ICTP 2015 Summer school

Neutrinos
selected topics

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Neutrinos:

1. neutral
2. extremely light
3. elusive

$Q_\gamma = 0$, $Q_c = 0$

$\sim 10^{-7}\ m_e$, $\sim 10^{-10}\ m_p$

the weak and gravitational interactions

fermion: spin $\frac{1}{2}$

One of the most abundant components of the Universe. They are everywhere

Play special and not completely understood role in evolution of the Universe being

probably connected to its ``Dark sector'' (Dark matter and Dark energy)

At one glance

Sources:

Sun

Atmosphere

Earth: Geo-ν

SN1987A

Cosmic Rays

Universe (indirectly)

Accelerators

Reactors

Rad. Sources

All well established/confirmed results are described by

Standard Model + **Mass and Mixing**

"3ν– paradigm"

of three neutrinos with rather peculiar pattern

and nothing more?

Introduction of neutrino mass and mixing may have negligible impact on the rest of SM

mass can be generated by $\frac{1}{\Lambda}$ L L H H

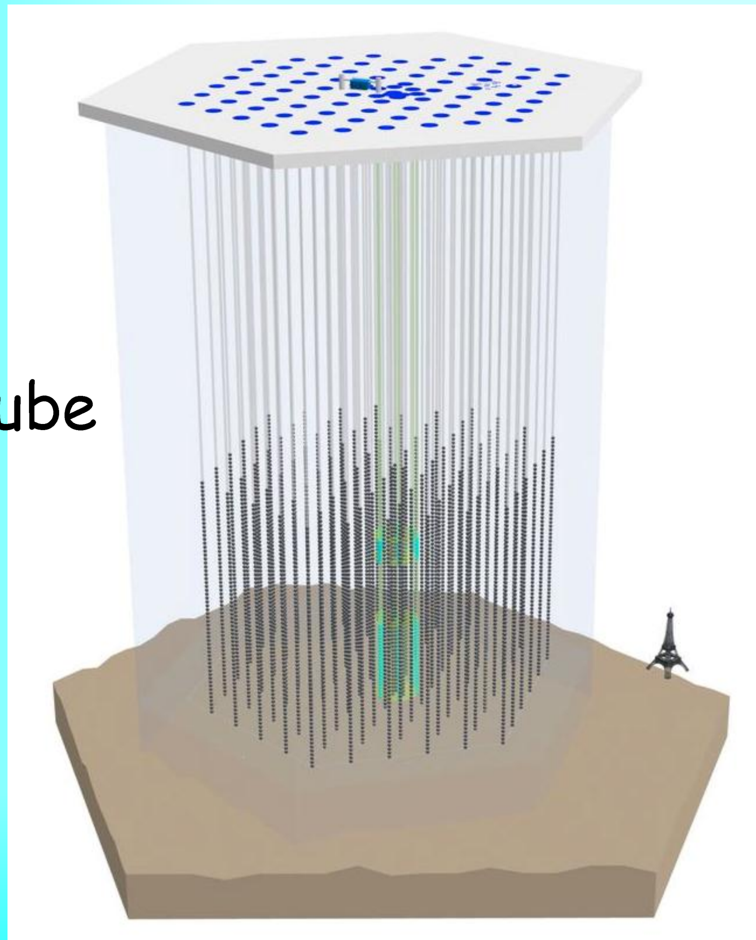
with however $\Lambda \ll M_{Pl}$ → BSM

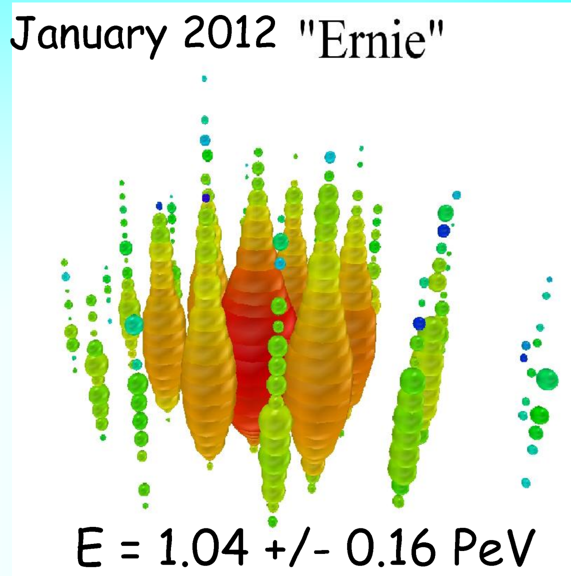
Hightlight

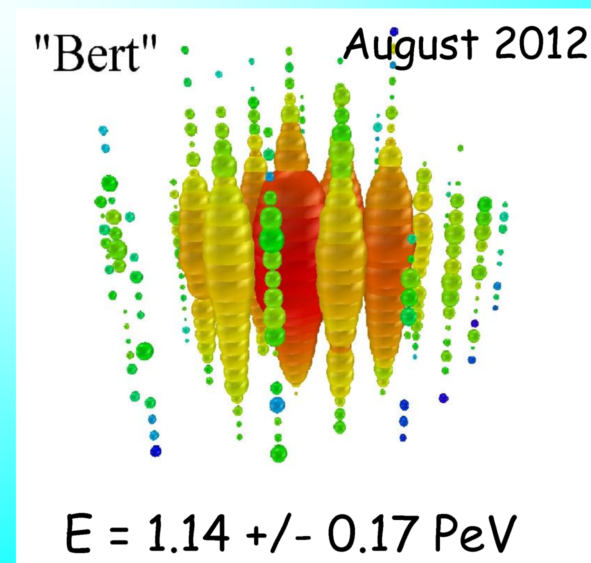
PeV cosmic neutrinos

M.G Aarsten, et al.
arXiv:1304.5356 [astro-ph.HE]

