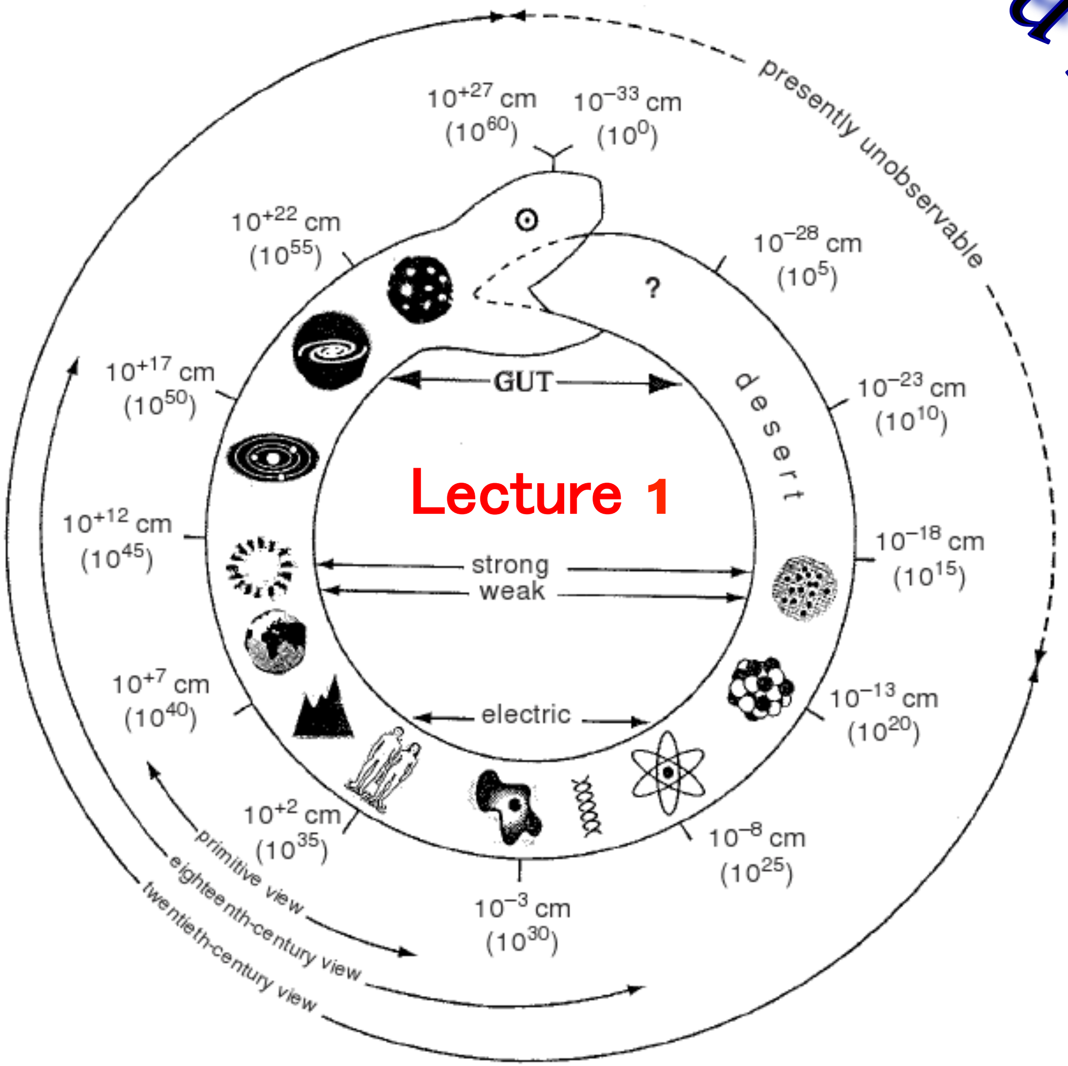
Physics of the early universe

Subir Sarkar



ICTP Summer School on Particle Physics, 15-26 June 2015, Trieste

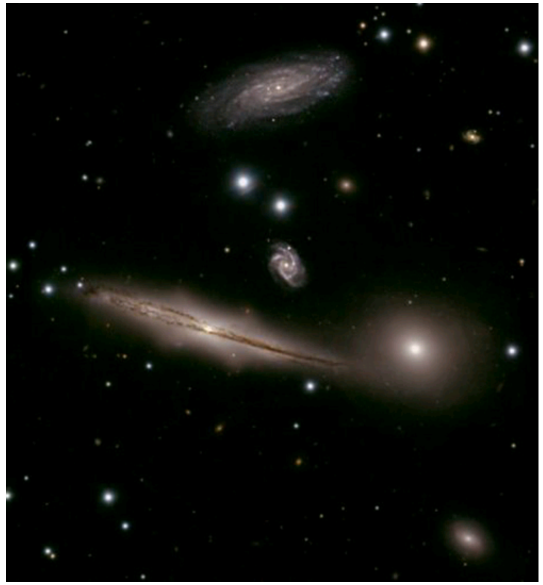
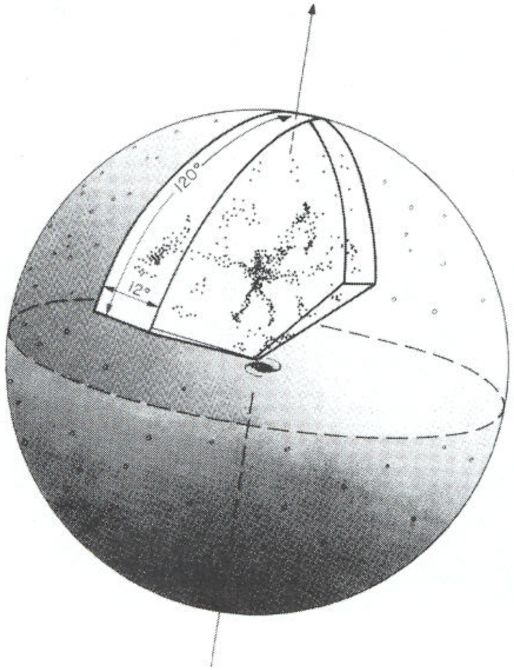
Oxford Physics

