Lecture 3

ICTP 2015 Summer school

Neutrinos

selected topics

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MAX-PLANCK-GESELLSCHAFT

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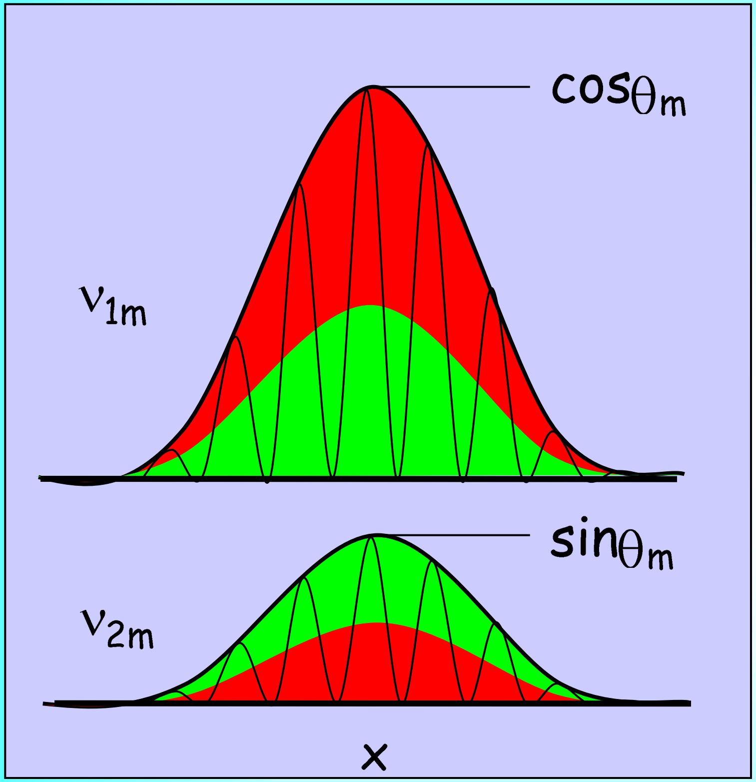
FÜR KERNPHYSIK

Oscillations in matter



$H_0 \rightarrow H = H_0 + V$

$\nu_k \rightarrow \nu_{mk}$

eigenstates of H_0 eigenstates of H

$\theta \rightarrow \theta_m (n)$

$\phi \rightarrow \phi_m (n)$

Resonance - maximal mixing in matter - oscillations with maximal depth

$\theta_m = \pi/4$

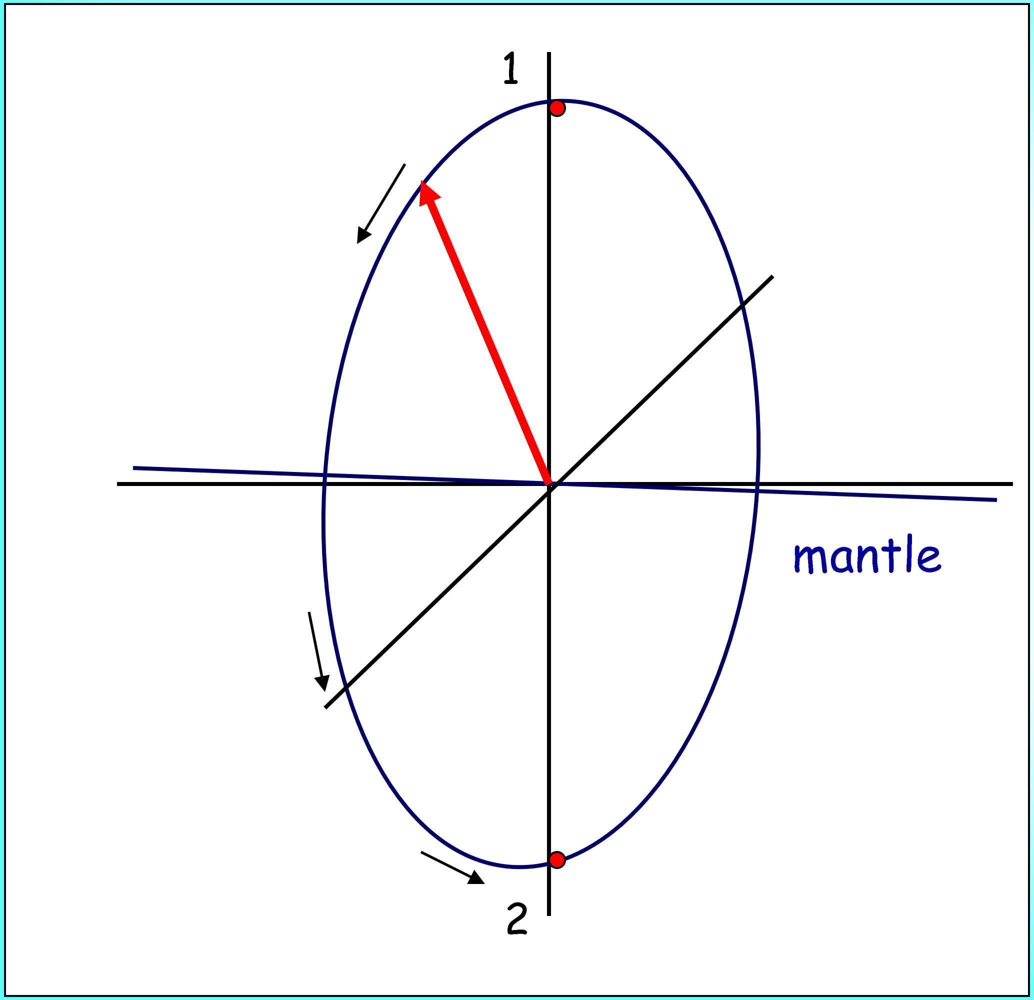
Resonance condition:

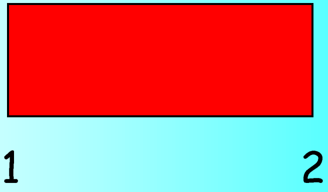
$$V = \cos 2\theta \frac{\Delta m^2}{2E}$$

