The Goldstone Boson Equivalence Theorem

After this introduction to LHC physics, I would like to take up a theoretical topic that plays an important role in the physics of high-energy colliders. To begin, recall some results from our discussion in the first lecture of vector boson polarizations. A massless vector boson has two polarization states, R and L, related in a clear way to the two possible values of the boson helicity. A massive vector boson has three polarization states. The third of these, the longitudinal polarization vector, has the strange-looking form (for $\vec{p} \parallel \hat{3}$)

$$\epsilon^\mu(p) = \left(\frac{p}{m}, 0\ 0\ \frac{E}{m}\right)^\mu$$

This polarization vector has the property that

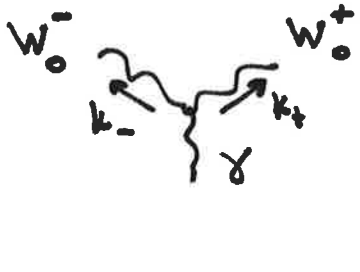
$$\epsilon^\mu(p) \xrightarrow[p^0/m \to \infty]{} \frac{p^\mu}{m}$$

and yet obeys

$$\epsilon(p) \cdot p = 0$$

as required for a polarization vector.

Potentially, the large size of the components of this vector can lead to large results for amplitidues correspoinding to the emission and absorption of vector bosons. For example, the W boson coupling to electromagnetism contains the term



$$\sim \quad ie\ \epsilon_-^* \cdot \epsilon_+^* (k_- - k_+)^\mu$$

similar to the coupling to scalar fields, but with an extra factor of $\epsilon^*(k_+) \cdot \epsilon^*(k_-)$. This seems to lead to the result

$$\sigma(e^+e^- \to W^+W^-) \quad \sim \quad \sigma(e^+e^- \to \phi^+\phi^-) \cdot \frac{k_+ \cdot k_-}{m_W^2}$$

$$\sim \quad \sigma(e^+e^- \to \phi^+\phi^-) \cdot \frac{s}{2m_W^2}$$

It is easy to find other examples in which individual diagrams involving longitudinal W bosons are enhanced over our simpler expectations by factors of E_W/m_W.

Are these enhancements real or spurious? The answer to this question is not simple. But there is an organizing principle that can be applied. A part of the logic of the Standard Model is the statement that a vector boson can obtain mass only through spontaneous symmetry breaking of its corresponding gauge symmetry. The Goldstone boson of the system with spontaneous symmetry breaking gives the extra degree of freedom needed to give the vector boson a third polarization state. We will see that the form of the longitudinal polarization vector given above is just what is needed so that, at high energy, the longitudinal polarization state retains the properties of this Goldstone boson.

The description of the degrees of freedom of a vector boson is gauge-dependent. In the Feynman-'t Hooft gauge, the vector boson has four polarization states with the propagator

$$\frac{-i g^{\mu\nu}}{p^2 - m^2}$$

and the Goldstone boson also appears in calculations as a scalar with mass m. The timelike polarization state of the gauge boson has negative metric

$$[a^0_p, a^{0\dagger}_{p'}] = -(2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}\,')$$

so it counts as a negative degree of freedom. The properties of the vector boson results from calcellations among the timelike and longitudinal polarization states and the Goldstone boson. In the unitary gauge, the Goldstone boson is gauged away, and the gauge boson propagator becomes

$$\frac{-i}{p^2-m^2}\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m_W^2}\right) = \frac{+i}{p^2-m^2}\sum_{i=R,0,L}\epsilon_i^\mu(p)\,\epsilon_i^{\nu *}(p)$$

The factor $p^\mu p^\nu/m^2$ expresses the growth of the longitudinal polarization state that we saw earlier.

For a vector boson at rest, the three polarization states are equivalent, related simply by spatial rotations. However, for a highly boosted vector boson, especially in the regime $p^0 \gg m$ in which we can ignore the boson mass, we might expect that the different character of the polarization states would become apparent. And, indeed, there is a theorem on this point, called the "Goldstone Boson Equivalence Theorem" (GBET):

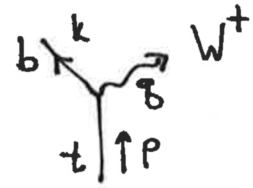
$$\mathcal{M}(X\to Y+W^+(p)) = \mathcal{M}(X\to Y+\pi^+(p))\left(1+\mathcal{O}\left(\frac{m_W}{E_W}\right)\right)$$

This result was first enuciated by Cornwall and Tiktopoulos and Vayonakis. You can find a careful proof of the theorem, applicable to the case in which a process emits several high-energy vector bosons, in a beautiful paper of Chanowitz and Gaillard (Nucl. Phys. B 261, 379 (1985)). In this lecture, I will not give a proof of the theorem, but I will give illustrative examples of different types.