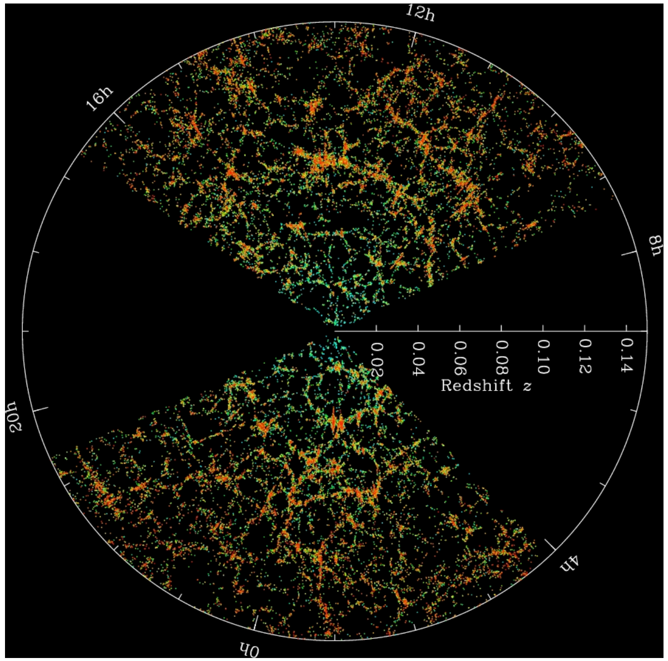
The Niels Bohr International Academy

The Abdus Salam International Centre for Theoretical Physics

The universe appears complex and structured on many scales …

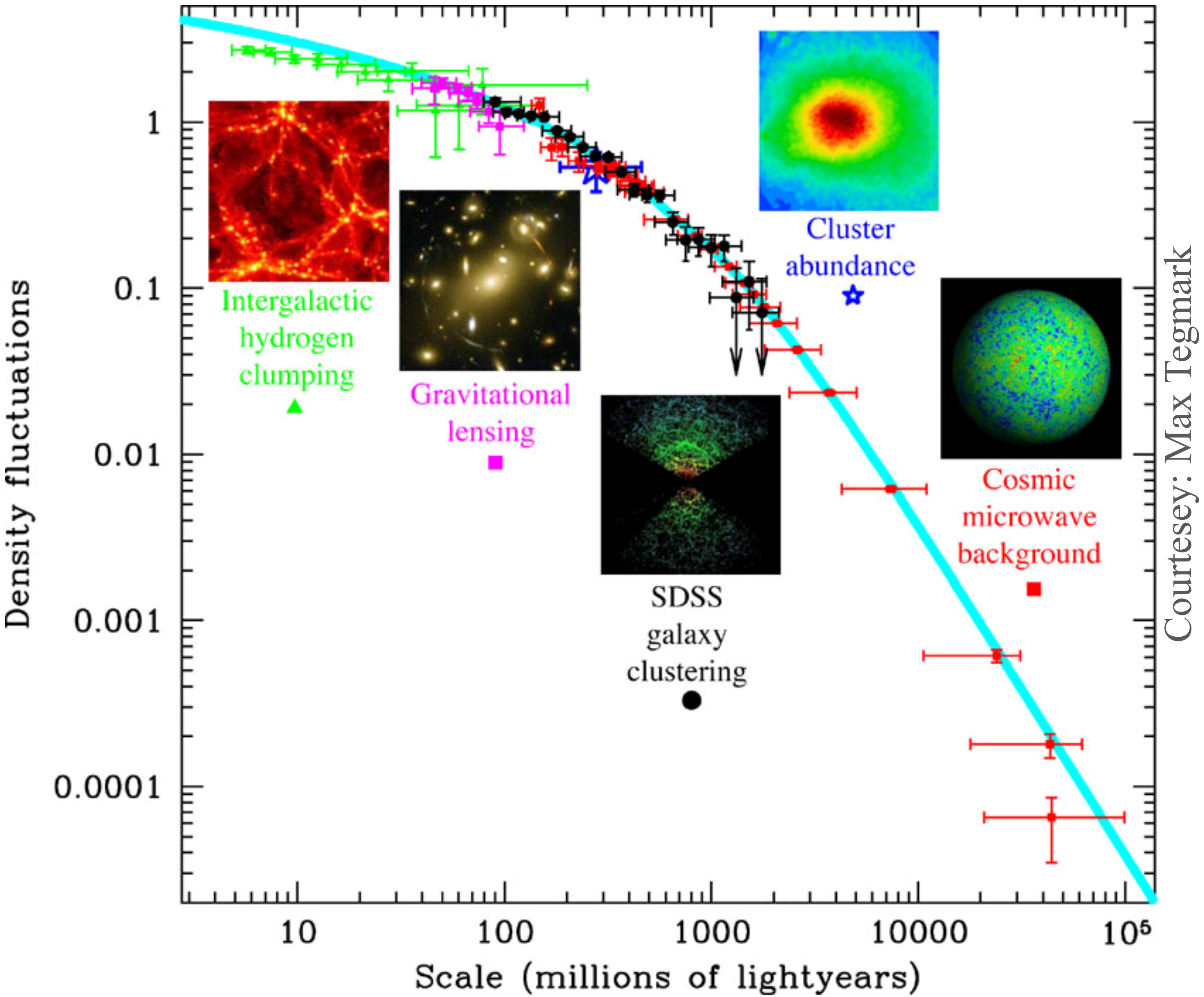


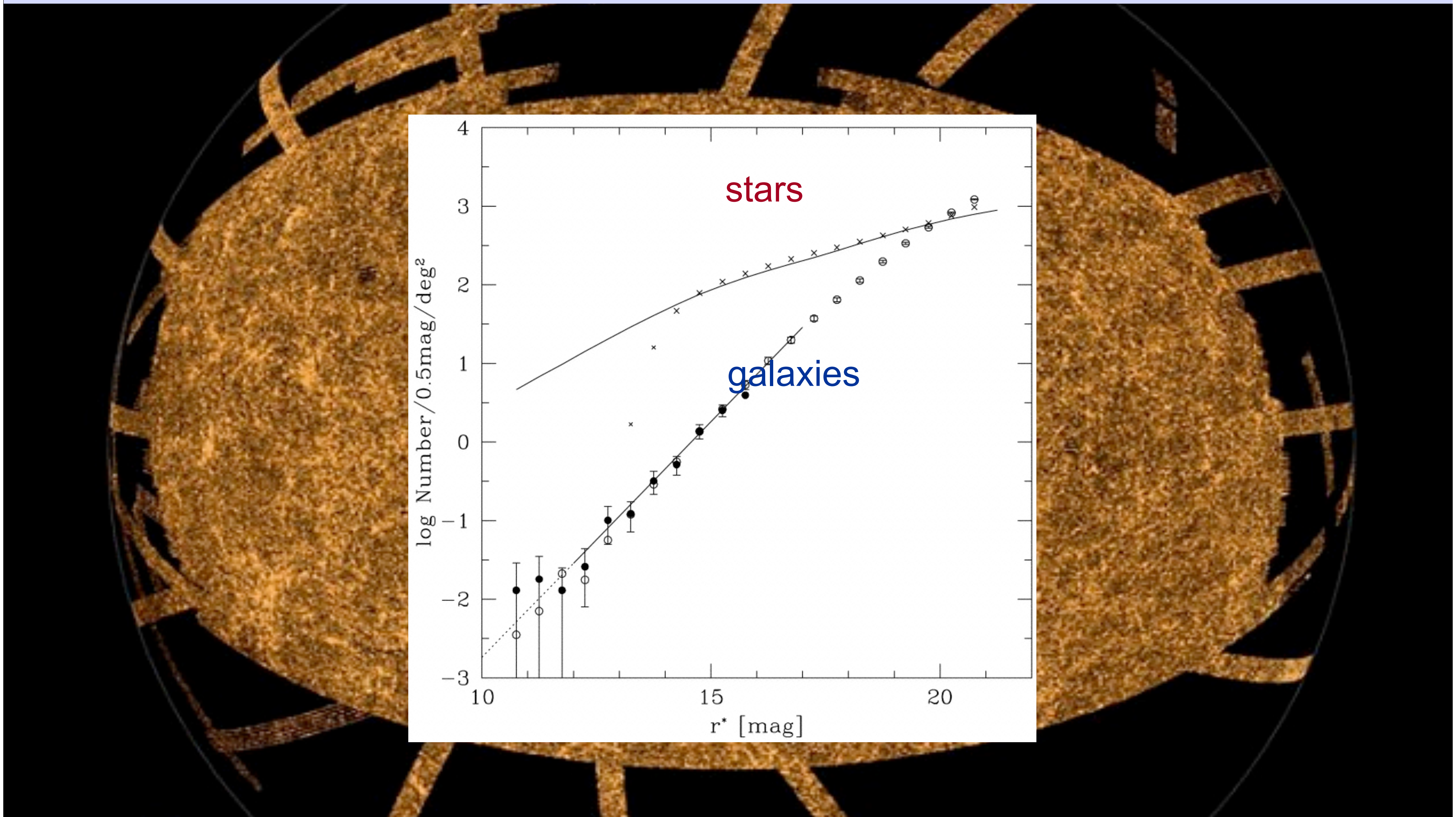


How can we describe it by a simple mathematical model?