Constant density medium:
the same dynamics

Mixing changed
phase difference changed

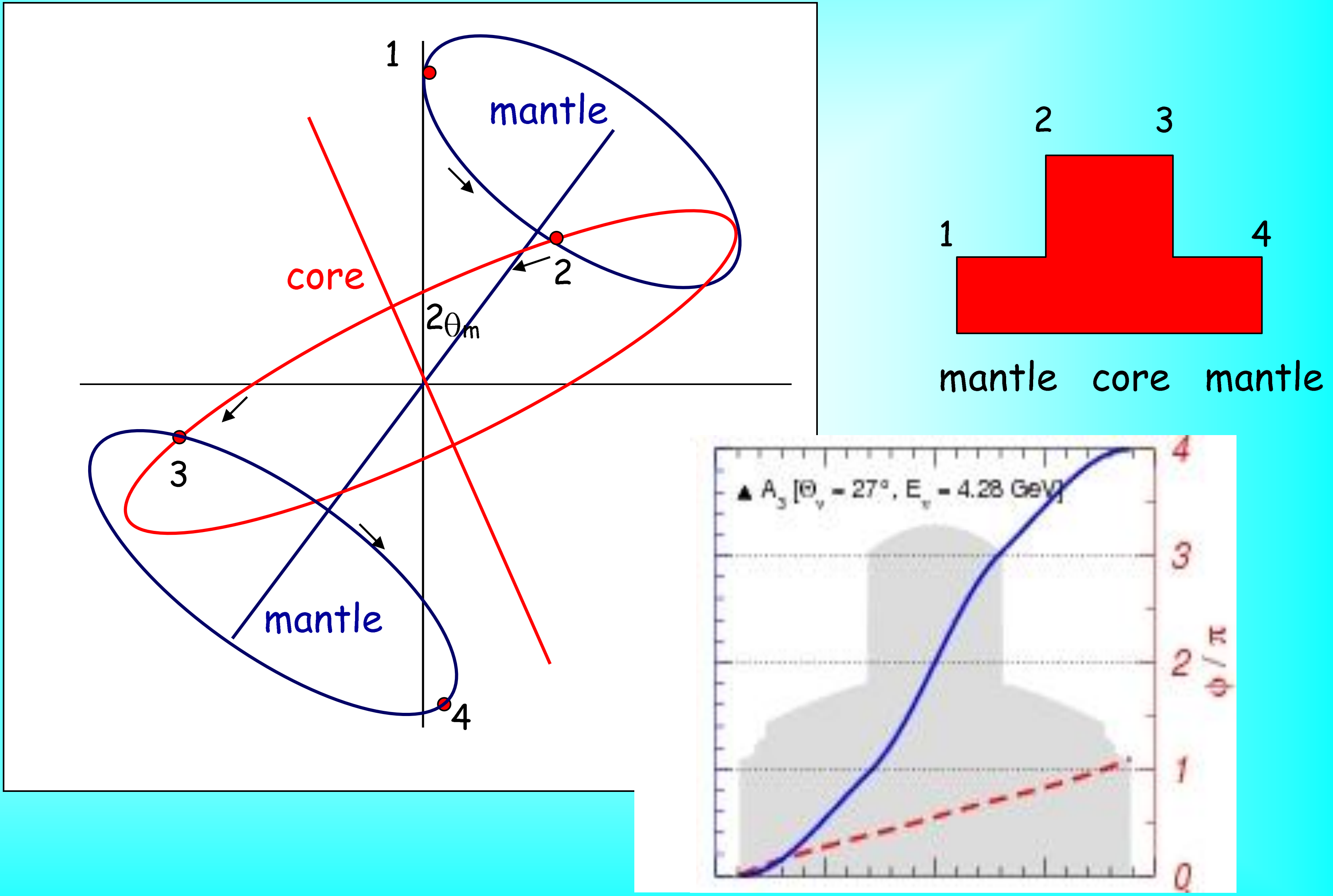
Resonance enhancement



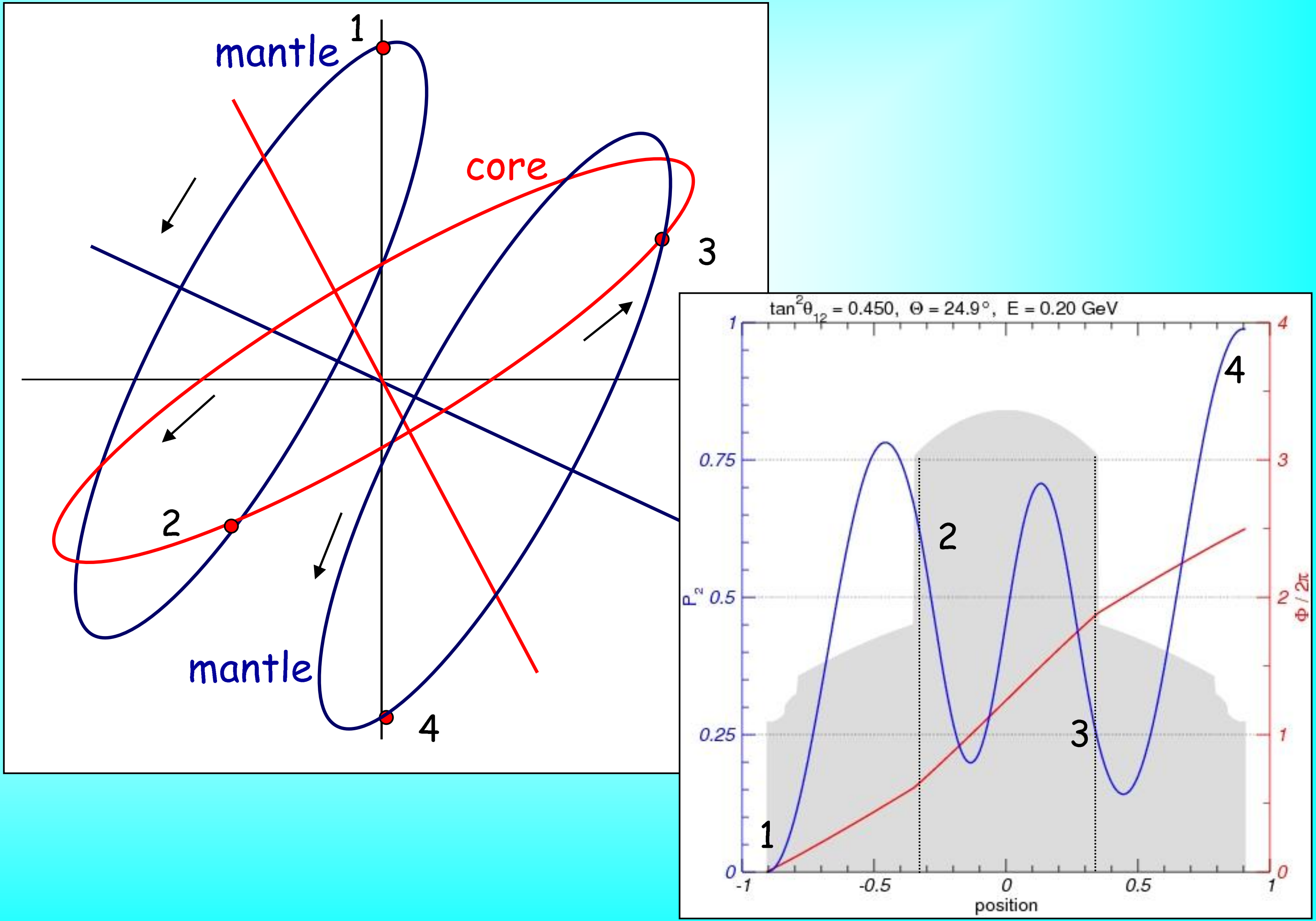


Mantle
of the Earth

Parametric enhancement of 1-3 mode



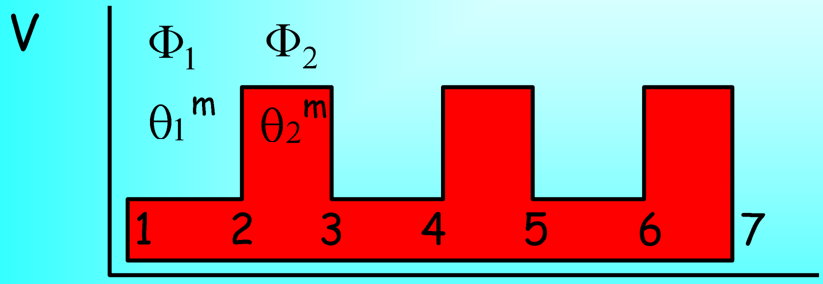
Parametric enhancement of 1-2 mode



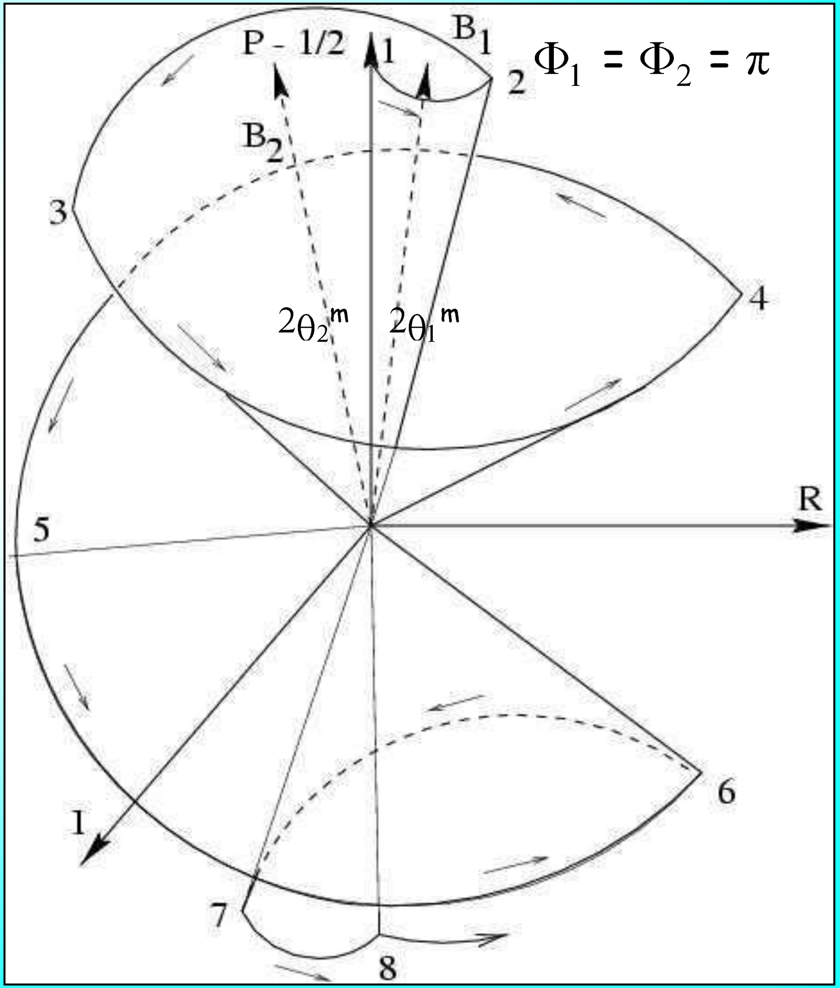
Parametric resonance

Enhancement is associated to certain conditions for the phase of oscillations

V. Ermilova V. Tsarev, V. Chechin
E. Akhmedov
P. Krastev, A.S., Q. Y. Liu,
S.T. Petcov, M. Chizhov



``Castle wall profile"



$\Phi_1 = \Phi_2 = \pi$

Adiabatic conversion

Varying density

Evolution equation for eigenstates

In non-uniform medium the Hamiltonian depends on time:

$$H_{tot} = H_{tot}(n_e(t))$$

$$\boxed{i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f}$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad \nu_m = \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

instantaneous
eigenstates
of the Hamiltonian

Inserting $\nu_f = U(\theta_m)\, \nu_m \qquad \theta_m = \theta_m(n_e(t))$

⇒

$$\boxed{i \frac{d}{dt} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & i \dfrac{d\theta_m}{dt} \\ -i \dfrac{d\theta_m}{dt} & H_{2m} - H_{1m} \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}}$$

off-diagonal
terms imply
transitios

$$\nu_{1m} \leftrightarrow \nu_{2m}$$

However

if $\boxed{\left| \dfrac{d\theta_m}{dt} \right| \ll H_{2m} - H_{1m}}$

off-diagonal elements can be neglected
no transitions between eigenstates
propagate independently

Adiabaticity

Adiabaticity condition $$\left|\frac{d\theta_m}{dt}\right| \ll H_{2m} - H_{1m}$$

External conditions (density) change slowly the system has time to adjust the changes

transitions between the eigenstates can be neglected

$\nu_{1m} \not\leftrightarrow \nu_{2m}$ → The eigenstates propagate independently

→ shape factors of the eigenstates do not change

A.C. crucial in the resonance layer:
- the mixing changes fast,
- level splitting is minimal

If vacuum mixing is small, A.C.: $\Delta r_R > l_R$

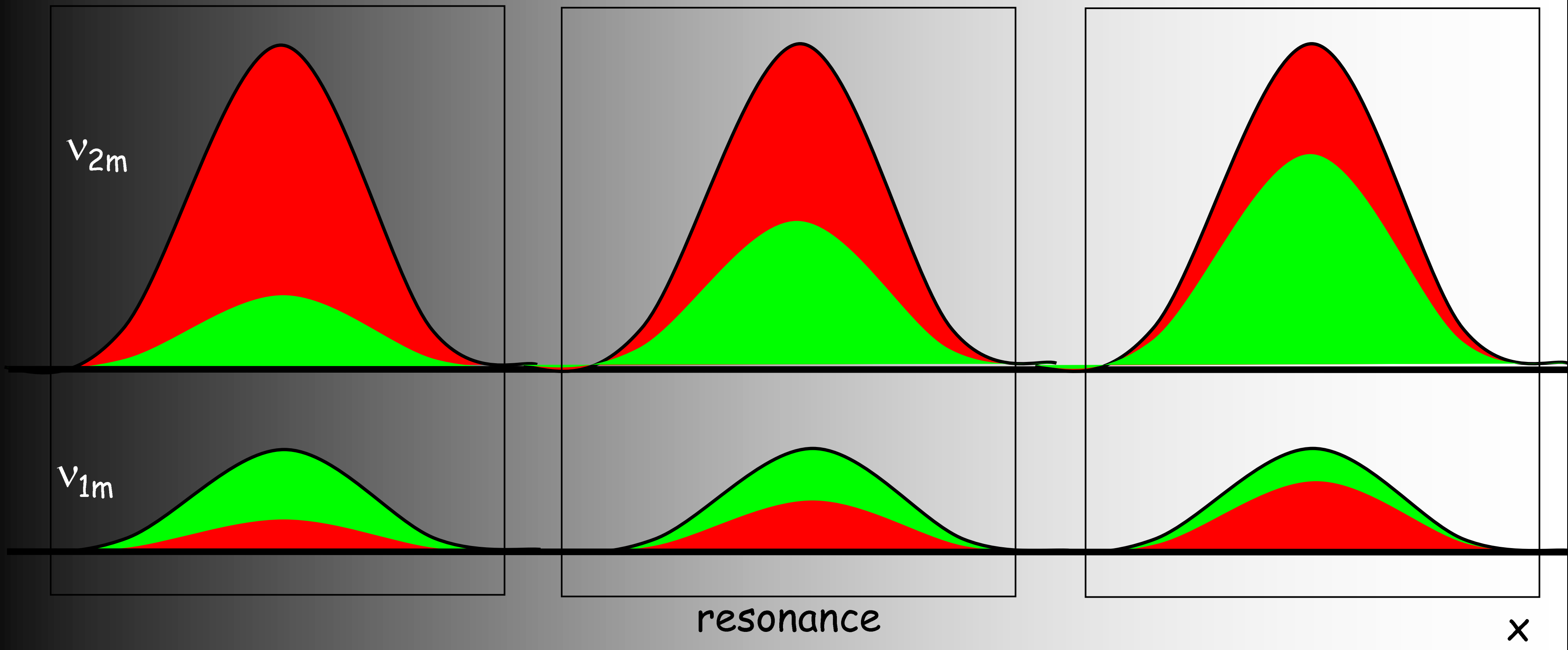
width of the resonance layer

$$\Delta r_R = n_R \tan 2\theta \,/\, (dn/dx)_R$$

oscillation length in resonance

$$l_R = l_\nu / \sin 2\theta$$

Adiabatic conversion



if density
changes
slowly

- the amplitudes of the wave packets do not change
- flavors of the eigenstates being determined by mixing angle follow the density change

Adiabatic conversion probability

Sun, Supernova

Initial state: $\nu(t_0) = \nu_e = \cos\theta_m^0\, \nu_{1m}(t_0\,) + \sin\theta_m^0\, \nu_{2m}(t_0)$

Mixing angle in matter in initial state

Adiabatic evolution to the surface of the Sun:

$V \rightarrow 0$

$\nu_{1m}(t_0) \;\rightarrow \nu_1$

$\nu_{2m}(t_0) \;\rightarrow \nu_2$

Final state: $\nu(t_f) = \cos\theta_m^0\, \nu_1 + \sin\theta_m^0\, \nu_2\; e^{i\phi}$

Probability to find ν_e averaged over oscillations

$$P_{ee} = |\langle\nu_e|\nu(t_f)\rangle|^2 = (\cos\theta\, \cos\theta_m^0)^2 + (\sin\theta\, \sin\theta_m^0)^2$$

