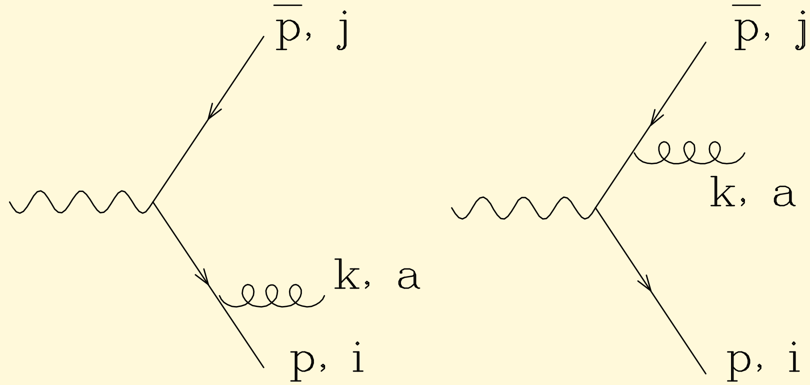
Soft gluon emission



$$A = \bar{u}(p)\epsilon(k)(ig)\frac{-i}{\not p+\not k}\Gamma^\mu v(\bar p)\,\lambda^a_{ij} + \bar u(p)\,\Gamma^\mu \frac{i}{\not{\bar p}+\not k}(ig)\epsilon(k)v(\bar p)\,\lambda^a_{ij}$$
$$= \left[\frac{g}{2p\cdot k}\bar u(p)\epsilon(k)\,(\not p+\not k)\Gamma^\mu v(\bar p) - \frac{g}{2\bar p\cdot k}\bar u(p)\,\Gamma^\mu(\not{\bar p}+\not k)\epsilon(k)\,v(\bar p)\right]\lambda^a_{ij}$$

$p\cdot k = p_0\, k_0\, (1-\cos\theta) \Rightarrow$ singularities for collinear ($\cos\theta\to 1$) or soft ($k_0 \to 0$) emission

Collinear emission does not alter the global structure of the final state, since its preserves its "pencil-like-ness". **Soft emission** at large angle, however, could spoil the structure, and leads to strong interferences between emissions from different legs. So soft emission needs to be studied in more detail.

In the soft ($k_0\to 0$) limit the amplitude simplifies and ***factorizes*** as follows:

Exercise:

$$A_{soft} = g\lambda^a_{ij}\left(\frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar p\cdot\epsilon}{\bar p\cdot k}\right) A^0_{Born} \quad \text{where} \quad A_{Born} \equiv \delta_{ij} A^0_{Born}$$

Factorization: it is the expression of the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

Exercise: verify the following relations

Normalization convention and T_F definition: $$\mathrm{tr}\,\lambda^a\lambda^b = \frac{\delta^{ab}}{2} \equiv T_F\,\delta^{ab}$$

Follows from previous convention, def of C_F $$\sum_a (\lambda^a\lambda^a)_{ij} = \frac{N_c^2-1}{2N_c}\delta_{ij} \equiv C_F\delta_{ij}$$

$$\sum_a \lambda^a_{ij}\lambda^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \quad \Rightarrow \sum_{a,i,j}\lambda^a_{ij}\lambda^a_{ji} = \frac{N_C^2-1}{2}$$

$$A_{soft} = g\lambda^a_{ij} \quad \left(\frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k}\right)\ A^0_{Born}$$

$$\sum_{col} |A_{Born}|^2 = \delta_{ij}\delta_{ji}|A^0_{Born}|^2 = N_c|A^0_{Born}|^2$$

$$\sum_{pol}\left|\frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k}\right|^2 = \frac{p^\mu p^\nu + \bar{p}^\mu\bar{p}^\nu - 2p^\mu\bar{p}^\nu}{(pk)(\bar{p}k)}\sum \epsilon_\mu\epsilon^*_\nu = \frac{2(p\bar{p})}{(pk)(\bar{p}k)}$$

$$\sum_{col}|A_{Soft}|^2 = C_F\,g^2\ \frac{2(p\bar{p})}{(pk)(\bar{p}k)}\ |A_{Born}|^2$$