Ice Cube







Scales of new physics

28 orders of magnitude

GUT - Planck mass

$$\frac{V_{EW}^2}{m_\nu}$$

High scale seesaw
Quark- lepton symmetry /analogy
GUT

Unification

Electroweak - LHC

Looking under the lamp

Low scale seesaw, radiative mechanisms, RPV, high dimensional operators

m_ν

eV- sub-eV

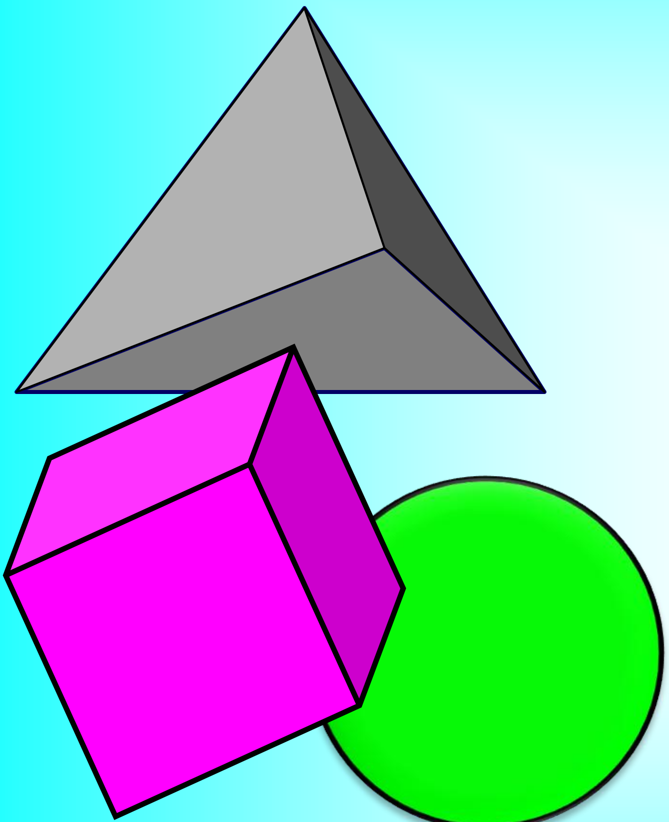
Scale of neutrino masses themselves
Relation to dark energy, MAVAN?

Spurious scale?

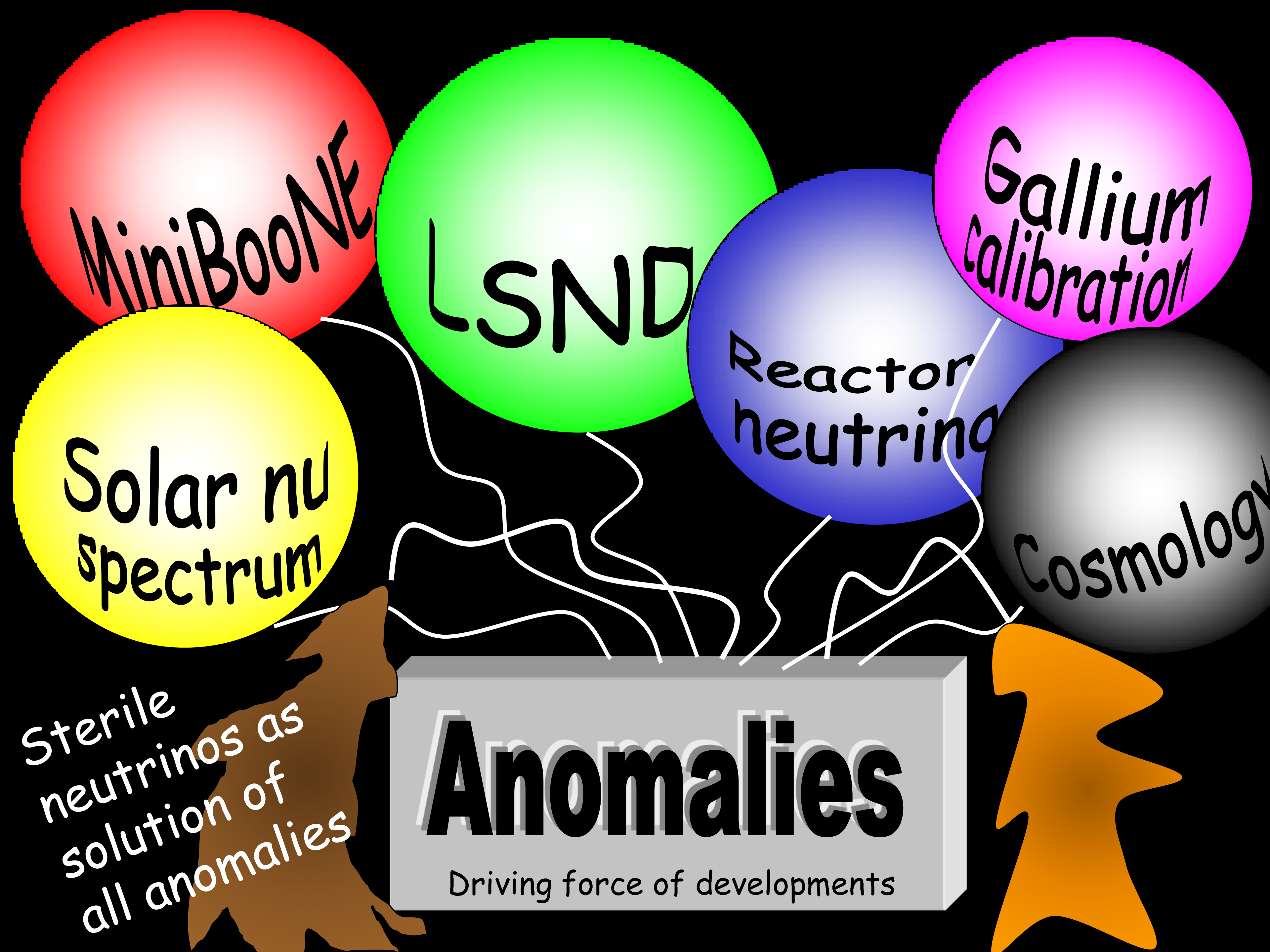
Neutrino mass itself is the fundamental scale of new physics

