QCD for (future) hadron colliders

ICTP Summer School

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Lecture 1

Michelangelo L. Mangano

TH Unit, Physics Department, CERN

michelangelo.mangano@cern.ch

QCD: quantum chromodynamics

Understanding the physics of proton-proton collisions requires the understanding of the proton structure, and of the interactions among its elementary constituents, quarks and gluons.

These interactions are described by quantum chromodynamics, a gauge theory, with SU(3) symmetry group, whose charge is called **color**

Gluons: force carriers, in the adjoint representation, **8**, of SU(3)

Quarks: matter fields, in the fundamental representation, **3**, of SU(3)

QCD is characterized by

confinement: the potential grows linearly at large distance, confining colored objects into color-singlet systems (hadrons)

asymptotic freedom: the strength of the coupling constant decreases at short distance, where colored partons can be treated as free particles

QCD is to particle physics what EM and QM are to chemistry: it's omnipresent

- hadronic spectroscopy and transitions (scattering, decays, etc)
- EW properties of quarks ("CKM physics"): K, D, B decays
- proton structure (DIS, polarized DIS, diffraction, ...)
- $e^+ e^-$ to hadrons (determination of α_S, non-PT effects, ...)
- jet physics (in ee, ep and pp(bar) collisions)
- quark-gluon plasma (relativistic heavy-ion collisions)
- ....

.. things I'll give for granted you know ..

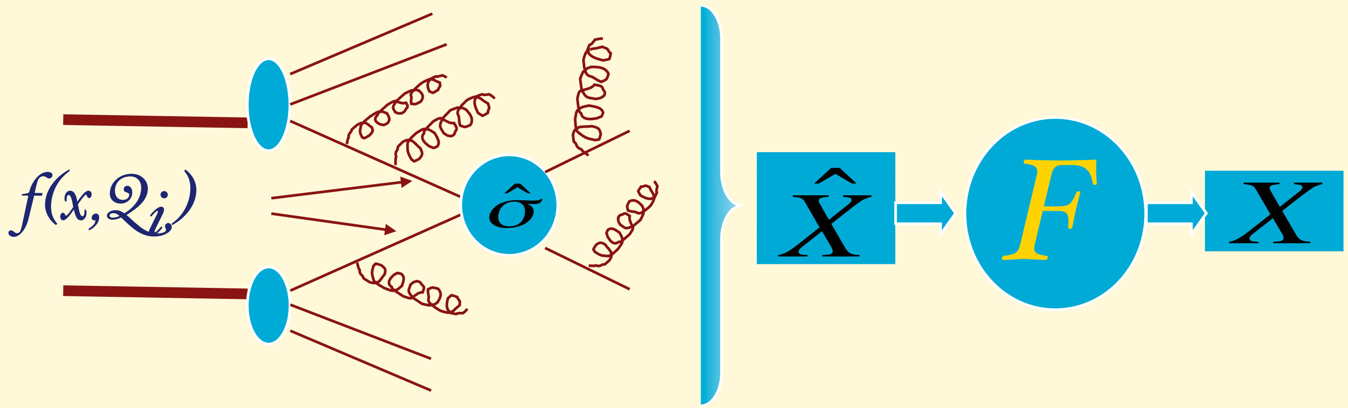
- quarks and gluons
- mesons and baryons
- asymptotic freedom
- Feynman diagrams and Feynman rules
- basic knowledge of what high-energy hadronic collisions are about:
  - production and study of jets, heavy quarks (bottom, top)
  - production and study of W/Z bosons,
  - production and study of Higgs bosons,
  - search for phenomena beyond the Standard Model (supersymmetry, dark matter, new gauge forces, etc.)

Outline

1. Introduction to the theoretical principles of hadron collisions:
   1.1. Factorization, initial state evolution of PDFs
   1.2. Drell-Yan observables
2. Introduction to the theoretical principles of hadron collisions:
   2.1. final state evolution, turning quarks and gluons into hadrons
   2.2. jet production
3. Phenomenological applications, review and interpretation of LHC data
4. QCD phenomena at future hadron colliders

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i)\, f_k(x_2, Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i, Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$f_j(x, Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

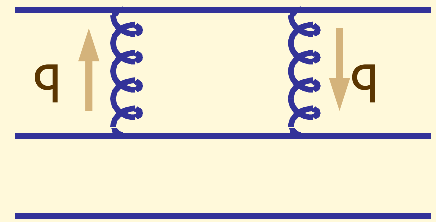
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i, Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

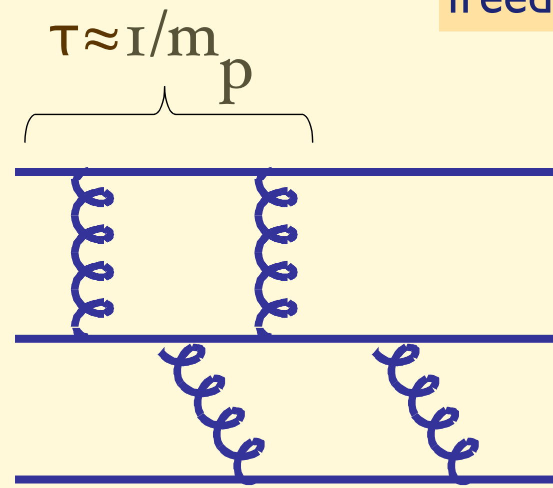
Universality of parton densities and factorization, an intuitive picture

1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\overset{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

