Mixing in Oceans and Seas

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Ocean Climate Modelling:
Physical and biogeochemical **dynamics** of semi-enclosed **seas**.

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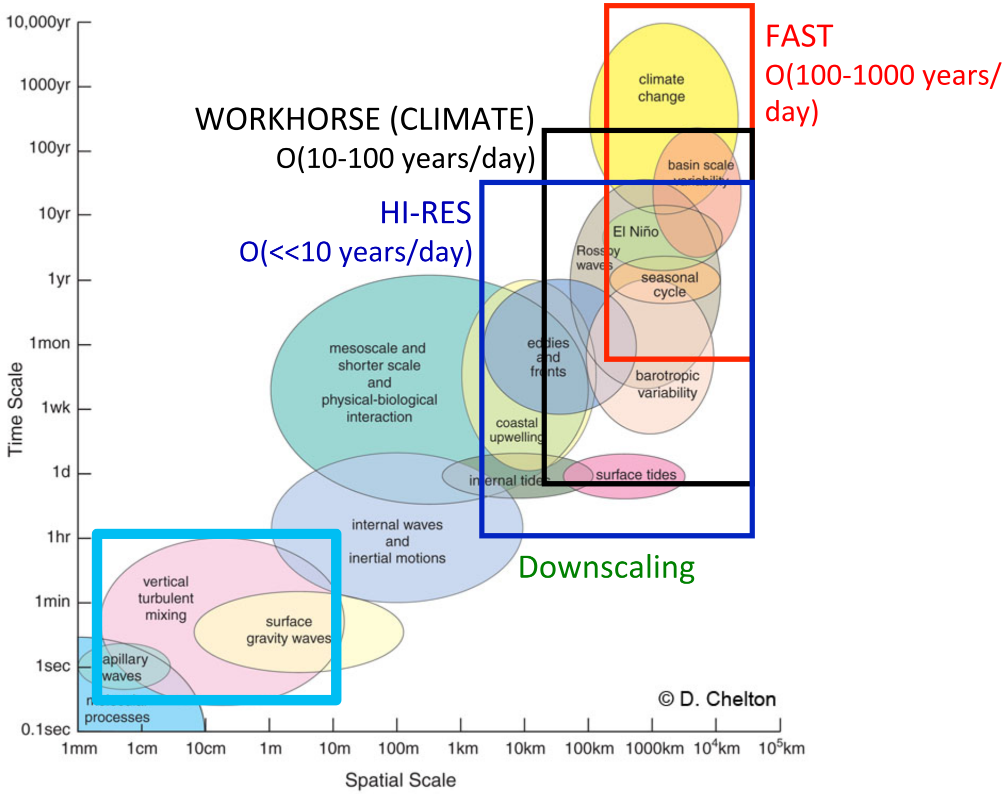
Ocean Modeling Challenges

Space – Time Scales and Global Ocean Models



Outline

1. **Non-linear mixing**
2. Turbulent Eddy Viscosity and Closures
3. Mixing Regimes in Oceans and Seas
4. The Surface Boundary Layer & Similarity Theory
5. The Convective Boundary Layer

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6. Modeling the Surface Boundary Layer
   --- Mixed Layer Models
   --- First Order Closures
7. The Viscous Surface Layer
8. A Practical Example: Diurnal Cycling

Non-linear terms of Navier-Stokes Equations ➔ [3-dimensional flow; $\underline{\boldsymbol{U}} = (U,V,W) = U_i$, i=1,3]

$$\partial_t U_i = - U \partial_x U_i - V \partial_y U_i - W \partial_z U_i$$

$$= - \underline{\boldsymbol{U}} \bullet \underline{\nabla} U_i$$

$$= - \partial_x (U U_i) - \partial_y (V U_i) - \partial_z (W U_i) \quad \text{(flux form)}$$

$$+ U_i [\partial_x U + \partial_y V + \partial_z W]$$

0 (continuity, incompressible)

Reynolds' Decomposition (Mean + Fluctuation)

For any state variable: $Q = \{U, V, W, T, S, P, \rho(T,S,P)\}$

$Q = \mathrm{Q} + q \qquad : \qquad \langle Q \rangle = \mathrm{Q} \; ; \quad \langle q \rangle = 0$

= Mean + fluctuation

Products : e.g. $(WQ) = (\mathrm{W} + w)(\mathrm{Q} + q)$

$= (\mathrm{WQ}) + w\mathrm{Q} + \mathrm{W}q + wq$

Average $\langle WQ \rangle = (\mathrm{WQ}) + \langle wq \rangle$

(eddy fluxes)

Navier-Stokes Equations ➔ Primitive Equations

Consider mean, U - momentum equation: $\partial_t U$ =

$- U \partial_x U - V \partial_y U - W \partial_z U$, Advection (non-linear)

$+ f V$, Coriolis (earth's rotation)

$- \partial_x P / \rho$, Pressure gradient

$- \partial_z \langle wu \rangle$, Turbulent vertical mixing (NL)

$- \partial_z \langle uu \rangle - \partial_z \langle vu \rangle$, Lateral mixing (NL)

$+ \upsilon_m \partial_{zz} U$, Molecular viscosity (Damping)

Numerical Model Decomposition:

Q = Resolved + Unresolved (sub-grid-scale)

Q + q

Use of same equations implicitly assumes equivalence to Reynolds' Mean + fluctuation decomposition

BUT in general < Unresolved, q > $\neq$ 0

Questions: What about <wQ> and <Wq> terms ?

Is <w> in upwelling regions identically zero ?

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Eddy Viscosity :

Reynolds' Stress Tensor, **$\underline{\sigma}$**

σ_{ij} = $-\langle u_i \; u_j \rangle$ = $-\,1/3\;\delta_{ij}$ - $\langle u_k \; u_k \rangle$ + d_{ij}

Eddy viscosity is a **<u>parameterization</u>** of the deviatoric component, d_{ij} ,
in terms of resolved shear $e_{kl} = ½\,[\nabla_k \; U_l + \nabla_l \; U_k\,]$:

$d_{ij} = T_{ijkl} \; e_{kl}$, where T_{ijkl} is a $3^4 = 81$ element tensor

Symmetries $T_{ijkl} = T_{jikl} = T_{ijlk} = T_{klij}$ leave only 21 independent

Eddy Viscosity in Ocean Models :

Usually specify only two viscosities: a vertical, K_u

a horizontal (lateral), υ_H

Some specify three : a vertical K_u

a downstream lateral, A_H

a cross-stream lateral, B_H

$$T_{1111} = T_{2222} = 2\,\upsilon_H = A_H + B_H$$

$$T_{3333} = K_u + \upsilon_H = K_u + A_H$$

$$T_{1212} = \upsilon_H = B_H$$

$$T_{1313} = T_{2323} = K_u = K_u$$

$$T_{1133} = T_{2233} = \upsilon_H - K_u = B_H - K_u$$

Large , et. al., 2001, J. Phys. Oceanogr.)

Cartesian Constant Coefficients, A_H & B_H

$$\partial_t U = -\partial_x\langle uu\rangle - \partial_y\langle vu\rangle - \partial_z\langle wu\rangle + \ldots\ldots..$$

$$= A_H\,\partial_{xx}U + B_H\,\partial_{yy}U + K_u\,\partial_{zz}U + \ldots\ldots..$$

$$\partial_t V = -\partial_x\langle uv\rangle - \partial_y\langle vv\rangle - \partial_z\langle wv\rangle + \ldots\ldots..$$

$$= B_H\,\partial_{xx}V + A_H\,\partial_{yy}V + K_u\,\partial_{zz}V + \ldots\ldots..$$

Low B_H allows currents to grow
High A_H keeps numerics stable

Turbulence Closures :

The order (n^{th}) of a turbulence closure is given by the moment (n^{th}) of the highest prognostic equation

$\partial_t U = - \partial_z \langle wu \rangle$ U is a 1^{st} moment; 1^{st} order)

$\partial_t \langle uw \rangle$ = terms with 3^{rd} moments (e.g. $\langle uwq \rangle$) (2^{nd} order)

$\partial_t \langle uwx \rangle$ = terms with 4^{th} moments (e.g. $\langle uwq^2 \rangle$) (3^{rd} order)

….. And so on and so on and so on …………

The Fundamental Problem of Turbulence :

Consider a system of $Q = \{\ U, W, T\ \}$:

Order	1st	2nd	3rd	4th ….
Unknowns	U W T	uu ww tt uw ut wt	uuu www ttt uuw uut uwt utt uww wwt wtt	$N_4 > 10$

	Equations	Unknowns	Excess Unknowns
1st order	3	3+6 = 9	6
2nd order	9	9+10 = 19	10
3rd order	19	19 + N_4	$N_4 > 10$

As the order of the closure (number of prognostic equations) increases the number of unknowns increases faster and solutions don't converge.