Mixing:

theta-13



From symmetry
to anarchy
and randomness



MiniBooNE
LSND
Reactor neutrino
Gallium calibration
Solar nu spectrum
Cosmology
Anomalies
Driving force of developments
Sterile neutrinos as solution of all anomalies

Content:

1. Theory of propagation
2. Phenomenology and neutrino parameters
3. Toward understanding neutrino mass and mixing
4. Beyond 3nu paradigm, Sterile neutrinos

1. Masses, mixing, and theory of propagation

Flavors and mixing

Flavor neutrino states:

ν_e ν_μ ν_τ

e μ τ

- correspond to certain charged leptons
- interact in pairs

flavor is characteristic of interactions

$n \rightarrow p + e^- + \bar{\nu}_e$

$\pi \rightarrow \mu + \nu_\mu$

Mass eigenstates

ν_1 ν_2 ν_3

m_1 m_2 m_3

Mixing

Flavor states $\neq$ Mass eigenstates

Neutrinos in SM

SM definition of flavor states may differ from physical one

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$I_W = 1/2$
$I_{3W} = 1/2$

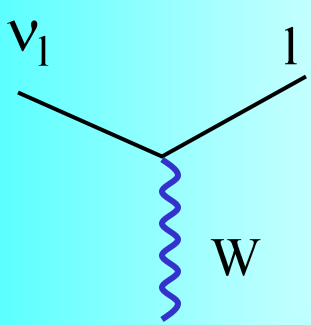
Chiral components

$$\nu_L = \tfrac{1}{2}(1 - \gamma_5)\,\nu$$

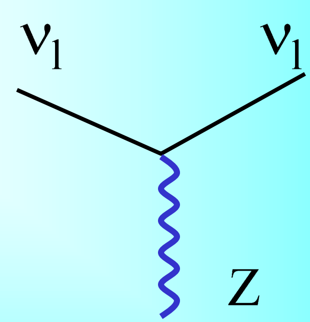
$$\nu_R = \tfrac{1}{2}(1 + \gamma_5)\,\nu$$

?

$\nu_e \quad \nu_\mu \quad \nu_\tau$ neutrino flavor states, form doublets (charged currents) with definite charged leptons,



$l = e\,,\ \mu\,,\ \tau$



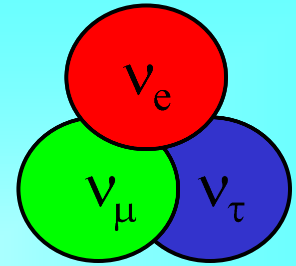
Neutral current interaction

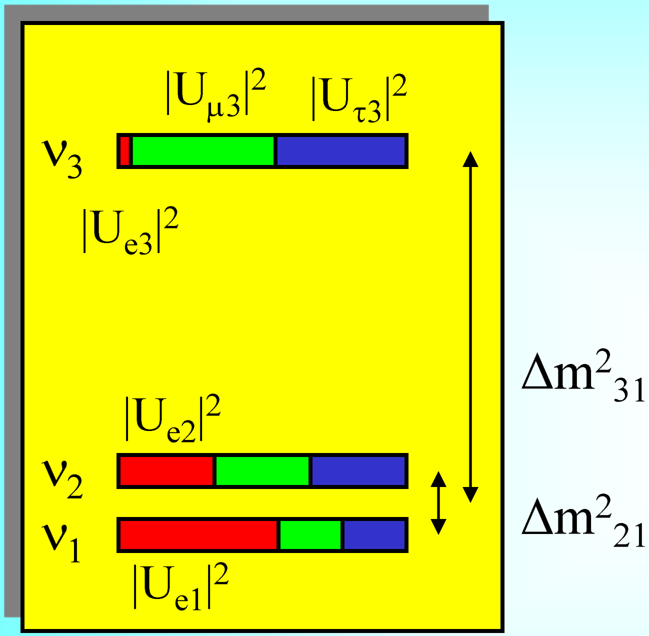
$$\frac{g}{2\sqrt{2}}\,\bar{l}\,\gamma^\mu(1 - \gamma_5)\nu_l\, W^+_\mu \;+\; \text{h.c.}$$

$$\frac{g}{4}\,\bar{\nu}_l\,\gamma^\mu(1 - \gamma_5)\nu_l\, Z_\mu$$

Conservation of lepton numbers L_e, L_μ, L_τ

Mixing angles





flavor

Normal mass hierarchy

$\Delta m^2_{31} = m^2_3 - m^2_1$

$\Delta m^2_{21} = m^2_2 - m^2_1$

Mixing determines the flavor composition of mass states

Mixing parameters

$\tan^2\theta_{12} = |U_{e2}|^2 \;/\; |U_{e1}|^2$

$\sin^2\theta_{13} = |U_{e3}|^2$

$\tan^2\theta_{23} = |U_{\mu 3}|^2 \;/\; |U_{\tau 3}|^2$

Mixing matrix:

$\nu_f = U_{PMNS}\, \nu_{mass}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$

Parameterization

$U_{PMNS} = U_{23} \; I_\delta \; U_{13} \; I_{-\delta} \; U_{12}$

$I_\delta = \mathrm{diag}\,(1,\; 1,\; e^{i\delta})$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & - s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$, etc.