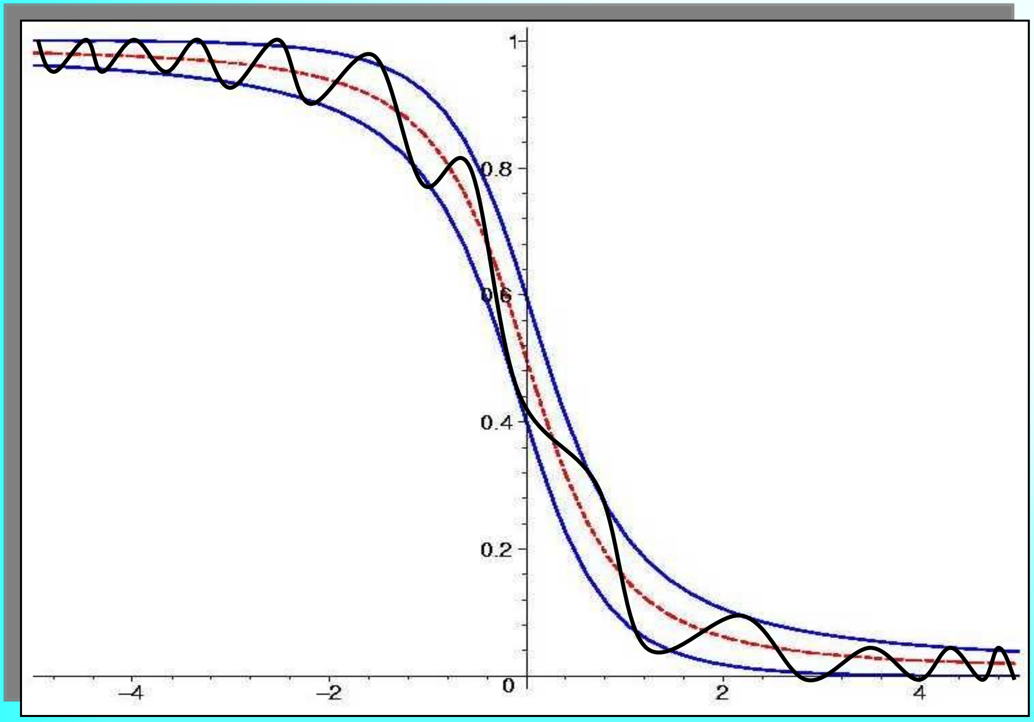
$$= \;0.5[\,1 + \cos2\theta_m^0\, \cos2\theta\,]$$

or

$$P_{ee} = \sin^2\theta \;+ \cos 2\theta\, \cos^2\theta_m^0$$

Spatial picture

Adiabatic conversion

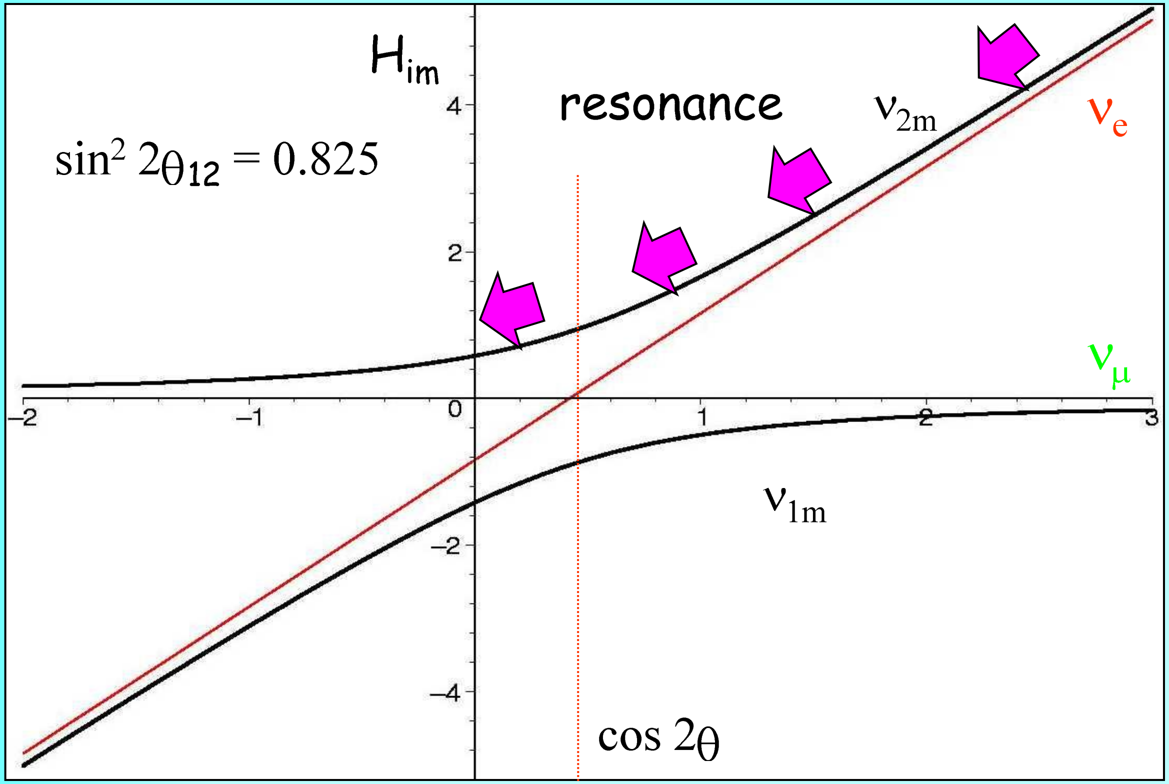


distance

Level crossing

Large mixing

ν_e



$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

l_ν / l_0 n_e

Crossing point - resonance

- the level split is minimal
- the oscillation length is maximal

Adiabaticity violation

If density $n_e(t)$ changes fast $\left| \frac{d\theta_m}{dt} \right| \sim | H_{2m} - H_{1m} |$

the off-diagonal terms in the Hamiltonian can not be neglected

→ transitions $\nu_{1m} \Leftrightarrow \nu_{2m}$

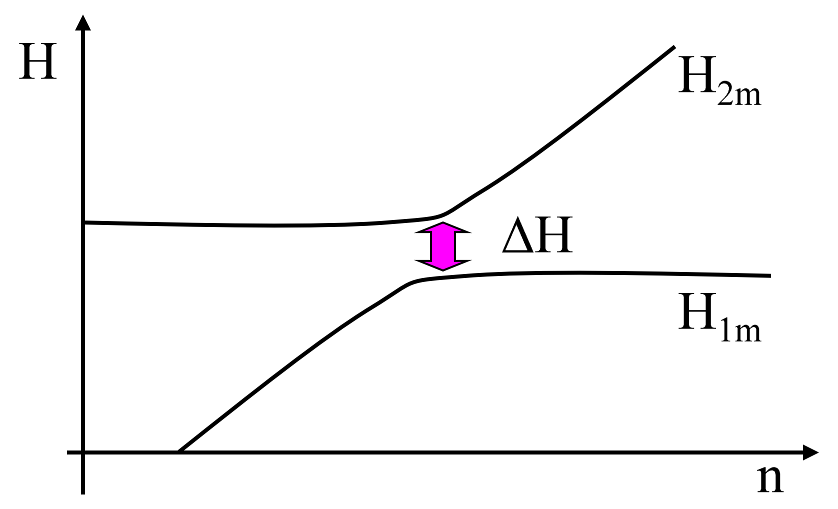
SN shock waves

If sterile neutrinos with small mixing exist

Admixtures of ν_{1m} ν_{2m} in a given neutrino state change

``Jump (flop) probability''
penetration under barrier

$$P_{12} = e^{-\frac{\Delta H}{E_n}}$$



$$E_n \sim 1/h_n$$

is the energy associated to change of density

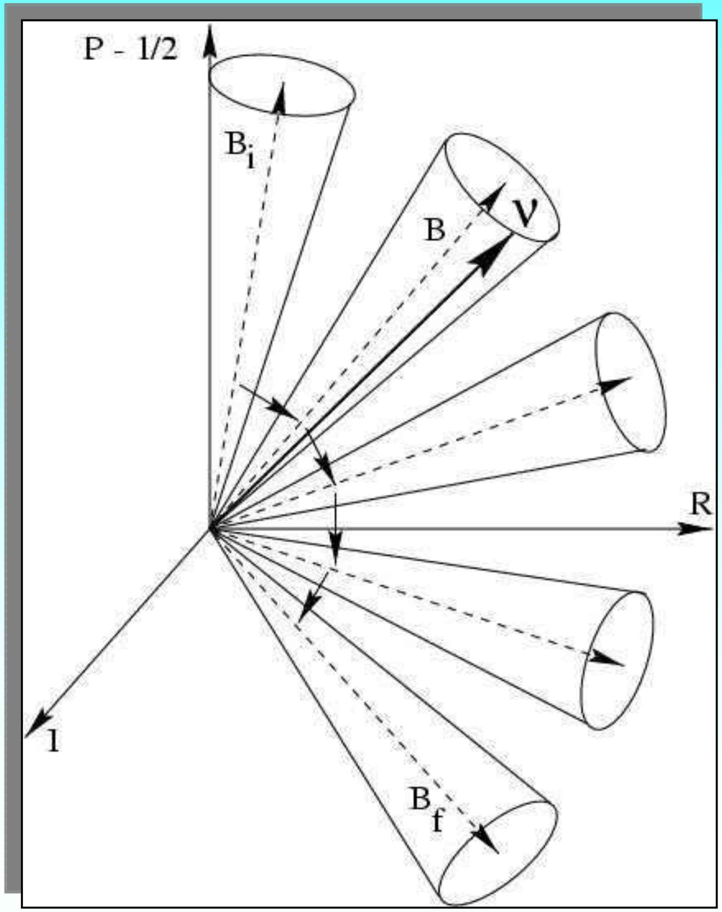
$$P_{12} = e^{-\pi\kappa_R/2}$$

Landau-Zener

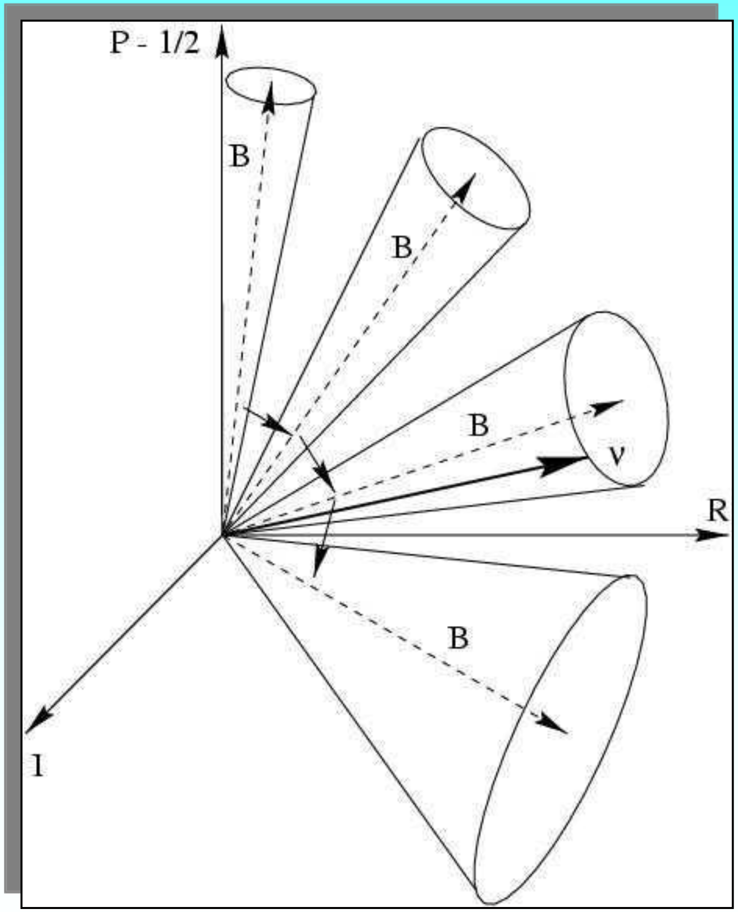
$\kappa_R = \Delta r_R / l_R$ adiabaticity parameter

Adiabatic conversion

Adiabatic conversion



Violation of adiabaticity



Oscillations versus MSW

Different degrees of freedom

Oscillations

Vacuum or uniform medium with constant parameters

Phase difference increase between the eigenstates

$\phi(t)$

$\theta_m(E)$

Mixing does not change

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

Change of mixing in medium → change of flavor of the eigenstates

$\theta_m(t)$

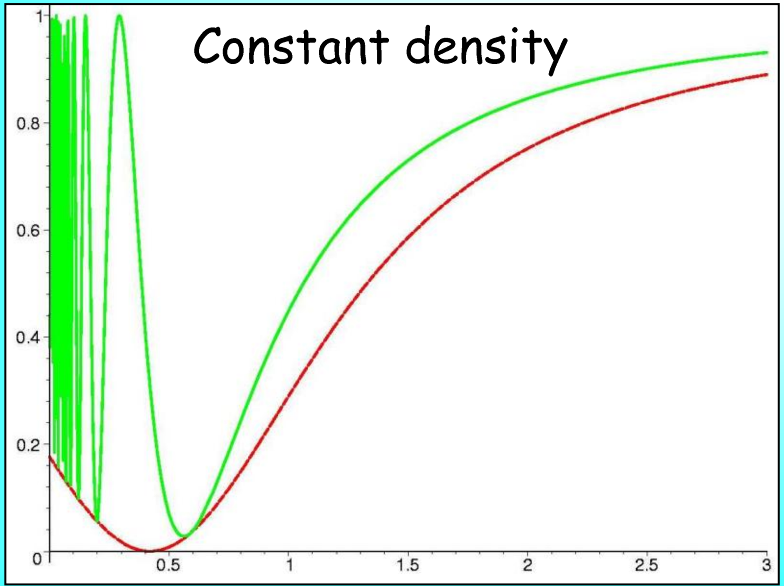
Phase is irrelevant

In non-uniform medium: interplay of both processes

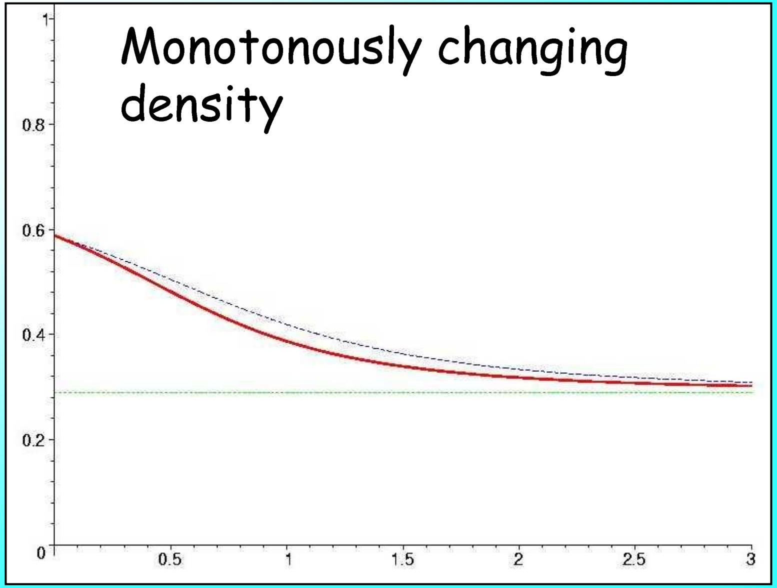
Resonance oscillations vs. adiabatic conversion