One of the nicest examples of the GBET comes in the theory of the decays of the top quark. In lecture 4, we saw that the top quark decays to an on-shell W boson and a b quark, leading to a variety of final states with 3 fermions. It is interesting to compute the decay width of the top quark for the various possible polarization states of the final W. In this discussion, I will ignore the mass of the b quark for simplicity.

The matrix element for t decay is

$$i\mathcal{M} = i\frac{g}{\sqrt{2}}\, u_L^\dagger(k)\,\bar{\sigma}\cdot\epsilon^*(q)\,u_L(p)$$



For a top quark at rest, we have

$$u(p) = \sqrt{m}\begin{pmatrix}\xi \\ \xi\end{pmatrix} \quad \text{or} \quad u_L = \sqrt{m}\,(\xi)$$

It is easiest to work in the frame in which the W boson is produced in the $\hat{3}$ direction. The results can be interpreted by rotating the top quark spin to the $\hat{3}$ direction and the W to the polar angle θ. Then the setup is

with 2-component spinors

$$u_L(k) = \sqrt{2k}\begin{pmatrix}-1 \\ 0\end{pmatrix} \qquad u_L(p) = \sqrt{m}\begin{pmatrix}\cos\theta/2 \\ -\sin\theta/2\end{pmatrix}$$

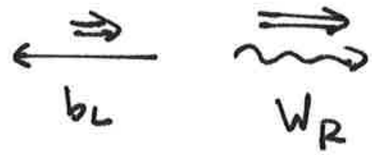
and W polarization vectors

$$\varepsilon^*(q) = \begin{array}{ll} \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & -i & 0\end{pmatrix} & R \\ \begin{pmatrix}\frac{p}{m} & 0 & 0 & \frac{E}{m}\end{pmatrix} & 0 \\ \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & i & 0\end{pmatrix} & L \end{array}$$

The amplitude for the decay $t \to W_R b$ is given by

$$i\frac{g}{\sqrt{2}}\sqrt{2km}\,\begin{pmatrix}-1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0 \\ 2 & 0\end{pmatrix}\begin{pmatrix}\cos\theta/2 \\ -\sin\theta/2\end{pmatrix} = 0$$

We might have expected this vanishing because the state $W_R b_L$ has spin $\frac{3}{2}$



For the decay to $W_L b$, we find

$$i\frac{g}{\sqrt{2}}\sqrt{2km}\;(-1\;\;0)\,\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 2\\ 0 & 0\end{pmatrix}\begin{pmatrix}\cos\theta/2\\ -\sin\theta/2\end{pmatrix} = -i\,g\sqrt{2km}\,\sin\theta/2$$

For the decay to the longitudinal polarization state W_0, we find

$$i\frac{g}{\sqrt{2}}\sqrt{2km}\;(-1\;\;0)\,\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{p+E}{m_W} & 0\\ 0 & \frac{p-E}{m_W}\end{pmatrix}\begin{pmatrix}\cos\theta/2\\ -\sin\theta/2\end{pmatrix}$$

$$p+E=m_t$$

$$= -i\frac{g}{\sqrt{2}}\sqrt{2km_t}\;\frac{m_t}{m_W}\;\cos\theta/2$$

Summing these contributions, the total decay rate is

$$\Gamma = \frac{1}{2m_t}\,\frac{1}{8\pi}\,\langle \cos^2\theta/2 \text{ or } \sin^2\theta/2\rangle\; g^2\; 2km_t\left(\frac{2k}{m_t}\right)\left(1+\frac{m_t^2}{2m_W^2}\right)$$

$$= \frac{\alpha_w}{8}\,m_t\left(\frac{2k}{m_t}\right)^2\left(1+\frac{m_t^2}{2m_W^2}\right)$$

or, finally,

$$\Gamma_t = \frac{\alpha_w}{16}\,\frac{m_t^3}{m_W^2}\left(1+\frac{2m_W^2}{m_t^2}\right)\left(1-\frac{m_W^2}{m_t^2}\right)^2$$

Something is odd here. We expected a result of the order of

$$I_t \sim \alpha_w m_t$$

but instead we found the dependence

$$I_t \sim \alpha_w m_t \frac{m_t^2}{m_W^2}$$

It is not so hard to trace the origin of the enhancement. It is associated with the large values of the components of the longitudinal W polarization vector. If we had summed over polarizations at an earlier stage, using

$$\sum_i \varepsilon_i^\mu(q)\,\varepsilon_i^{\nu *}(q) = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2}\right)$$

the large factor would have come from the term in this expression

$$\frac{q^\mu q^\nu}{m_W^2} \sim \frac{m_t^2}{m_W^2}$$

But, is the enhancement really there?

