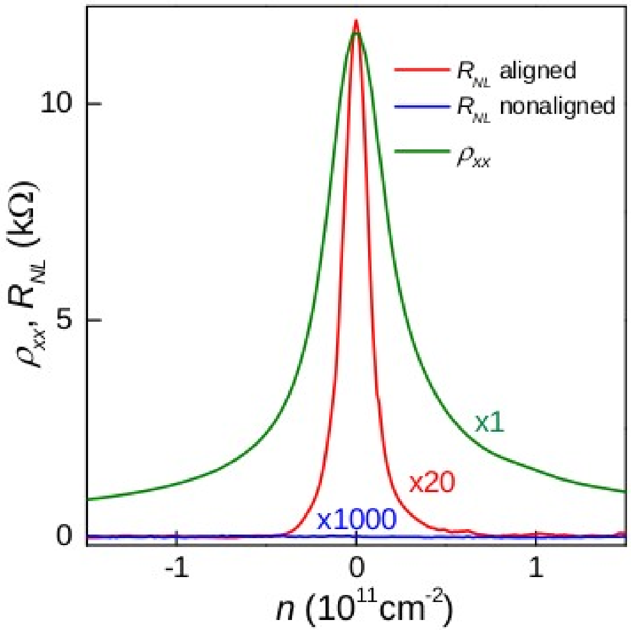


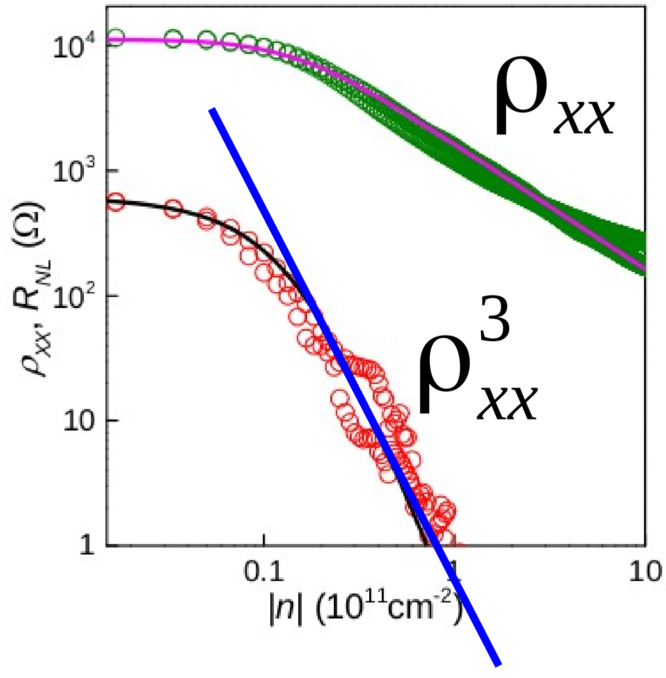


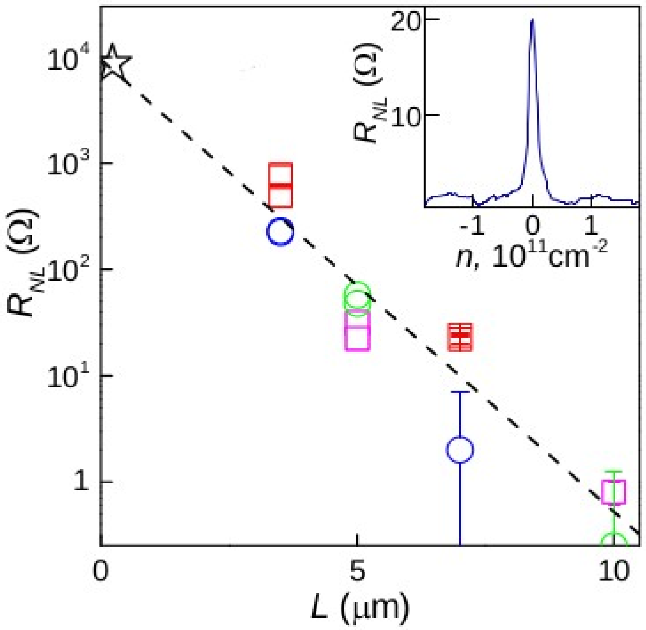


Checklist

- ☑ 1) Stray ohmic currents too small, peaks in density dependence line up w Berry pockets; mediated by long-range neutral currents
- ☑ 2) Observed at B=0, excludes energy and spin (prev work)
- ☑ 3) Good quantitative agreement w/ topo valley currents for Berry curvature induced by gap opening
- ☑ 4) Seen in aligned G/hBN devices, never in nonaligned devices
- ☑ 5) Scales as cube of ρ_{xx} as expected for valley currents