Passing through the matter filter ν

$\frac{F(E)}{F_0(E)}$



E/E_R



E/E_R

Conclusion:

Adiabatic conversion is effect of change of mixing angle in matter in medium with slowly enough density change on the way of neutrino propagation

Conversion without oscillations

Resonance enhancement of oscillations occures in certain energy range in matter with constant density

nearly constant

Original papers:

L. Wolfenstein, Phys. Rev. D17 (1978) 2369

L. Wolfenstein, in ``Neutrino-78'', Purdue Univ. C3, 1978.

adiabaticity

L. Wolfenstein, Phys. Rev. D20 (1979) 2634

V. D. Barger, K. Whisnant, S. Pakvasa, R.J.N. Phillips,
Phys.Rev. D22 (1980) 2718

enhancement of oscillations

S.P. Mikheev, A.Yu. Smirnov, Sov. J. Nucl.Phys. 42 (1985) 913-917,
Yad.Fiz. 42 (1985) 1441-1448

Resonance, Adiabaticity Solar nu

S.P. Mikheev, A.Yu. Smirnov, Nuovo Cim. C9 (1986) 17-26

S.P. Mikheev, A.Yu. Smirnov, Sov. Phys. JETP 64 (1986) 4-7,
Zh.Eksp.Teor.Fiz. 91 (1986) 7-13, arXiv:0706.0454 [hep-ph]

adiabatic formulas

S.P. Mikheev, A.Yu. Smirnov, 6th Moriond workshop, Tignes, Jan.
1986 p. 355

Earth matter effects, day night, atmospheric

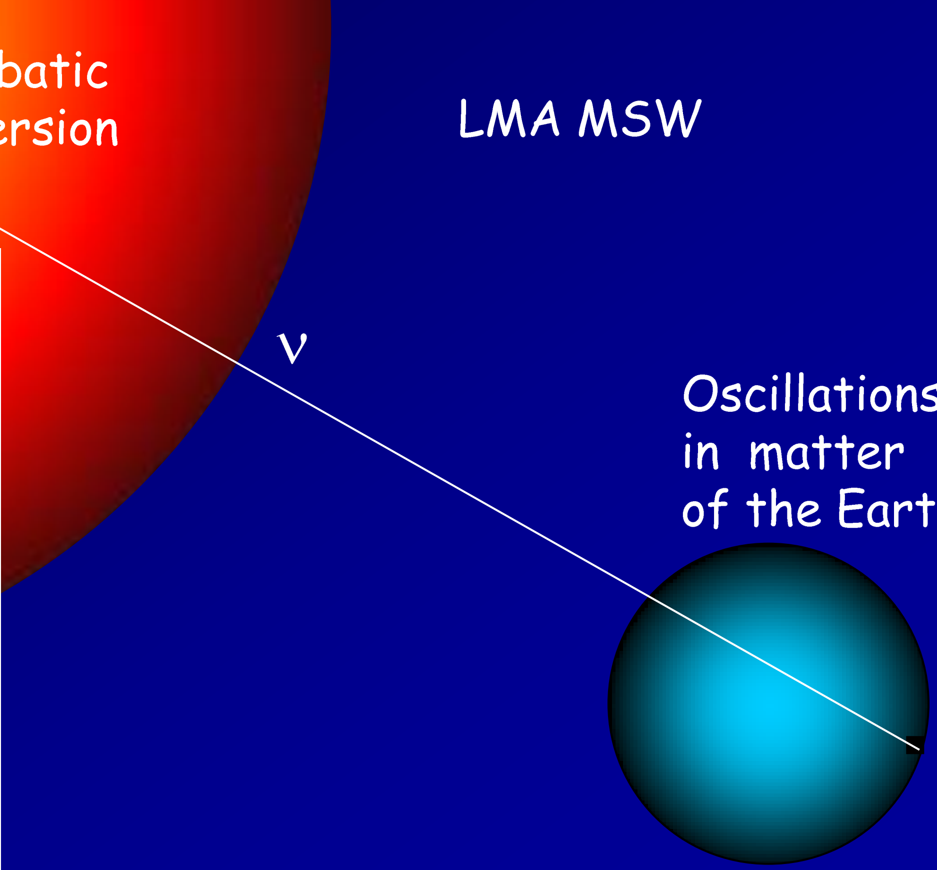
Phenomenology

fluxes, detection, results

from major neutrino
experiments

Solar neutrinos

$4p + 2e^- \rightarrow {}^4He + 2\nu_e + 26.73\ \text{MeV}$

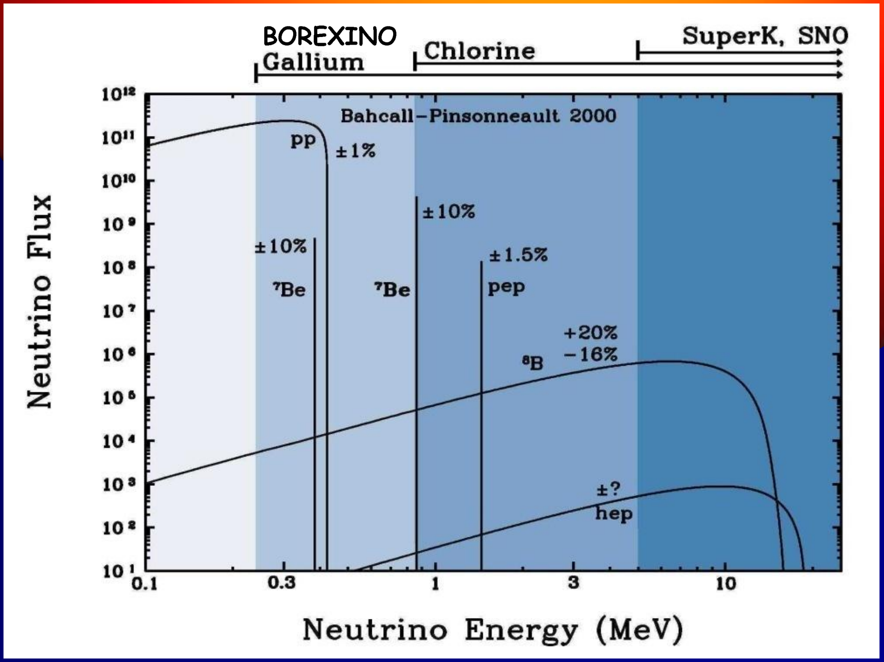


Adiabatic conversion

LMA MSW

ν

Oscillations in matter of the Earth



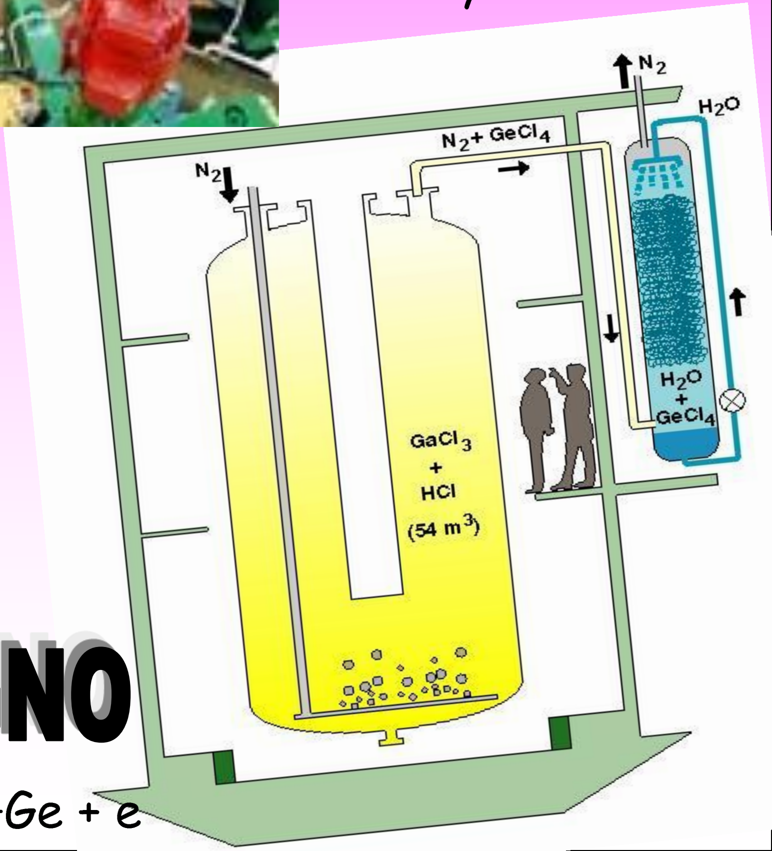
Homestake

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e$

SAGE

Solar neutrino fluxes

Ga-calibration experiments anomaly

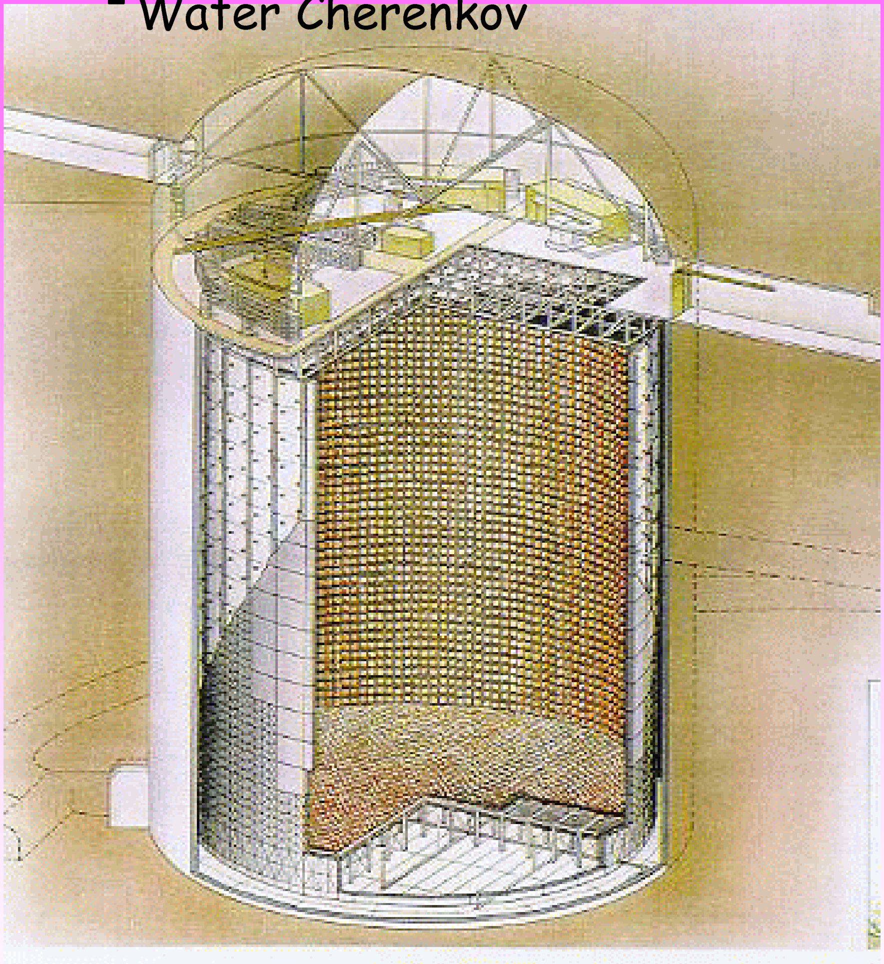


Gallex, GNO

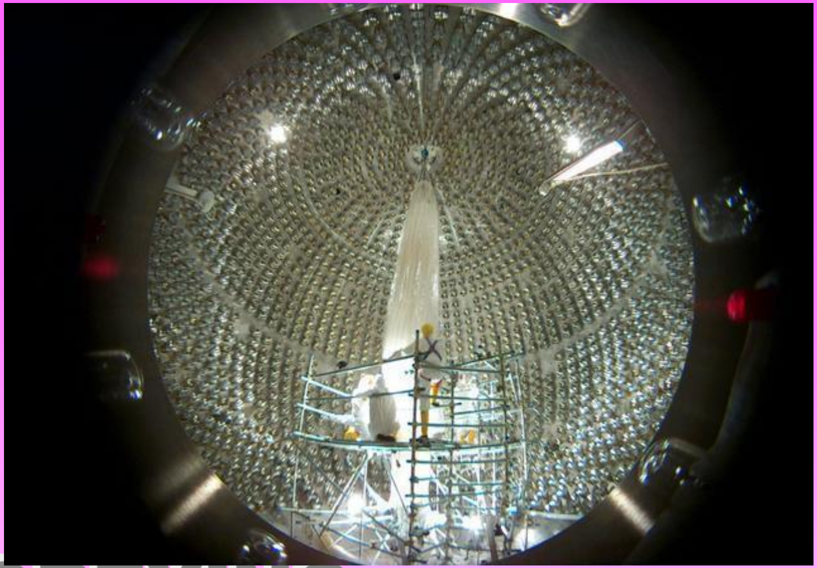
$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e$