We can get some insight by evaluating the right-hand side of the GBET. The Goldstone boson of the Higgs field couples to the top quark through the Yukawa coupling

$$\Delta\mathcal{L} = -y_t\,\overline{Q}_L\cdot\phi\, t_R + h.c.$$

with

$$\phi = \begin{pmatrix} \pi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi^0) \end{pmatrix}$$

The coefficient y_t is the coupling that generates the top quark mass through

$$m_t = \frac{y_t v}{\sqrt{2}}$$

This vertex gives rise to a top quark decay amplitude

$$\begin{aligned} i\mathcal{M}(t \to b\pi^+) &= -iy_t \, u_L^\dagger(k)\, u_R(p) \\ &= -iy_t \sqrt{2km_t} \cdot \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix} \\ &= iy_t \sqrt{2km_t} \cos\theta/2 \end{aligned}$$

This is exactly what we found in the direct calculation of the amplitude for $t \to W_0 b$, since the prefactor in that expression can be simplified to

$$\frac{g}{\sqrt{2}} \frac{m_t}{m_W} = \frac{g}{\sqrt{2}} \frac{y_t v/\sqrt{2}}{gv/2} = y_t \qquad !$$

So, the enhancement of the top quark decay rate by the factor m_t^2/m_W^2 has a physical origin. it reflects the fact that the longitudinal W boson has a Higgs-sector coupling, rather than a gauge-sector coupling, to the top quark.

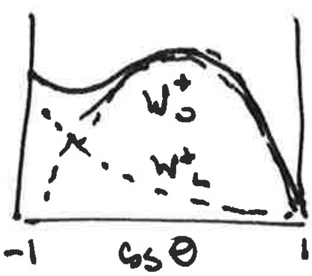
This argument makes a quite specific prediction for the polarization of W bosons in top quark decays. This polarization can be measured. In lecture 4, I explained that it is straightforward to collect a sample of $pp \to t\bar{t}$ events in which the top quark decays are fully reconstructed. If we then boost the W bosons to their rest frames, the distribution of the polar decay angle θ indicates the polarization state in the way that we analyzed in lecture 1. The polar angle is most readily determined by studying the leptonic W decay in a final state

$$t\bar{t} \to b W^+ \bar{b} W^- \to b\, q\bar{q}\, \bar{b}\, \ell^- \nu$$

The above analysis implies that the W bosons appear only in the 0 and L polarization states, with

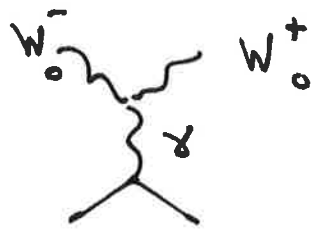
$$\mathrm{Prob}(W_0^+) = \frac{m_t^2/m_W^2}{m_t^2/m_W^2 + 2} = 70\%$$

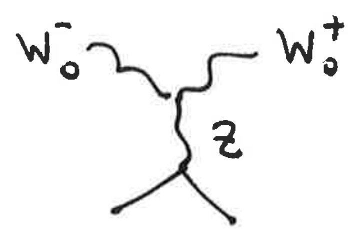
From the angular distributions given in lecture 1, we expect that the θ distribution of the W boson in top decay has the form



The measurements from the CDF experiment at the Tevatron and the ATLAS expeiment at the LHC are shown in Figures 1 and 2. The prediction works very well.

Another possibility for the behavior of the longitudinal polarization state of the W is illustrated by the process $e^+e^- \to W_0^+ W_0^-$. I pointed out earlier that the diagrams





contain an extra, enhancing, factor of $s/2m_W^2$. Let us now analyze this process more carefully.

Consider first the production from $e_R^- e_L^+$. For longitudinally polarized Ws, the diagrams with virtual γ and Z taken the form

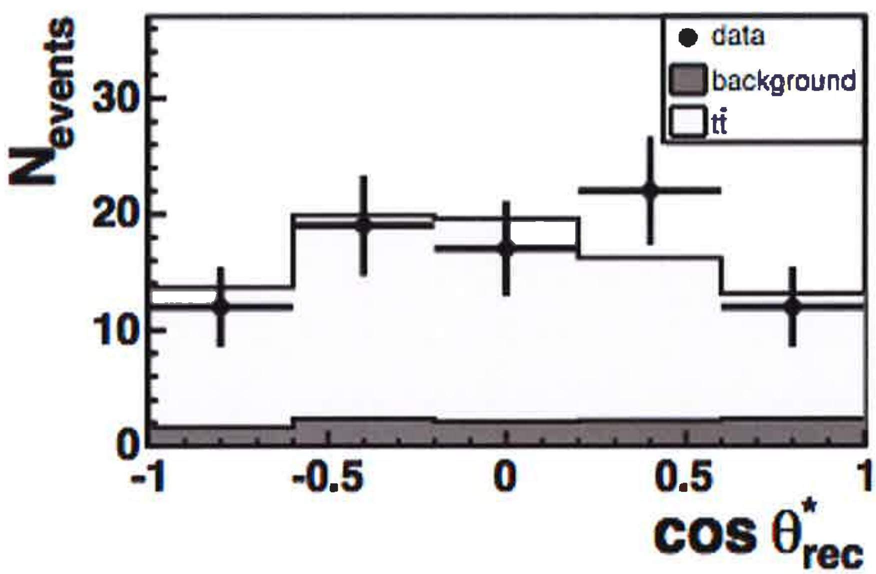


Fig 1 Distribution of $\cos\Theta$ in W decays from top quarks, measured by the CDF experiment at the Tevatron, arXiv: 0612011, Phys. Rev D75 052001 (2007).