δ is the Dirac CP violating phase

θ_{12} is the ``solar" mixing angle

θ_{23} is the ``atmospheric" mixing angle

θ_{13} is the mixing angle determined by T2K, Daya Bay, CHOOZ, DC…

Mixing and mass matrices

Origin of mixing:
off-diagonal mass matrices

$M_l \neq M_\nu$

Diagonalization: → Mixing matrix → Mass spectrum

$M_l = U_{lL} m_l^{diag} U_{lR}^+$

$m_l^{diag} = (m_e, m_\mu, m_\tau)$

$M_\nu = U_{\nu L} m_\nu^{diag} U_{\nu L}^T$

for Majorana neutrinos

$m_\nu^{diag} = (m_1, m_2, m_3)$

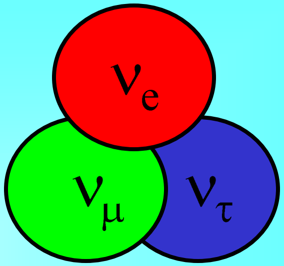
CC in terms of mass eigenstates: $\bar{l} \gamma^\mu (1 - \gamma_5) U_{PMNS} \nu_{mass}$

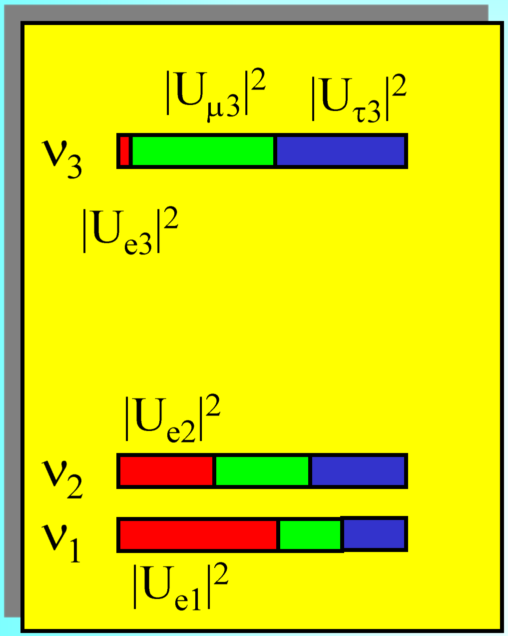
→ $U_{PMNS} = U_{lL}^+ U_{\nu L}$

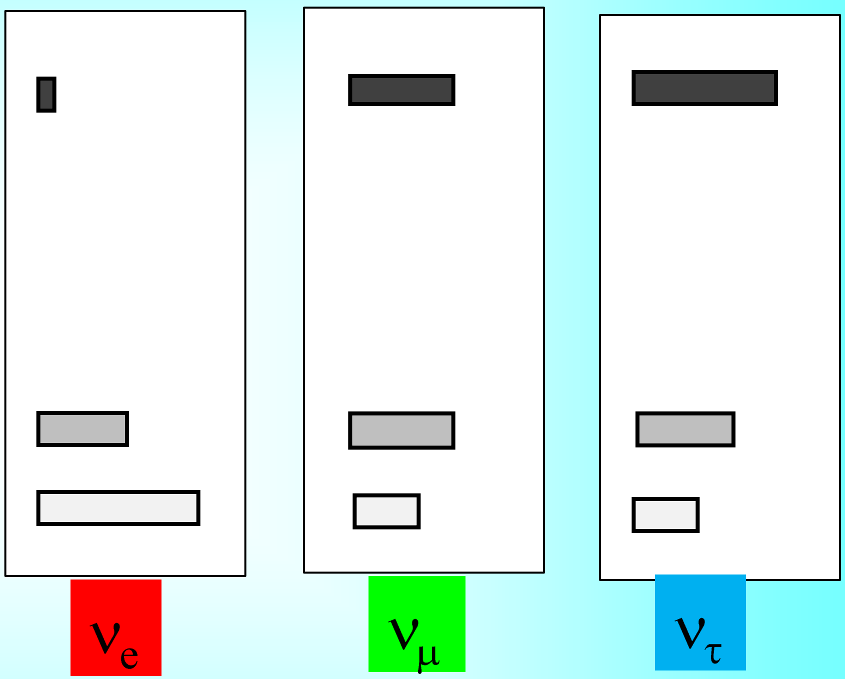
Flavor basis: $M_l = m_l^{diag}$ $U_{PMNS} = U_{\nu L}$

Mixing

Dual role







Flavor content of mass states

$$\nu_{mass} = U_{PMNS}^{+} \nu_f$$

Mass content of flavor states

$$\nu_f = U_{PMNS} \nu_{mass}$$

Who mixes neutrinos?

Mixing in CC → mixing in produced states

Non-trivial interplay of

Charged current weak interactions

Kinematics of specific reactions

Difference of the charged lepton masses

ν_e → $\beta-$ decays, energy conservation

ν_μ → $\pi-$ decays, chirality suppression

ν_τ → Beam dump, D - decay

Energy interval selection, loss of coherence

What about neutral currents?