Although the universe is lumpy, it seems to become smoother and smoother when averaged over larger and larger scales …



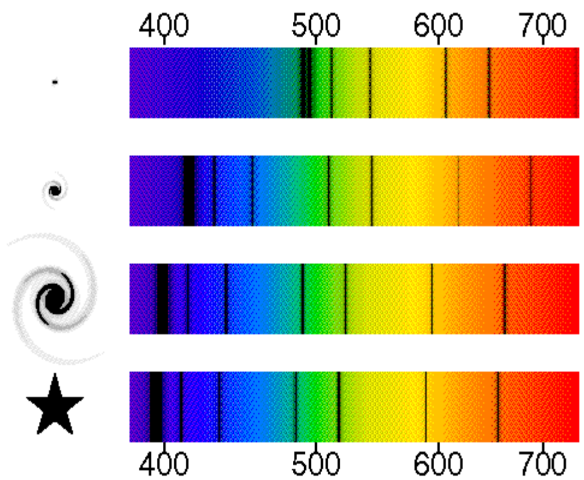
Hubble (1926) showed that the distribution of galaxies is **homogeneous**, i.e. $N\,(>S) \propto S^{-3/2} \Rightarrow N\,(<m) \propto 10^{0.6m}$, where $m \equiv -2.5 \log\,(S/S_0)$

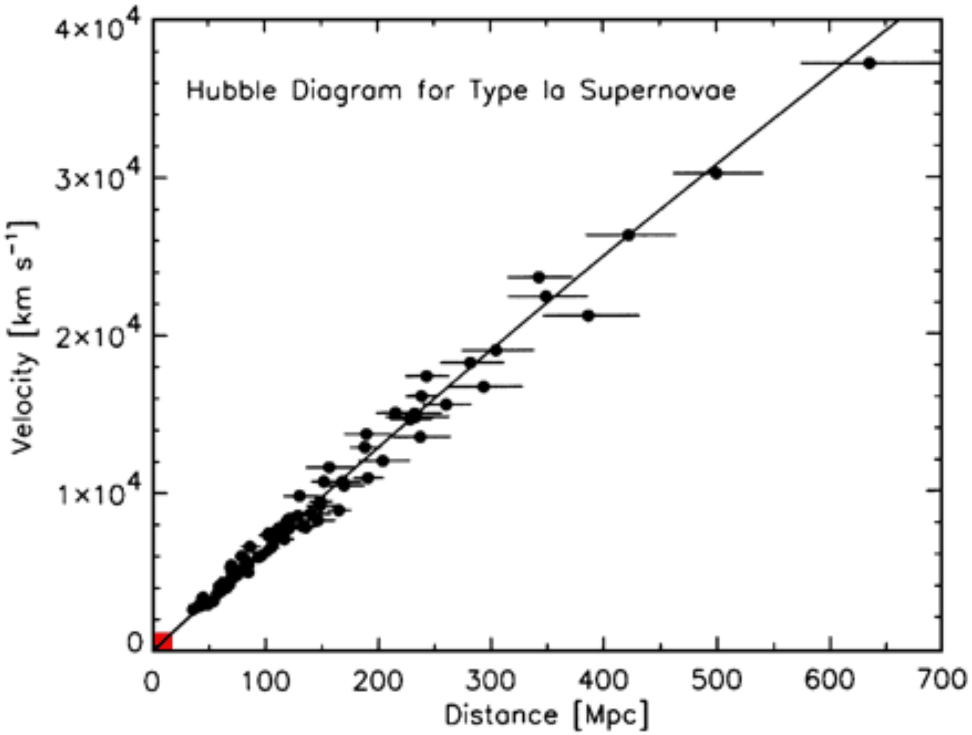


Here is the test done on galaxies in the **Sloan Digital Sky Survey**

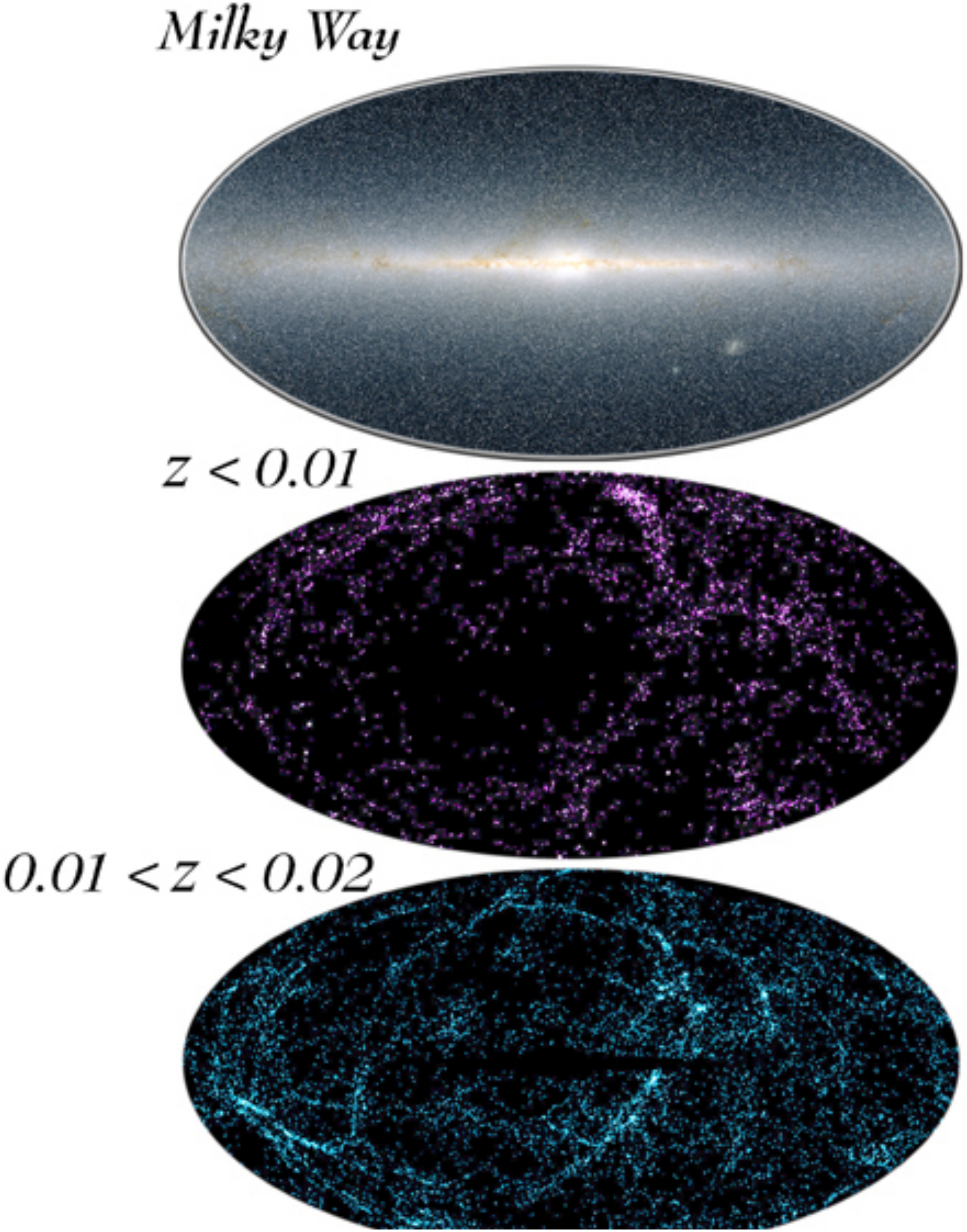
NB: For stars, $N\,(<m) \propto 10^{0.4m}$, reflecting their 2D distribution