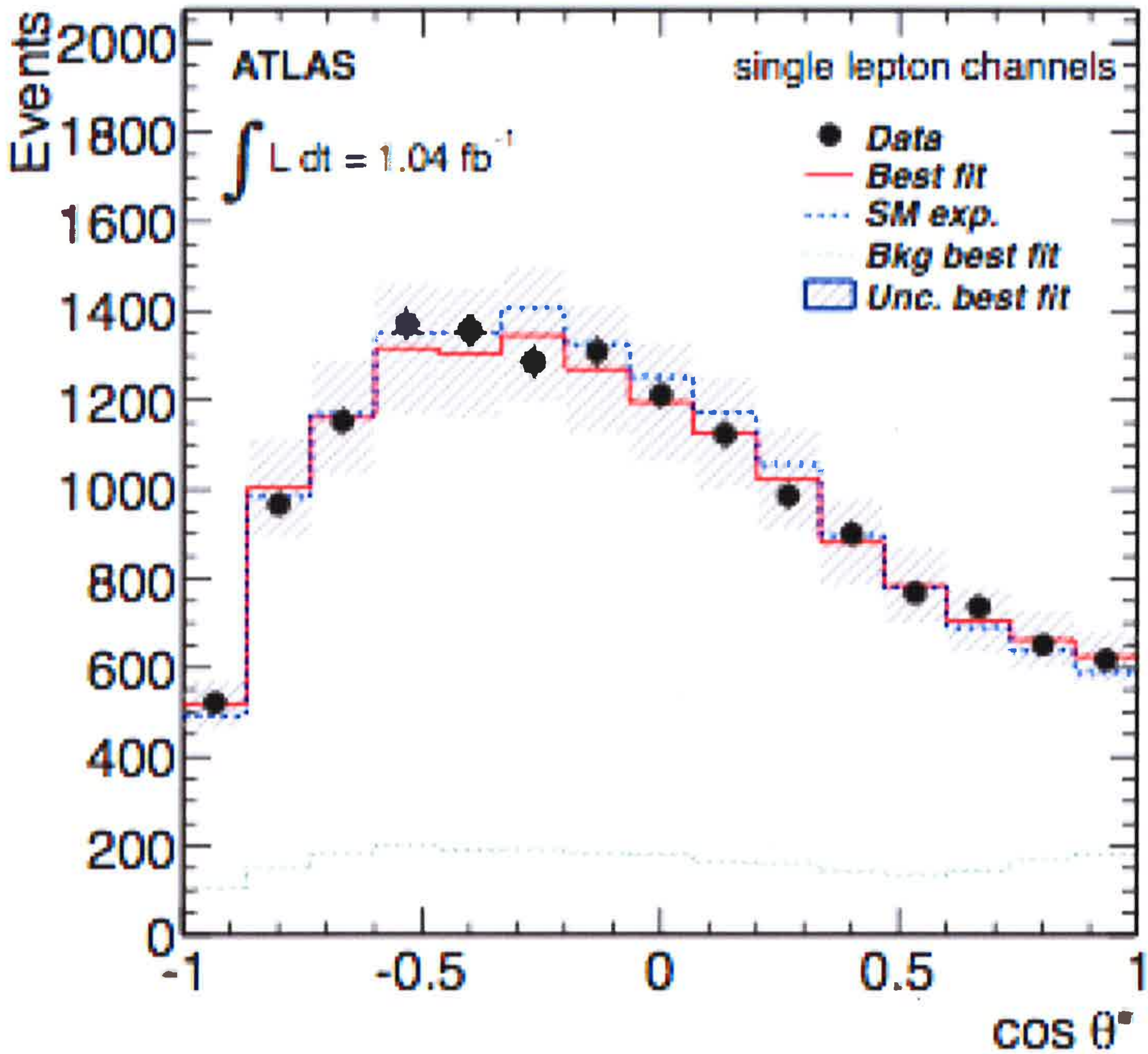


Fig. 2 Distribution of cos Θ in W decays from top quarks, measured by the ATLAS experiment at 7 TeV, arXiv: 1205.2484, JHEP 1206 088(2012).

$$i\mathcal{M} = (-ie)(ie)\; 2\sqrt{2}\, p\, \varepsilon_R^\mu(p)$$
$$\cdot \left[\frac{-i}{s} + \frac{-i}{s-m_Z^2}\left(\frac{-s_w^2}{c_w s_w}\right)\left(\frac{c_w}{s_w}\right)\right]$$
$$\cdot \left[\varepsilon_-^* \cdot \varepsilon_+^* (k_- - k_+)_\mu + \varepsilon_{-\mu}^* (-q-k_-)\cdot \varepsilon_+^* + \varepsilon_{+\mu}^* (q+k_+)\cdot \varepsilon_-^* \right]$$