SuperKamiokande

Water Cherenkov



$\nu + e \rightarrow \nu + e$



BOREXINO

Scintillator

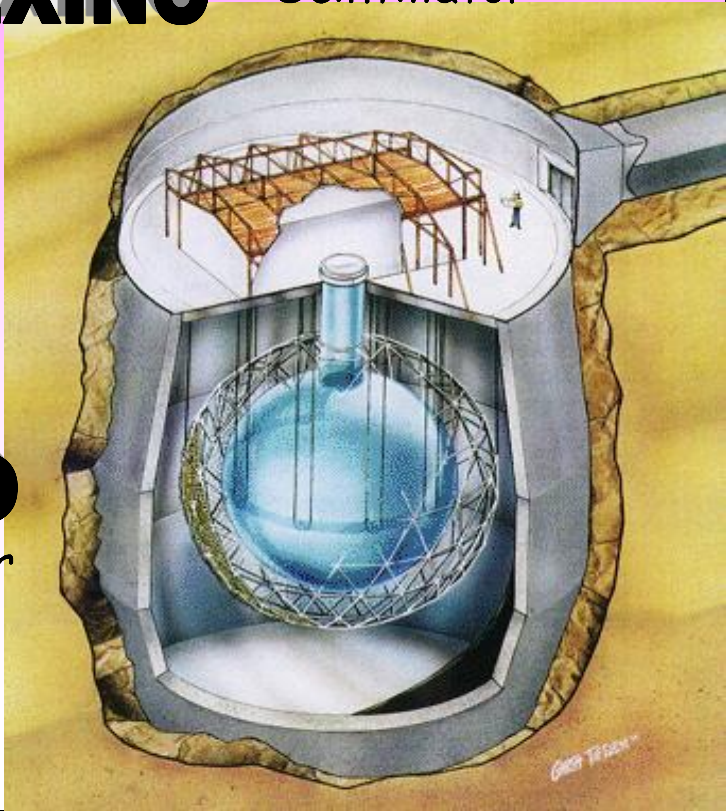
SNO

Heavy water

$\nu_e + d \rightarrow e + p + p$

NC: $\nu + d \rightarrow \nu + n + p$

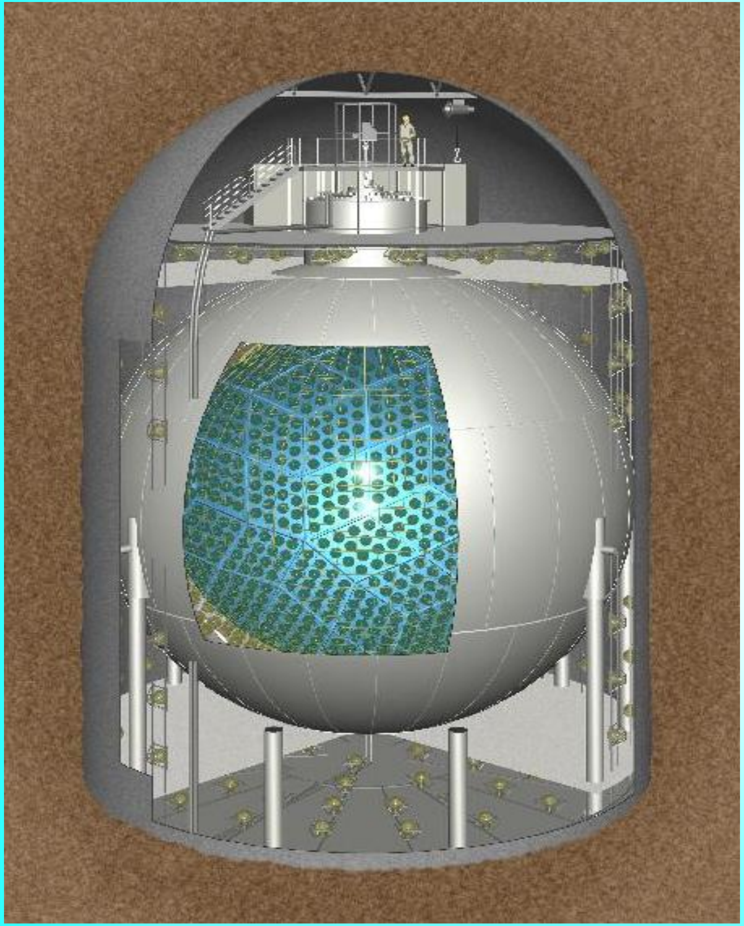
$\nu_e + e \rightarrow \nu_e + e$



KamLAND

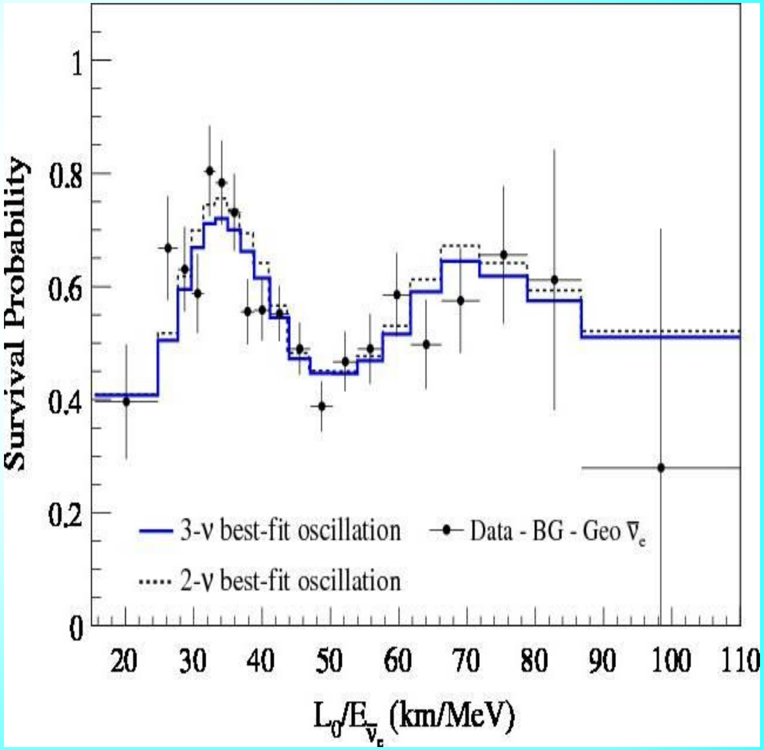
Decisive experiment

scintillator



reactors

<L> ~ 180 km

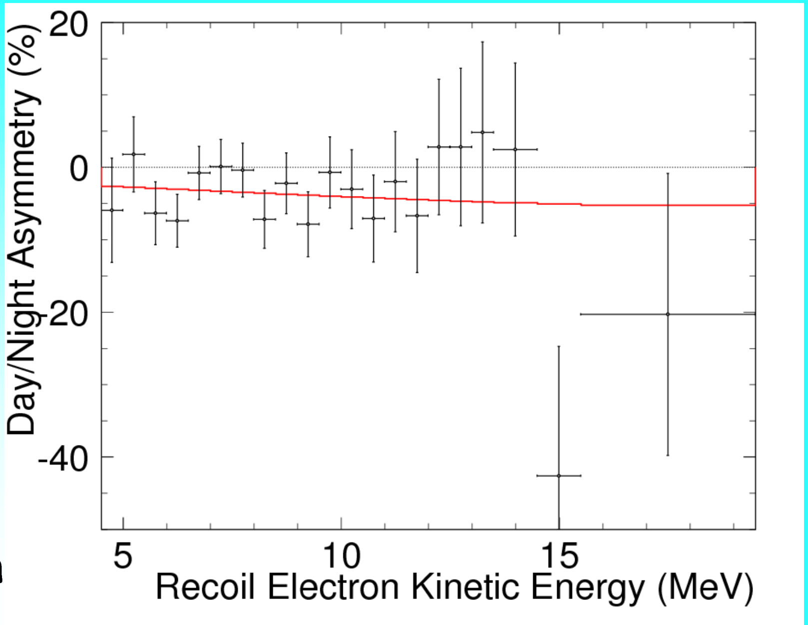


$\bar{\nu}_e + p \rightarrow e^+ + n$

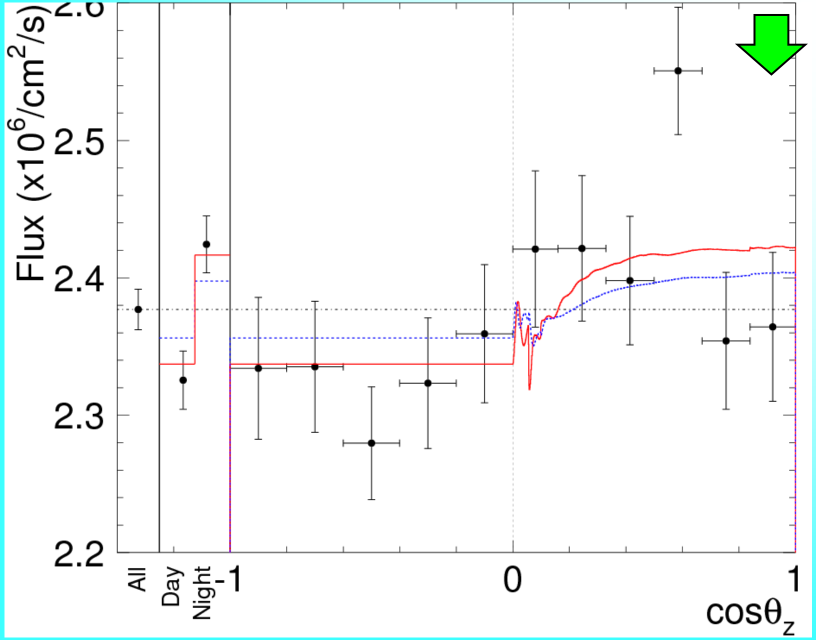
Day-Night effect

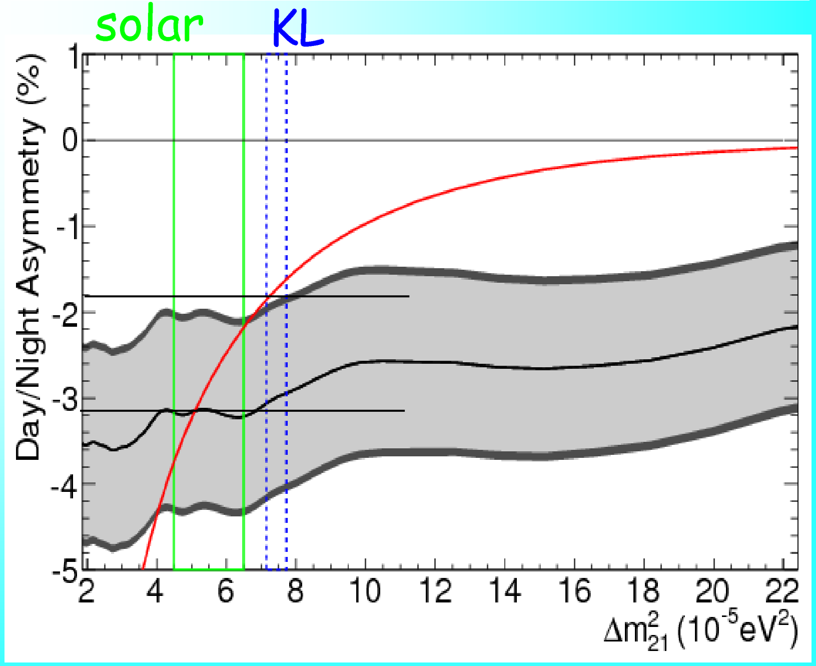
First Indication of Terrestrial Matter Effects on Solar Neutrino Oscillation