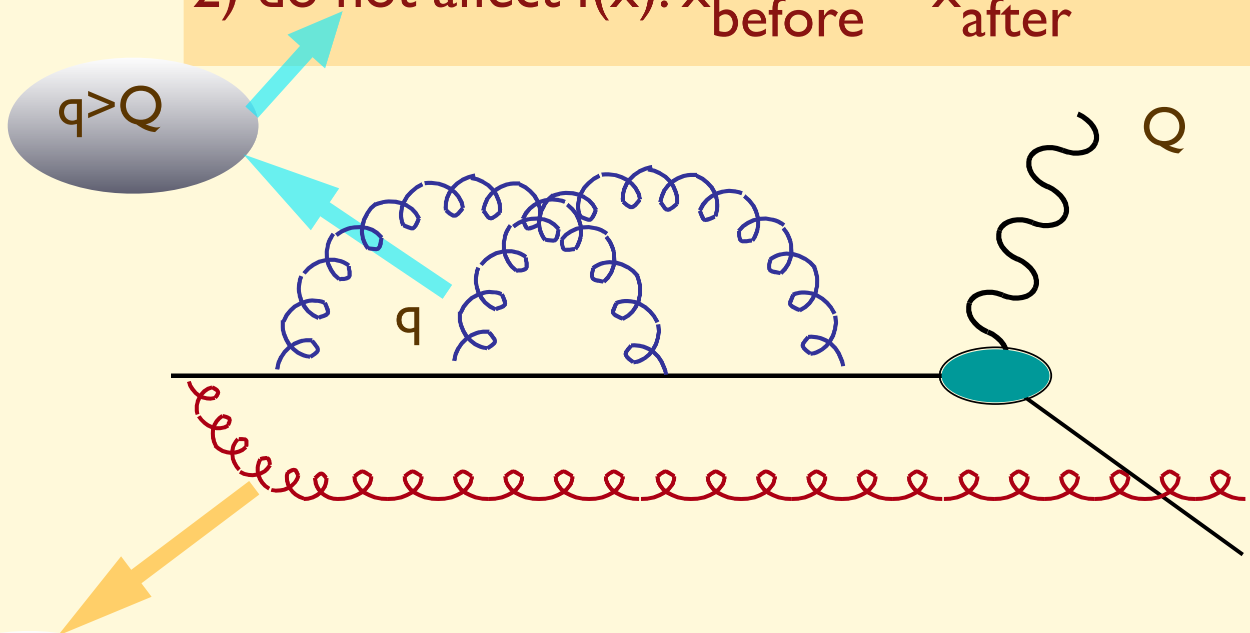


2) Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau = \gamma/m_p = E/m_p^2$)

3) If a hard probe ($Q >> m_p$) hits the proton, on a time scale $= 1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



This gluon cannot be reabsorbed because the quark is gone

q<Q

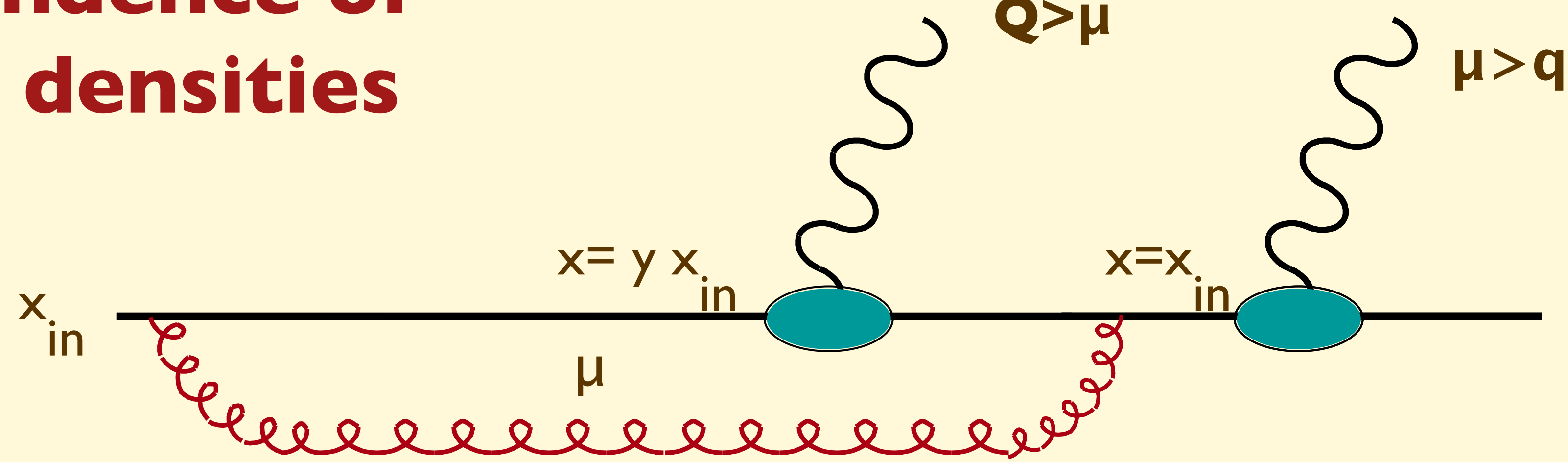
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ **Universality of f(x)**

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \boxed{\frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)}$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} P(x)$$

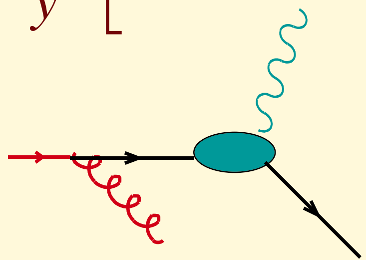
and finally (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi DGLAP equation):

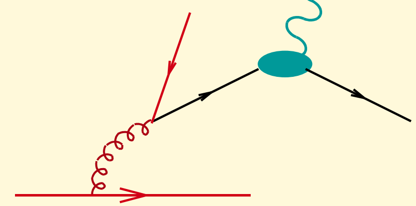
$$\boxed{\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)}$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q (t=logQ^2):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

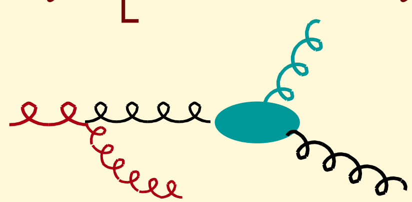


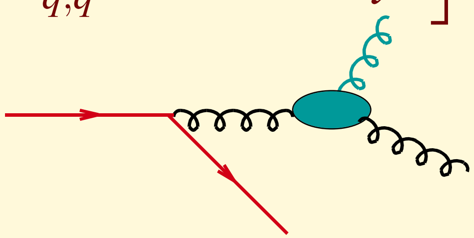


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,q} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$