Putting

$$\varepsilon_- \sim \frac{k_-}{m_W} \qquad \varepsilon_+ \sim \frac{k_+}{m_W}$$

so that

$$\varepsilon_+ \cdot (k_- + q) \cong \frac{2k_+ \cdot k_-}{m_W} \qquad \varepsilon_- \cdot (k_+ + q) \cong \frac{2k_- \cdot k_+}{m_W}$$

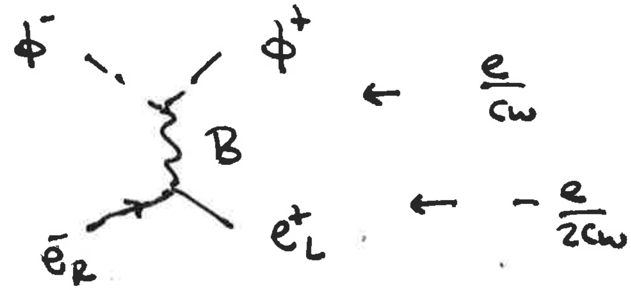
we find

$$i\mathcal{M} = -ie^2\, 2\sqrt{2}\, p\, \varepsilon_R^\mu(p)\, (k_- - k_+)_\mu \left(\frac{1}{s} - \frac{1}{s-m_Z^2}\right)$$
$$\cdot \left(\frac{s}{2m_W^2} - \frac{s}{m_W^2}\right)$$
$$= -ie^2\, 2\sqrt{2}\, p\, \varepsilon_R(p)\cdot(k_- - k_+) \left(-\frac{s}{2m_W^2}\right)\left(\frac{-m_Z^2}{s^2}\right)$$

The final result is . $(m_W^2 = m_Z^2 c_W^2)$

$$= -ie^2\, 2\sqrt{2}\, p\, \varepsilon_R(p)\cdot(k_- - k_+) \cdot \frac{1}{s} \cdot \frac{1}{2c_W^2}$$

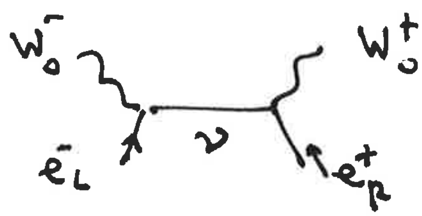
which is just the amplitude for the production of $\phi^+\phi^-$ in an unbroken $U(1)$ gauge theory.



For e_L^-, the analysis is a little more complicated. In the same approximation as above, the γ and Z diagrams give

$$i\mathcal{M} = (-ie)(ie)\, 2\sqrt{2}\, p\, \varepsilon_L(p)\cdot(k_- - k_+)\left(-\frac{s}{2m_W^2}\right) \cdot \left[\frac{-i}{s} + \frac{-i}{s-m_Z^2}\,\frac{(\frac{1}{2}-s_w^2)}{c_w s_w}\cdot\frac{c_w}{s_w}\right]$$

so that the cancellation of the leading terms for large s is not complete. However, for e_L^-, there is one more diagram



This contribution equals

$$i\mathcal{M} = \left(\frac{ig}{\sqrt{2}}\right)^2 v_R^\dagger\, \bar{\sigma}\cdot\varepsilon_+^*\, \frac{i\,\sigma\cdot(p-k_-)}{(p-k_-)^2}\, \bar{\sigma}\cdot\varepsilon_-^*\, u_L(p)$$

Putting

$$\varepsilon_- \cong \frac{k_-}{m} \qquad \varepsilon_+ \cong \frac{k_+}{m}$$

we can simplify

$$\frac{\sigma\cdot(p-k_-)}{(p-k_-)^2}\,\frac{\bar\sigma\cdot k_-}{m_W}\,u(p) = \frac{\sigma\cdot(p-k)\,\bar\sigma\cdot(k-p)}{(p-k)^2\,m_W}\,u(p) = -\frac{u(p)}{m_W}$$

and then we find $\quad k_+ = \frac{1}{2}(k_+ - k_- + k_+ + k_-) = \frac{1}{2}(k_+ - k_- + p + \bar p)$

$$i\mathcal{M} = \frac{-ig^2}{2}\, v_p^\dagger\, \bar\sigma\cdot\frac{k_+}{m_W}\left(-\frac{1}{m_W}\right) u_L$$

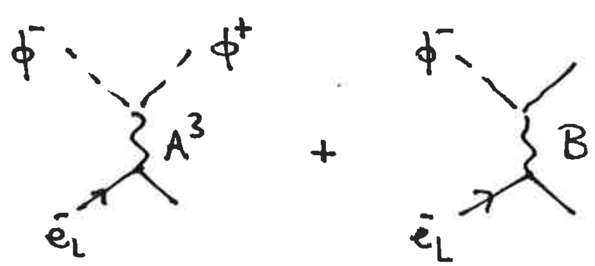
$$= \frac{+ig^2}{2\cdot 2}\, v_p^\dagger\, \bar\sigma\cdot(k_+ - k_-)\, u_L\cdot\frac{1}{m_W^2}$$