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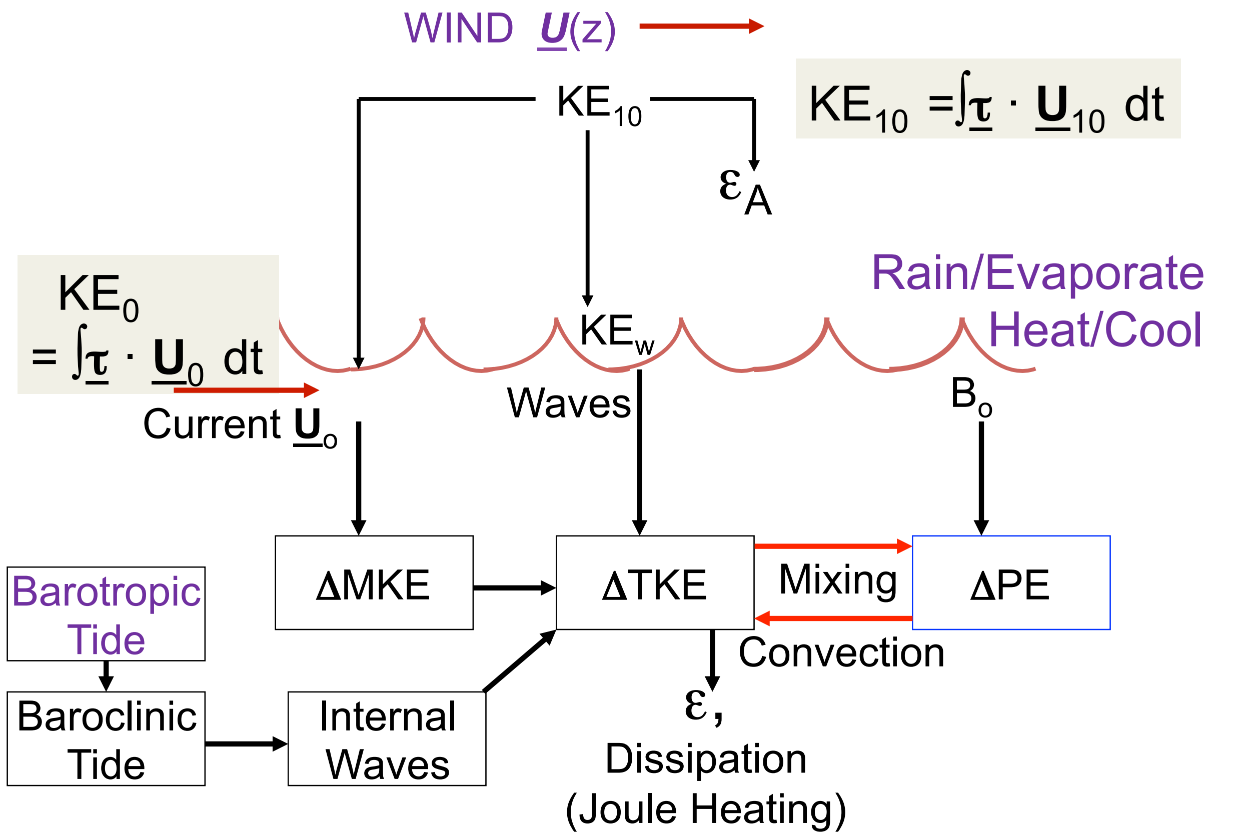
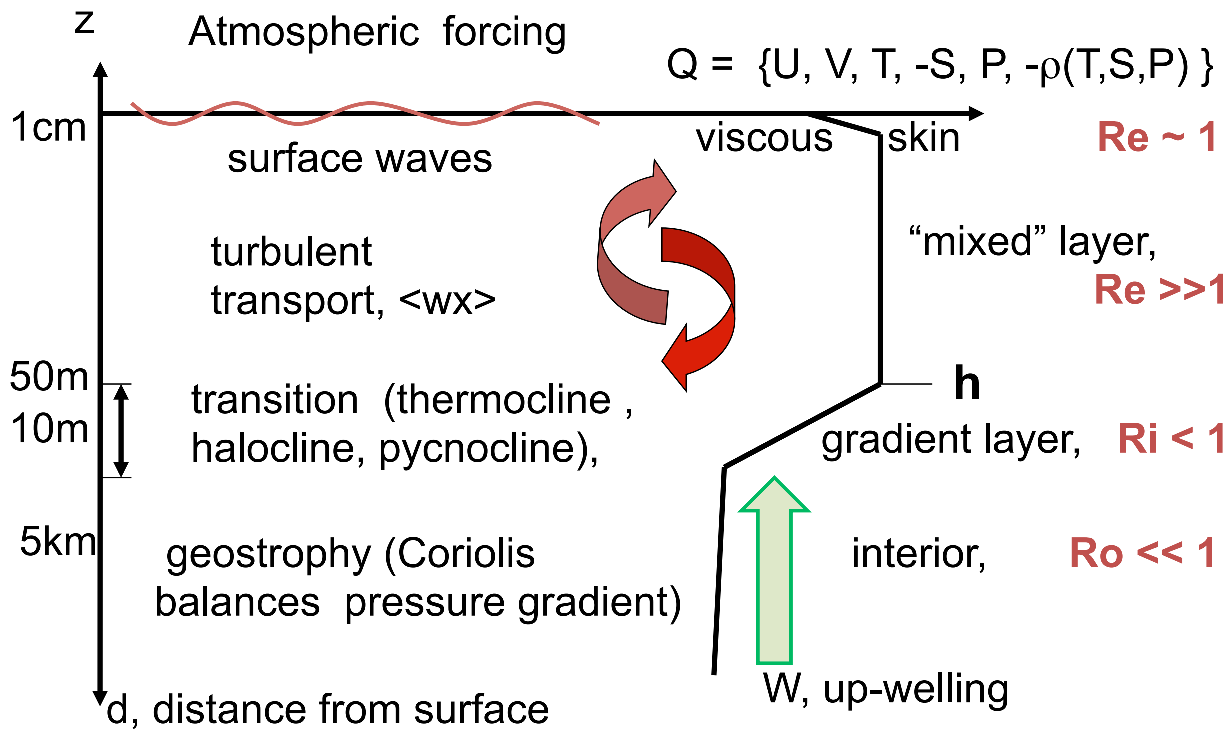
Mixing Energetics (Sources)



Turbulent Mixing Regimes
(changing balance of terms with, d)



Interior (small Rossby Number)

Rossby Number, $Ro = \frac{\text{non-linear}}{\text{Coriolis}} = \frac{U}{f\ d}$,

Where $f = 2\ \Omega \sin(\text{latitude})$ is the vertical Coriolis parameter $\approx 10^{-4}\ s^{-1}$

Therefore , far from the boundary there will be a geophysical fluid interior , characterized by $Ro \ll 1$:

geostrophic flow ===>> [$f U = - \partial_y P / \rho$; $f V = \partial_x P / \rho$]

Vertical velocity given by continuity : $\partial_z W = - [\partial_x U + \partial_y V]$

$W(\text{bottom}) = 0$

Pressure, $P = \int_{-d}^{0} \rho(T,S,P)\ dz$

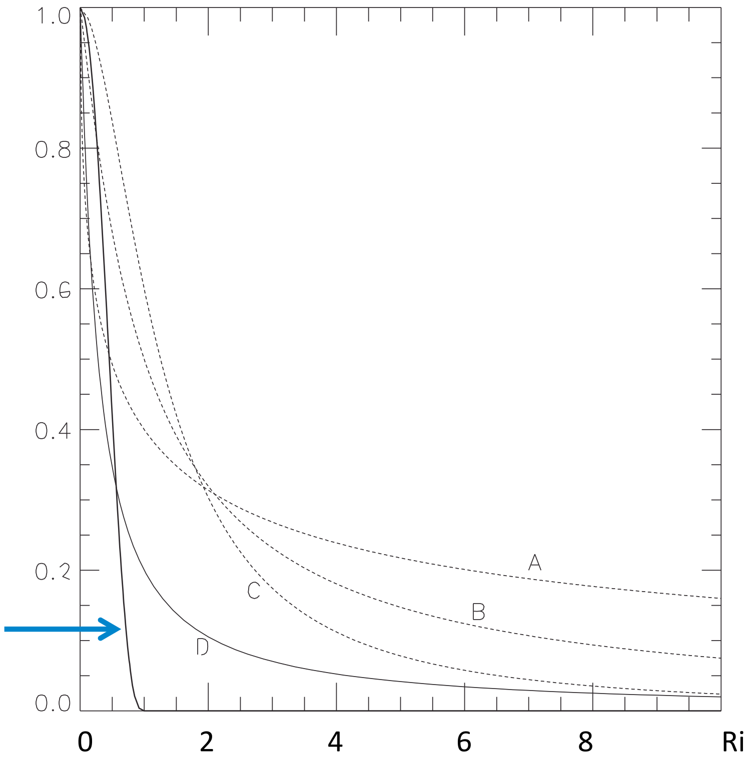
Interior (Vertical) Mixing

	Viscosity, K_u	$Pr = \frac{\text{Viscosity}}{\text{Diffusivity}}$
• Dynamic instability (K-H)	K_o^{sh} Ƒ(Ri)	1
• Double diffusion instability	$K_q^{dd}(R_\rho)$	1
• Internal wave breaking	K_o^{iw}	10

- Tidal Energy (Bryan and Lewis, JGR, 1979)
  (Simmons et. al., Ocn. Mod. 2004)

$K_o^{iw} = 1\ cm^2/s$; $K_o^{sh} = 50\ cm^2/s$; $K_q^{dd}(R_\rho) < 5\ cm^2/s$

Functions of Ri, Ƒ(Ri)



Equatorial Undercurrent
Pacanowski-Philander, 81

K_V = .1 + 50(cm^2/s) Ƒ(Ri)

Low Richardson Number, Ri <1

Stratified Shear Flow : Buoyancy $B(z) = -g\,\rho(z) / \rho_o$ [m/s^2]

Stratification gives Buoyancy Frequency, N, $N^2 = \partial_z B > 0$ [s^{-2}]

Shear is $\partial_z \mathbf{V}$ [s^{-1}] for $\mathbf{V} = (U, V)$, high shear is unstable
lots of kinetic energy, KE