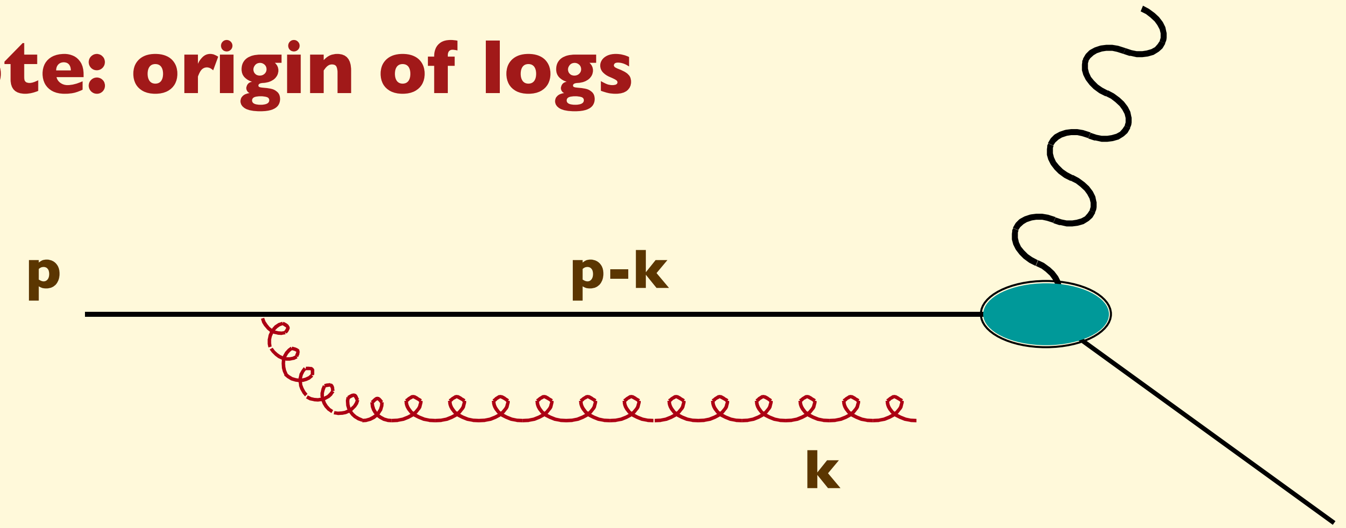




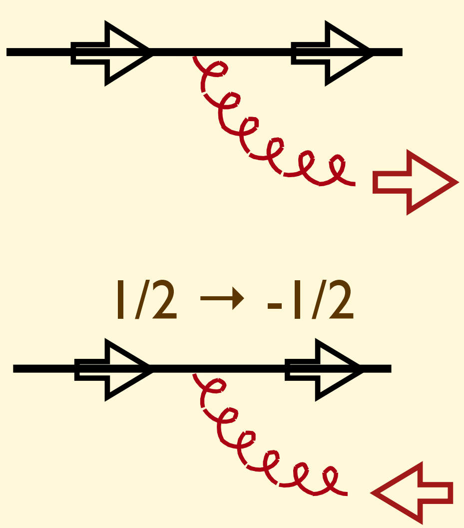
$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Note: origin of logs



$$(p-k)^2 \;=\; -2\,p^0\,k^0\,(\,1-\cos\theta_{pk}\,)$$



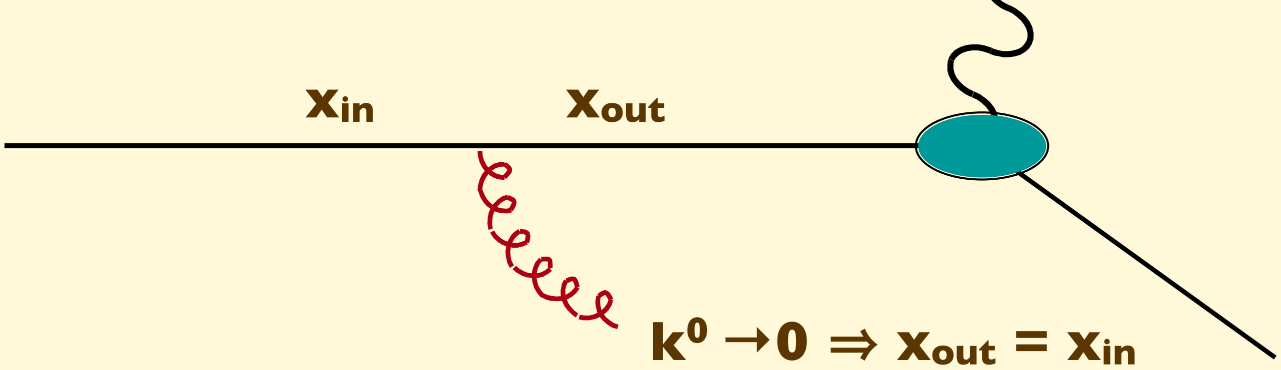
Helicity conservation ~ p · k

$$|M|^2 \sim \left[\frac{1}{(p-k)^2}\right]^2 \times (p\cdot k) \quad\longrightarrow\quad \frac{1}{p^0}\frac{dk^0}{k^0}\,\frac{d\theta}{\theta}$$

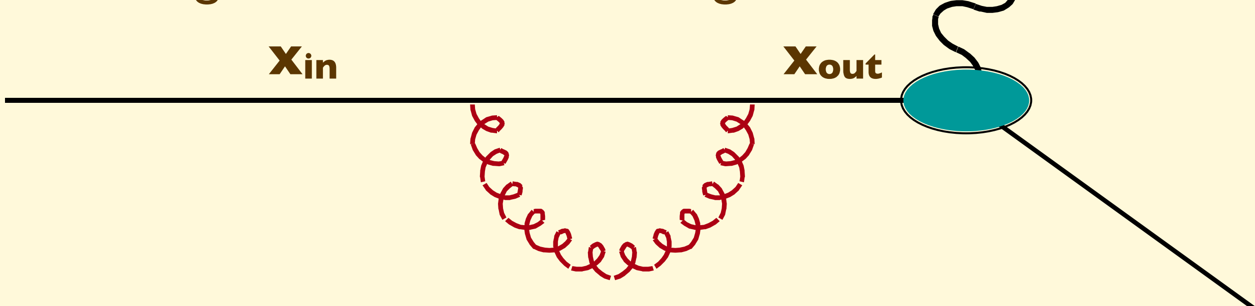
Soft divergence

Collinear divergence

Soft emission cannot lead to a physical divergence, however, since it is not observable

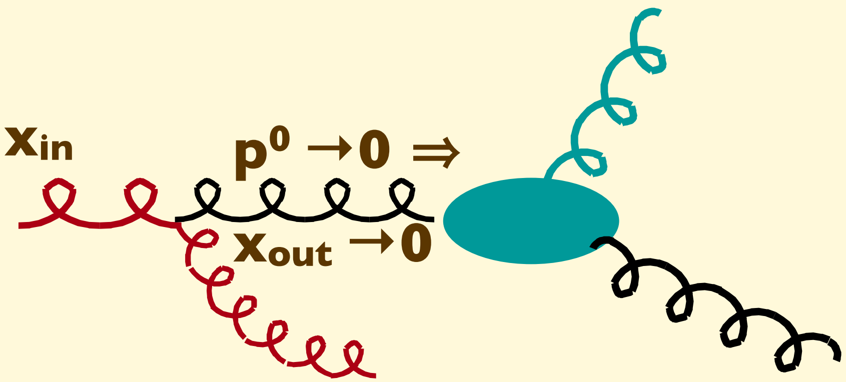


The soft-emission divergence must cancel against the IR divergence of the virtual diagram