The leading terms in s/m_W from this and the previous set of diagrams are

$$i\mathcal{M} = -ie^2\, 2\sqrt{2}\,p\;\varepsilon_L\cdot(k_- - k_+)$$

$$\left[-\frac{s}{2m_W^2}\,\frac{1}{s}\cdot\frac{1/2}{c_w s_w}\,\frac{c_w}{s_w} + \frac{1}{4 s_w^2 m_W^2}\right]$$

Again we find a complete cancellation of these terms. With a little more work, using a more complete expression for the polarization vectors, it is possible to show that the amplitude for $e^-_L e^+_R \to W^+_0 W^-_0$ indeed becomes equal to the sum of diagrams



at high energy.

The cancellation I have shown here is a very delicate one that sensitively tests the W couplings predicted by the Standard Model. It is interesting to compare this prediction to the value of the W pair production total cross section measured at LEP. The result is shown in Figure 3; there is excellent agreement. In the figure, the

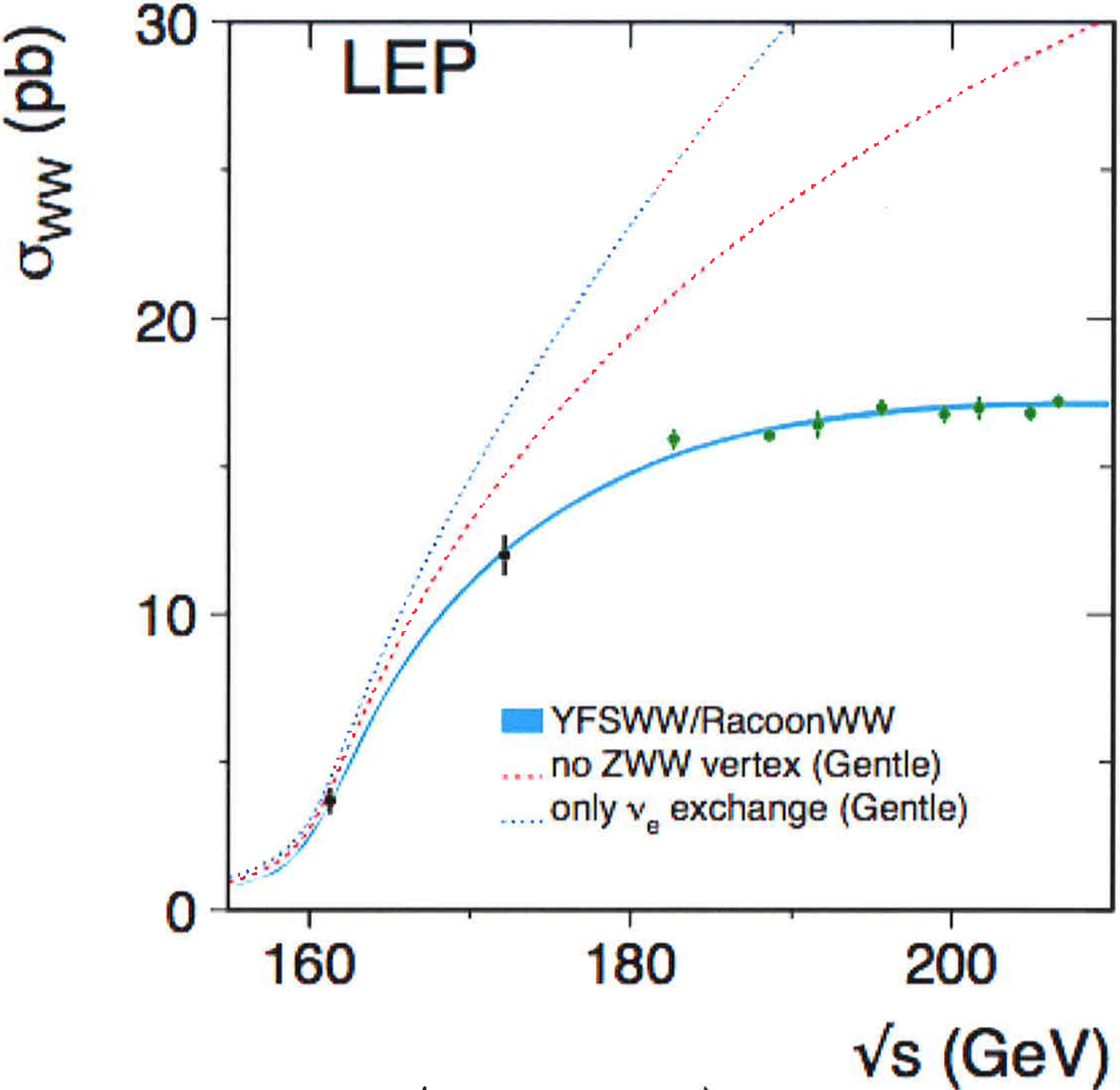
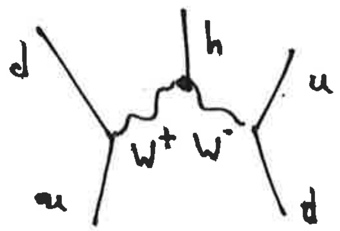