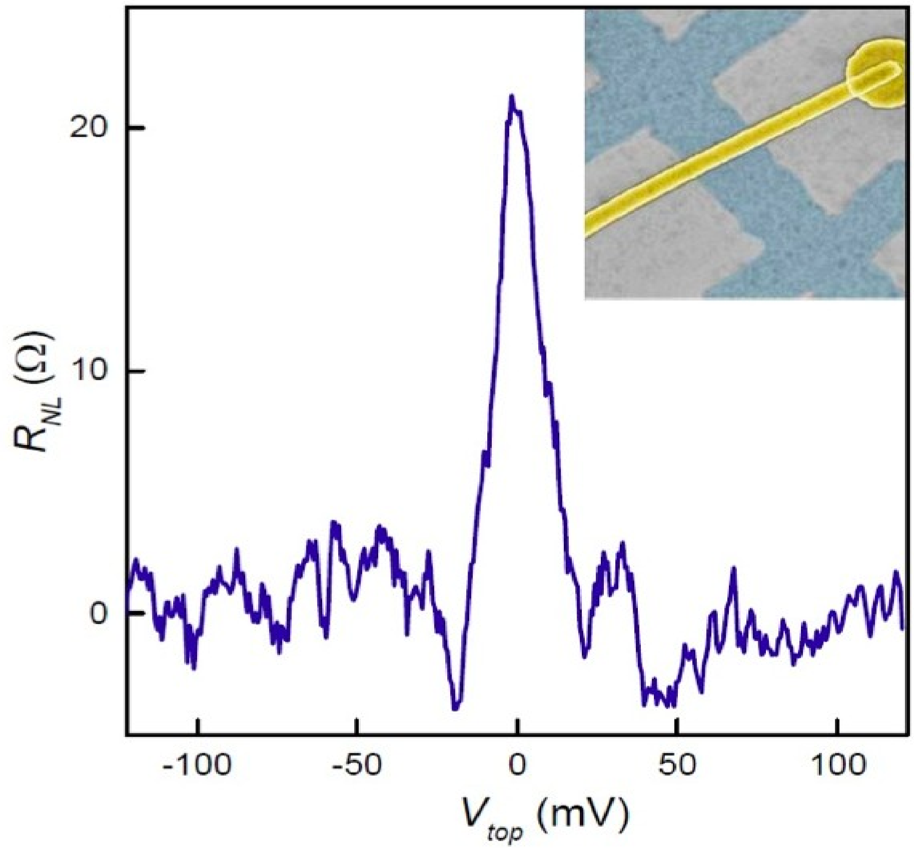




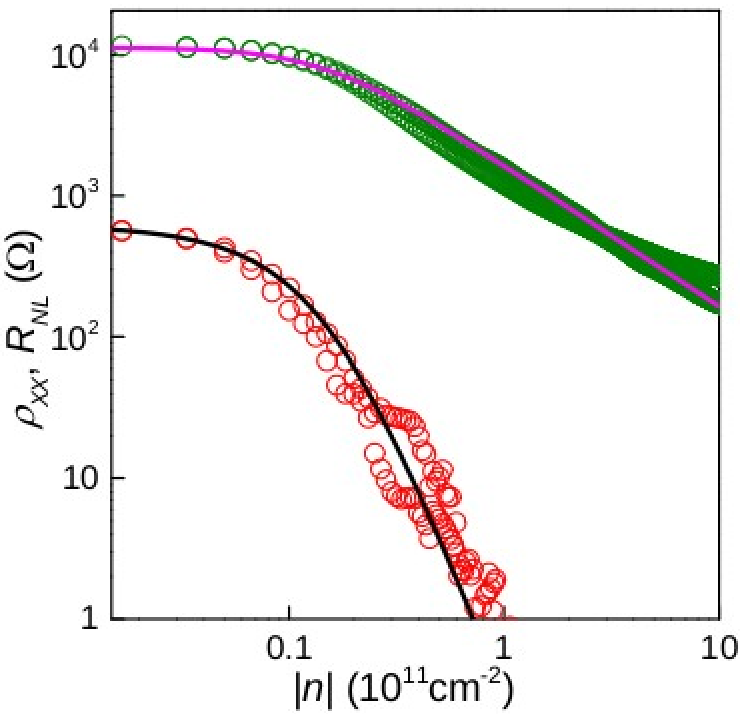


Valley transistor: proof of concept

1) Full separation of valley and charge current
2) ~140 mV/decade
3) Gate-tunable valley current



Modulation > 100 fold



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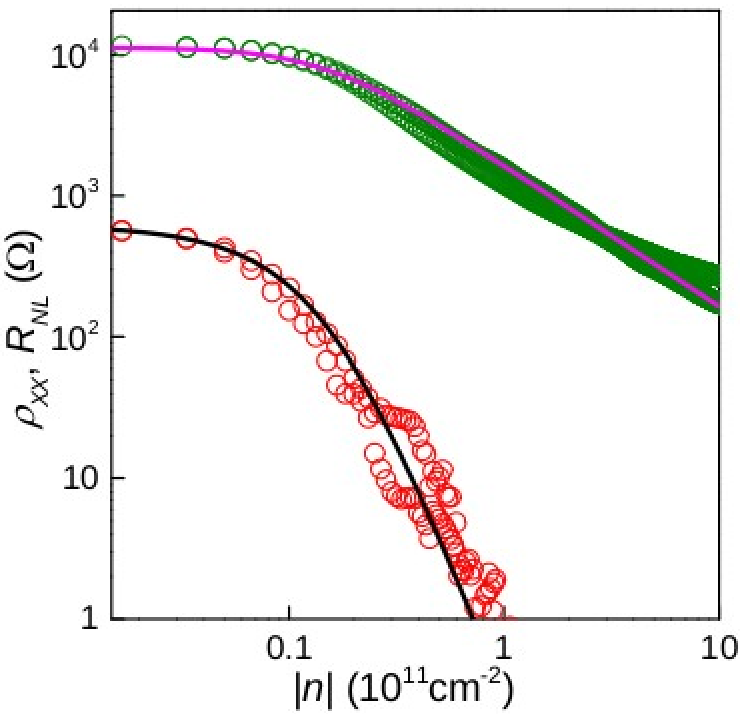
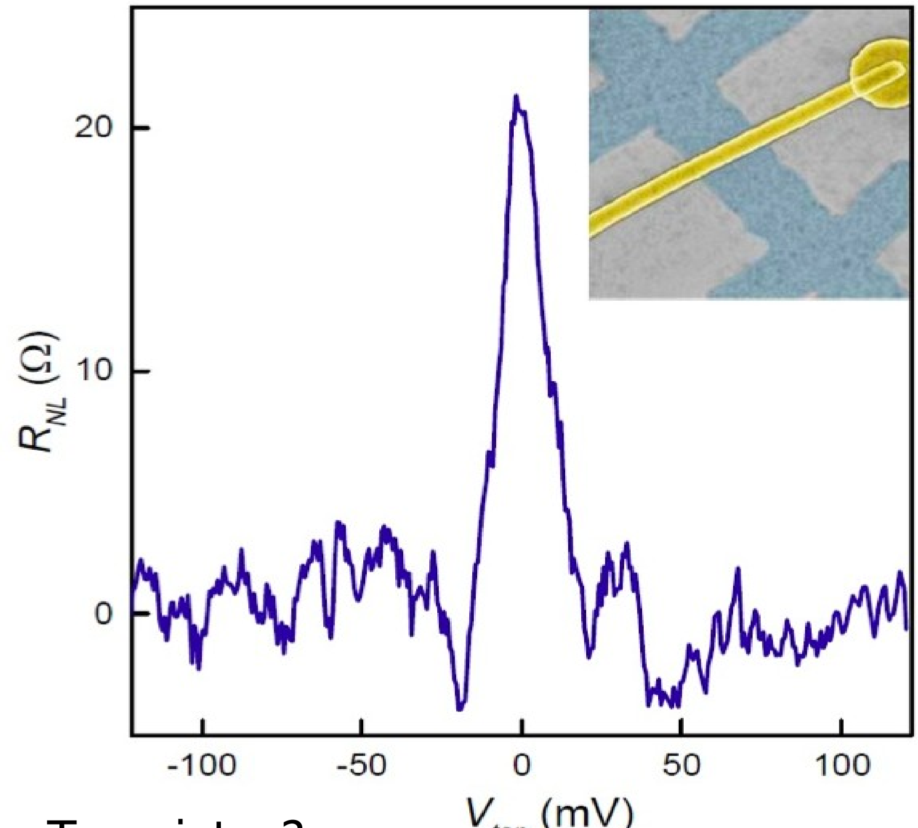
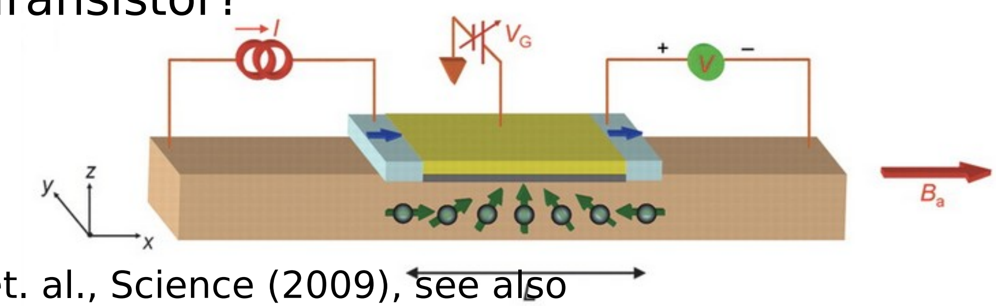