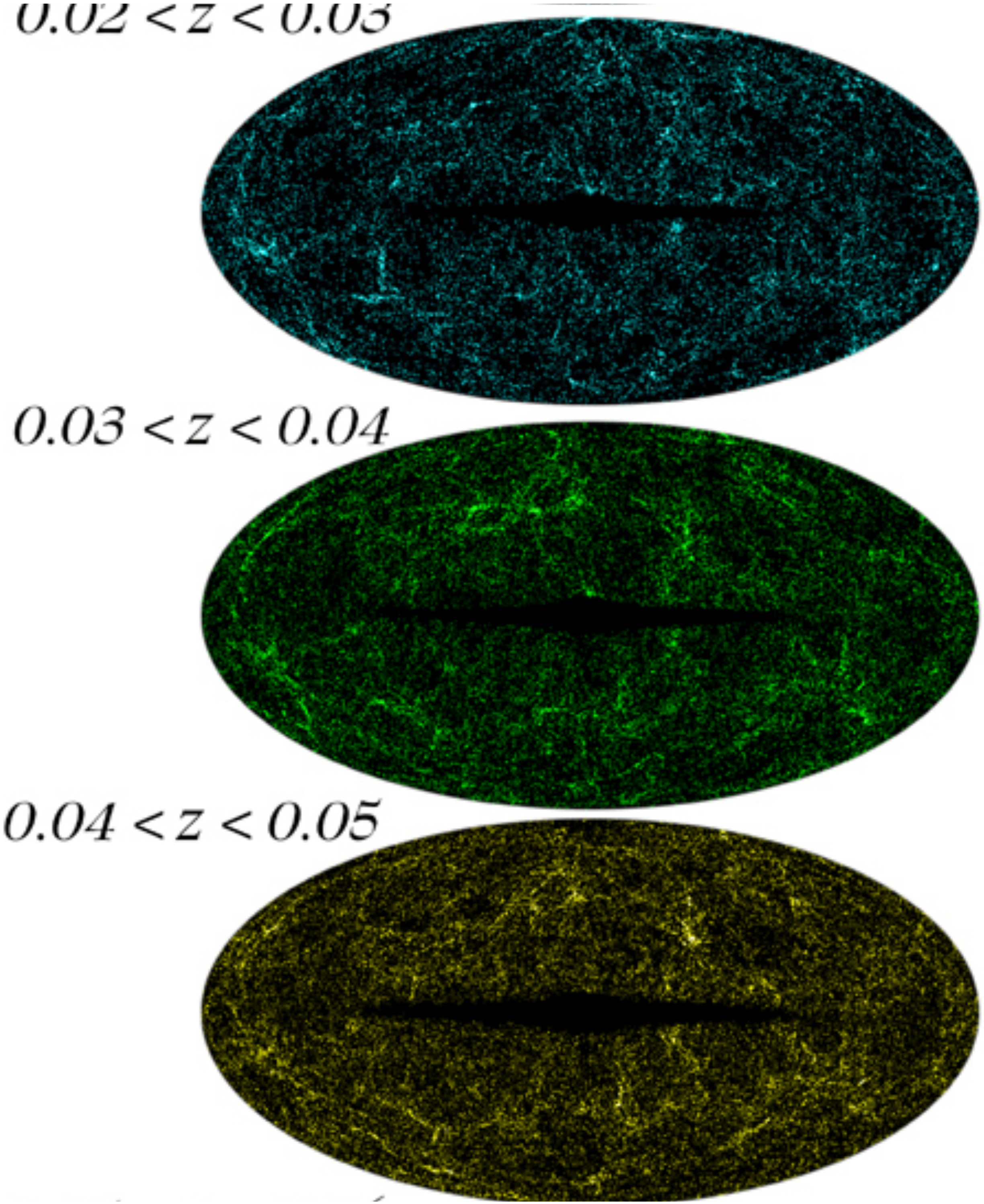
Red Shift: $z = \frac{\lambda_{\text{emitted}} - \lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \simeq v/c$, for $z \ll 1$





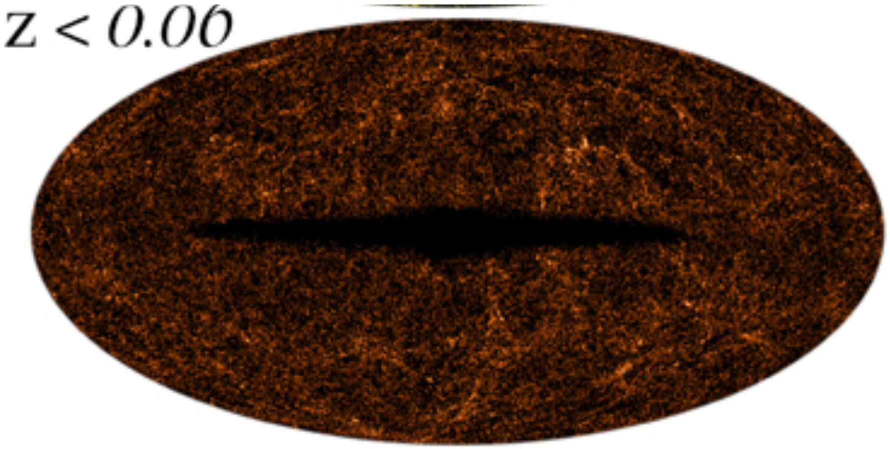
Velocity (redshift) is *proportional* to distance, so $z = 0.1 \Rightarrow d \sim 500$ Mpc