Fig. 3 Cross section for $e^+e^- \to W^+W^-$ measured at LEP, illustrating the "unitarity cancellation" of diagrams, from LEP Electroweak Working Group arXiv:1302.3415 Phys. Repts. 532, 119 (2013).

red curve is the prediction for the γ and ν diagrams only, and the blue curve is the prediction for the ν diagram only. These alternative predcitions have a completely different dependence on the center of mass energy and are completely excluded even at LEP energies.

In the next lecture, we will discuss the production mechanisms for Higgs bosons at the LHC. Among these is the WW fusion process,

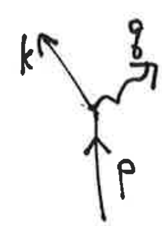


In this process, two W bosons are radiated almost collinearly from initial quarks, and these annihilate to produce a W boson. More generally, we can use the collinear radiation of W bosons from the initial beams to produced other types of new particles. It would be especially nice if we could produce longitudinally polarized W bosons. But the GBET, these would act as particles in the Higgs sector and would access heavy particles by Higgs sector couplings rather than by gauge sector couplings. But, the initial quarks in the proton are light quarks, with very little coupling to the Higgs sector. Is it possible that could produce longitudinally polarized Ws?

To analyze this question, I will compute the splitting functions for massless quarks to radiate W bosons of the various possible polarizations. It is straightforward to do this using the methods of lecture 3.

Set up the kinematics as we did before for radiation from the initial state. The W boson is off-shell, so the three vectors are

$$p = (E, 0, 0, E)$$
$$k = \left((1-z)E, -p_T, 0, (1-z)E - \frac{p_T^2}{2(1-z)E}\right)$$
$$q = \left(zE, p_T, 0, zE + \frac{p_T^2}{2(1-z)E}\right)$$



The denominator of the W boson propagator is

$$q^2 - m_W^2 = -p_T^2 - \frac{z p_T^2}{(1-z)} - m_W^2 = -\left(\frac{p_T^2}{(1-z)} + m_W^2\right)$$

The matrix element for the emission process is

$$i\mathcal{M} = i\frac{g}{\sqrt{2}} \; u^\dagger(k)\,\vec{\sigma}\cdot\vec{\varepsilon}_W^{\,*}\, u(p)$$

For massless quarks, only the initial and final quarks must be left-handed. The polarization spinors are

$$u(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad u_k = \sqrt{2(1-z)E}\begin{pmatrix} p_T/2(1-z)E \\ 1 \end{pmatrix}$$

The polarization vectors for the emitted W bosons are

$$\varepsilon_{R,L}^{\mu *} = \frac{1}{\sqrt{2}}\left(0, 1, \mp i, -\frac{p_T}{zE}\right)$$

$$\varepsilon_0^{*\mu} = \left(\frac{q}{m_W}, \frac{p_T}{m_W}, 0, \frac{zE}{m_W}\right)$$

The calculation of the matrix element for the left- and right-handed W polarization states goes through just as before, and we find

$$i\mathcal{M} = i\frac{g}{\sqrt{2}}\,\frac{2E}{\sqrt{2}}\sqrt{1-z}\;\frac{p_T}{z(1-z)}\cdot\begin{cases} 1 & L \\ (1-z) & R \end{cases}$$

For the longitudinal W polarization, we find

$$i\mathcal{M} = i\frac{g}{\sqrt{2}}\, 2E\sqrt{1-z}\, \left(\frac{p_T}{2(1-z)E} \quad 1\right)\begin{pmatrix} \frac{q+zE}{m_W} & \frac{p_T}{m_W} \\ \frac{p_T}{m_W} & \frac{q-zE}{m_W} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with $$= i\frac{g}{\sqrt{2}}\, 2E\sqrt{1-z}\, \frac{q-zE}{m_W}$$

$$q - zE = \left((zE)^2 - m_W^2\right)^{1/2} - zE = -\frac{m_W^2}{2zE}$$

Then

$$i\mathcal{M} = \frac{ig}{\sqrt{2}}\, 2E\sqrt{1-z}\cdot\left(-\frac{m_W}{2zE}\right)$$

In all, the matrix elements are

$$i\mathcal{M} = ig\left\{\begin{array}{lll} \sqrt{1-z}\;\frac{p_T}{z(1-z)} & & L \\ \sqrt{1-z}\;\frac{p_T}{z(1-z)}\,(1-z) & & R \\ -\frac{1}{\sqrt{2}}\sqrt{1-z}\;\frac{m_W}{z} & & 0 \end{array}\right.$$