Super-Kamiokande collaboration
(Renshaw, A. et al.)
Phys.Rev.Lett. 112 (2014) 091805
arXiv:1312.5176



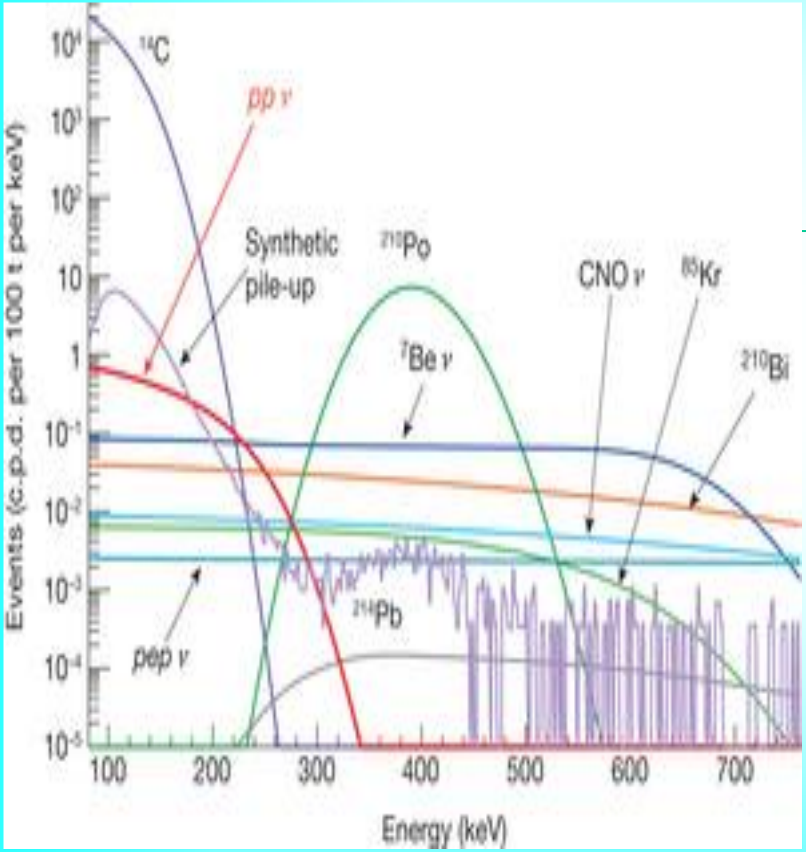
Attenuation effect

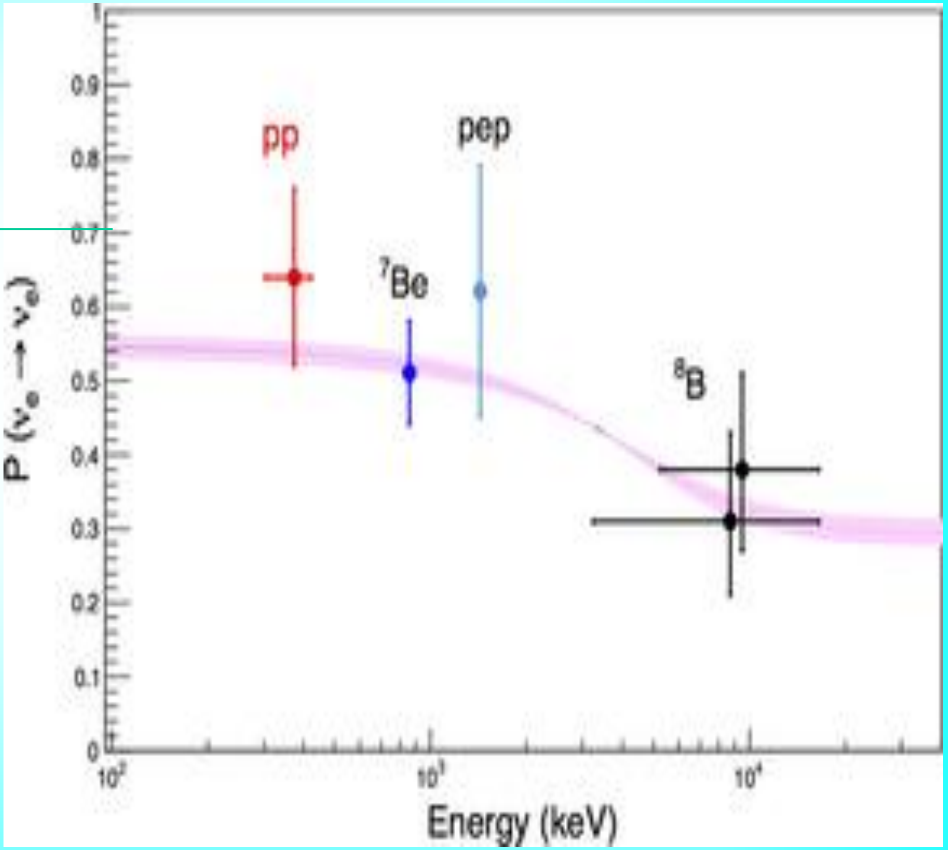




Solar pp-neutrinos

Neutrinos from the primary pp-reactions in the Sun
BOREXINO Collaboration
(G. Bellini et al.)
Nature 512 (2014) 7515, 383

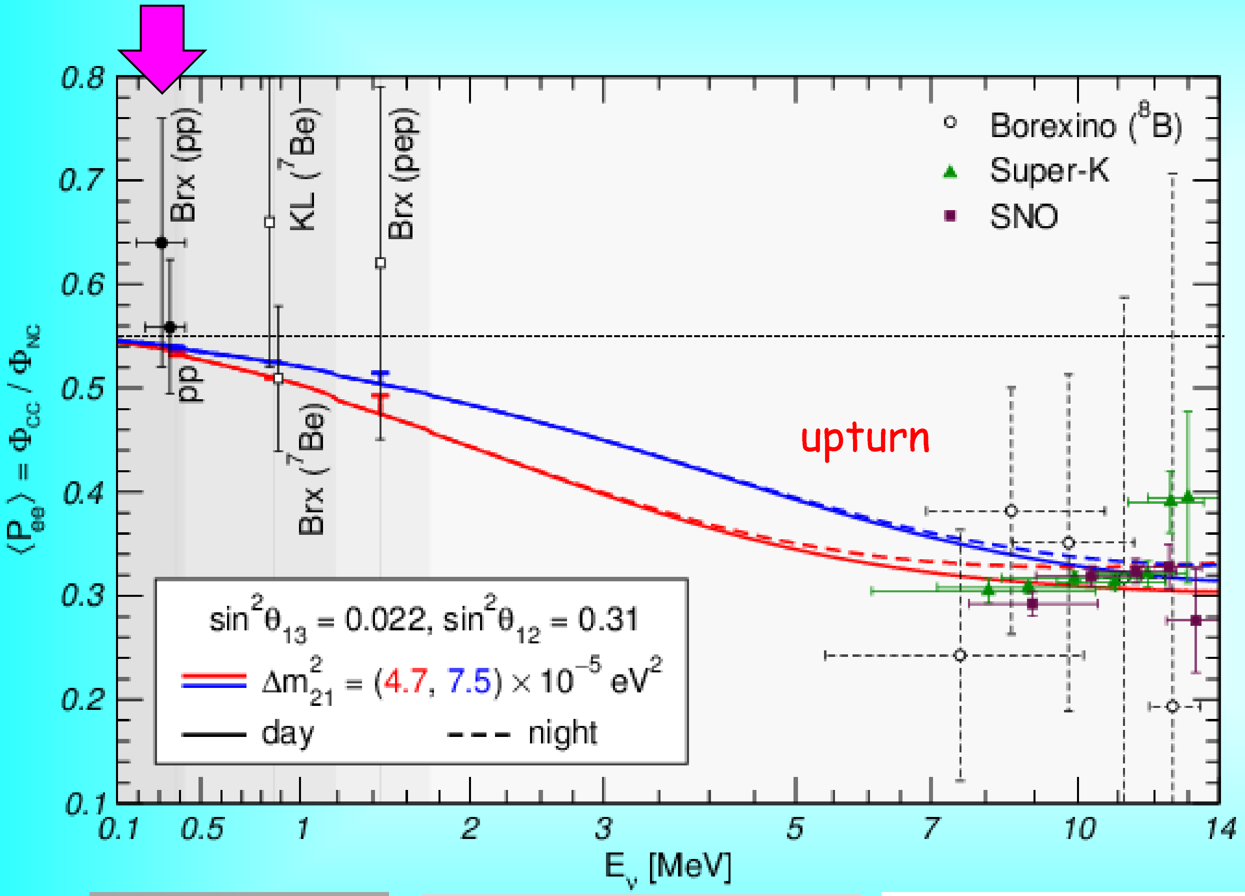




$\frac{1}{2}\cos^4\theta_{13}\ \sin^2 2\theta_{12}$

Results

M. Maltoni, A.Y.S.
to appear



upturn

for two different values of $\Delta m_{21}{}^2$

best fit value from solar data

best global fit

Vacuum dominated

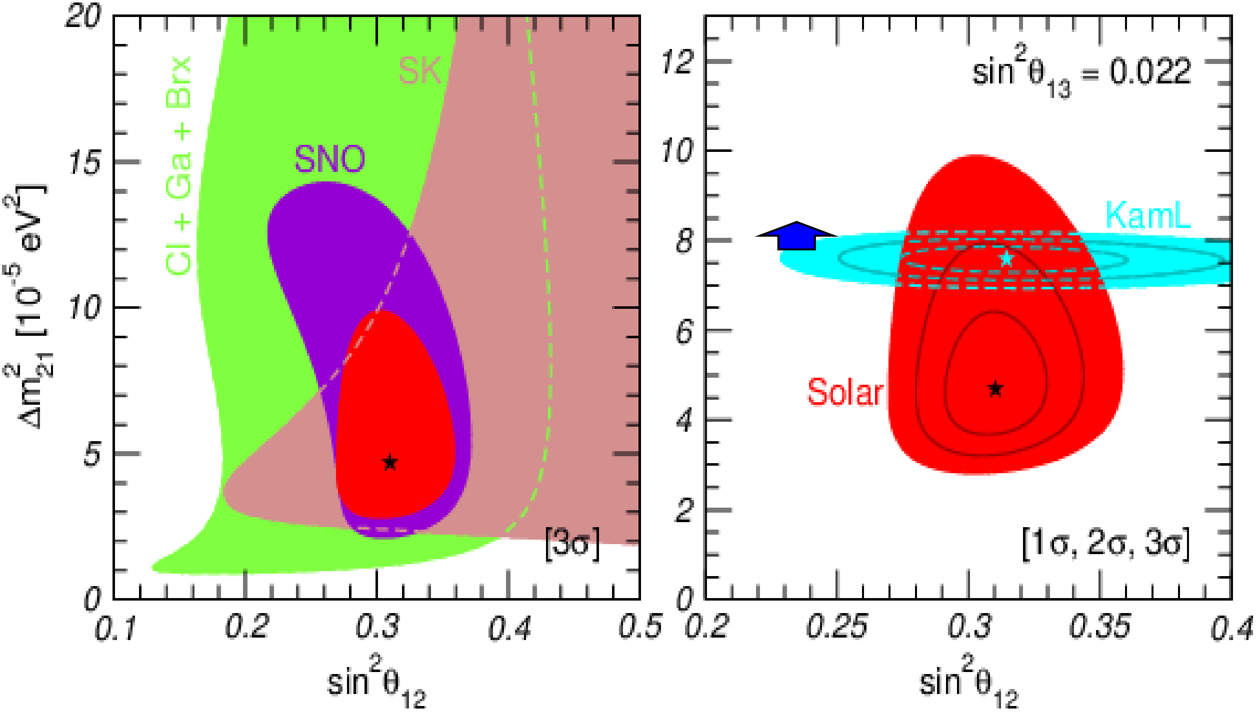
Transition region resonance turn on

Matter dominated region

Reconstructed exp. points for SK, SNO and BOREXINO at high energies

Neutrino parameters

Solar neutrinos
Vs. KamLAND



M. Maltoni, A.Y.S.
to appear

Δm^2_{21} : about 2σ descrepancy of the KL and solar values

Red regions: all solar neutrino data

also restrictions from individual experiments

$\sin^2\theta_{13}$ as fit parameter then marginalized

b.f.: $\sin^2\theta_{13} = 0.017$

$\sin^2\theta_{13}$ fixed by reactor experiments

KamLAND data reanalized in view of reactor anomaly (no front detector) bump at 4 -6 MeV

Δm^2_{21} increases by 0.5 10^{-5} eV^2

Problems, future

Absence of upturn of the spectrum

Large D-N asymmetry

Difference of values of Δm^2_{21} extracted from solar and KamLAND data

Large value of matter potential extracted from global fit

at about 3σ - level

Another reactor anomaly or new physics in solar neutrinos?