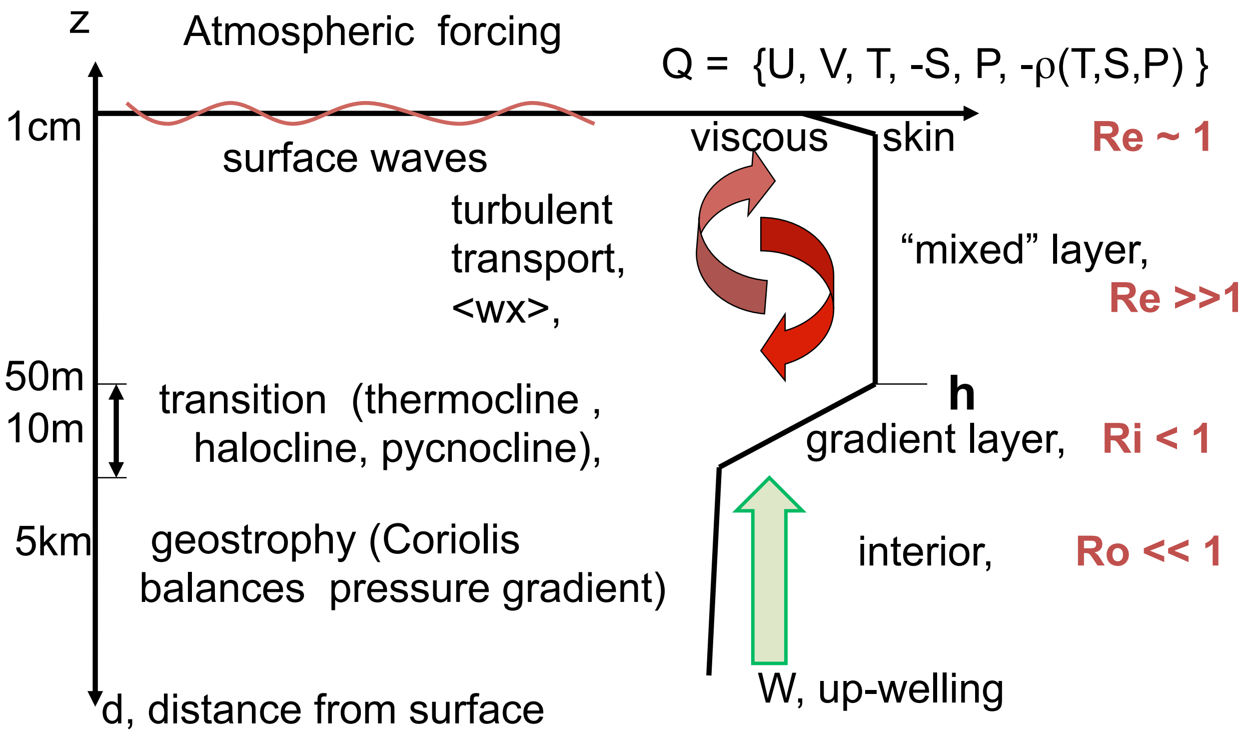
High stratification means stable (negative) potential energy, PE

$$Ri = \frac{N^2}{(\partial_z \mathbf{V})^2} = \frac{\text{stable PE}}{\text{available KE}} < 0.25$$

(empirical) ===>> local turbulent mixing (K-H) (Kelvin-Helmholtz)

NON - DIMENSIONAL

Turbulent Mixing Regimes (changing balance of terms with, d)



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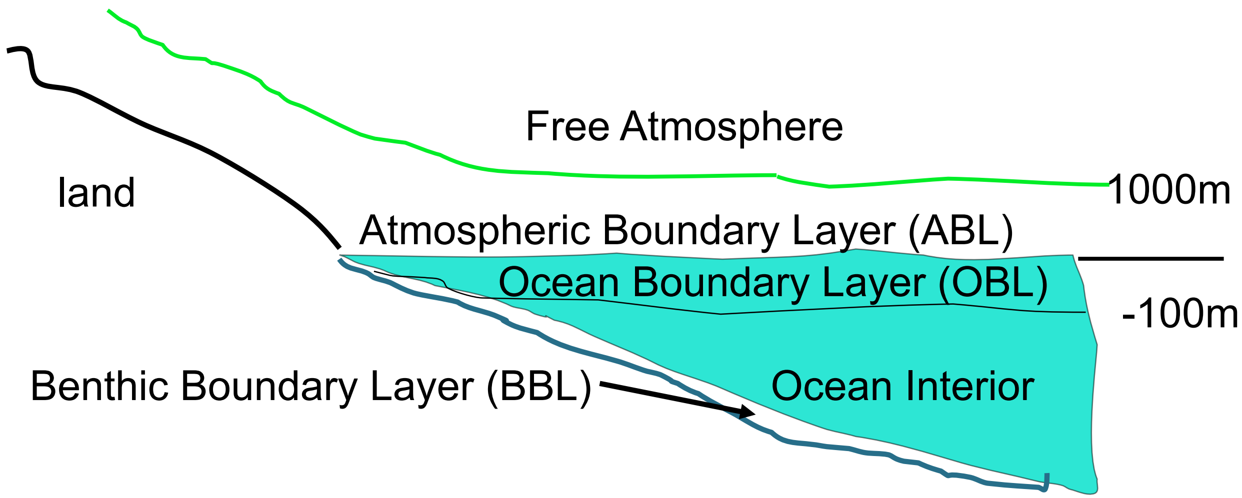
The Sea Surface Boundary Layer

-The portion of an ocean or sea that is directly influenced (forced) by the boundary

-Geophysical fluids "feel" the earth's rotation, $\Omega = 7.3 \times 10^{-5}\ s^{-1}$



The free atmosphere and ocean interior connect through the OBL and ABL

Turbulent Surface Forcing

Wind Stress, $\underline{\boldsymbol{\tau}}_o = (\tau_x , \tau_y)_o$

Freshwater flux, F_o = P , Precipitation, > 0
+ E , Evaporation, usually < 0
+ M , Melt; Sea-ice. Ice-bergs

Surface heat flux, Q_o
= Q_{nsol} , non-solar heat fluxes < 0
+ SW_{net} (0) , net surface solar radiation > 0
- SW_{net} (d_s), , solar not driving the OBL

In limit of $d_s = 0$, solar radiation does not drive OBL,
Clearly d_s should not be beyond the OBL

3) Surface Kinematic Fluxes

$$|\langle \mathbf{v}\, w \rangle_o| = |\boldsymbol{\tau}_o| \;/\; \rho_o = u^* u^* = u^{*2}$$

$$\langle w\, t \rangle_o = -Q_o \;/\; (\rho_o C_p) = u^* \; t^*$$

$$\langle w\, s \rangle_o = F_o \; S_o \;/\; \rho_o = u^* \; s^*$$

Surface buoyancy flux $B_o = -g\,(\alpha \langle w\, t \rangle_o - \beta \langle w\, s \rangle_o)$

$\alpha = (2 \rightarrow 4) \times 10^{-4}\ C^{-1}$; $\beta = 3.5 \times 10^{-4}\ (\text{psu})^{-1}$

Monin-Obukhov Length, $L = u^{*3} \;/\; (\kappa\ B_o)$; < 0 unstable

Depth where wind power (= Force x Velocity = u^{*3})
equals PE loss (gain) due to $B_o>0$ ($B_o< 0$) $= \kappa\, B_o\, L$

3) Monin-Obukhov Similarity Theory

Near the surface of a boundary layer, but away from the surface roughness elements, the ONLY important turbulence parameters are the distance, d, and the surface kinematic fluxes.

$$| \langle \mathbf{v}\, w \rangle_o | = |\boldsymbol{\tau}_o| / \rho_o = u^* u^* = u^{*2}$$

$$\langle w\, t \rangle_o = -Q_o / (\rho_o C_p) = u^*\, t^*$$

$$\langle w\, s \rangle_o = F_o\, S_o / \rho_o = u^*\, s^*$$

Monin-Obukhov Length, $L = u^{*3} / (\kappa\ B_o)$; < 0 unstable

Depth where wind power (= Force x Velocity = $\kappa\, u^{*3}$) equals PE loss (gain) due to $B_o > 0$ ($B_o < 0$) = $B_o\, L$

3) Monin-Obukhov Similarity Theory

Near the surface of a boundary layer, but away from the surface roughness elements, the ONLY important turbulence parameters are the distance, d, and the surface kinematic fluxes.