Propagation effects

Solar neutrinos

KamLAND

Atmospheric neutrinos

Double Chooz

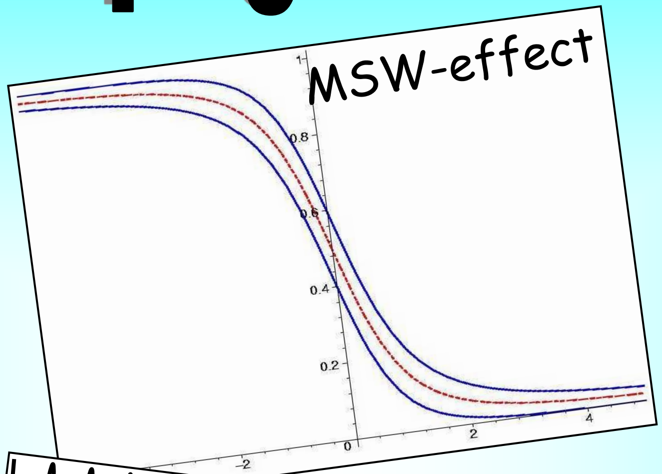
Daya Bay

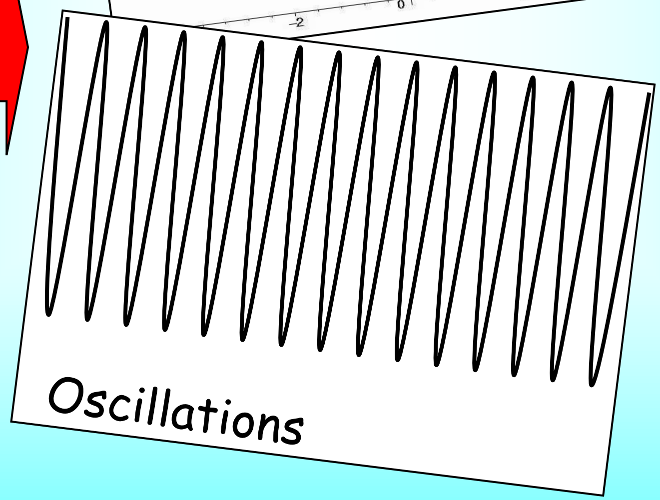
RENO

MINOS

K2K

T2K





Δm^2

θ

Can be resonantly enhanced in matter

Oscillations

Periodic (or quasiperiodic) process of transformation of one neutrino species into another in the process of propagation

58 years ago…

Pisa, 1913

B. Pontecorvo

``Mesonium and antimesonium''

Zh. Eksp.Teor. Fiz. 33, 549 (1957)
[Sov. Phys. JETP 6, 429 (1957)] translation

mentioned a possibility of neutrino mixing and oscillations

Oscillations imply non-zero masses (mass squared differences) and mixing

???

Proposal of neutrino oscillations was motivated by rumor that Davis sees effect in Cl-Ar detector from atomic reactor

... and now:

Oscillation effects have been observed in many experiments

Results are well described by the standard oscillation formula

Naïve derivation (in most of textbooks) in few lines

Still debates on validity of the formula and correctness of its derivation, possible deviations

Should not be oversimplified

Naive derivation ➡ Questions & Paradoxes

Based on

[1] Neutrino production coherence and oscillation experiments.
E. Akhmedov, D. Hernandez, A. Smirnov, JHEP 1204 (2012) 052, arXiv:1201.4128 [hep-ph]

[2] Neutrino oscillations: Entanglement, energy-momentum conservation and QFT.
E.Kh. Akhmedov, A.Yu. Smirnov, Found. Phys. 41 (2011) 1279-1306
arXiv:1008.2077 [hep-ph]

[3] Paradoxes of neutrino oscillations.
E. Kh. Akhmedov, A. Yu. Smirnov Phys. Atom. Nucl. 72 (2009) 1363-1381
arXiv:0905.1903 [hep-ph]

[4] Active to sterile neutrino oscillations: Coherence and MINOS results.
D. Hernandez, A.Yu. Smirnov, Phys.Lett. B706 (2012) 360-366
arXiv:1105.5946 [hep-ph]

[5] Neutrino oscillations: Quantum mechanics vs. quantum field theory.
E. Kh. Akhmedov, J. Kopp, JHEP 1004 (2010) 008
arXiv:1001.4815 [hep-ph]

In principle:

Lagrangian

$$\frac{g}{2\sqrt{2}}\,\bar{l}\,\gamma^{\mu}(1-\gamma_5)\nu_l\,W^{+}_{\mu}$$

$$-\tfrac{1}{2}\,m_L\,\nu_L{}^{T}C\nu_L$$

$$-\,\bar{l}_L\,m_l\,l_R\;+\;h.c.$$

Starting from
the first principles

QFT

QM

Amplitudes,
probabilities
of processes

Observables,
number of
events, etc..

Actually not very simple

Quantum mechanics at macroscopic distances

What is the problem?