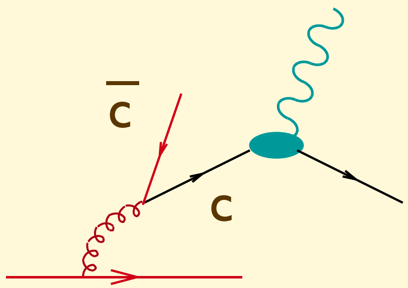
The cancellation cannot take place in the case of collinear divergence, since $\mathbf{x_{out} \neq x_{in}}$, so virtual and real configurations are not equivalent

Things are different if $\mathbf{p^0 \to 0}$. In this case, again, $\mathbf{x_{out} \neq x_{in}}$, no virtual-real cancellation takes place, and an extra singularity due to the $\mathbf{1/p^0}$ pole appears



These are called **small-x** logarithms. They give rise to the double-log growth of the number of gluons at small **x** and large **Q**

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

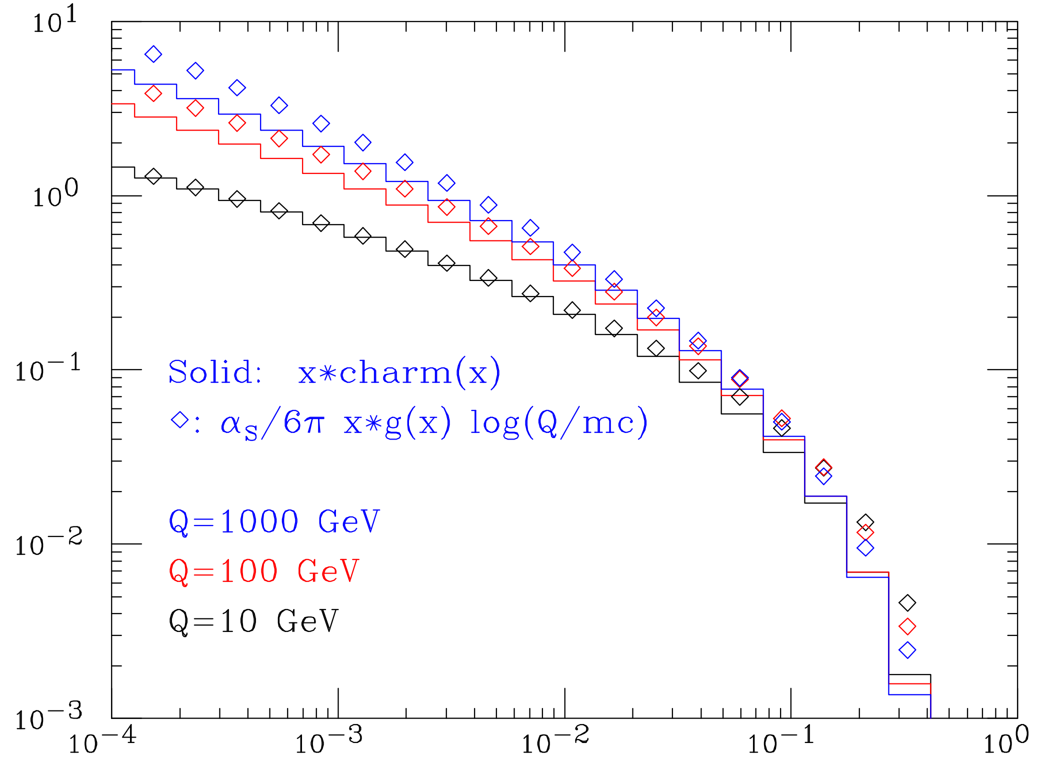
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}\left[y^2 + (1-y)^2\right] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

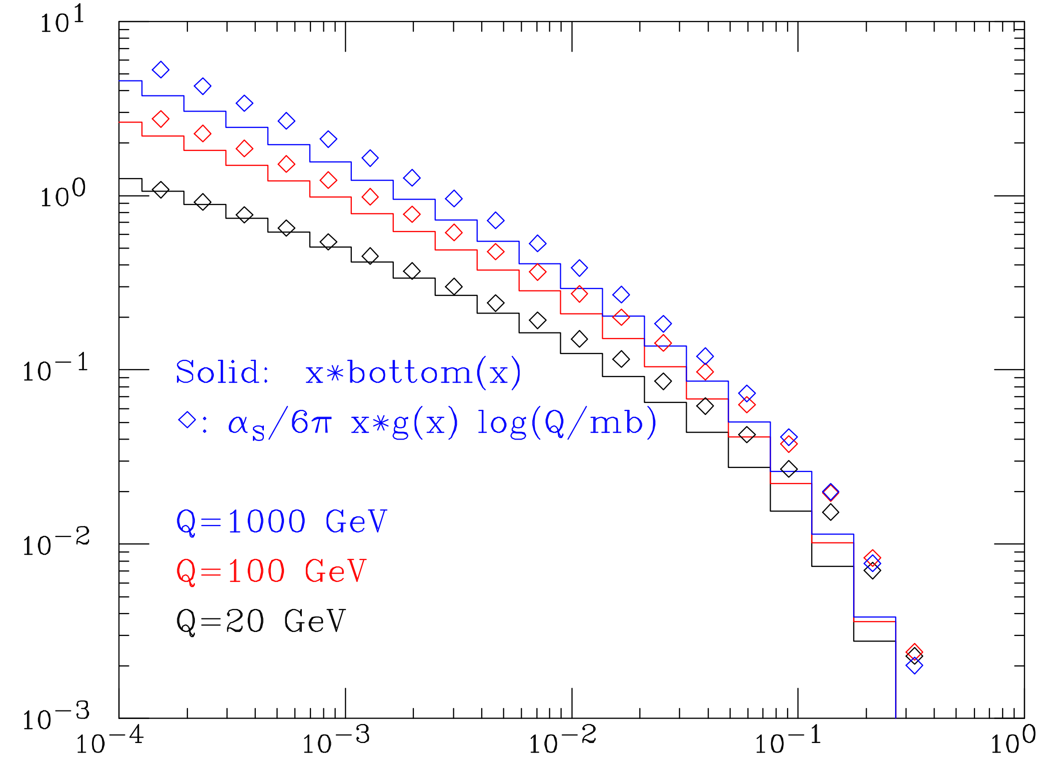
and therefore:

$$\boxed{c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)}$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

Numerical example





Excellent agreement, given the simplicity of the approximation!