KEY : Dimensional Analysis
5 parameters (u^*, t^*, s^*, d, L)

4 units (m, s, ºK, psu)

Non-dimensional groups are functions of (d/L), the stability parameter (< 0, unstable)

3) Dimensional Analysis (d = -z)

Non-dimensional gradients : $-\partial_z Q \;\; d \; / \; q^* \;\; \propto \;\; \phi_q (d/L)$,

Empirically $\kappa = 0.4$, von Karman constant ,
makes ϕ_q (0) = 1 in neutral (wind only) forcing ($B_o = 0$, $L \rightarrow \infty$)

$\kappa \, \partial_z Q \;=\; q^* / z$ ➔ neutral logarithmic profiles, Q(z)

Let : $\langle w\, q\rangle_o \;=\; u^* q^* \;=\; -K_q \;\; \partial_z Q$, defines diffusivity, K_q

Near the surface of a PBL similarity theory (MOS) says

$$K_q \rightarrow -\frac{u^* \;\; x^*}{\partial_z Q} \;\;=\;\; \frac{\kappa \, d \, u^*}{\phi_q \; (d/L)} \;\; \underset{\text{neutral}}{\longrightarrow} \;\; \kappa \;\; u^* \, d_-$$

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The convective SBL ($B_o < 0$)

Surface buoyancy flux, $B_o = -\langle wb \rangle_o < 0$

Wind stress, $\tau_o = 0$; $u^* = 0$

Convective Velocity Scale , $w^* = (-B_o\ h)^{1/3}$,

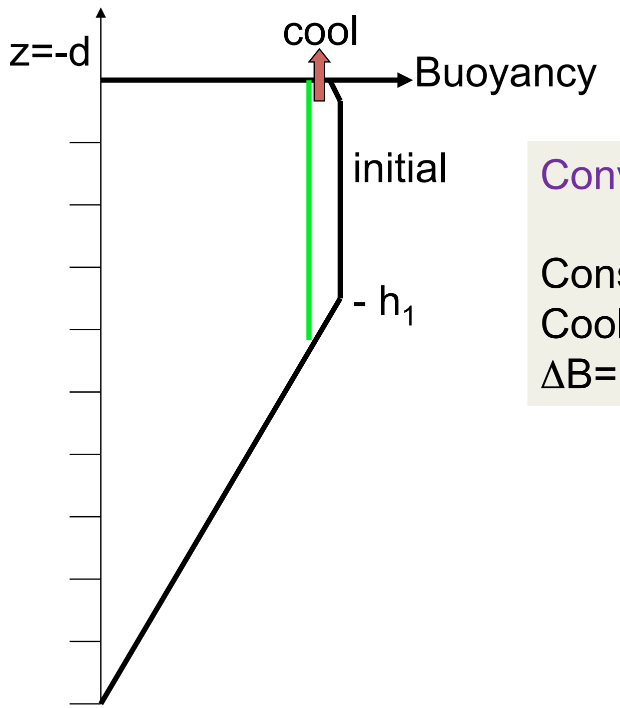
Where h is boundary layer depth : $\sigma = d / h$

$$d/L = \kappa\ d\ B_o\ /\ u^{*3} \longrightarrow -\infty$$

$$\phi_q\ (d/L) \longrightarrow (1 - c\ d/L)^{-1/3} = (1 - c\,\sigma\, h/L)^{-1/3}$$

$$w_q = u^*/\phi_q \longrightarrow (u^{*3} - c\,\kappa\, d\, B_o)^{1/3} \longrightarrow (c\,\kappa\ \sigma)^{1/3}\ w^*$$

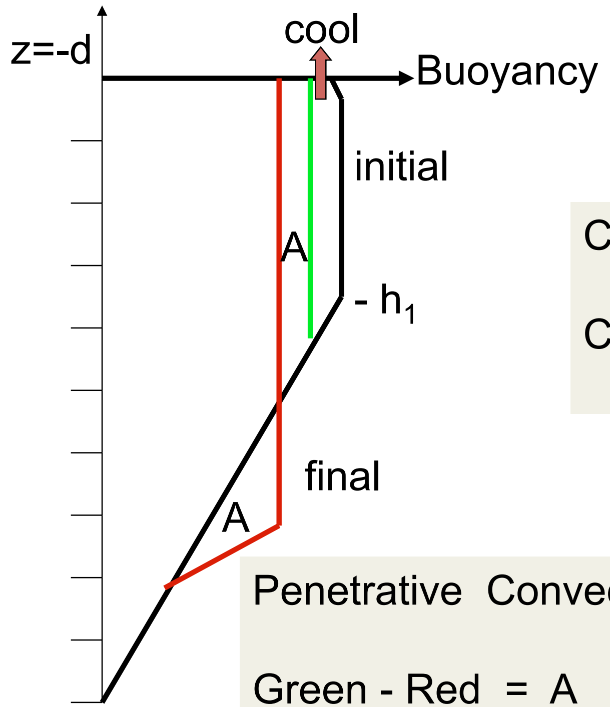
Simple Cooling (Buoyancy Loss)



Convective Adjustment

Conservation :
Cool = Initial-Green
$\Delta B = \Delta t\, \partial_z - \langle wb \rangle$

Penetrative Convection



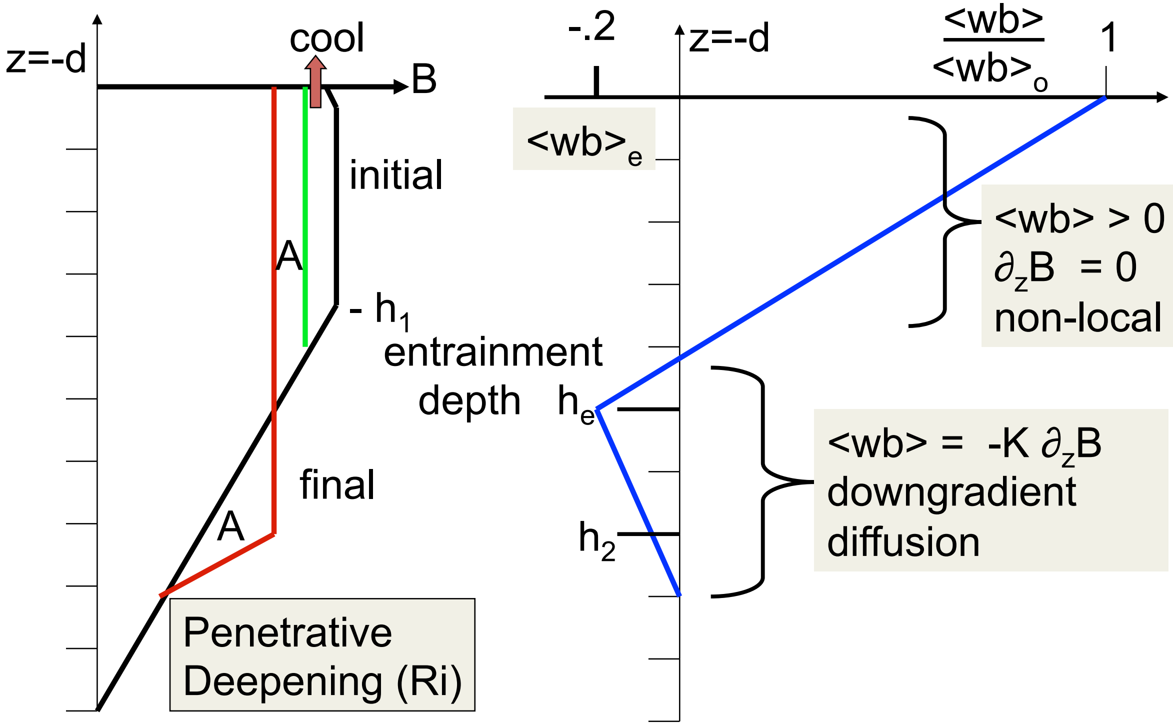
Conservation :

Cool = Initial-Green

Penetrative Convection

Green - Red = A

Convective SBL Turbulent Fluxes:



Questions ?

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Modeling the SBL

Solve 1st order conservation equations
(prognostic variables are 1st moments, or means)

1-D in vertical : $\partial_t T = - \partial_z \langle wt \rangle$; $\partial_t S = - \partial_z \langle ws \rangle$

$\partial_t U = - f V - \partial_z \langle wu \rangle$; $\partial_t V = f U - \partial_z \langle wv \rangle$

The surface boundary conditions are the kinematic fluxes

Linearize density, $\rho = \rho_o \, (1 - \alpha (T\text{-}To) + \beta(S - So))$
e.g. $\rho_o = 1025$ kg/m^3 ; $T_o = 17$ C ; $S_o = 36$ psu
$\alpha = .00023 \ C^{-1}$; $\beta = .00075 \ psu^{-1}$

How to solve a system of 4 equations in 8 unknowns ???????
Poorly constrained, therefore many options !!!!!!!

SBL Models/Schemes

Mixed-Layer Models

- Prognostic, $\partial_t h$
  - TKE budget
    - Kraus-Turner (1984)
    - Niiler (1975)
    - Garwood (1978)
    - Gaspar (1988)
- Diagnostic, h
  - Price-Weller-Pinkel

Turbulence Closure Models

- 1^{st} order
  - Ekman
  - Pacanowski-Philander
  - K-Profile (KPP)
- 2^{nd} order — Canuto (2000s)
  - Mellor-Yamada (1984)
  - Gaspar et al. (1990)

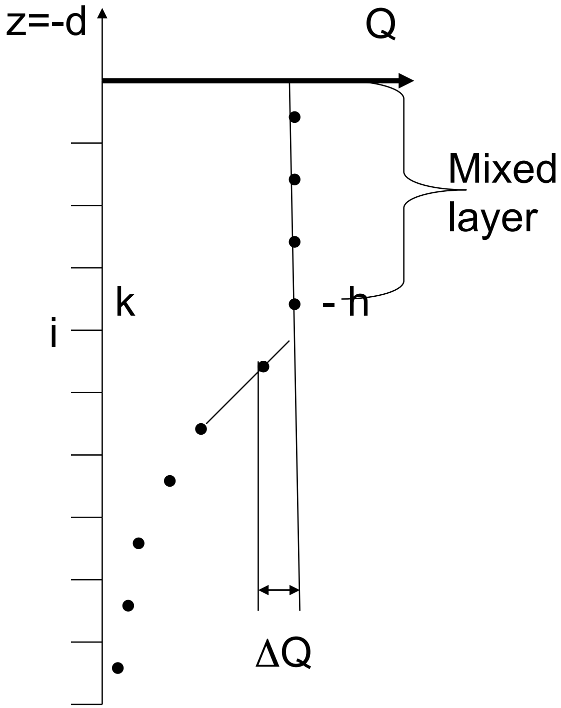
Mixed - Layer Models

ASSUME : a well mixed, homogenous, layer of depth, h
diagnose or compute evolution of h with time

ISSUES :