Gorbachev, Song, et. al. , Science (2014)



Spin Transistor?



Koo, et. al., Science (2009), see also
Wunderlich, et. al. , Science (2010)
Original Proposal: Datta, Das, APL (1990)

Summary

- **Valley currents:** charge-neutral, can mediate long-range electrical response
- Excited and detected using the Valley Hall effect

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- Topological domains, protected edge modes

Electron Viscosity and Nonlocal Response

Is hydrodynamics ever relevant?

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- In one-comp fluid or gas a hydrodynamic approach works b/c one has local conservation of energy and momentum
- All transport properties governed by just 3 quantities: the shear viscosity (η), the second viscosity (ζ), and the thermal conductivity (κ)

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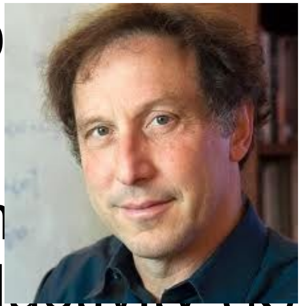

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PRL **106,** 256804 (2011)   PHYSICAL REVIEW LETTERS   week ending 24 JUNE 2011

Hydrodynamic Description of Transport in Strongly Correlated Electron Systems

A. V. Andreev,[1] Steven A. Kivelson,[2] and B. Spivak[1]

[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*
[2]*Department of Physics, Stanford University, Stanford, California 94305, USA*
(Received 16 November 2010; published 24 June 2011)

We develop a hydrodynamic description of the resistivity and magnetoresistance of an electron liquid in a smooth disorder potential. This approach is valid when the electron-electron scattering length is sufficiently short. In a broad range of temperatures, the dissipation is dominated by heat fluxes in the electron fluid, and the resistivity is inversely proportional to the thermal conductivity, κ. This is in striking contrast to the Stokes flow, in which the resistance is independent of κ and proportional to the fluid viscosity. We also identify a new hydrodynamic mechanism of spin magnetoresistance.

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- Elec… with the lattic…
- High… s e.g. some…



$$\rho_{2D} = \frac{1}{2e^2}\left\langle \frac{T}{\kappa}(\delta s_0)^2 + (\eta + \zeta)\left(\nabla \frac{1}{n_0}\right)^2 \right\rangle$$

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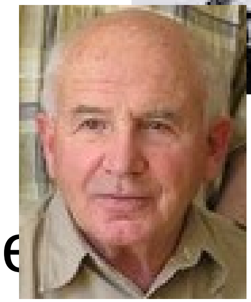
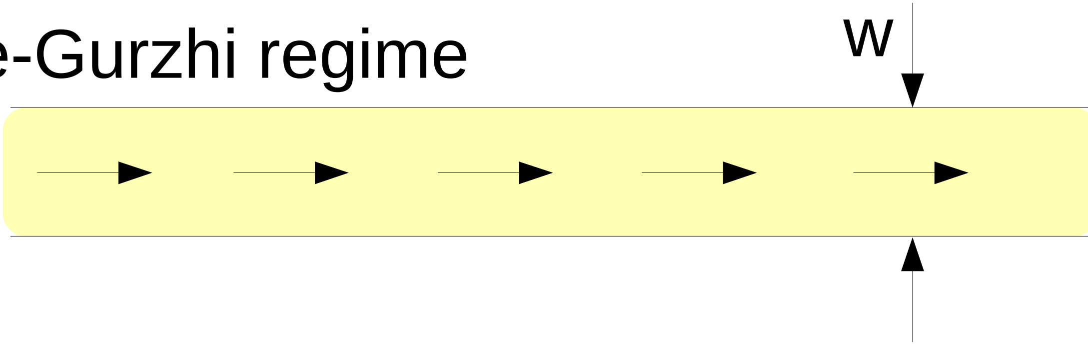
Collisions of ballistic carriers