Can be improved by tuning the argument of the log (threshold onset), including a better parameterization of g(x), etc....

General properties of the PDF evolution

Definition of n-th moment: $$g_n = \int_0^1 \frac{dx}{x}\, x^n\, g(x)$$

In moment space, the evolution eqs become coupled linear differential equations

$$\frac{df_i^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{qq}^{(n)} f_i^{(n)} + P_{qg}^{(n)} f_g^{(n)}]$$

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{gg}^{(n)}\, f_g + P_{gq}^{(n)} f_i^{(n)}]$$

exercise!

or, equivalently:

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, \left[P_{gq}^{(n)}\, \Sigma^{(n)} + P_{gg}^{(n)}\, f_g^{(n)}\right]$$

$$\frac{d\Sigma^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, \left[P_{qq}^{(n)}\, \Sigma^{(n)} + 2n_f\, P_{qg}^{(n)}\, f_g^{(n)}\right]$$

$$\frac{dV^{(n)}}{dt} = \frac{\alpha_s}{2\pi}\, P_{qq}^{(n)}\, V^{(n)}$$

where we define "singlet" and "valence" distributions as:

$$\Sigma(x) = \sum_i f_i(x) + \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

$$V(x) = \sum_i f_i(x) - \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

Valence sum rule

$$V^{(1)} = \int_0^1 dx \sum_q (f_q(x) - f_{\bar{q}}(x)) = N(\text{valence quarks}) = constant$$

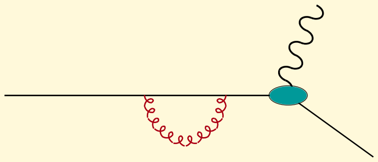
Thus $$\frac{dV^{(1)}}{dt} \equiv 0 \Rightarrow \frac{\alpha_s}{2\pi} P_{qq}^{(1)} V^{(1)} = 0$$

Since $V^{(1)}=3$, we must have $P_{qq}^{(1)}=0$, i.e. $$\int_0^1 dz\, P_{qq}(z) = 0$$

This requires to modify $P_{qq}(z)$ as follows:

$$P_{qq}(z) \to \left(\frac{1+z^2}{1-z}\right)_+ \equiv \frac{1+z^2}{1-z} - \delta(1-z)\int_0^1 dy \left(\frac{1+y^2}{1-y}\right)$$

Subtraction of the virtual singularity



$$\int_0^1 dx\, f(x)\, [g(x)]_+ \equiv \int_0^1 dx\, [f(x) - f(1)]\, g(x)$$

Momentum sum rule (exercise)

$$\int_0^1 dx\, x \left[\sum_{i,\bar{i}} f_i(x) + f_g(x) \right] \equiv \Sigma^{(2)} + f_g^{(2)} = 1$$

This implies

(1) $\quad P_{qq}^{(2)} + P_{gq}^{(2)} = 0$

(2) $\quad P_{gg}^{(2)} + 2n_f \, P_{qg}^{(2)} = 0$

(1) is trivially true (check!)

(2) requires a modification of P_{gg} to subtract soft virtual singularity (verify!):

$$P_{gg} \to 2C_A \left\{ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right\} + \delta(1-x) \left[\frac{11C_A - 2n_f}{6} \right]$$

Subtraction of gluon loop in virtual diagrams

Subtraction of quark loop in virtual diagrams

General solution of the PDF evolution

$$V^{(n)}(Q^2) \overset{*}{=} V^{(n)}(\mu^2) \left[\frac{\log Q^2/\Lambda^2}{\log \mu^2/\Lambda^2}\right]^{P_{qq}^{(n)}/2\pi b_0} = V^{(n)}(\mu^2) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)}\right]^{P_{qq}^{(n)}/2\pi b_0}$$

* see footnote next page

Verify that all moments $P_{qq}^{(n)}$ are negative. Therefore as Q grows, all moments decrease. The valence distribution becomes softer and softer.

For $\Sigma^{(n)}$ and $g^{(n)}$ one needs to diagonalize the 2x2 matrix. In the case of n=2, corresponding to the momentum fraction carried by gluons and quarks, simple asymptotic solutions ($Q^2 \to \infty$) can be obtained (exercise!):

$$\begin{cases} P_{qq}^{(2)}\,\Sigma^{(2)} + 2n_f\,P_{qg}^{(2)} f_g^{(2)} = 0 & (\Sigma^{(2)} \to \text{constant at large Q}) \\ \Sigma^{(2)} + f_g^{(2)} = 1 & (\text{sum rule}) \end{cases}$$

$$\begin{cases} \Sigma^{(2)} = \dfrac{1}{1+\frac{4C_F}{n_f}} \\ f_g^{(2)} = \dfrac{4C_F}{4C_F+n_f} \end{cases}$$

$$\frac{g^{(2)}}{\Sigma^{(2)}} = \frac{4C_F}{n_f} = \frac{16}{3n_f}$$

Footnote: α_s running

$$\frac{d\alpha_s(\mu^2)}{d\log\mu^2} = \beta(\alpha_s)$$

$$\beta(\alpha_s) = -b_0\alpha_s^2\,(1 + b'\alpha_s + \ldots)$$

LO NLO

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

$$b' = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{2\pi(11C_A - 2n_f)}$$

$\beta(\alpha)$ for QCD is known up to NNNLO (4-loops)