- boundary layers are not homogenous (skin, transition, MOS)
- the implicit diffusivity is infinite, contrary to MOS
- discontinuities in Q or ∂_z Q at h that are not observed
- parameters are not always found to be universal

Price-Weller-Pinkel (1986)



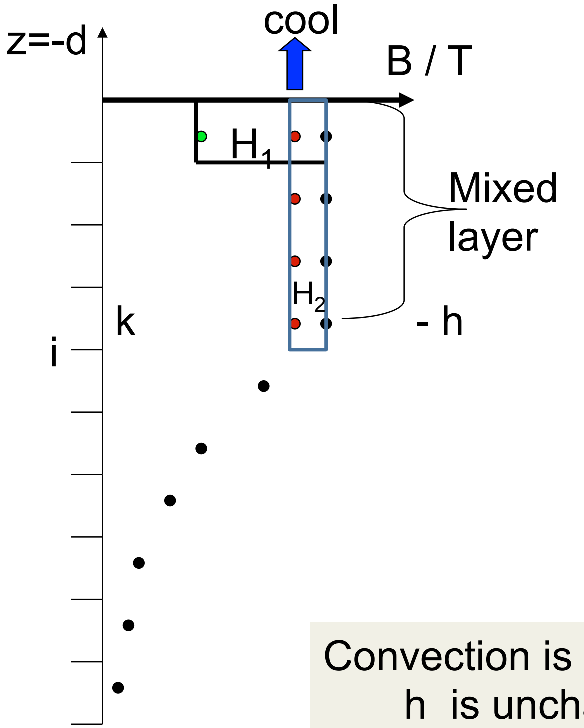
Turbulent fluxes across interfaces, i, are diagnosed to change grid point values Q_k to satisfy 3 stability criteria

A) $\partial_z \rho < 0$, static stability

B) $Ri_b = \dfrac{g \, \Delta\rho \, h}{\rho_o \, |\Delta \mathbf{V}|^2} > 0.6$ "tuned"

C) $Ri_g = \dfrac{N^2}{(\partial_z \mathbf{V})^2} > 0.25$

PWP Convection



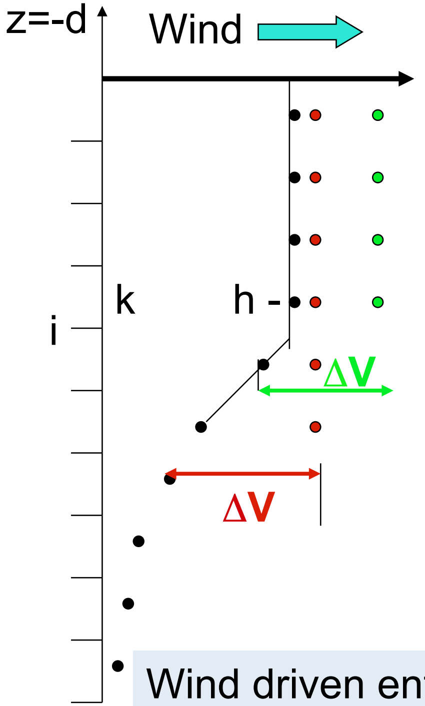
1) From initial (black) profile

2) Remove Heat (Buoyancy) equivalent to area H_1 from upper layer (green) .

3) "Mix" to final profile (red) such that area $H_2 = H_1$ and
 A) $\partial_z \rho < 0$, is satisfied

Convection is non-penetrative
h is unchanged !!!

PWP Entrainment



1) From initial profile (black). distribute momentum uniformly across mixed-layer (green).

2) "Mix" until profile satisfies

B) $R_b = \dfrac{g \, \Delta\rho \, h}{\rho_o \, | \, \Delta \mathbf{V}|^2} > 0.6$

3) As h increases so does ΔB (ΔT), but any large "jumps" are smoothed by

(C) $R_g = \dfrac{N^2}{(\partial_z \mathbf{V})^2} > 0.25$

Wind driven entrainment can be penetrative, h increases !

Ekman 1^{st} order closure (local):

ASSUME : analogy to molecular diffusion $\langle wq \rangle = - K_v \partial_z X$

: K_v = CONSTANT

: Produces Ekman Transport and distribution (Spiral)

BUT non-zero scalar fluxes are **observed** in regions of zero local gradient.

Therefore, the analogy is known to be wrong for scalars in a PBL, and can't be corrected by any choice of constant K_V, but may be good enough for some dynamic problems.

Pakanowski -- Philander (1st order local)

<wq> = - $K_V \ \partial_z X$

$$K_v = K_o^{iw} + K_o^{sh}\, \mathcal{F}(Ri) \quad :$$

K_o^{iw} = 10 cm^2/s Pr = 10

K_o^{sh} = (40-100) cm^2/s Pr = 1

K_V(Ri) formulations are popular despite being local

e.g. Pakanowski and Philander,JPO 1981) studied the Equatorial Pacific, but in their 25m upper laver, K_V is effectively infinite.

K-Profile Parameterization, KPP (1st order, non-local)

Temperature variance equation (2nd order) says

$$<wq> = - K_q \, (\partial_z Q - \gamma_q)$$

OBL of depth, h for $0 < (\sigma = d/h) < 1$

$$K_q(\sigma) = h \; w_q \; G(\sigma)$$

Non-local - K_q knows about h, σ, and surface forcing

- γ_q gives non-zero flux for $\partial_z Q = 0$.

KPP (Vertical Profile)

$K_q(\sigma) = h \quad w_q \quad G(\sigma)$

$G(\sigma) = a_0 + a_1\sigma + a_2\sigma^2 + a_3\sigma^3$

$a_0 = 0$; turbulent eddies don't pass through the surface

$a_1 = 1$; then for $\sigma \rightarrow 0$; $G(\sigma) \rightarrow d/h$; $K_q \rightarrow w_q\, d$

Consistent with MOS $\quad w_q = \dfrac{\kappa \; u^*}{\phi_q(d/L)}$

a_2 & a_3 match $G(\sigma)$ and its slope at $\sigma=1$; to interior

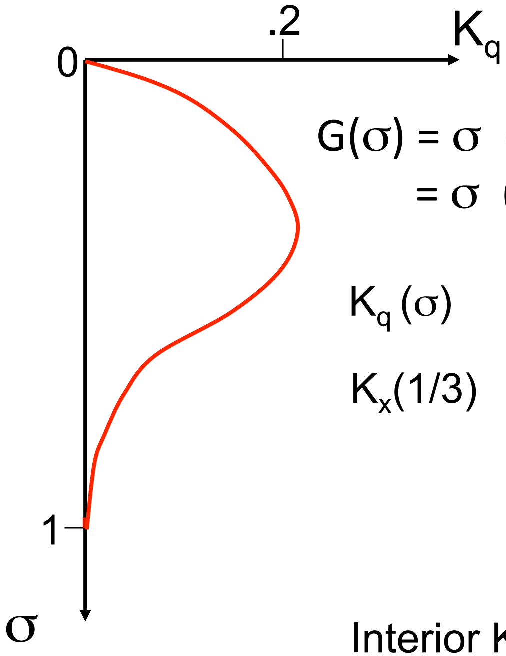
KPP at h, (σ =1)

a_2 and a_3 used to match K_x and $\partial_z K_x$ at h, (σ =1)

for $K_x = \partial_z K_x = 0$ at h

$a_2 = -2$; $a_3 = 1$

KPP Magnitude



$G(\sigma) = \sigma \ (1 - 2\sigma + \sigma^2)$

$= \sigma \ (1 - \sigma)^2$

$K_q(\sigma) \quad = \quad h \qquad w_q \qquad G(\sigma)$

$K_x(1/3) \quad = \ 100 \text{ m} \quad 10^{-3} \text{ m/s} \quad .2$

$= \ 200 \text{ cm}^2/\text{s}$

Interior K_q = (0.1→ 10) cm^2/s

5) KPP -- non local transport

Empirically : $\gamma_u = \gamma_v = 0$

q = scalar

$$\gamma_q = 0 \quad ; \quad d/L > 0 \text{ (stable forcing)}$$

$$\gamma_q = \frac{C \, \langle wq \rangle_o}{w_q \, h} \quad ; \quad d/L < 0 \text{ (convective forcing)}$$

$$\text{non-local } \langle wq \rangle = K_q \, \gamma_q = G(\sigma) \, C \, \langle wq \rangle_o$$

C is order 10

5) KPP -- boundary layer depth, h

$$Ri_b(d) = \frac{(B(0) - B(d))\ d}{|\mathbf{V}(0) - \mathbf{V}(d)|^2 + V_t^2}$$

in a convective OBL : $Ri_b(d) = \frac{(B(0) - B(d))\ d}{V_t^2}$

so V_t^2 is formulated to make $\langle wb \rangle_e = -0.2 \langle wb \rangle_o$

Empirically
h is shallowest depth where $Ri_b(h) = 0.3$

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Low Reynolds' Number, Re order 1

$\partial_t U = - U \partial_x U - V \partial_y U - W \partial_z U$, Advection (non-linear)

$+ f V$, Coriolis (earth's rotation)

$- \rho \partial_x P$, Pressure gradient

$- \partial_z \langle wu \rangle$, Vertical turbulent mixing (non-linear)