Milky Way
z < 0.01
0.01 < z < 0.02

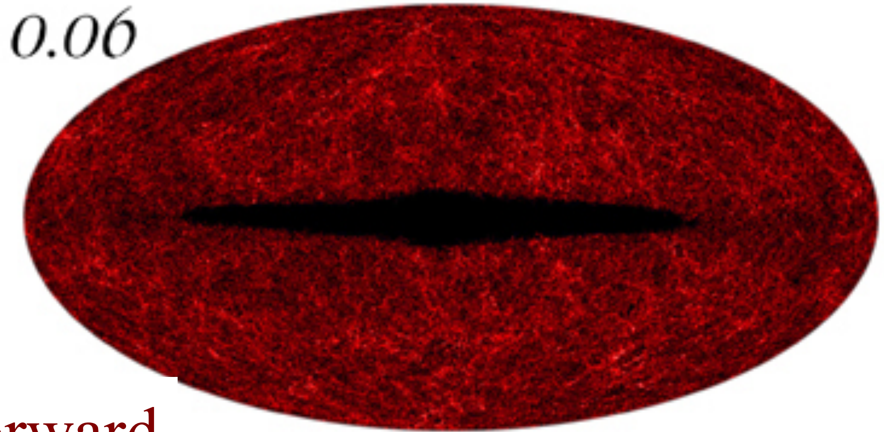


$0.02 < z < 0.03$
$0.03 < z < 0.04$
$0.04 < z < 0.05$

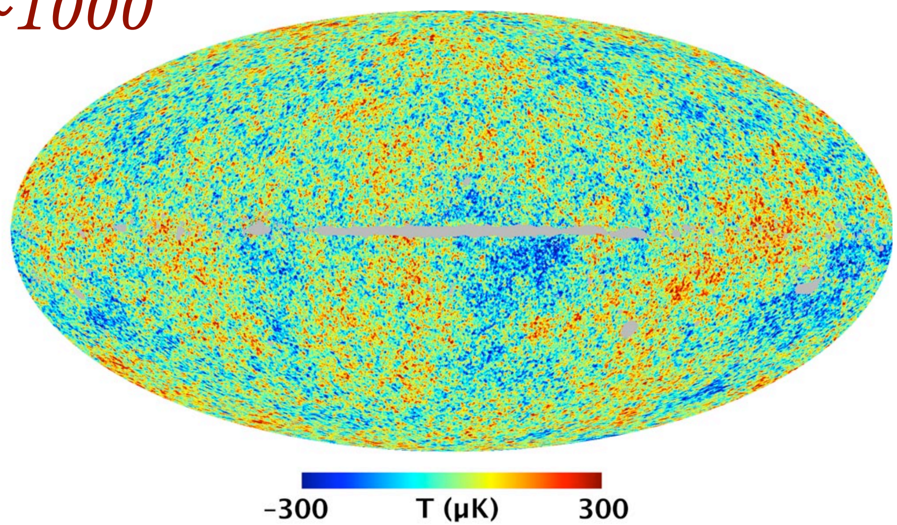
$0.05 < z < 0.06$



$z > 0.06$

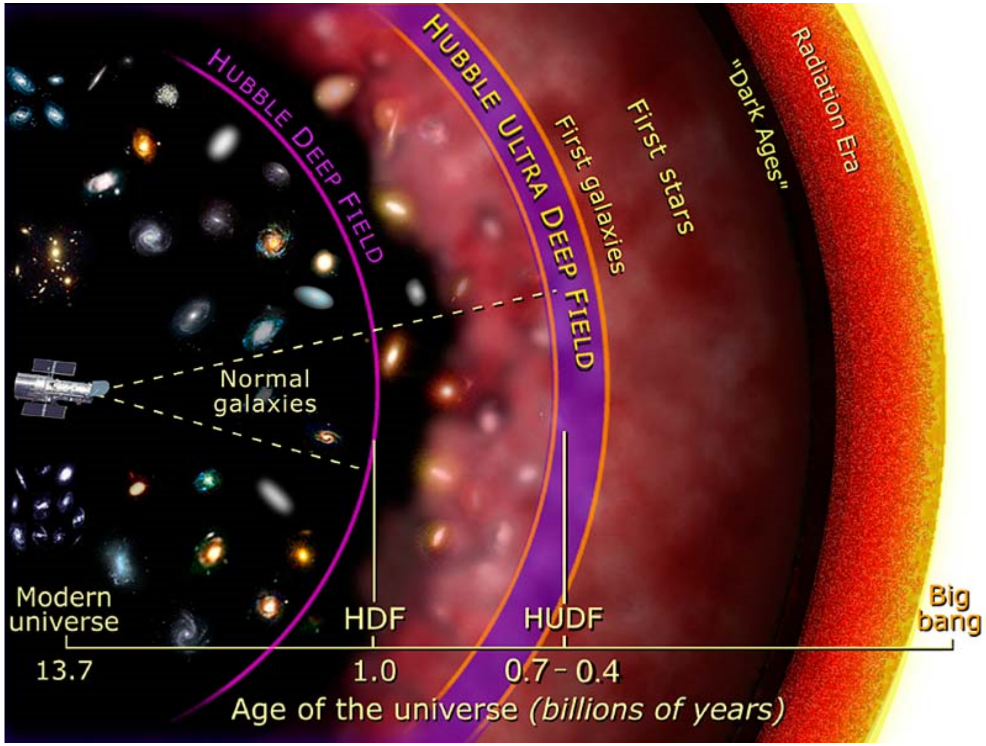


… fast forward to: *z ~1000*



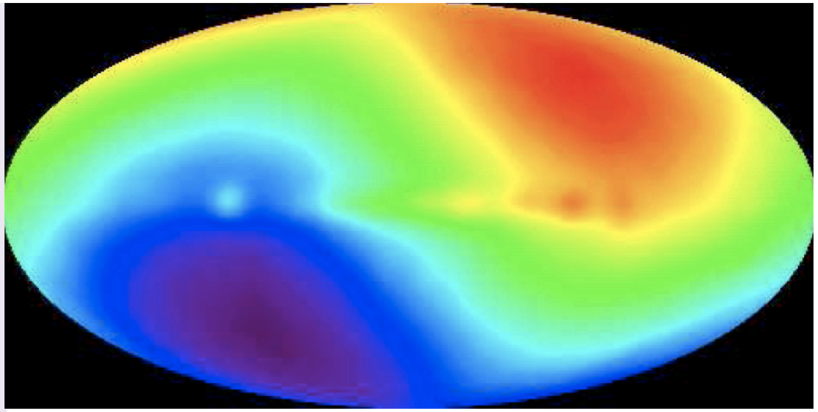
The **cosmic microwave background** (blackbody with T=2.7255 ± 0.0006 K) is isotropic to 1 part in ~10^5

When we look out in distance, we look *back in time* so what we see makes sense if the universe was denser (hence hotter) when it was younger … back to the Big Bang:

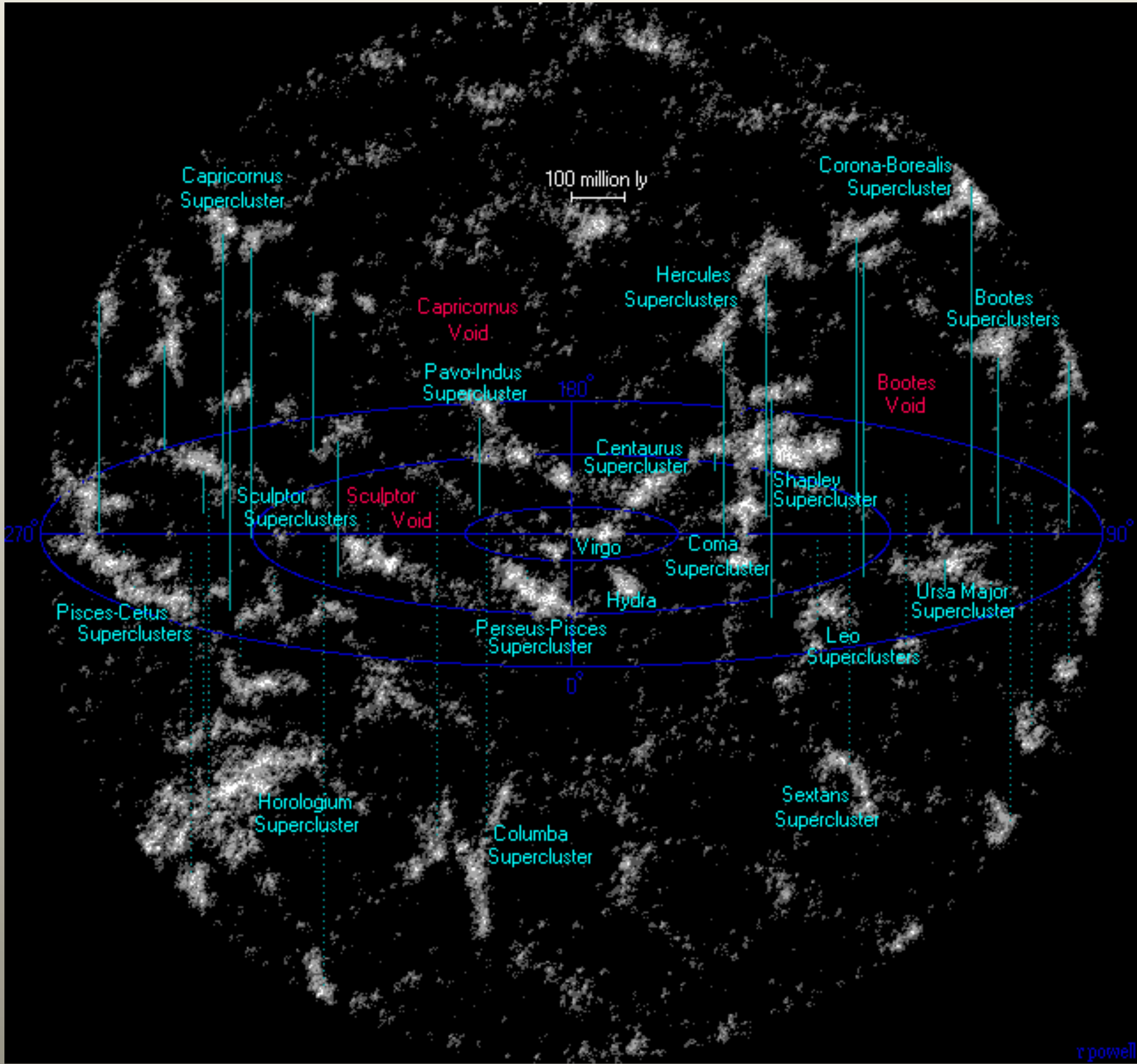


Open Question: The CMB exhibits a dipole anisotropy with amplitude 3.355±0.008 mK, due to our local 'peculiar' motion with $v = 369.0 \pm 0.9$ km/s towards the Shapley supercluster. It is *after* we boost to this frame that we see the CMB as isotropic. However using SNe Ia to trace the Hubble flow, convergence to the CMB frame has not occured even as far out as $z \sim 0.07$ (~300 Mpc)!