- Temperature-dependent scattering time τ_{ee}~(E_F/T)^2
- Sample width ***w* << *l*ee** (low T) Knudsen-Fuchs regime
- ***w* >> *l*ee** (higher T) Poiseuille-Gurzhi regime
- Control value τ_{ee} by current



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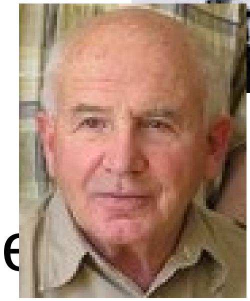

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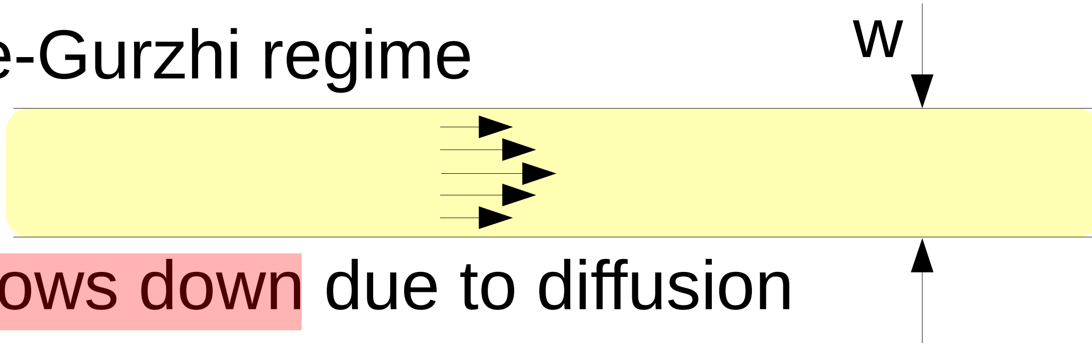
- R=dV/dI vs. I first grows then decreases



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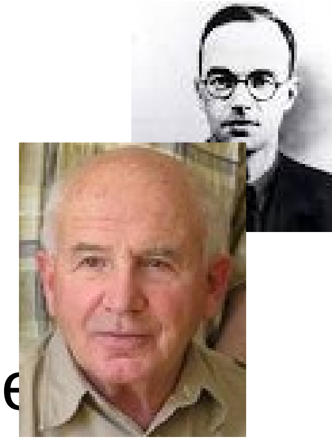

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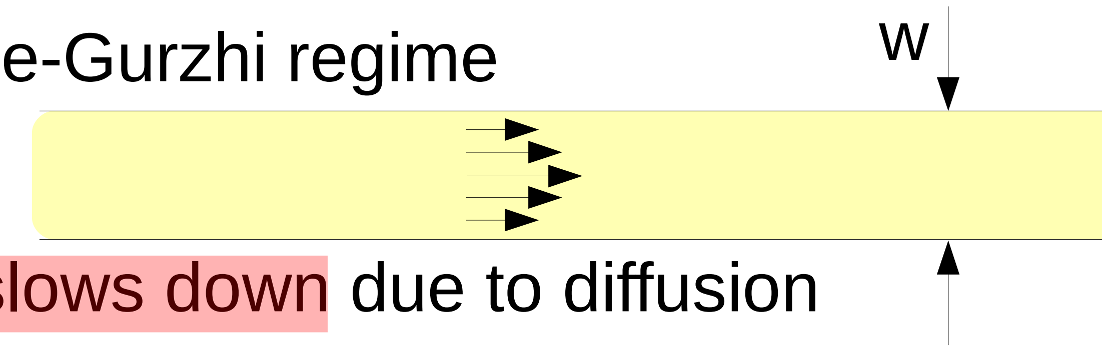
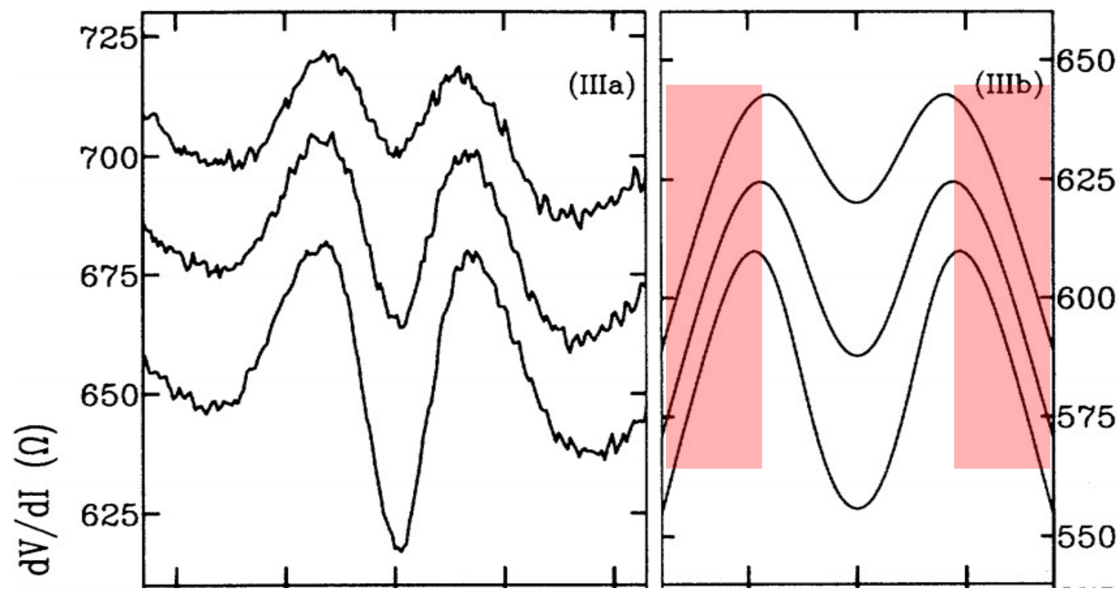