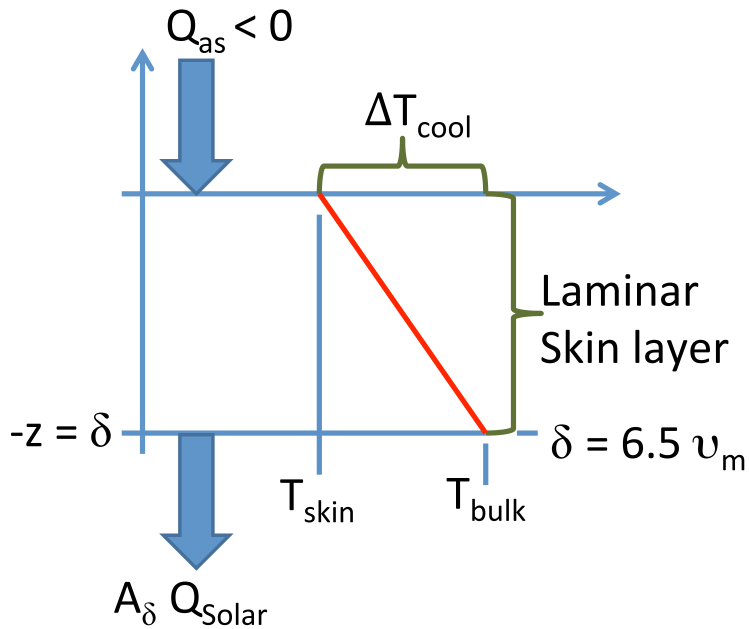
$+ \upsilon_m \partial_{zz} U$, Molecular viscosity

Reynolds Number, $Re = \frac{\text{non-linear}}{\text{viscous}} = \frac{d\, u^*}{\upsilon_m} = \frac{10^{-2}\ 10^{-4}}{10^{-6}} = 1$

At small d (< 1 cm) there is a viscous sub-layer !!!

Deeper there is a (Re>>1) turbulent (3-d) layer !!!

Viscous Surface Layer Thickness, δ



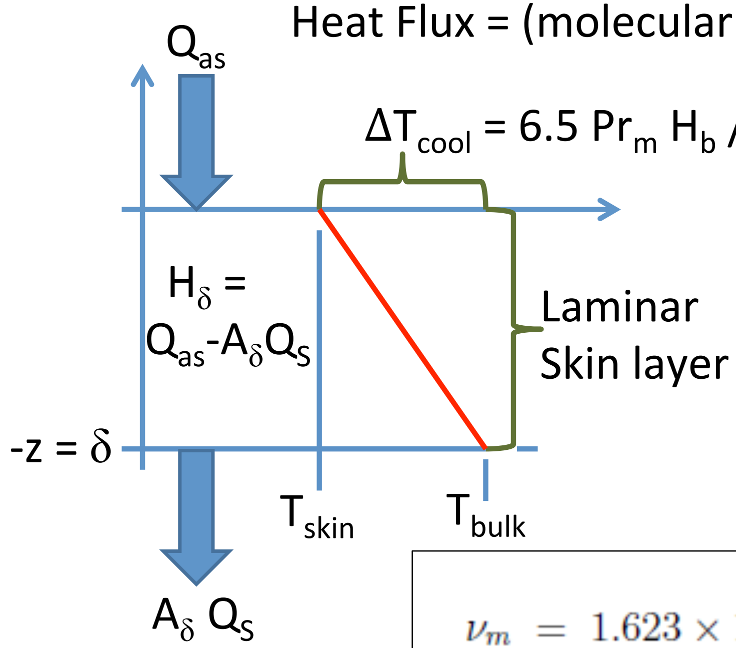
$\delta = 6.5\, \upsilon_m / u^* = \text{Order } 0.5 \text{ cm}$

C.E. Fairall, et. al., 1996, J. Geophys. Res.)

Sea Surface Cool Skin

Heat Flux = (molecular diffusivity) ΔT_{cool} / δ

$\Delta T_{cool} = 6.5 \, Pr_m \, H_b / u^* =$ Order (-0.1°C)



$$\nu_m = 1.623 \times 10^{-6} (m^2/s) \; e^{(-T_{bulk}/45.2^\circ C)}$$

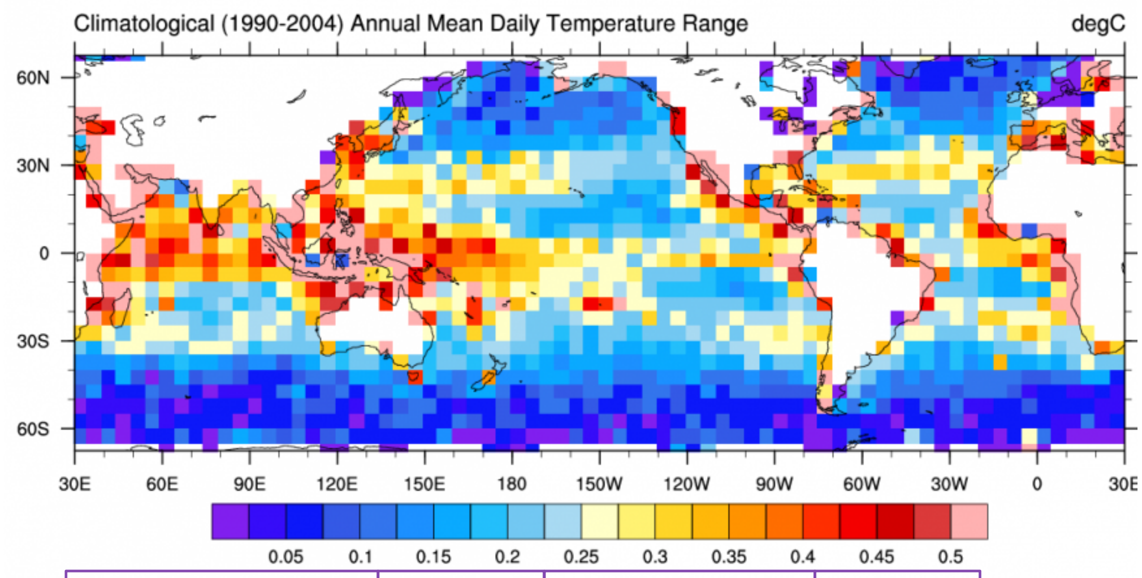
$$Pr_m = 11.64 \; e^{(-T_{bulk}/40.7^\circ C)} \quad .$$

Outline

1. Non-linear mixing
2. Turbulent Eddy Viscosity and Closures
3. Mixing Regimes in Oceans and Seas
4. The Surface Boundary Layer & Similarity Theory
5. The Convective Boundary Layer
6. Modeling the Surface Boundary Layer
   --- Mixed Layer Models
   --- First Order Closures
7. The Viscous Surface Layer
8. **A Practical Example: Diurnal Cycling**

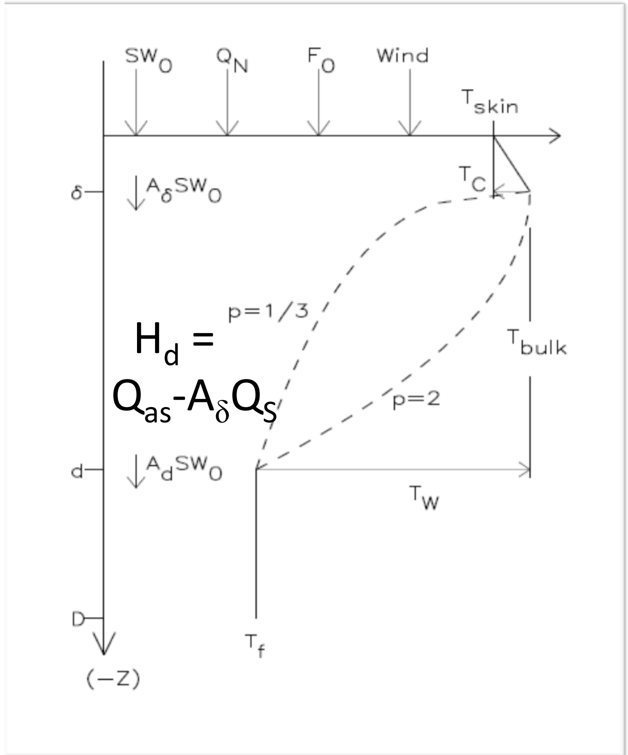
Mean Diurnal Warming

T_{skin} (14:00h) – T_{skin} (Night)