Detection of CNO neutrinos to shed some light on the problem of the SSM: controversy of helioseismology data and abundance of heavy elements

High precision measurements of the pp- and Be- neutrino fluxes

Detailed study of the Earth matter effect

Experiments:

SNO+

SK

JUNO

Hyper-K

Oscillations inside the Earth

Oscillations in multilayer medium

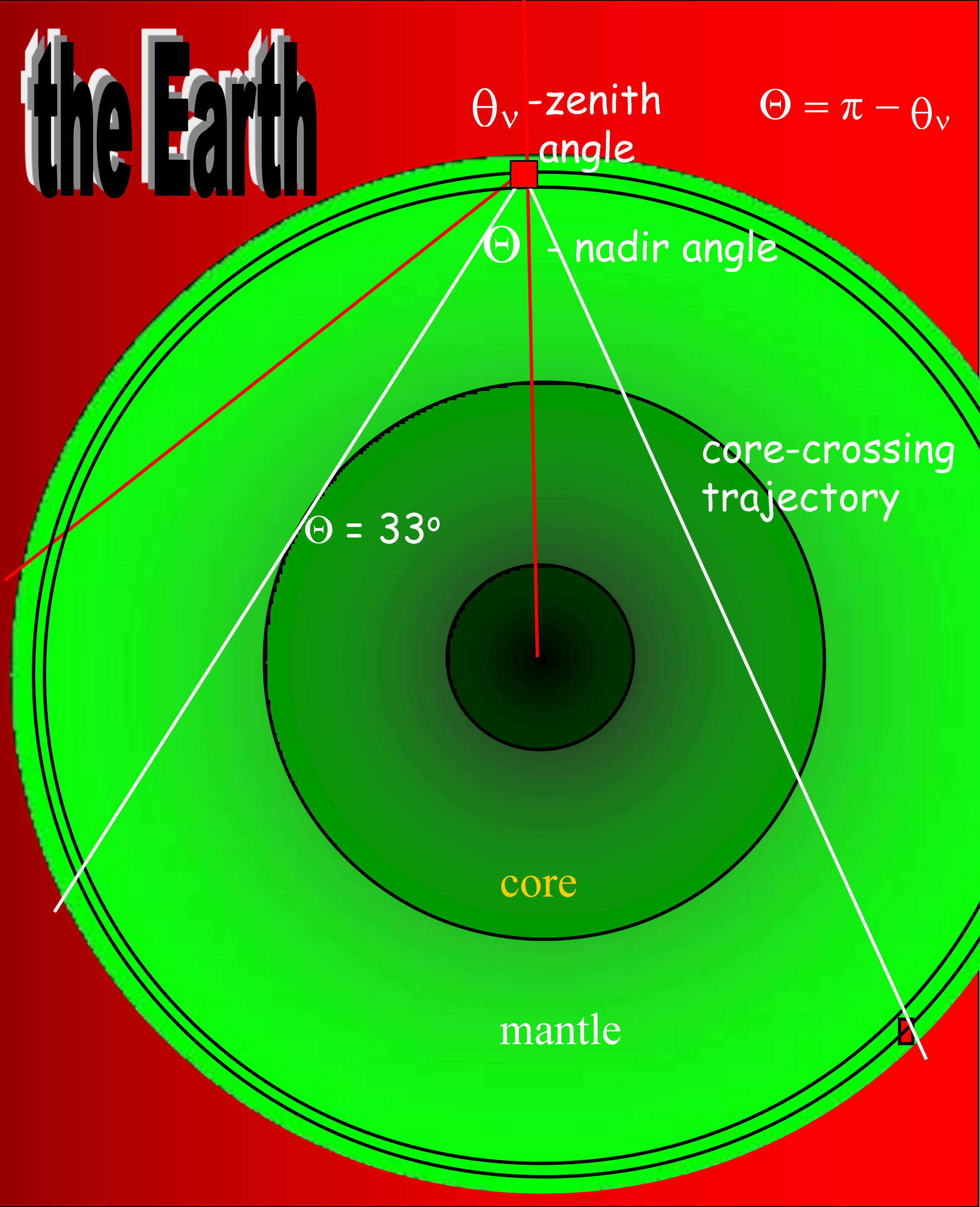
Applications:

flavor-to-flavor transitions

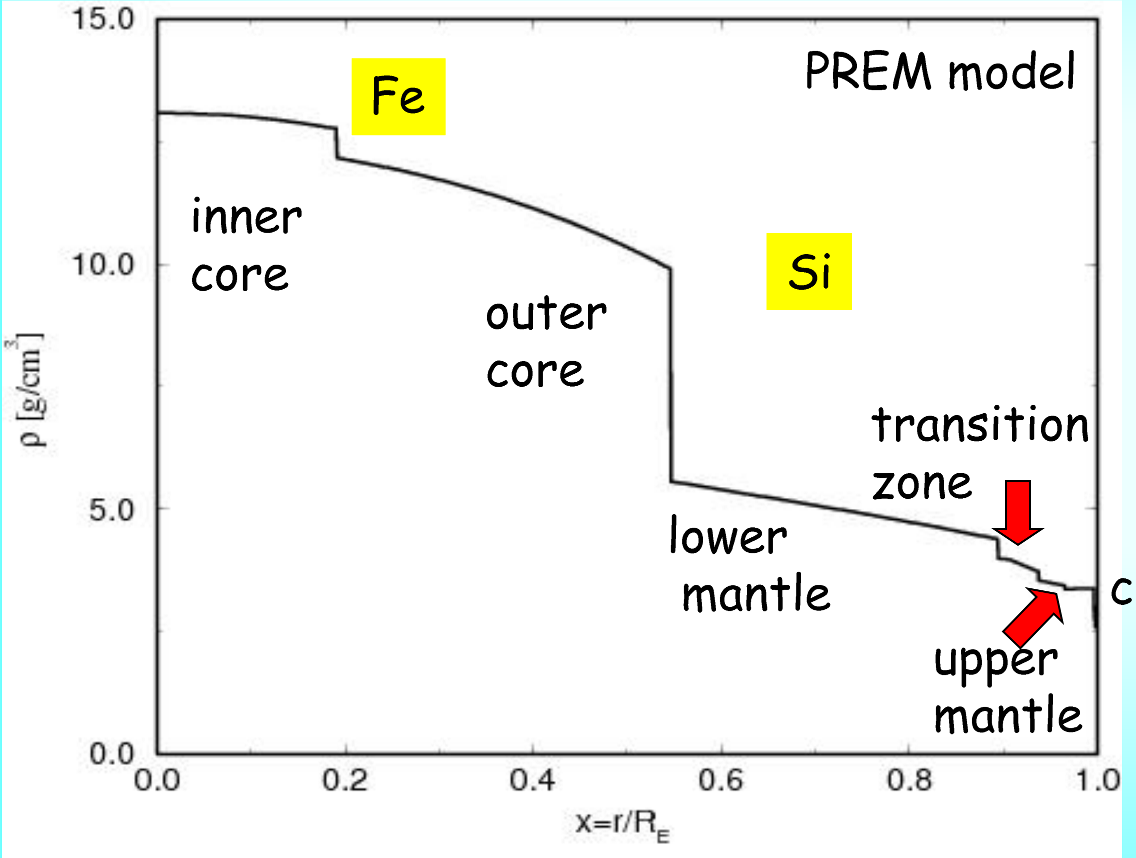
- accelerator
- atmospheric
- cosmic neutrinos

mass-to-flavor transitions

- solar neutrinos
- supernova neutrinos



The earth density profile



PREM model

Fe

Si

inner
core

outer
core

lower
mantle

transition
zone

upper
mantle

crust

A.M. Dziewonski
D.L Anderson 1981

(phase transitions in silicate minerals)