Set-up

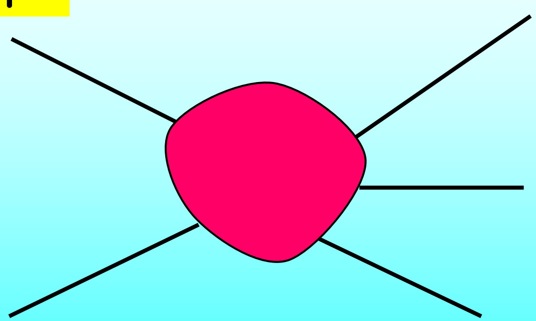
Formalism should be adjusted to specific physics situation

Initial conditions

Recall, the usual set-up

asymptotic states described by plane waves

- enormous simplification



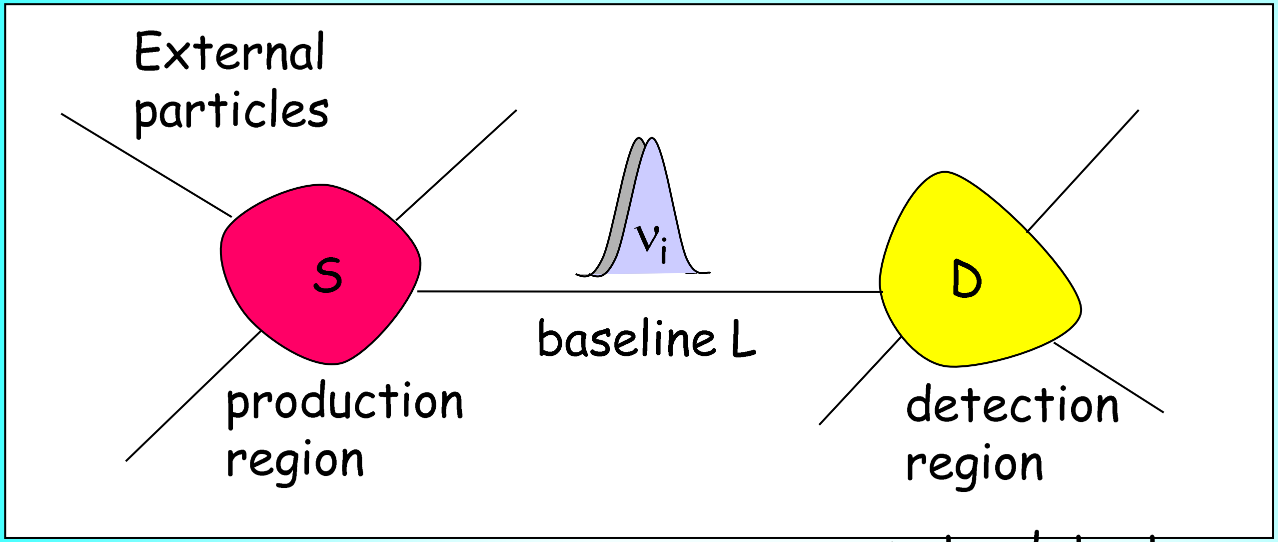
single interaction region

Approximations

Approximations, if one does not want to consider whole history of the Universe to compute signal in Daya Bay

Truncating the process

Oscillation set-up



not a detector

E. Akhmedov, A.S.

Finite space and time phenomenon

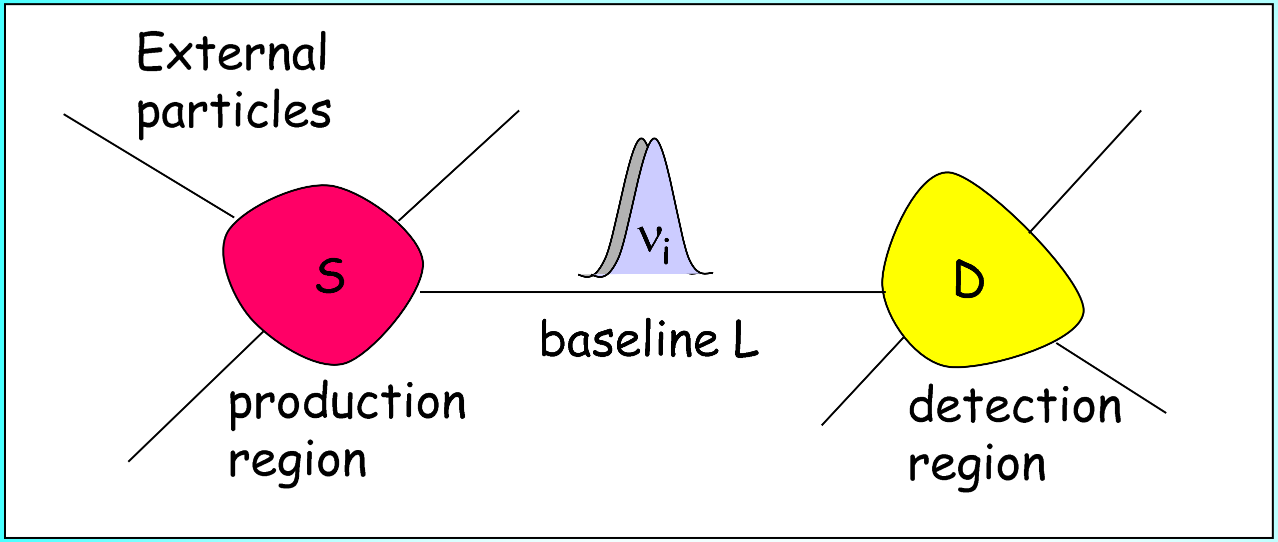
Two interaction regions in contrast to usual scattering problem

Neutrinos: propagator

QFT formalism should be adjusted to these condition

Localization

Where to truncate, how external particles should be described?