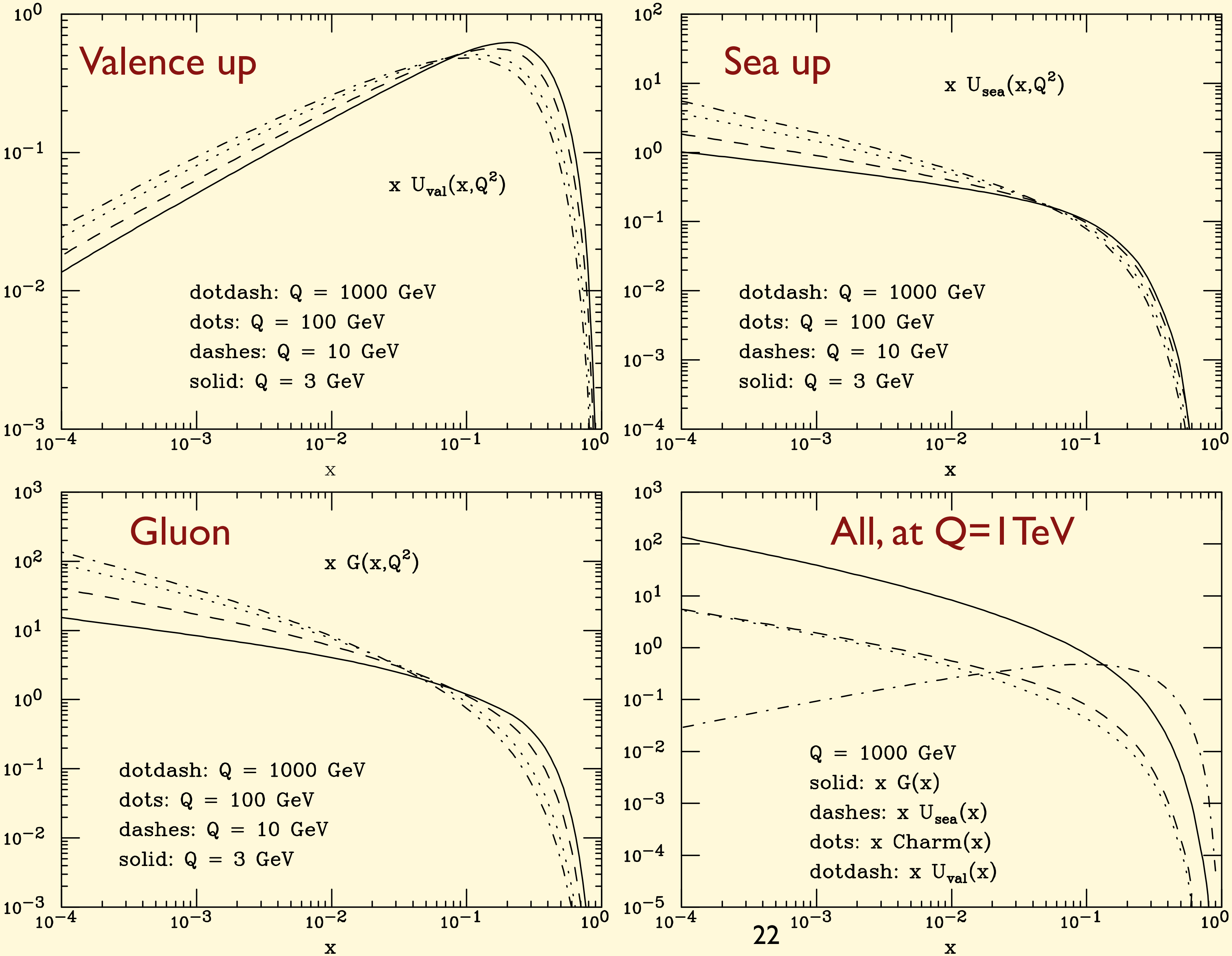
At LO

$$\alpha_s(\mu^2) = \frac{1}{b_0\,\log\mu^2/\Lambda^2}$$

At NLO

$$\alpha_s(\mu^2) = \frac{1}{b_0\,\log\mu^2/\Lambda^2}\left[1 - \frac{b'}{b_0}\frac{\log\log\mu^2/\Lambda^2}{\log\mu^2/\Lambda^2}\right]$$

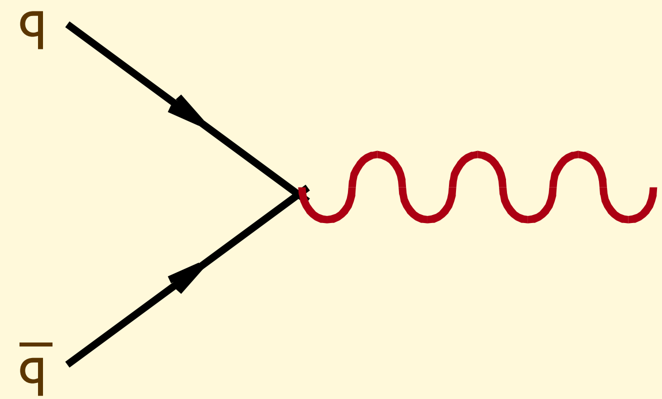
Examples of PDFs and their evolution



Note:
sea ≈10% glue

Note:
charm≈up at high Q

Example: Drell-Yan processes



$$W \to \ell\nu$$

$$Z \to \ell^+\ell^-$$

Properties/Goals of the measurement:

- Clean final state (no hadrons from the hard process)
- Tests of QCD: $\sigma(W,Z)$ known up to NNLO (2-loops)
- Measure SM parameters: m(W), $\sin^2\theta_W$
- constrain PDFs (e.g. $f_{up}(x)/f_{down}(x)$)
- search for new gauge bosons: $q\bar{q} \to W', Z'$
- Probe contact interactions: $q\bar{q} \to e^+e^-$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2} \log \frac{E_W + p_W^z}{E_W - p_W^z}$

Pseudorapidity: $\eta = -\log(\tan \frac{\theta}{2})$

where:

$\tan \theta = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\tau = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\, E_{\mathrm{beam}} \\ p_W^z = (x_1 - x_2)\, E_{\mathrm{beam}} \end{cases} \Rightarrow y = \frac{1}{2} \log \frac{x_1}{x_2}$$

prove the following relations:

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad dx_1\, dx_2 = dy\, d\tau$$

$$dy = \frac{dx_1}{x_1} \qquad d\tau\, \delta(\hat{s} - m_W^2) = \frac{1}{S}$$

LO Cross-section calculation

$$\sigma(pp \to W) = \sum_{q,q'} \int dx_1 dx_2 \, f_q(x_1,Q)\, f_{\bar{q}'}(x_2,Q)\, \frac{1}{2\hat{s}} \int d[PS] \overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2$$

where:

$$\overline{\sum_{spin,col}} |M(qq' \to W)|^2 = \frac{1}{3}\frac{1}{4} 8 g_W^2 \, |V_{qq'}|^2 \hat{s} = \frac{2}{3} \frac{G_F m_W^2}{\sqrt{2}} |V_{qq'}|^2 \hat{s}$$

$$d[PS] = \frac{d^3 p_W}{(2\pi)^3 2 p_W^0} (2\pi)^4 \delta^4(P_{in} - p_W)$$

$$= 2\pi \, d^4 p_W \, \delta(p_W^2 - m_W^2)\, \delta^4(P_{in} - p_W) = 2\pi\, \delta(\hat{s} - m_W^2)$$

leading to:

Partonic Luminosity

$$\sigma(pp \to W) = \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \tau \int_\tau^1 \frac{dx}{x} f_i(x,Q)\, f_j(\frac{\tau}{x},Q) \equiv \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \tau L_{ij}(\tau)$$

where:

$$\frac{\pi A_{ud}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \tau = \frac{m_W^2}{S}$$