R_e = 6371 km

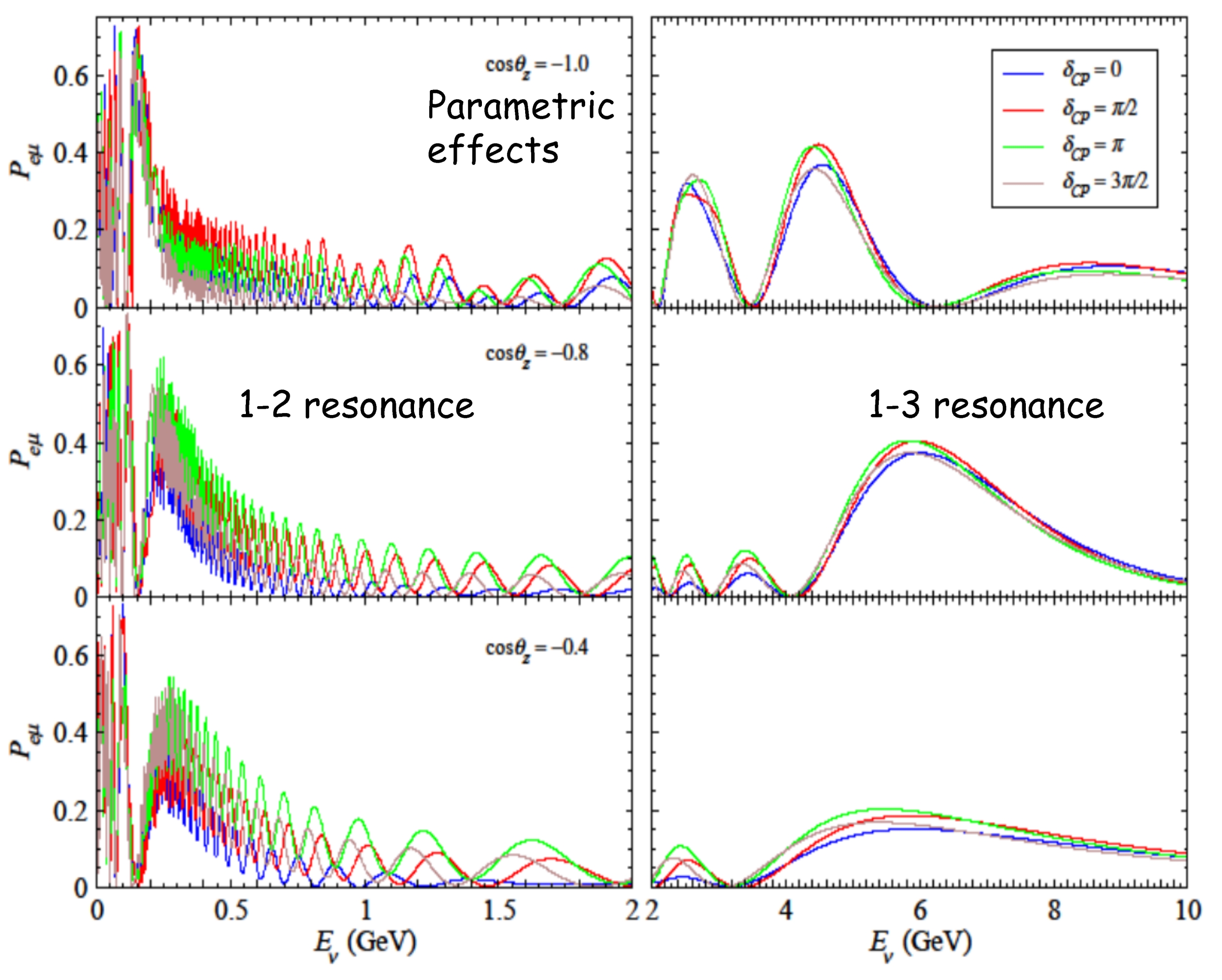
solid

liquid

Probabilities

S. Razzaque, A.Y.S.
arXiv: 1406.1407 hep-ph

$\nu_e \rightarrow \nu_\mu$ NH



Parametric effects

1-2 resonance

1-3 resonance

Large (10%) effect
at E ~ (0.5 – 1.5) GeV

The key: with
change of the phase

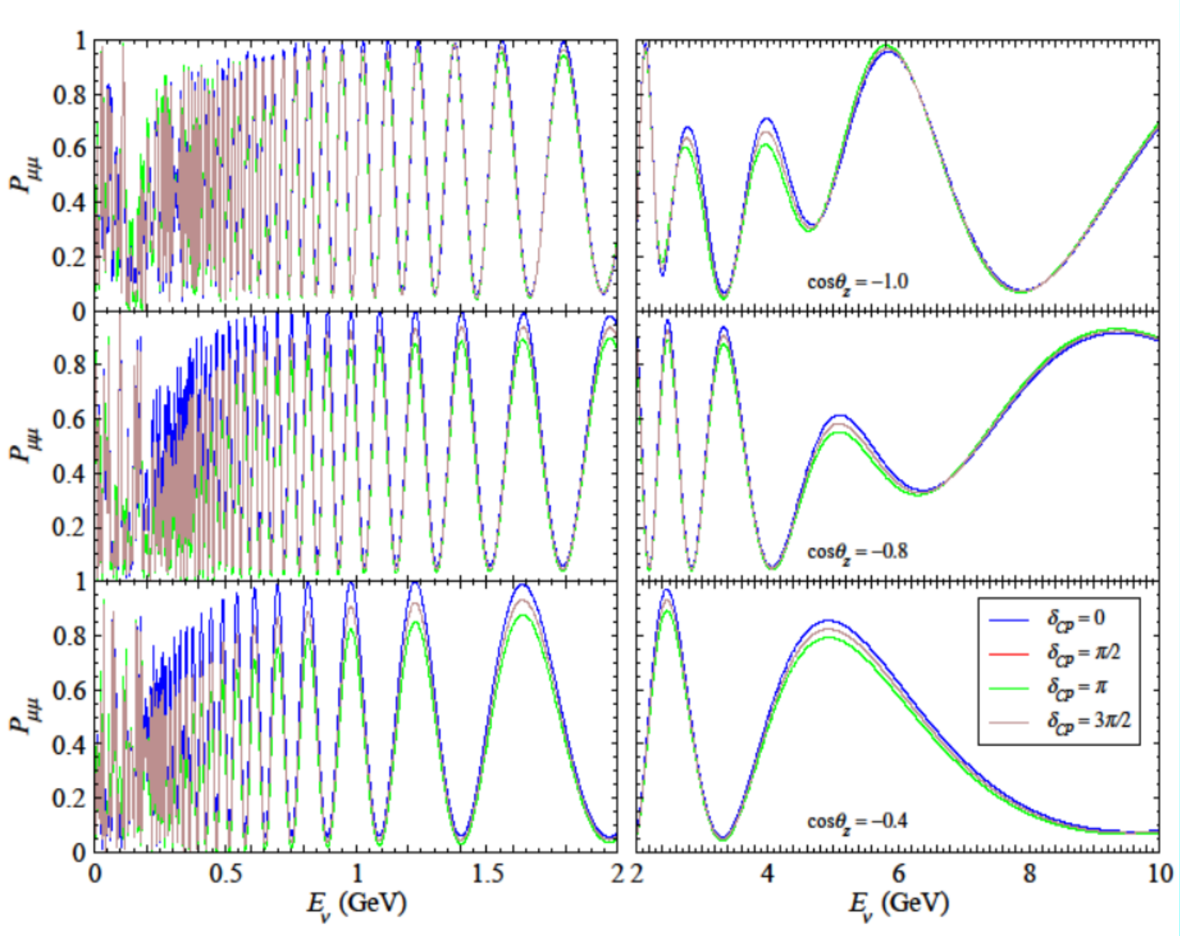
systematic shift
of curves,
the same for all zenith
angles in mantle

Averaging over
fast oscillations
and integration over
zenith angle does
not wash out CP
phase effect

Probabilities

S. Razzaque A Y S

$\nu_\mu \rightarrow \nu_\mu$



No phase shift

Effect is opposite to $\nu_e \rightarrow \nu_\mu$ with change of δ

Flavor suppression of effects for ν_μ events

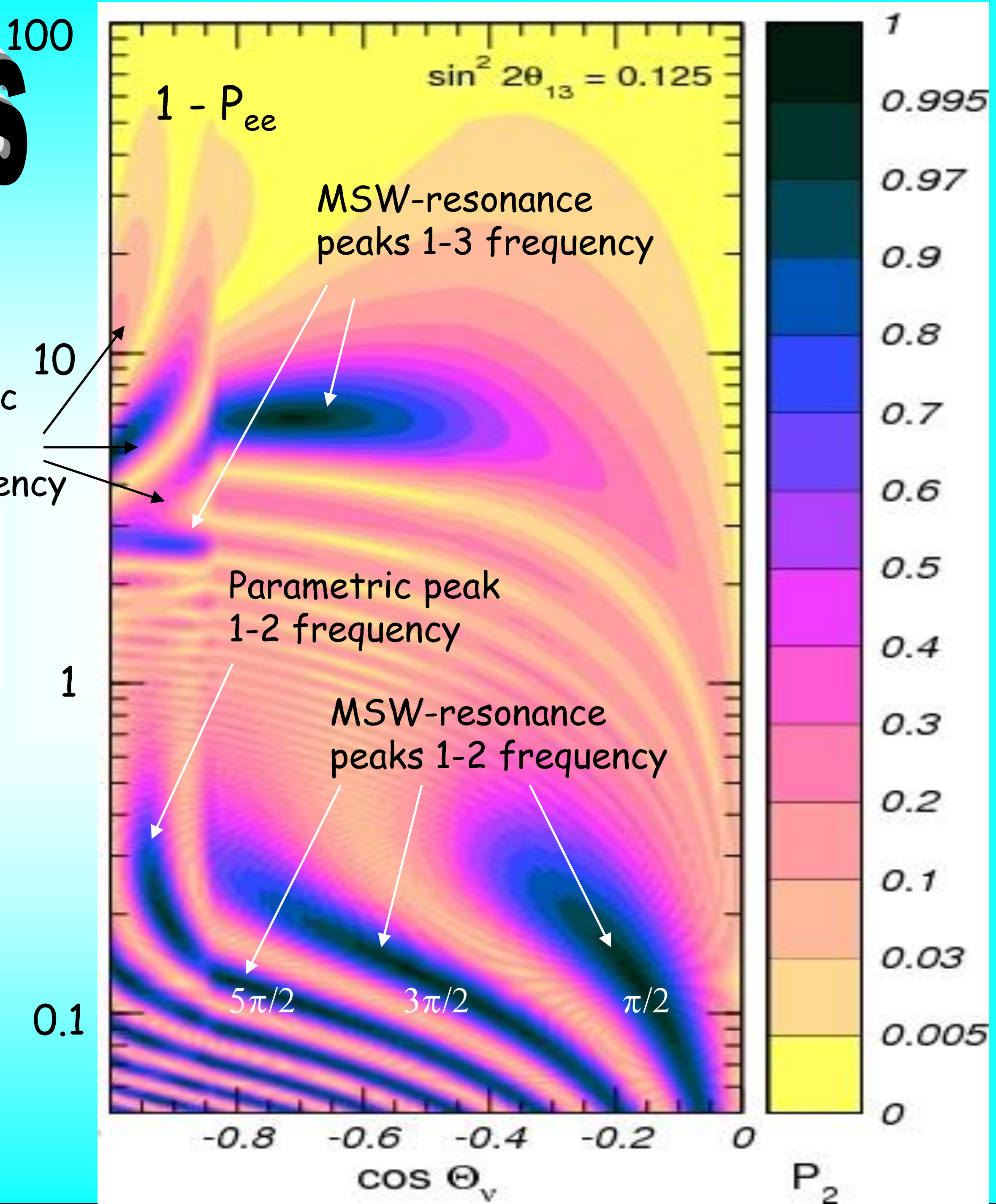
Flavor identification is crucial

Oscillograms of the Earth

Lines of equal Probability in the E – θ_z plane

$\nu_e \rightarrow \nu_\mu + \nu_\tau$

Parametric ridges 1-3 frequency



$1 - P_{ee}$

MSW-resonance peaks 1-3 frequency

Parametric peak 1-2 frequency

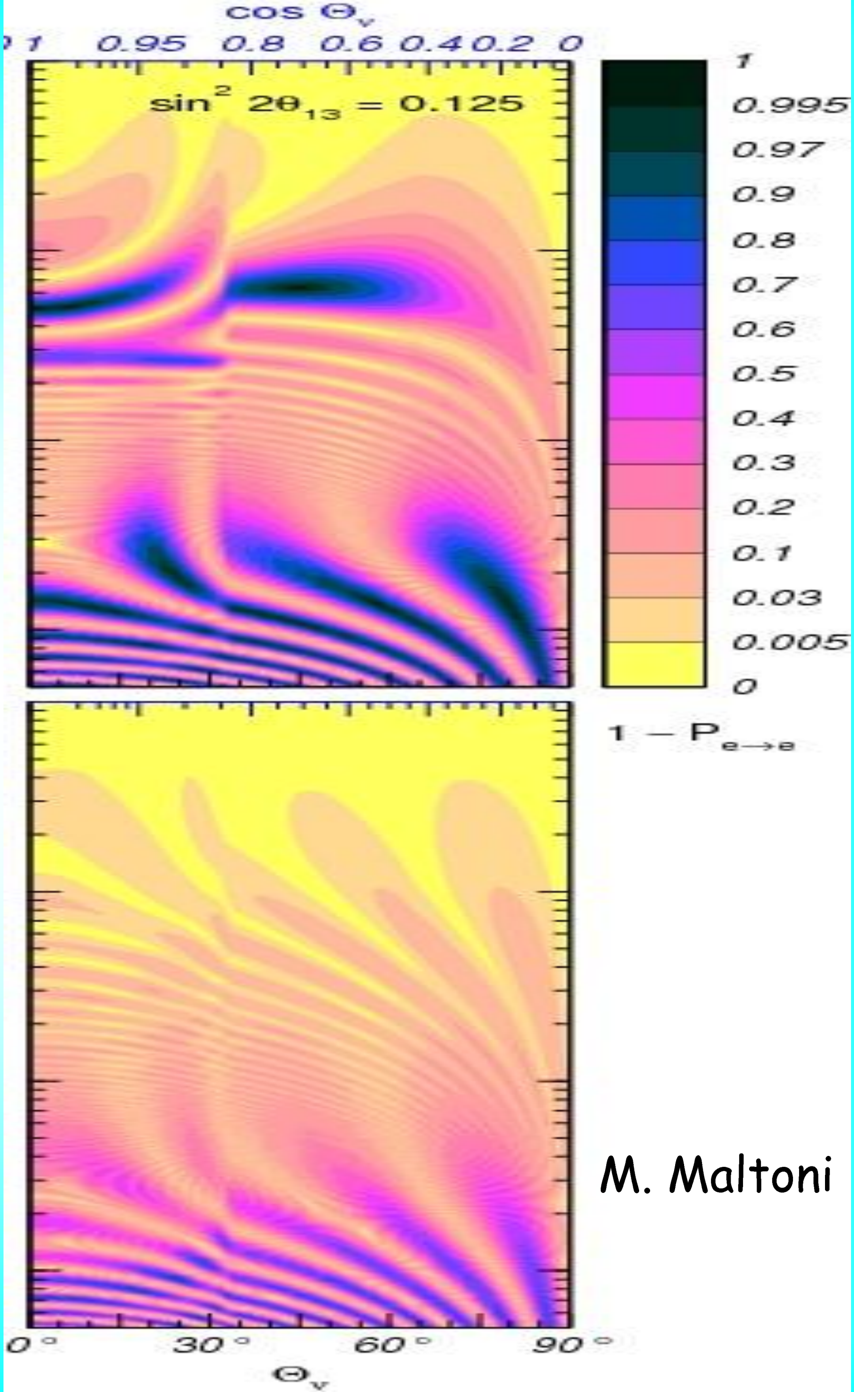
MSW-resonance peaks 1-2 frequency

$5\pi/2$ $3\pi/2$ $\pi/2$

Neutrinos and antineutrinos

Normal mass hierarchy

No resonances



M. Maltoni