Westerlies High latitudes	Trades	Doldrums	Tropics

Low Reynolds' Number, Re order 1

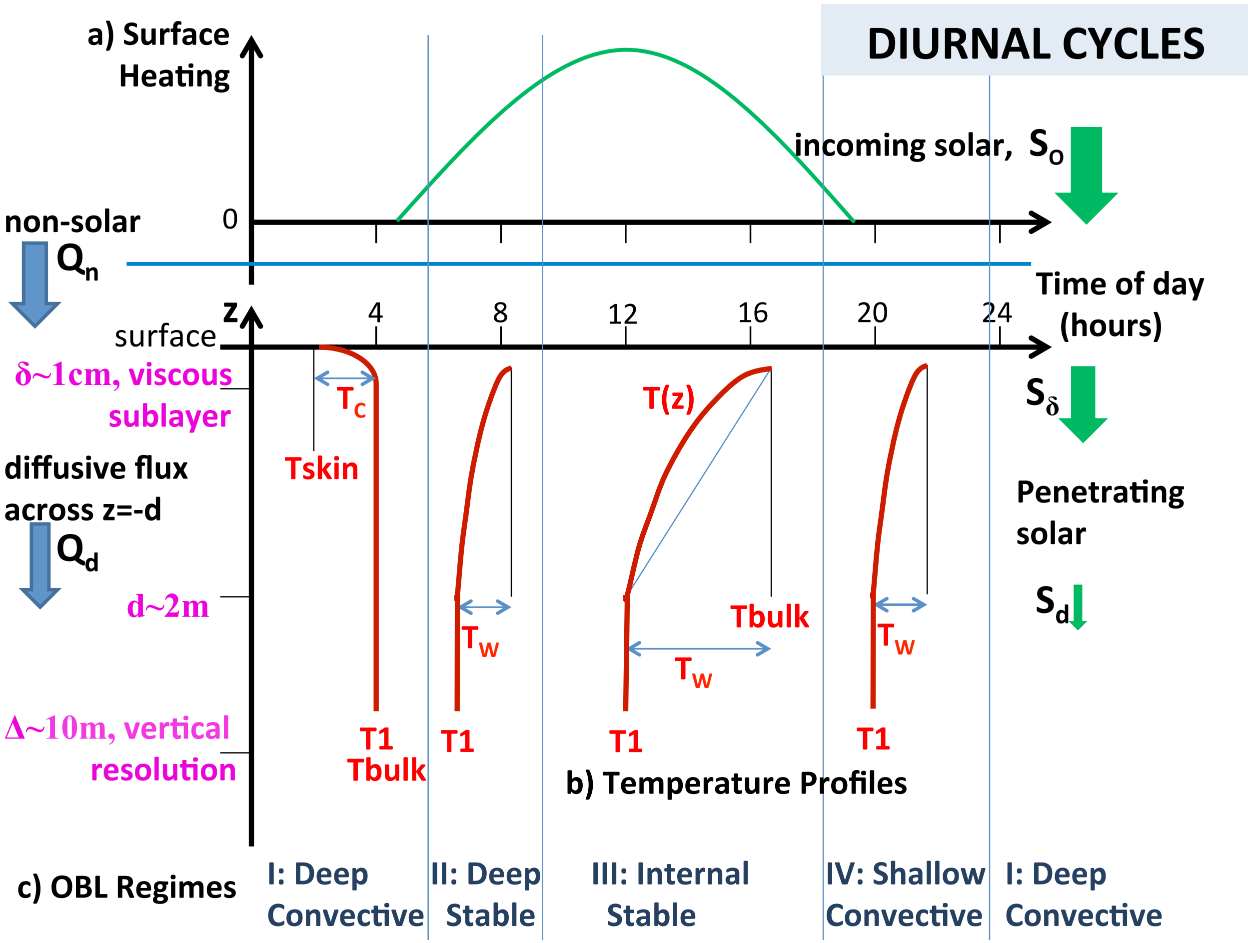


$$T(z) = T_{bulk} - \left[\frac{(z+\delta)}{(-d+\delta)}\right]^p T_W$$

$$\partial_t T_M = \frac{p}{(p+1)} \partial_t T_W$$

$$K_d \, \partial_z T(d) = K_d \, T_W \, \frac{p}{d}$$

$$\partial_t T_W = H_d \frac{(p+1)}{p \, d} - K_d \, T_W \frac{(p+1)}{d^2}$$



DIURNAL CYCLES
a) Surface Heating
incoming solar, S_o
non-solar
0
Q_n
Time of day (hours)
z
4
8
12
16
20
24
surface
δ~1cm, viscous sublayer
T_c
Tskin
T(z)
S_δ
Penetrating solar
diffusive flux across z=-d
Q_d
d~2m
T_w
Tbulk
T_w
T_w
S_d
Δ~10m, vertical resolution
T1
Tbulk
T1
T1
T1
b) Temperature Profiles
c) OBL Regimes
I: Deep Convective
II: Deep Stable
III: Internal Stable
IV: Shallow Convective
I: Deep Convective

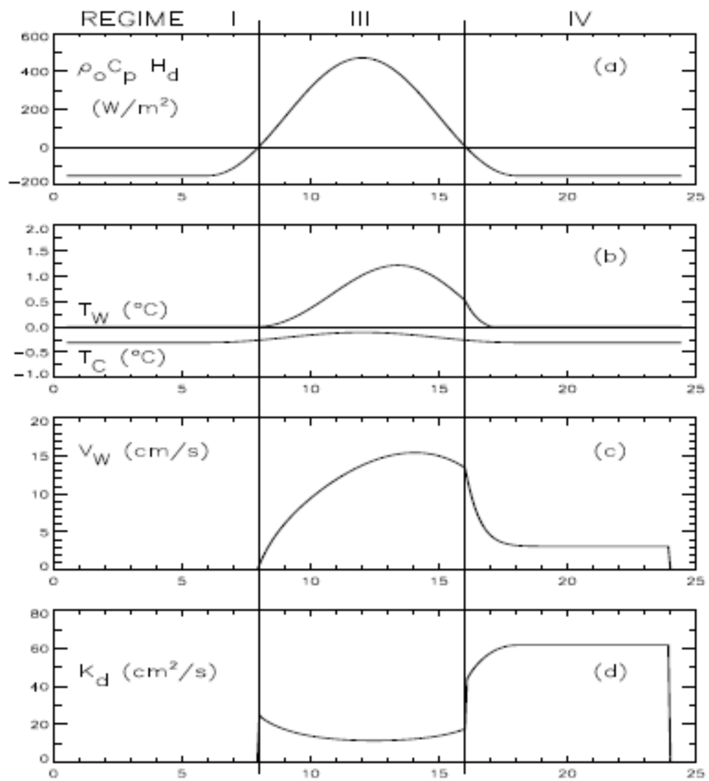
Diffusivity at −z = d, by Regime

REGIME	d/L	K_d
I Deep Convective	< 0	
II Deep Stable	0 → Λ ~1	$\frac{\kappa\, u^*\, d}{(1+5\, d/L)} \left(1 + a_2 \left[\frac{d}{\Lambda\, L}\right] + a_3 \left[\frac{d}{\Lambda\, L}\right]^2\right)$
III Shallow Stable	> 1	$\kappa_o + \nu_o\ \nu(Ri_d)$
IV Deepening Convective	< 0	$\kappa\, u^*\, d\, (1 - C_s\, d/L)^{\frac{1}{3}}$

$$Ri_d = g\,\frac{(\alpha\, T_W - \beta\, S_W)}{V_W^2}\ \frac{d}{p}$$

C_s ~ 7 ; Λ ~ 1

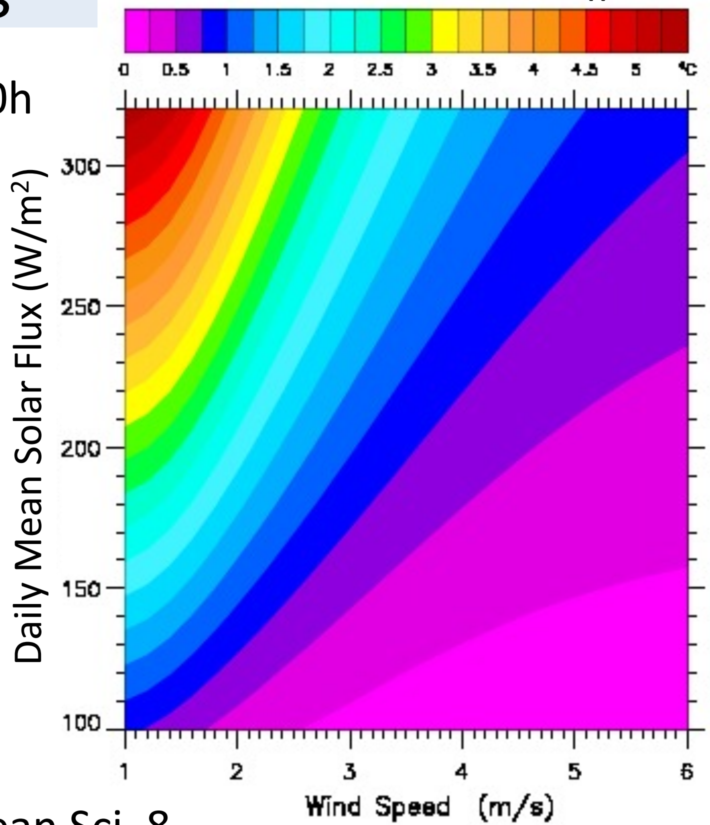
Diunal Cycles



Peak Warming

Typically at about 14:00h

Daily Maximum, T_W

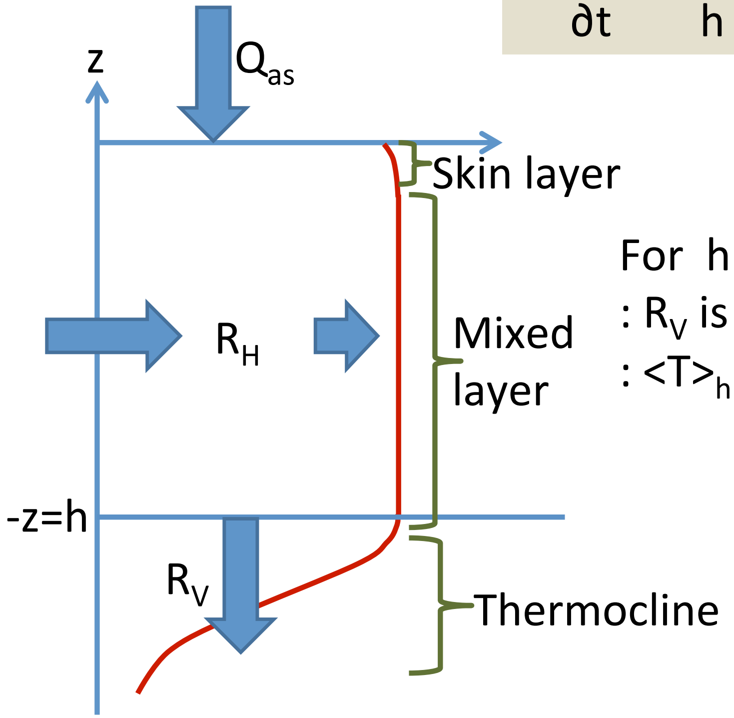


Filipiak, et. al., 2012, Ocean Sci, 8

Fin

III.2 A prognostic SST equation

$$\frac{\partial SST}{\partial t} = \frac{Q_{as}}{h\,(\rho c_p)_{ocn}} - \frac{R_V}{h} + \frac{\partial R_H}{\partial x}$$