Tested in ballistic wires
de Jong & Molenkamp
1995



Critical electron fluid in graphene

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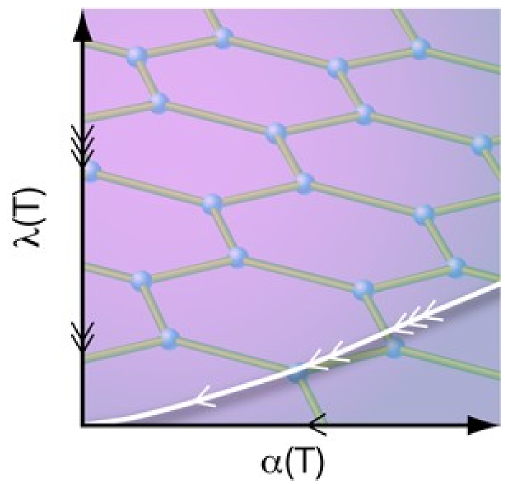

PRL **103**, 025301 (2009) PHYSICAL REVIEW LETTERS week ending 10 JULY 2009

Graphene: A Nearly Perfect Fluid

Markus Müller,[1] Jörg Schmalian,[2] and Lars Fritz[3]

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- Vanishing DOS but long-range interactions, strong coupling
- Fast p-conserving collisions, shear viscosity
- Low viscosity-to-entropy ratio η/s (near-perfect fluid)
- Comparable to universal low bound (AdS CFT, black holes)

PRL **94,** 111601 (2005) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2005

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics

P. K. Kovtun,[1] D. T. Son,[2] and A. O. Starinets[3]

PRL **103,** 025301 PHYSICAL REVIEW LETTERS week ending 10 JULY 2009

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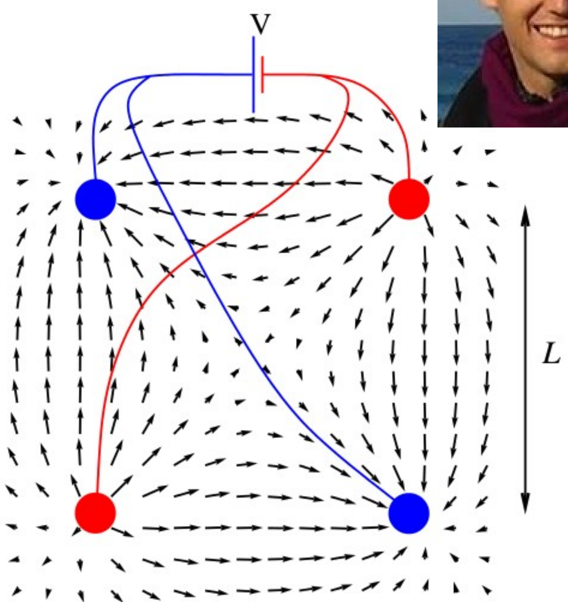
Measure viscosity?

- Scaling with system dimensions: R~L/W (Ohmic regime) vs. R~L/W^2 (Poiseuille-Gurzhi regime)



Mueller, Shmalian, Fritz

Measure viscosity?

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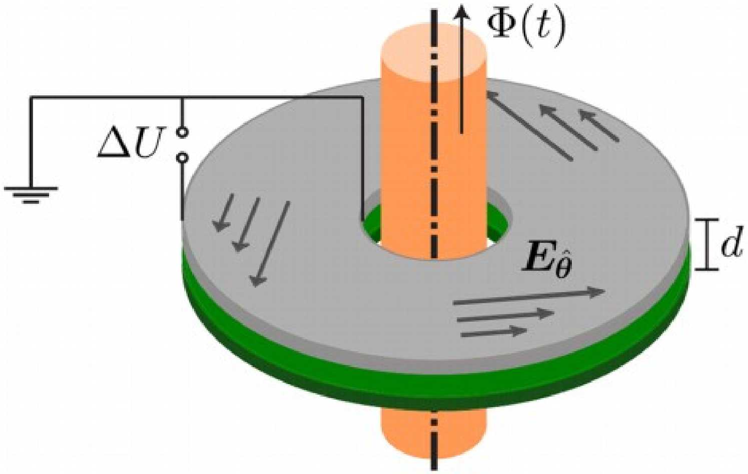
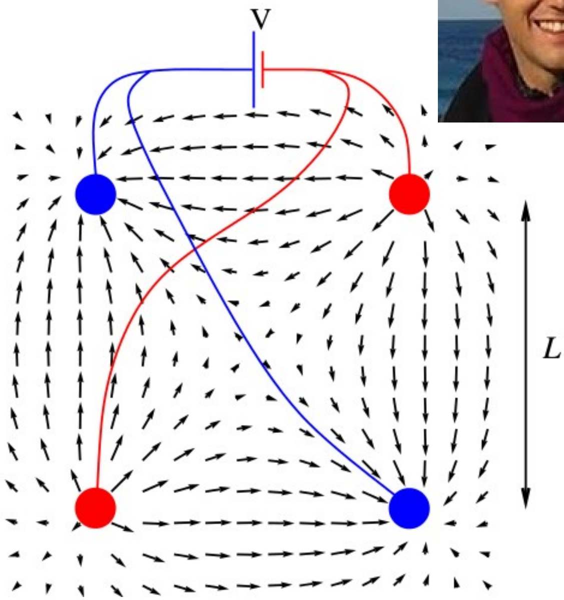
- Corbino geometry with a time-varying flux (Tomadin, Vignale, Polini)