Colin *et al*, MNRAS **414**:264,2011, Feindt *et al*, A&A **560**:A90,2013

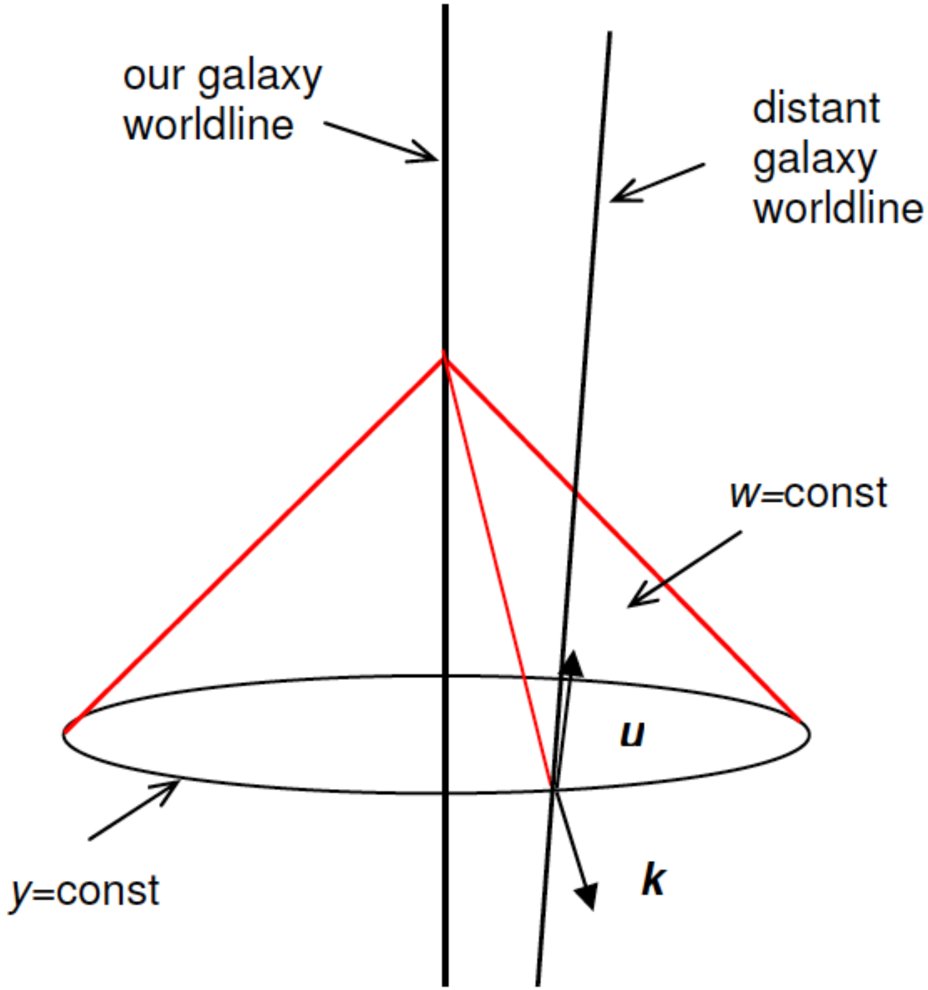


This is what our universe *actually* looks like out to ~300 Mpc



It is *not* clear where the transition to homogeneity occurs (usually quoted as ~100 Mpc)

Moreover all we can ever learn about the universe is contained within our past light cone



We *cannot* move over cosmological distances and check that the universe looks the same from other view points … so we must *assume* the validity of the 'Cosmological Principle' (Milne 1935) viz. our position is *typical*

Special relativity

$$\mathrm{d}s^2 = \sum g_{ij}\mathrm{d}x^i\mathrm{d}x^j \ldots \text{ interval between events } x^i \text{ and } x^j (i,j = 0,1,2,3)$$

$$g_{ij}(x) \equiv g_{ji}(x) \rightarrow 10 \text{ independent functions}$$

Minkowski metric

$$\eta_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \frac{\delta g_{ij}}{\delta x^k} = 0 \quad \Rightarrow \mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$$

… invariant under **Lorentz velocity transformations**, *i.e. equivalent* to **local inertial coordinates** of **Newtonian mechanics**

General relativity

Now g_{ij} is related to the **distribution of matter** … but $g_{ij} = \eta_{ij}$ is a solution in the *absence* of matter – contrary to **Mach's principle***!

* inertial frames are determined relative to the motion of the matter (distant stars) in the universe

Einstein (1919) saw two ways out:

* add suitable boundary conditions to eliminate anti-Machian solution, *viz.* let g_{ij} take some pathlogical form (rather than becoming η_{ij}) when far away from all matter ✗

… however de Sitter pointed out pheonomenological problems with this idea!

- Postulate that the matter distribution is **homogeneous** (in the average) and that matter causes space to curve so as to close in on itself (3D analogue of a 2D balloon)

→ Spatial volume finite but *no boundaries* and a non-singular metric everywhere ✔

Einstein's world model

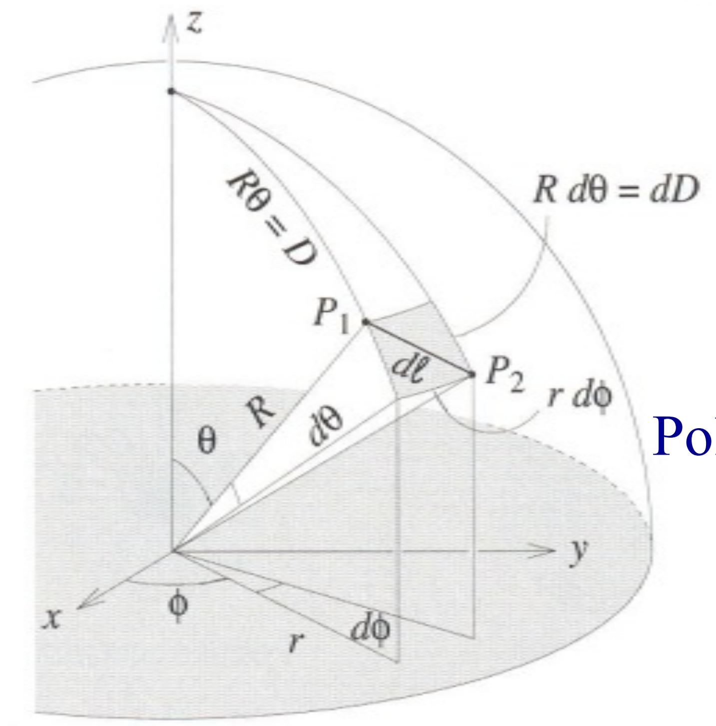
Homogeneity $\Rightarrow \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}m} \propto 10^{0.6m}$ … as observed later (Hubble 1926)

… incorporating Milne's Cosmological Principle

$\mathrm{d}s^2 = \mathrm{d}t^2 + g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ … synchronous gauge (dense set of comoving observers)

This is the 'standard model' we are still using *today* to interpret all observations

Picture the spatial part as **S**^{3} (3D analogue of balloon, embedded in flat 4D space)



Set of points defining **S**^{3}: $R^2 = x^2 + y^2 + z^2 + w^2$

where: $r^2 = x^2 + y^2 + z^2$

Line element: $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \mathrm{d}w^2$

i.e. $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$

Polar coordinates (z=$r\cos\theta$, x=$r\sin\theta\cos\varphi$, y=$r\sin\theta\sin\varphi$):

$$\mathrm{d}l^2 = \mathrm{d}r^2 + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$$
$$= \mathrm{d}r^2/(1 - r^2/R^2) + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

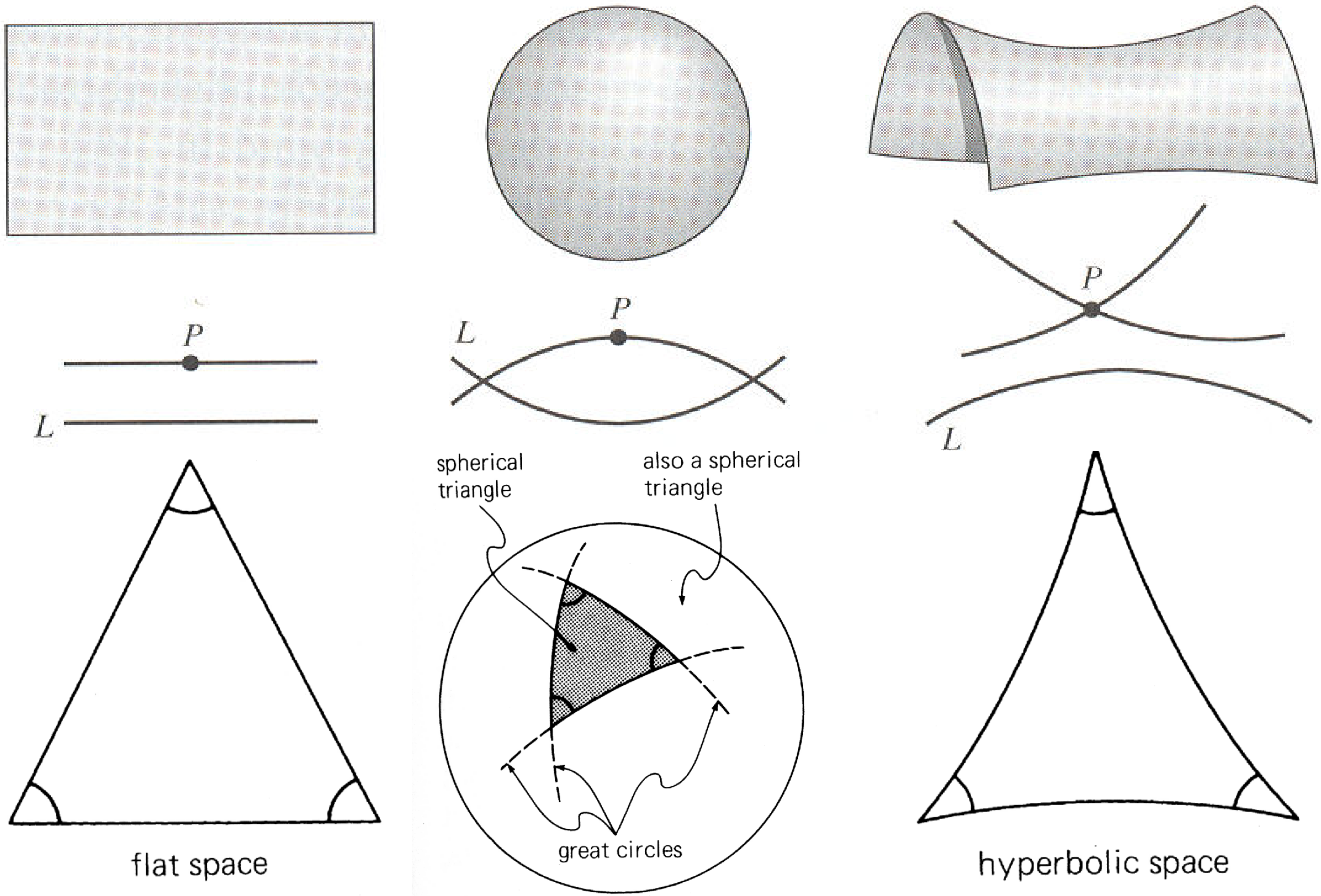
or, $\mathrm{d}s^2 = \mathrm{d}t^2 - R^2\,[\mathrm{d}\chi^2 + \sin^2\chi\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)]$,