Exercise: Study the function $\tau L(\tau)$

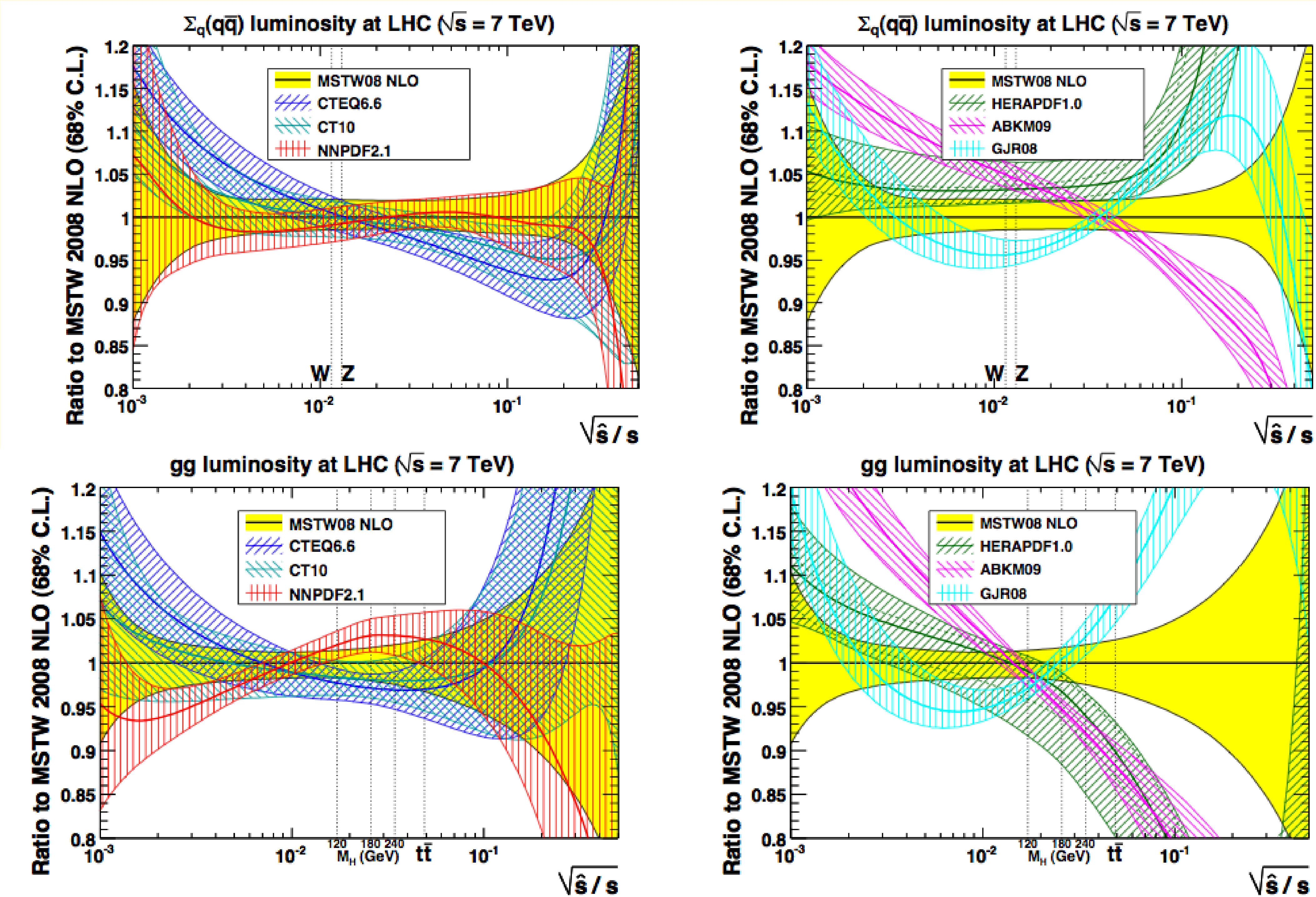
Assume, for example, that $$f(x) \sim \frac{1}{x^{1+\delta}}, \quad 0 < \delta < 1$$

Then: $$L(\tau) = \int_\tau^1 \frac{dx}{x} \frac{1}{x^{1+\delta}} \left(\frac{x}{\tau}\right)^{1+\delta} = \frac{1}{\tau^{1+\delta}} \log\left(\frac{1}{\tau}\right)$$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^\delta \log\left(\frac{S}{m_W^2}\right)$$

Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy.