Detection and production areas are determined by localization of particles involved in neutrino production and detection

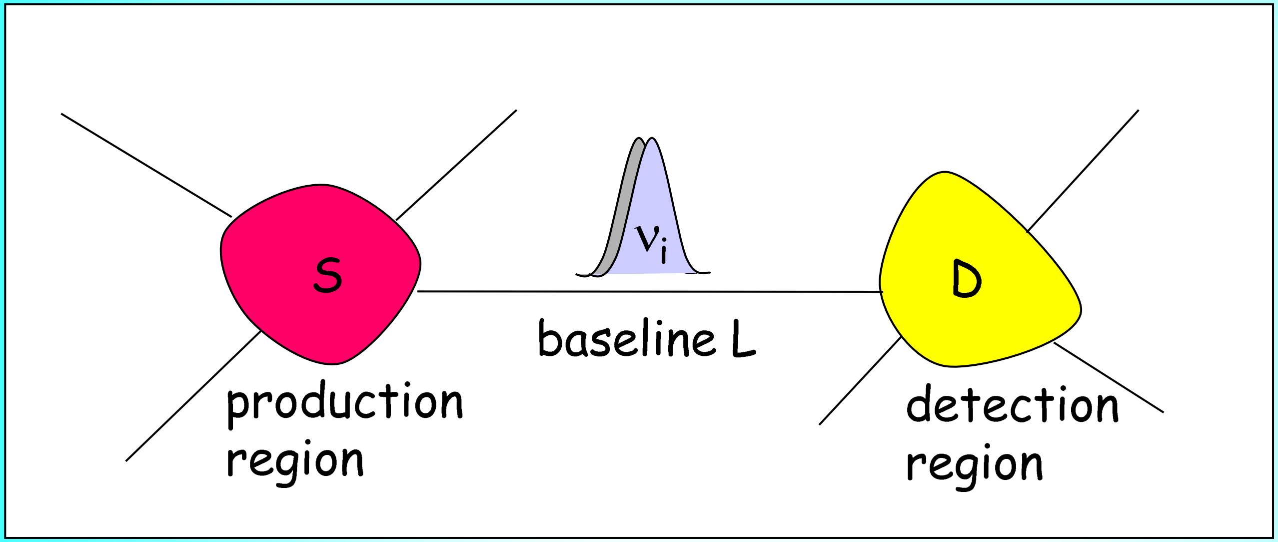
→ Wave packets for external particles

→ Finite space-time integration limits

→ describe by plane waves but introduce finite integration

Areas are not source/detector volumes (still to integrate over incoherently)

How to treat neutrinos?



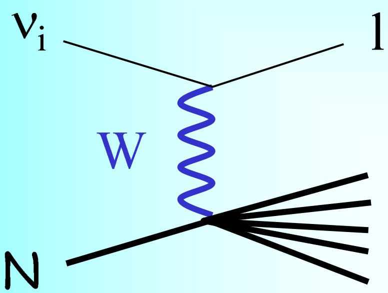
Unique process,
neutrinos with definite masses are described by propagators.
Oscillation pattern – result of interference of
amplitudes due to exchange of different mass eigenstates

Very quickly converge to mass shell
Real particles – described by wave packets

In terms of mass eigenstates

Without flavor states

Scattering

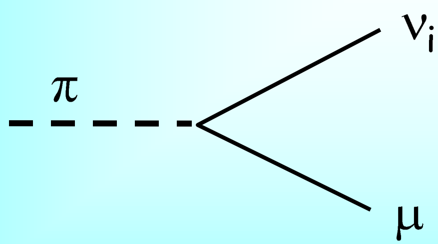


$$\frac{g}{2\sqrt{2}} U_{PMNS} \bar{l} \gamma^{\mu} (1 - \gamma_5) \nu_i W^{+}_{\mu} + h.c.$$

interaction constant

Eigenstates of the Hamiltonian in vacuum

$\pi \rightarrow \mu \ \nu_i$



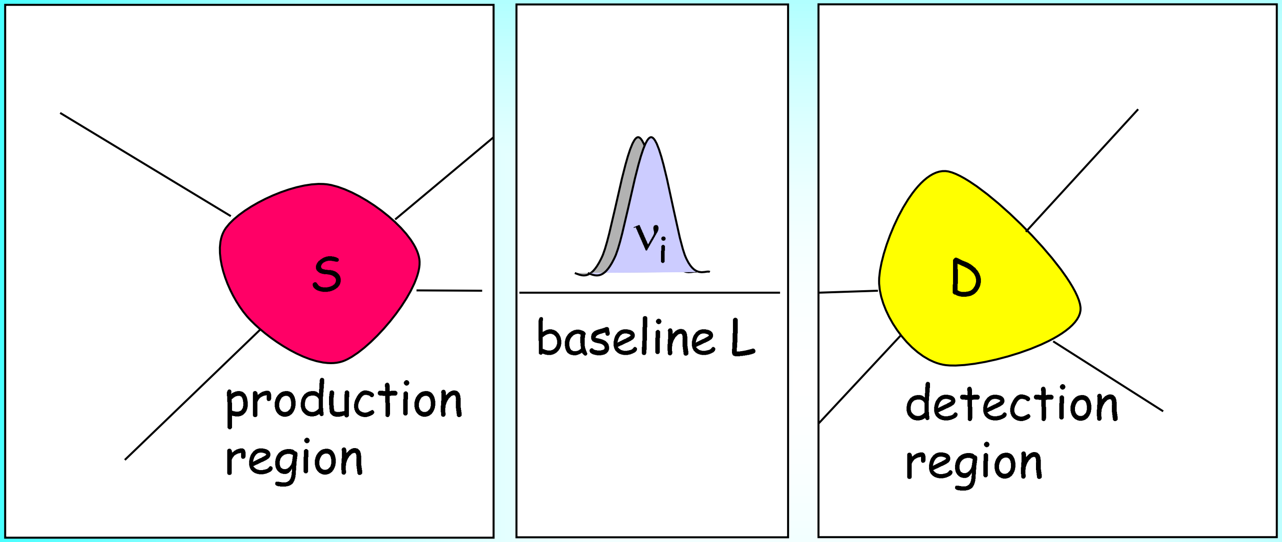
Lagrangian of interactions

wave functions of accompanying particles

compute the wave functions of neutrino mass eigenstates

Wave packets

Factorization



If oscillation effect in production/detection regions can be neglected

r_D , r_S << l_ν

⇒ **factorization**

Production, propagation and detection can be considered as three independent processes