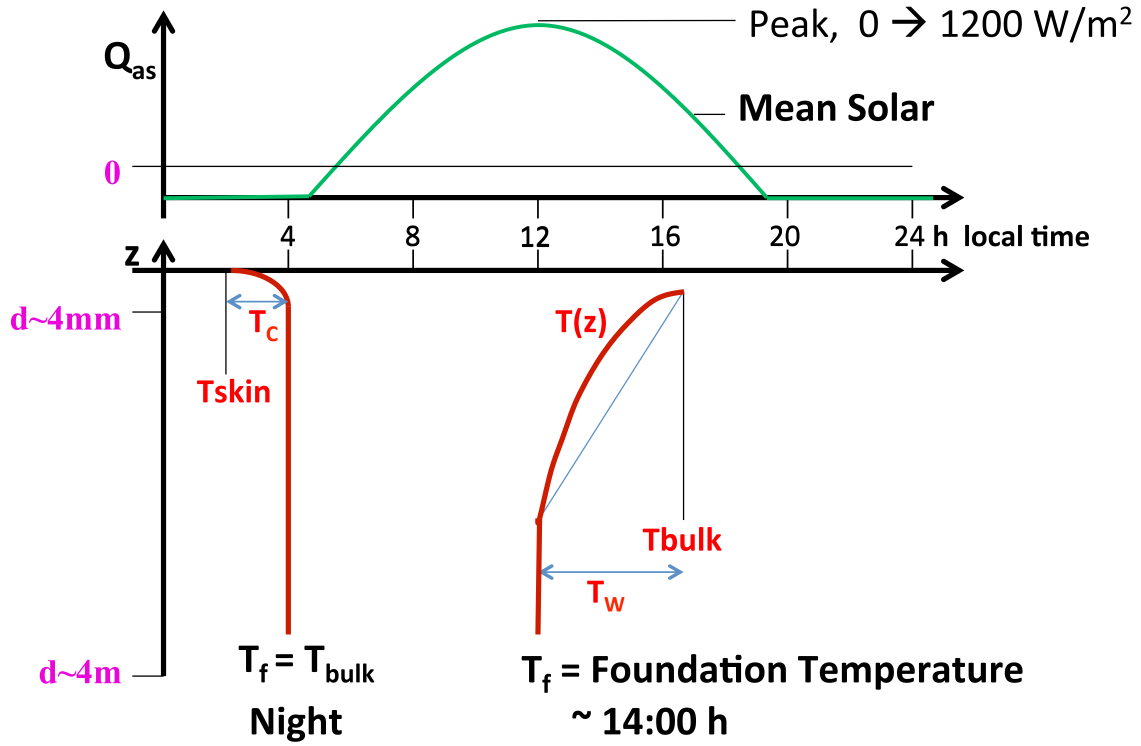


For h near thermocline top

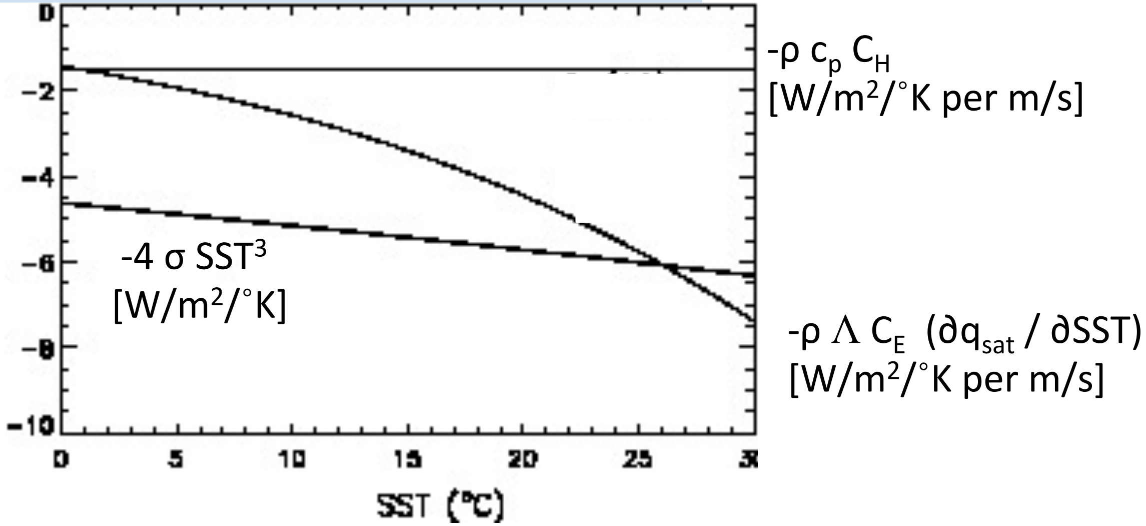
: R_V is tractable

: $\langle T\rangle_h \approx SST$

III.3 : Diurnal Cycling (T_{bulk})



III.3: Diurnal Heat Fluxes

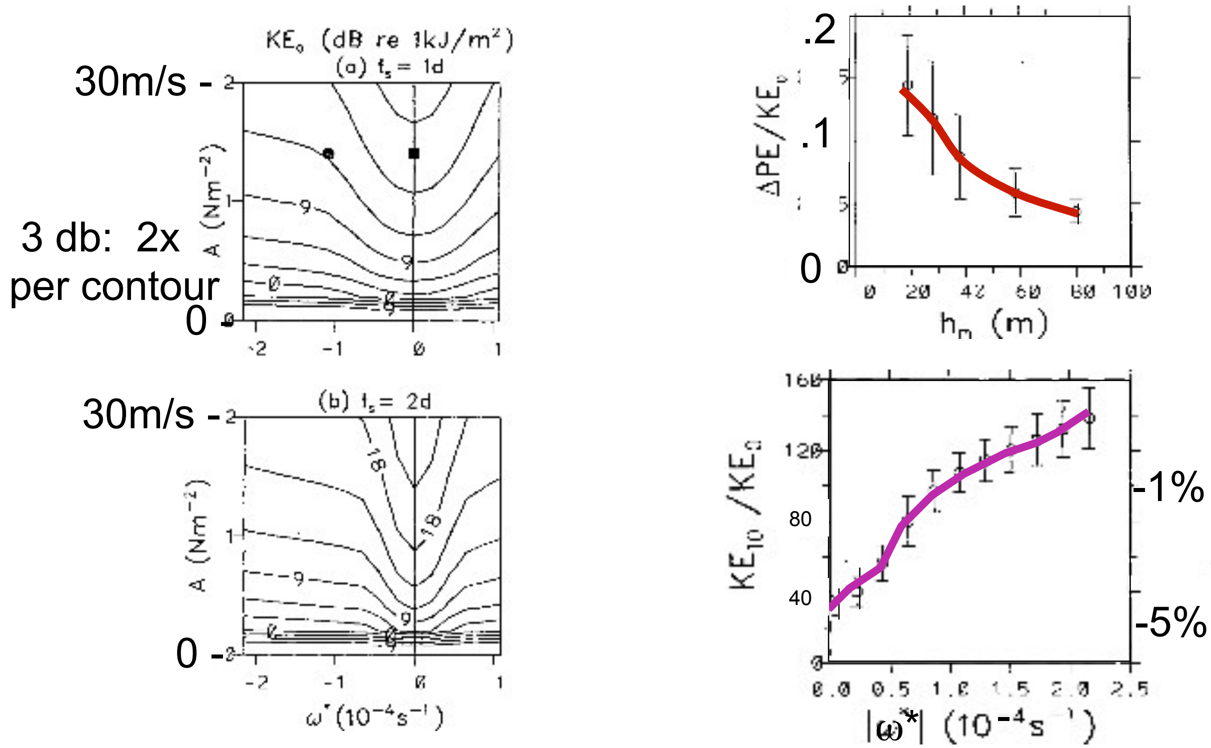


Warming 5°C @ 1m/s ; 2°C @ 3m/s

- $-Q_L$: ~ 25 → 3.0 W/m^2 ; 25 → 30 W/m^2
- $-Q_H$: ~7.5 W/m^2 ; 9 W/m^2
- $-Q_E$: ~7.5 → 35 W/m^2 ; 25 → 42 W/m^2

Inertial Resonance , KE_o : $\omega^* = (\omega - f)$

$\boldsymbol{\tau} = A \sin^n(\pi t/\Delta t) \quad \exp\{-j(\omega^*+f)\, t/\Delta t\}$



3) Evaporation (usually dominates Q_{nsol} variability)

For a surface evaporation, E < 0, that increases salinity by ΔS, there is a latent heat flux, $Q_E = \Lambda E$, that cools the ocean by ΔT (Λ is the latent heat of vaporization)

$$\frac{\Delta T}{\Delta S} = \frac{\Lambda}{S_o \; C_p} = \frac{2.5 \times 10^6 \; (j/kg)}{35psu \quad 4000 \; (j/kg/\,^oC)} = 17\,^oC/psu$$

Evaporation changes temperature more than salinity, And 90% of the buoyancy change is due to ΔT .

Other heat fluxes only change temperature
Precipitation changes Salinity

Turbulent Mixing Regimes
(changing balance of terms with, d)