where, $r = R\sin\chi$, $\chi \Rightarrow$ polar angle of hypersphere

Note interesting visual effects in curved space (when $r \sim R$), e.g. the angular size $\delta = D/R\sin\chi$ reaches minimum at $\chi = \pi/2$ and diverges to fill the entire sky when $\chi = \pi$ (this point is the just the 'Big Bang' – the antipodal point of the hypersphere)

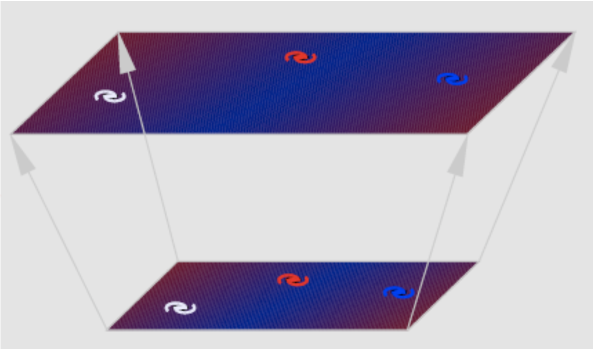
Also the parallax, $\varepsilon = A\cot\varphi/R$, *vanishes* at $\chi = \pi/2$

The 3 possible geometries of maximally-symmetric space



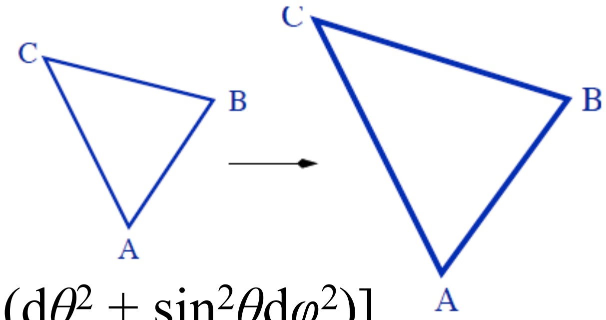
= 180°　　　> 180°　　　< 180°

The expanding universe (Friedmann 1922, Lemaitre 1931)



Generalise line element:

$R(t) = R_0\, a(t)$



$ds^2 = dt^2 - a^2(t)\, R_0^{\,2}\, [d\chi^2 + \sin^2\chi\, (d\theta^2 + \sin^2\theta d\varphi^2)]$

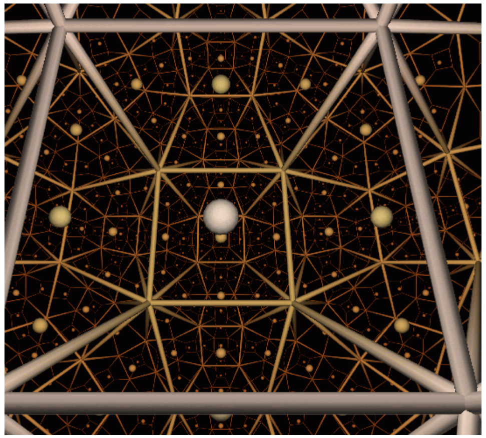
… a spatially **closed** expanding universe

To describe a spatially **open** expanding universe, change: $\chi \to i\chi$, $R_0 \to iR_0$, so

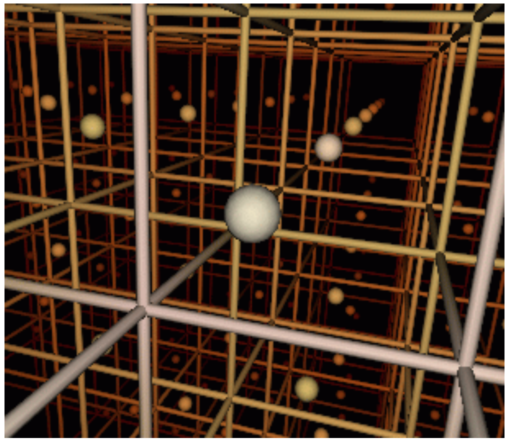
$$ds^2 = dt^2 - a^2(t)\, R_0^{\,2}\, [d\chi^2 + \sinh^2\chi\, (d\theta^2 + \sin^2\theta d\varphi^2)]$$

This is the Robertson-Walker line element (*maximally*-symmetric space-time):

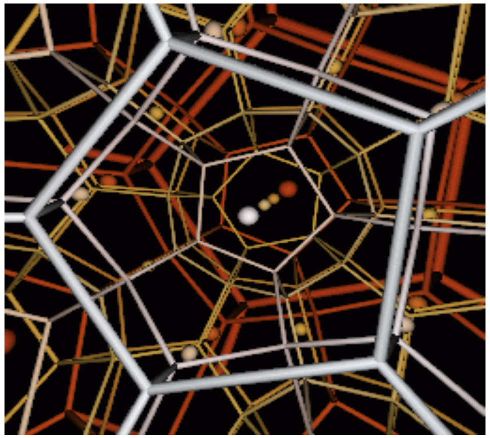
$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$