PDF luminosity uncertainties -- NLO -- 2011



G. Watt, http://arXiv.org/pdf/1106.5788

Example: gg->H cross section

G.Watt, http://arXiv.org/pdf/1106.5788

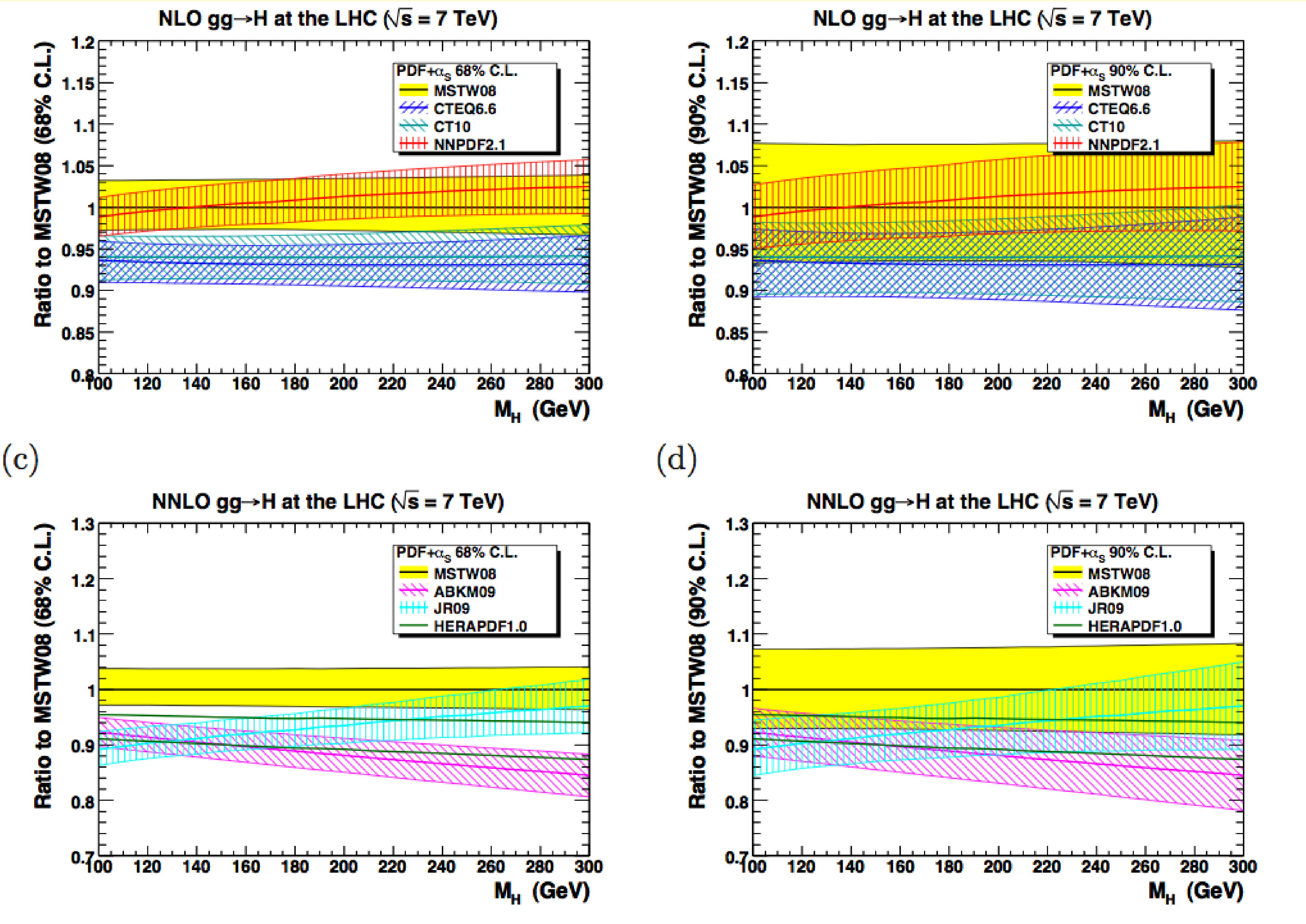
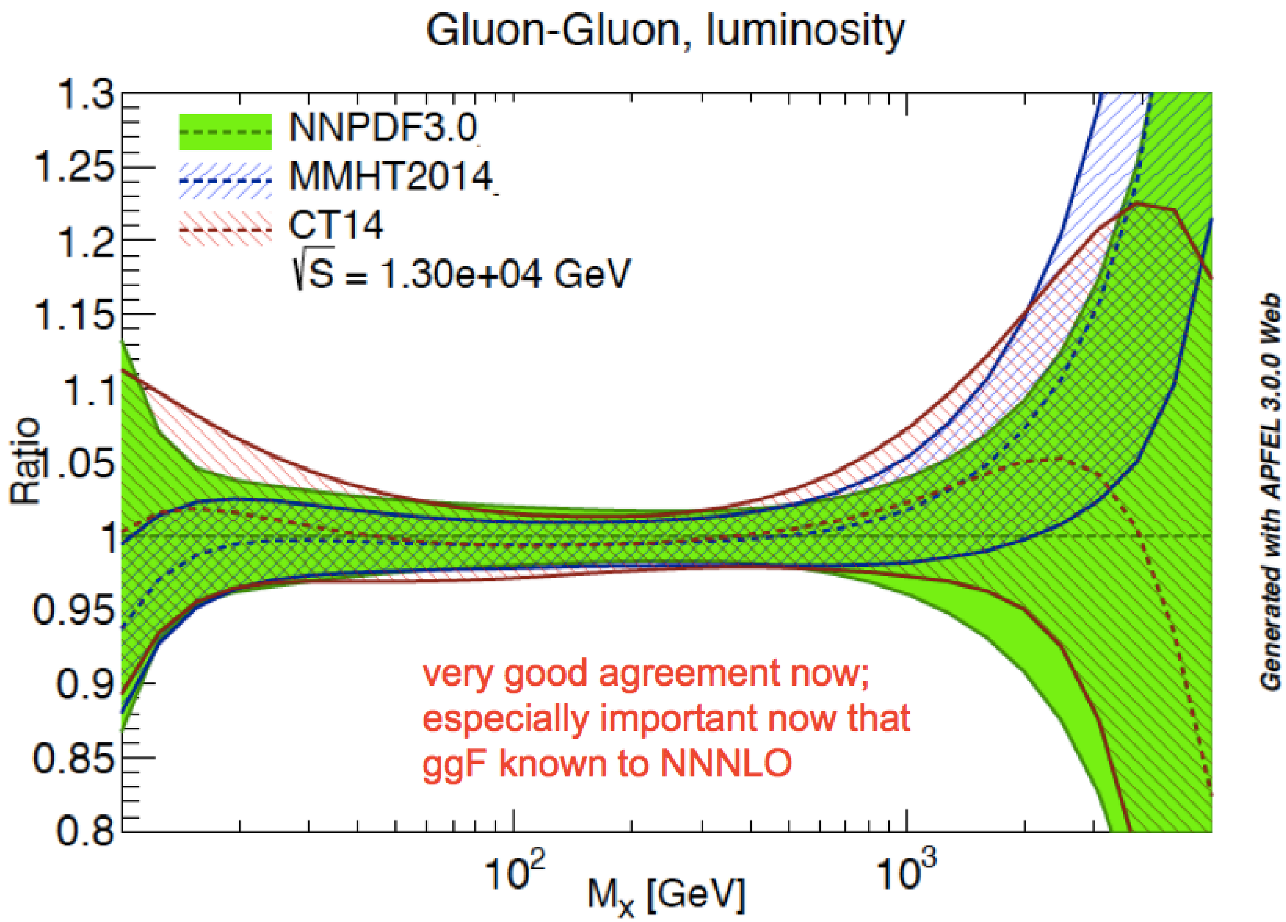


Figure 15. Ratio to the MSTW08 prediction for $gg \to H$ with PDF+α_S uncertainties for (a) NLO at 68% C.L., (b) NLO at 90% C.L., (c) NNLO at 68% C.L., (d) NNLO at 90% C.L.

PDF luminosity uncertainties -- NLO -- 2015

See PDF4LHC mtg, Apr 13 2015



Systematics for Higgs cross section

	CT14	MMHT2014	NNPDF3.0
8 TeV	18.66 pb -2.2% +2.0%	18.65 pb -1.9% +1.4%	18.77 pb -1.8% +1.8%
13 TeV	42.68 pb -2.4% +2.0%	42.70 pb -1.8% +1.3%	42.97 pb -1.9% +1.9%