Now consider a process with W emission from an initial state u quark. The full process is $uX \to dY$,

The cross section for this process is

$$\sigma(uX \to dY) = \frac{1}{2s}\int \frac{d^3k}{(2\pi)^3 2k} \int d\Pi_Y (2\pi)^4 \delta(p + p_X - k - p_Y)$$

$$\left| \mathcal{M}(u \to W^+ d) \frac{1}{(q^2 - m_W^2)^2} \mathcal{M}(W^+ X \to Y) \right|^2$$

Replacing, as we did before,

$$\frac{1}{2s}\int \frac{d^3k}{(2\pi)^3 2k} = \int \frac{dz\, E\, dp_T^2 \pi}{16\pi^3 E(1-z)} \frac{z}{2\hat{s}} = \int \frac{dz\, dp_T^2}{2\pi \cdot 4\pi} \frac{z}{1-z} \frac{1}{2\hat{s}}$$

this becomes

$$\sigma(uX \to dY) = \int dz \int \frac{d^2 p_T}{4\pi \cdot 4\pi} \left\{ \frac{1}{2\hat{s}} \int d\Pi\, 2\pi \delta(q + p_X - p_Y) \left| \mathcal{M}(W^+ X \to Y) \right|^2 \right\}$$

$$\frac{z}{1-z} \cdot \frac{1}{\left[\frac{p_T^2}{1-z} + m_W^2\right]^2} \cdot \left| \mathcal{M}(u \to W d) \right|^2$$

With the matrix elements for the transverse W polarizations, this reads

$$\sigma(uX \to dY) = \int dz \int \frac{d^2 p_T}{4\pi \cdot 4\pi} \sigma(W^+ X \to Y)$$

$$\frac{(1-z)^2}{[p_T^2 + (1-z) m_W^2]^2} \frac{z}{1-z} g^2 (1-z) \frac{p_T^2}{z^2 (1-z)^2} [1 + (1-z)^2]$$

and simplifies to

$$\sigma(uX \to dY) = \int dz \frac{d^2 p_T\, p_T^2}{[p_T^2 + (1-z) m_W^2]^2} \frac{\alpha_w}{4\pi} \frac{1 + (1-z)^2}{z}$$

$$\cdot \sigma(W^+ X \to Y)$$

Similarly, using the matrix element for emission of a longitudinally polarized W, the cross section becomes

$$\sigma(uX \to dY) = \int dz \int \frac{d^2 p_T}{4\pi \cdot 4\pi} \, \sigma(W^+X \to Y)$$

$$\frac{(1-z)^2}{[p_T^2 + (1-z) m_W^2]^2} \, \frac{z}{1-z} \, \frac{g^2 (1-z)}{2} \, \frac{m_W^2}{z^2}$$

or

$$\sigma(uX \to dY) = \int dz \int \frac{dp_T^2 \, m_W^2}{[p_T^2 + (1-z) m_W^2]^2} \, \frac{\alpha_w}{4\pi} \, \frac{(1-z)^2}{z}$$
$$\cdot \, \sigma(W^+X \to Y)$$

If we interpret this equation as a W parton distribution convolved with a hard cross section,

$$\sigma(uX \to dY) = \int dz \, f_W(z) \, \sigma(W^+X \to Y)$$

we find the W pdfs for the three polarization states

$$f_W(z) = \begin{cases} \dfrac{\alpha_w}{4\pi} \displaystyle\int \frac{dp_T^2 \, p_T^2}{[p_T^2 + (1-z) m_W^2]^2} \, \frac{1}{z} & L \\[2ex] \dfrac{\alpha_w}{4\pi} \displaystyle\int \frac{dp_T^2 \, p_T^2}{(p_T^2 + (1-z) m_W^2)^2} \, \frac{(1-z)^2}{z} & R \\[2ex] \dfrac{\alpha_w}{8\pi} \displaystyle\int \frac{dp_T^2 \, m_W^2}{(p_T^2 + (1-z) m_W^2]^2} \, \frac{(1-z)^2}{z} & 0 \end{cases}$$

For the transverse polarization states, the integral over p_T is logarithmic as before and the pdf again takes the form

$$f_W(z) = \frac{\alpha_w}{4\pi} \log\left(Q^2/m_W^2\right) \frac{1 + (1-z)^2}{z}$$

For the longitudinal polarization state, note that the integral is finite, with a characteristic p_T of order m_W. Performing the integral, we find

$$f_W(z) = \frac{\alpha_W}{8\pi} \frac{1-z}{z}$$

Although the emission rate is not logarithmically enhanced, it can be substantial.

Thus, at least to a certain extent, hadron colliders do naturally provide initial states with the couplings to new particles characteristic of Higgs boson. This will be an important tool in the search for new physics at the LHC.