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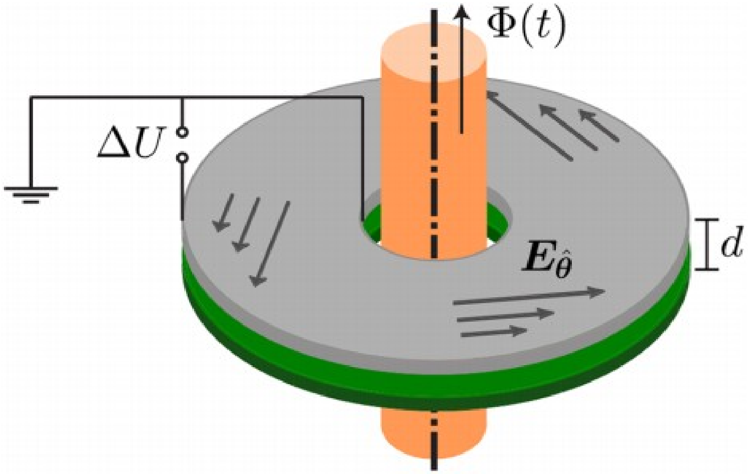
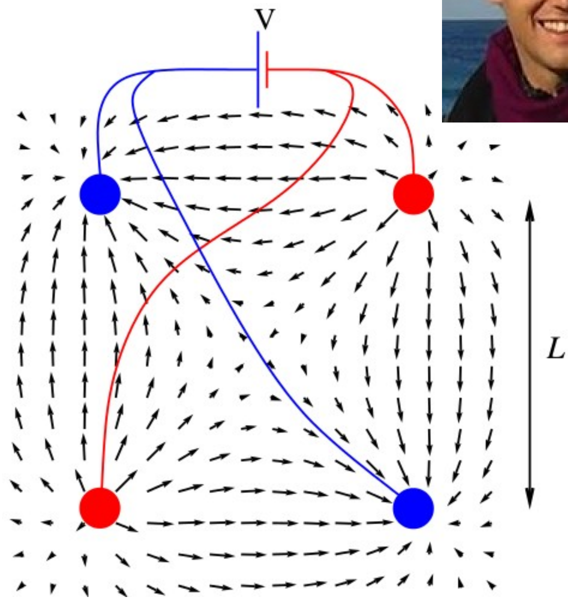


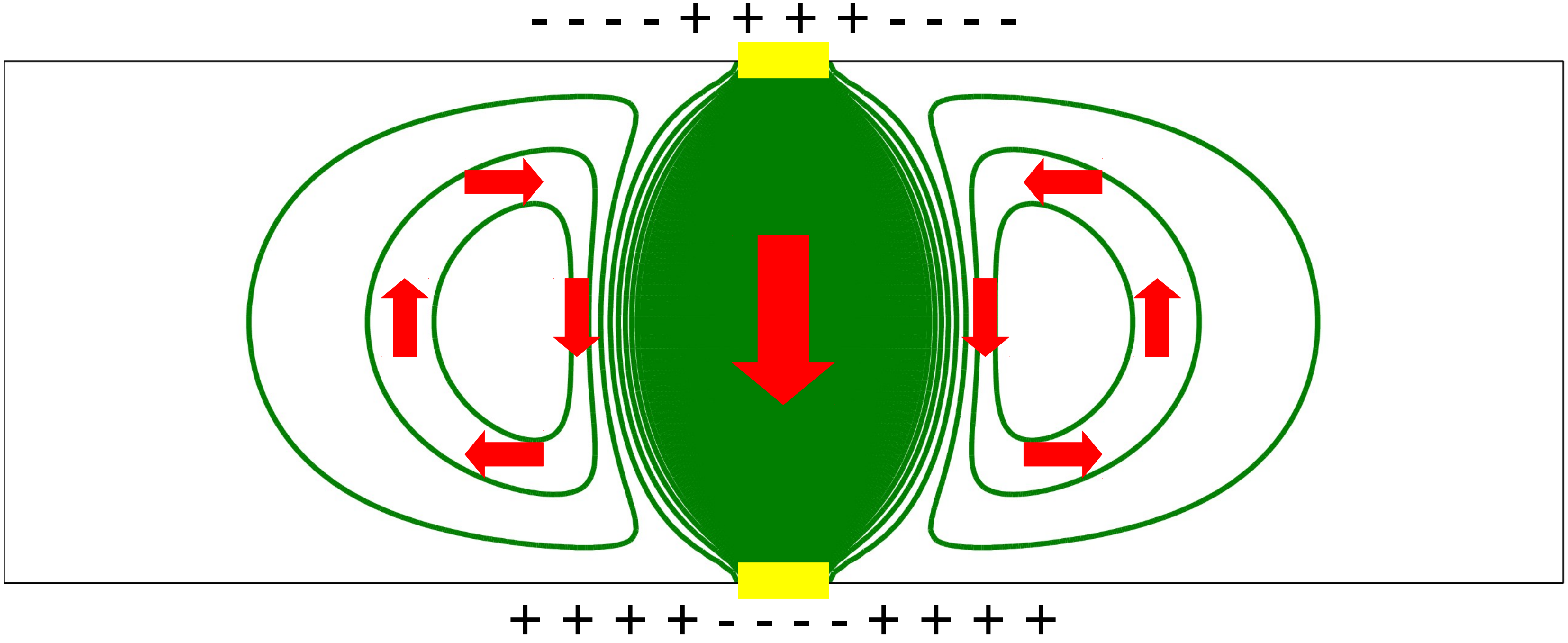


Mueller, Shmalian, Fritz

Challenge: do it w/ 1) one device; 2) linear response (no heating); 3) a DC measurement (no time dependence)