Wave packets and oscillations

Suppose ν_α is produced in the source centered at x = 0, t = 0

After formation of the wave packet (outside the production region)

$$|\nu_\alpha(x,t)\rangle = \Sigma_k\, U_{\alpha k}^* \,\Psi_k(x,t)|\nu_k\rangle$$

$$\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$$ - WF of k-mass state

$$E_k(p) = \sqrt{p^2 + m_k^2}$$ - dispersion relation

$f_k(p - p_k)$ - the momentum distribution function peaked at p_k - the mean momentum

Expanding around mean momentum

describes spread of the wave packets

$$E_k(p) = E_k(p_k) + (dE_k/dp)\Big|_{p_k}(p - p_k) + (dE_k^2/dp^2)\Big|_{p_k}(p - p_k)^2 + \ldots$$

$$v_k = (dE_k/dp)\Big|_{p_k} = (p/E_k)\Big|_{p_k}$$ - group velocity of ν_k

Shape factor and phase factor

$E_k(p) = E_k(p_k) + v_k(p - p_k)$

(neglecting spread of the wave packets)

Inserting into $\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t}\, g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$\phi_k = p_k x - E_k t$

Depends on mean characteristics p_k and corresponding energy:

$E_k(p_k) = \sqrt{p_k^2 + m_k^2}$

Shape factor

$$g_k(x - v_k t) = \int dp\, f_k(p)\, e^{ip(x - v_k t)}$$

Depends on x and t only in combination $(x - v_k t)$ and therefore describes propagation of the wave packet with group velocity v_k without change of the shape

Mixing & mixed states

One needs to compute the state which is produced
i.e. compute

the shape factors $g_k(x - v_k t)$

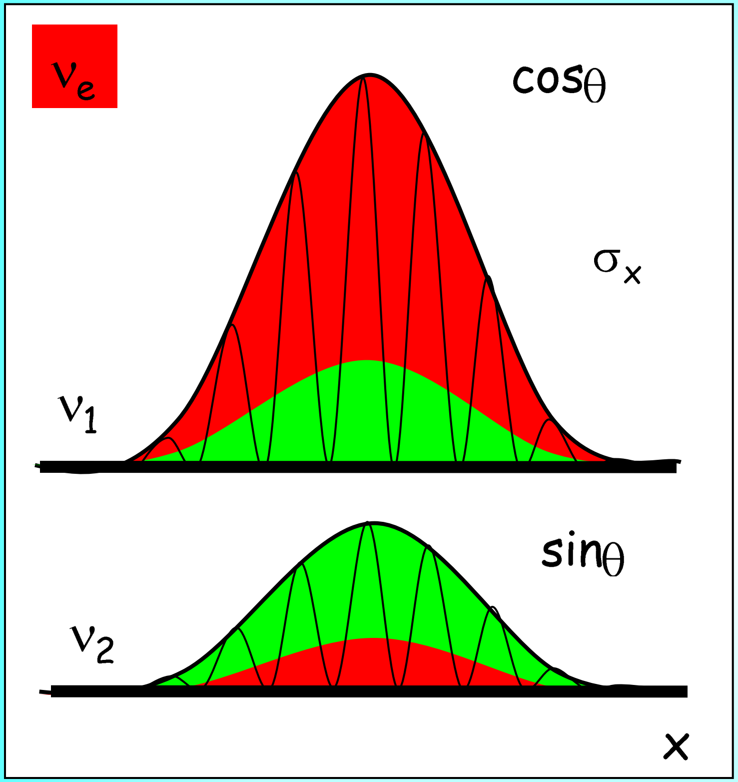
mean momenta p_k

- Fundamental interactions
- Kinematics
- characteristics of parent and accompanying particles

Process dependent

If heavy neutrinos are present but can not be produced for kinematical reasons, flavor states in Lagrangian differ from the produced states, etc..

Wave packet picture



$\phi = 0$

$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$

$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$

opposite phase

$\nu_1 = \cos\theta\ \nu_e - \sin\theta\ \nu_\mu$

$\nu_2 = \cos\theta\ \nu_\mu + \sin\theta\ \nu_\varepsilon$

Interference of the same flavor parts

Main, effective frequency

$$|\nu(x,t)\rangle = \cos\theta\ g_1(x - v_1 t) e^{i\phi_1} |\nu_1\rangle + \sin\theta\ g_2(x - v_2 t) e^{i\phi_2} |\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

Changes with (x,t), for $\phi = 0$ components ν_μ will not cancel → appearance of ν_μ

Propagation of wave packets

What happens?

Phase difference change	Due to different masses (dispersion relations) → phase velocities	**Oscillations**
Separation of wave packets	Due to different group velocities	**Loss of coherence**
Spread of individual wave packets	Due to presence of waves with different momenta and energy in the packet	**Loss of coherence within within WP**