Negative nonlocal resistance

- Vortices launched by shear flow
- Reverse E field buildup due to backflow



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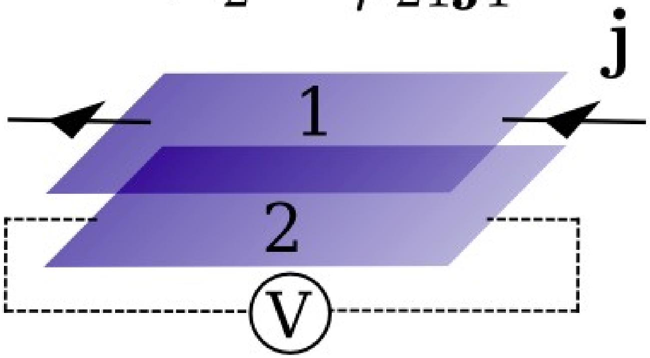
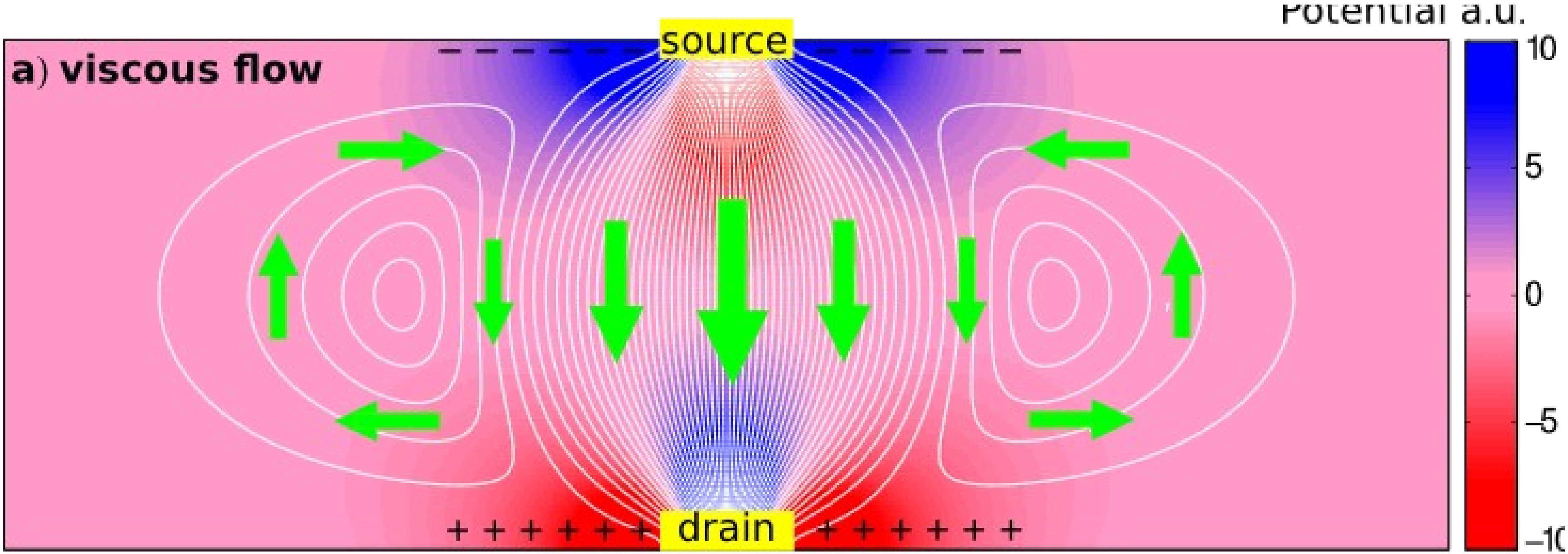
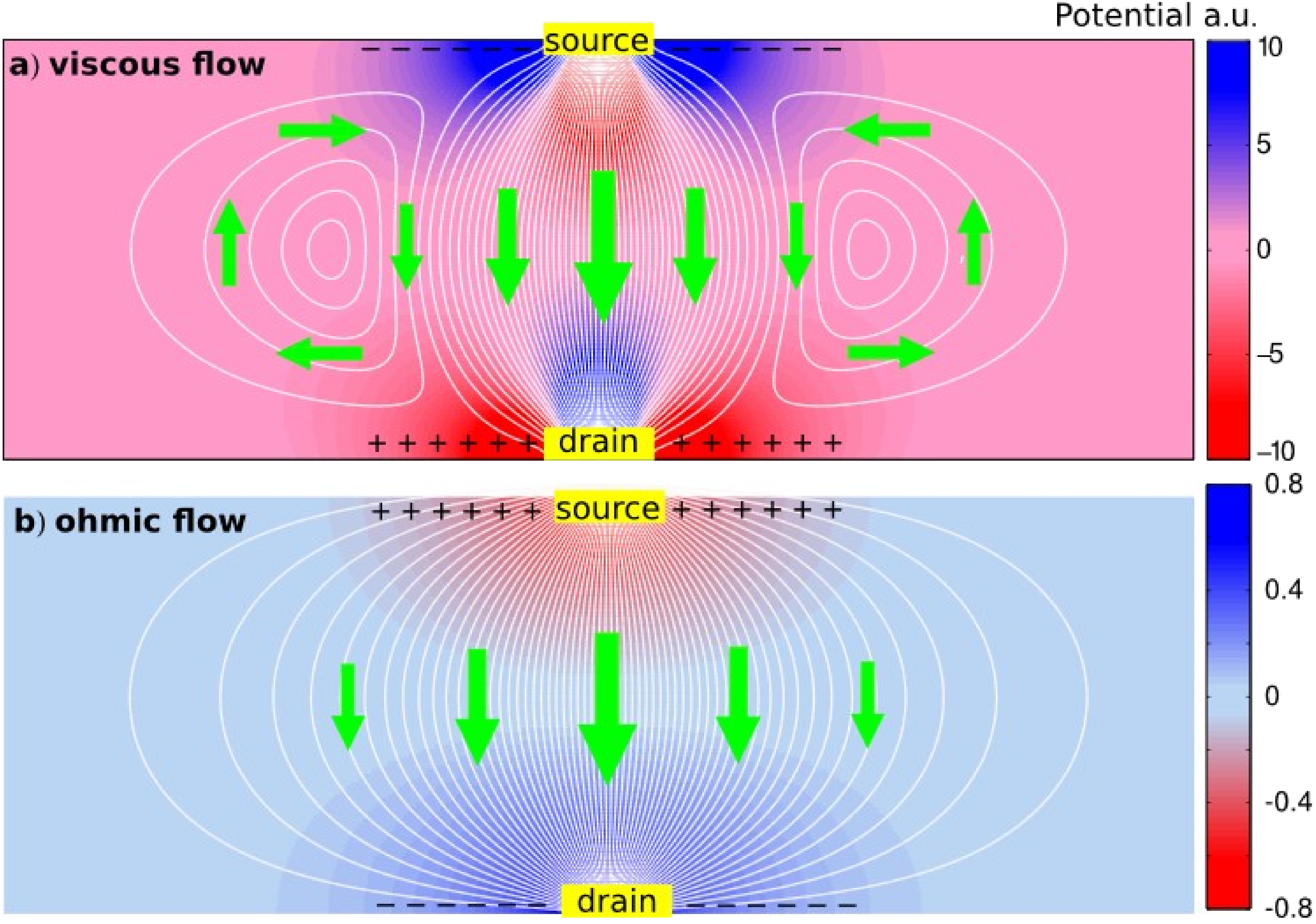
- Minus sign: analogous to Coulomb drag

$$\mathbf{E}_2 = \rho_{21}\mathbf{j}_1$$





Viscous vs. ohmic flow: a sign change



Minimal model

Continuity equation and momentum transport equation

$$\partial_t n + \vec{\nabla}(n\vec{v}) = 0, \quad \partial_t p_i + \nabla_i \sigma_{ij} = -\frac{1}{\tau} p_i$$

$$\sigma_{ij} = \frac{1}{2}\eta(\partial_i v_j + \partial_j v_i - \partial_k v_k \delta_{ij}) + (\zeta \partial_k v_k + P)\delta_{ij}$$

First and second viscosity (shear and dilation) $P = n(\mu + e\Phi)$

Boundary conditions:

continuity; realistic (partial slippage); idealized (no-slip)

$1: \; v_n = 0 \qquad 2: \; v_t = \alpha E_t \qquad 2': \; v_t = 0$

Solving for incompressible flow

Introduce a flow function: $\vec{v} = \nabla \times \psi \quad \left(\tau^{-1} = 0\right)$

$$\nu \nabla^2 \vec{v} = \nabla P \;\rightarrow\; \left(\nabla^2\right)^2 \psi = 0$$

$$\psi(x, y) = \sum_k e^{ikx}\left(\left(a + by\right)e^{ky} + \left(a' + b'y\right)e^{-ky}\right)$$

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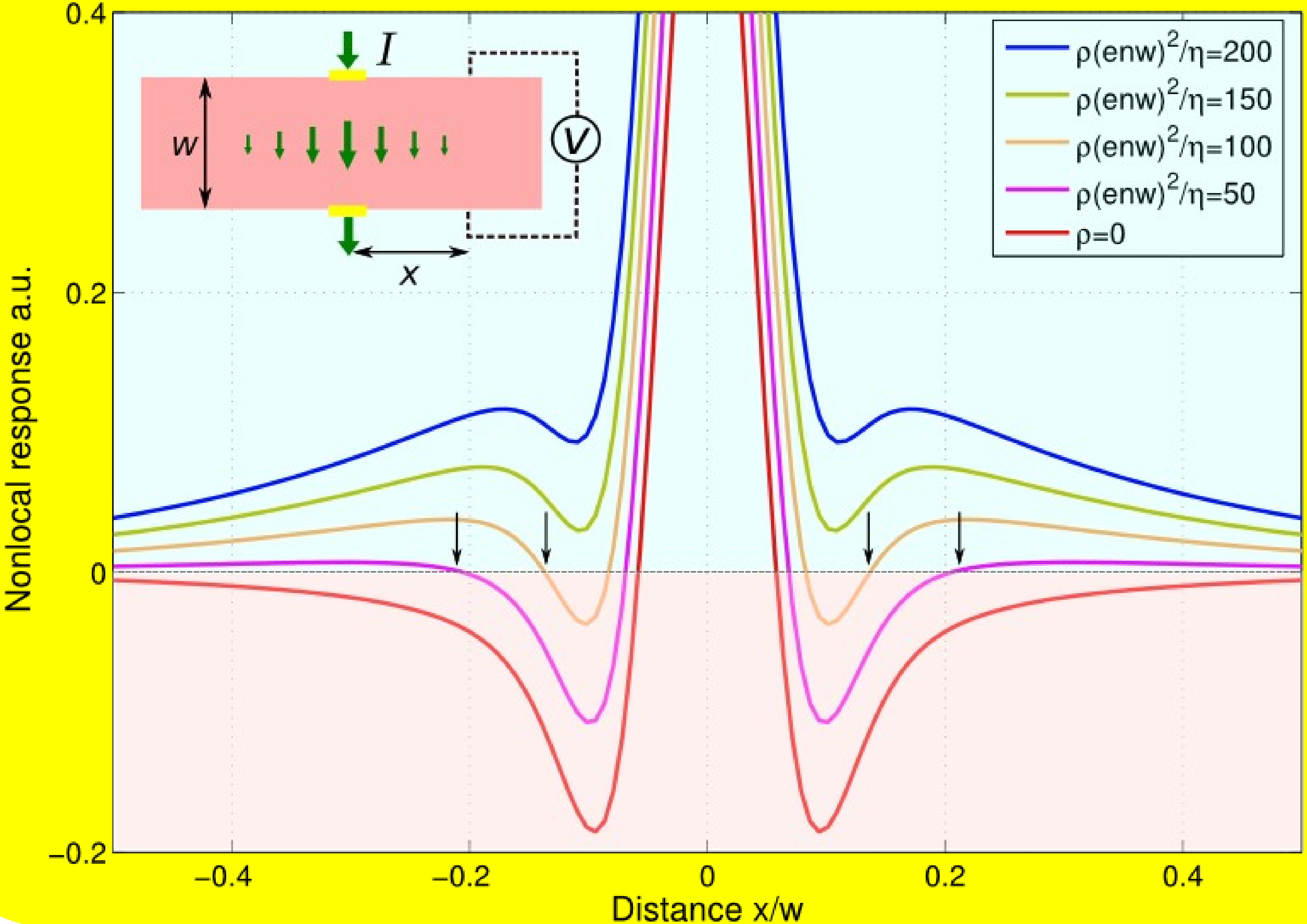
- Positive in Fourier space (**2 peaks**), **negative** in real space
- At very large L **positive** again due to residual ohmic effects
- Can use to **measure** the ν/ρ ratio

Solving for incompressible flow



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- **Next challenge:** measure second viscosity ζ (“string theory”: ζ=0 due to conformal symmetry)