$k = -1$

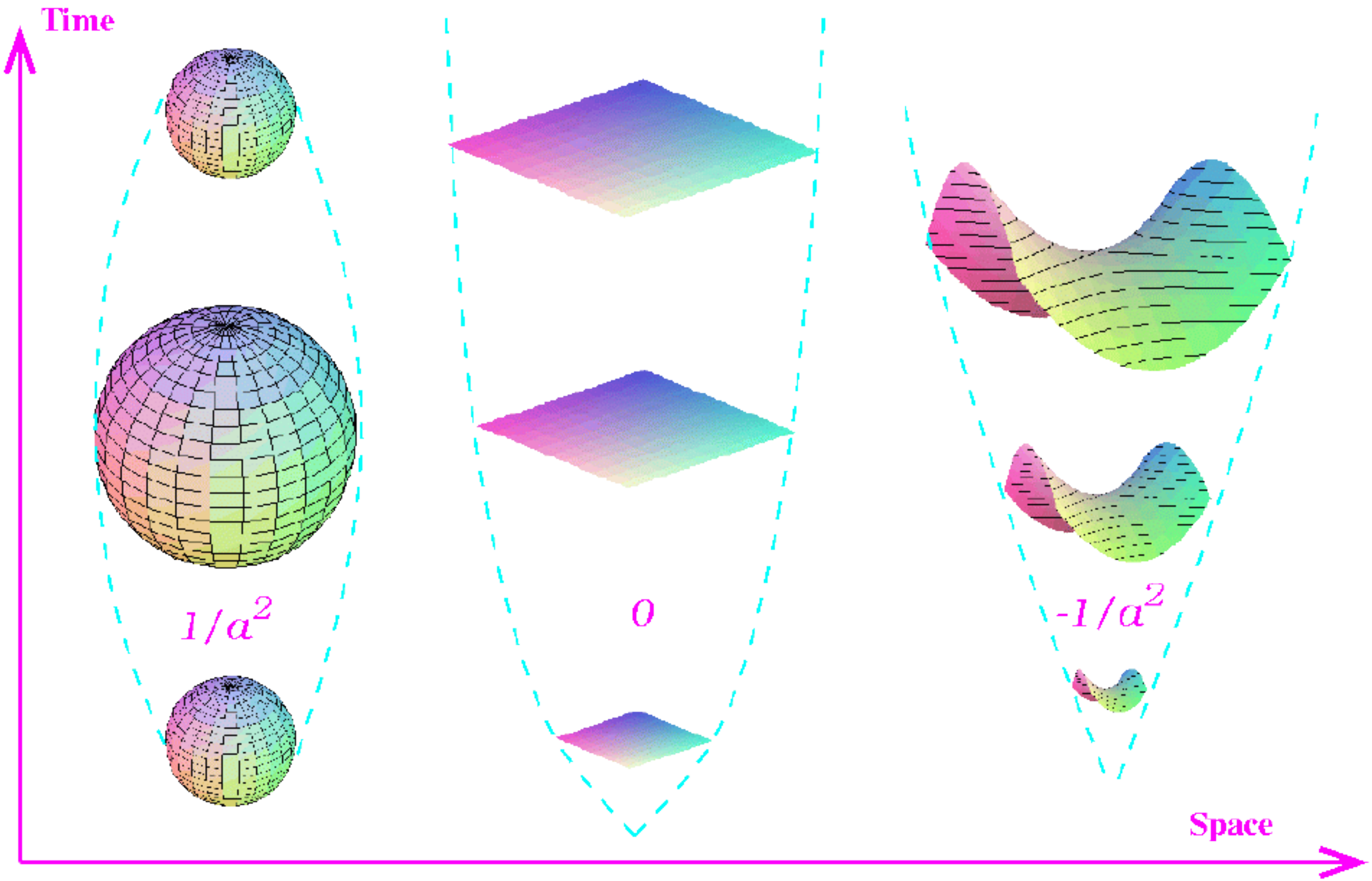


$k = 0$



$k = +1$

Homogeneous and isotropic world models



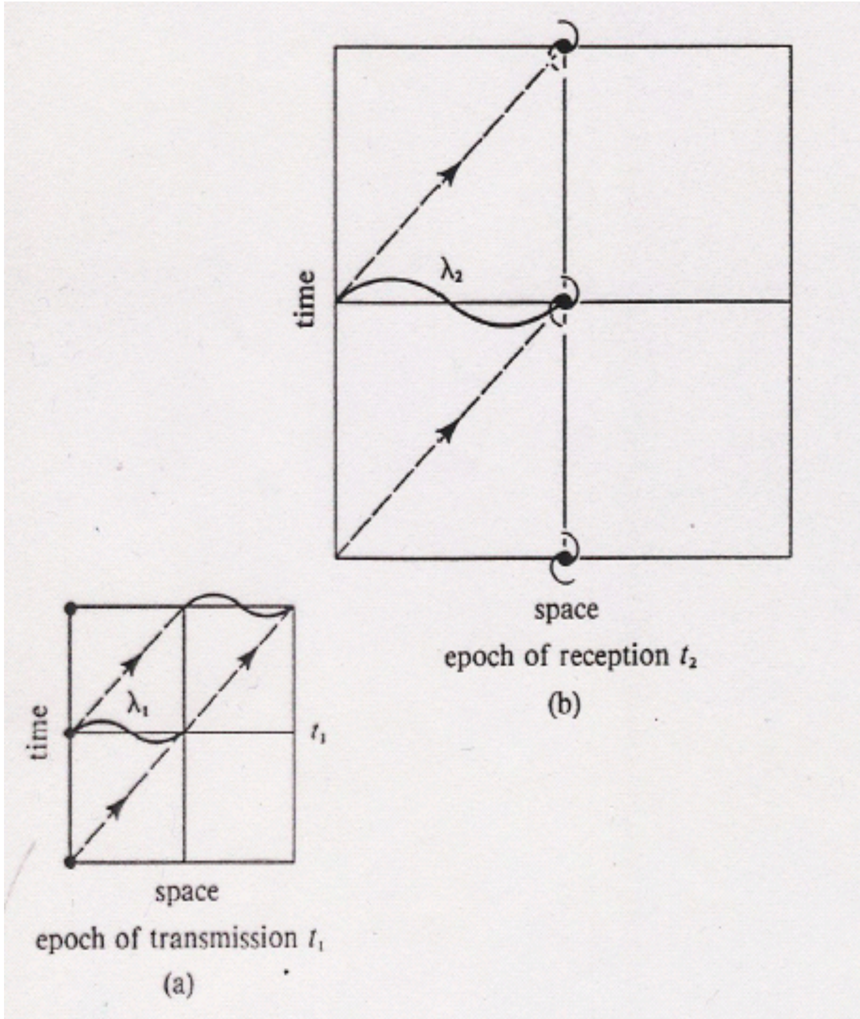
The redshift happens because, for null geodesics:

$$\int_t^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_0^r \frac{\mathrm{d}r}{\sqrt{1-kr^2}} = \text{const}$$

… for a galaxy (in co-moving coordinates), so crests of adjacent waves, separated by Δt at emission, will be received with separation, Δt_0:

$$\frac{\Delta t_0}{\Delta t} = 1 + \frac{\Delta\lambda}{\lambda_0} \equiv 1 + z = \frac{a(t_0)}{a(t)}$$

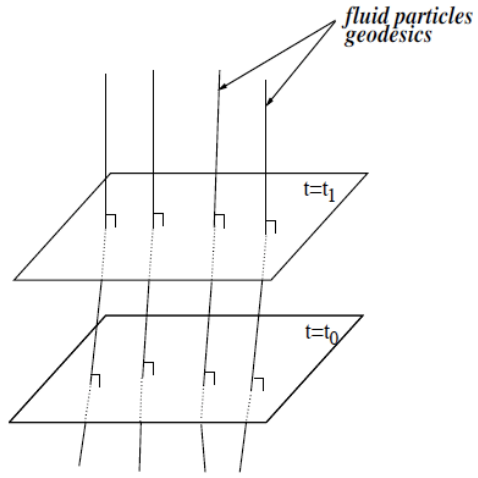
This is the cosmological time dilation or redshift: $z = \infty$ is the 'Big Bang' at $t = 0$ (the antipodal point of the hypersphere ⇒ the furthest we can look back in principle)



Everything is not expanding (how would we know?) … certainly not bound structures like atoms or planets or galaxies - it is only the large-scale *smoothed* space-time metric which is stretching with cosmic time (and there is no restriction on the rate!)

The 'expansion' is in a sense *illusory* … because we can always transform to a "comoving" coordinate system where *galaxies are at rest* wrt each other

Ideal fluid: $T_{ij} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$



Poisson's equation: $\nabla . g = -4\pi G_{\mathrm{N}}(\rho + 3p)$

Birkhoff's theorem: If $T_{ij} = 0$ in some region within a spherically symmetric distribution of matter, then the solution in the hole ⇒ flat space-time

Einstein's field equations

$$R_{ij} + \tfrac{1}{2} g_{ij} R_{\mathrm{c}} = 8\pi G_{\mathrm{N}} T_{ij}, \text{ where } R_{ij} \equiv g^{\lambda k} R_{\mu\nu\lambda k} \text{ and } R_{\mathrm{c}} \equiv g^{\mu\nu} R_{\mu\nu}$$

For the RW metric, the 00 and 11 components simplify to the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho - \frac{k}{a^2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_{\mathrm{N}}}{3}(\rho + 3p)$$

'Newtonian' Cosmology

Consider sphere of radius *l* embedded in homogeneous background (McCrea & Milne 1934):

$$\ddot{\ell} = -G_{\rm N}M/r^2 = -\tfrac{4\pi}{3}G_{\rm N}(\rho + 3p)\ell; \text{ also } {\rm d}U \equiv \rho {\rm d}V + V{\rm d}\rho = -p{\rm d}V$$

$$\Rightarrow \dot{\rho} = -(\rho + p)\frac{\dot{V}}{V} = -3(\rho + p)\frac{\dot{\ell}}{\ell} \text{ ... energy eq. for ideal fluid}$$

$$\text{So, } \ddot{\ell} = \tfrac{8\pi}{3}G_{\rm N}\rho\ell + \tfrac{4\pi}{3}G_{\rm N}\dot{\rho}\frac{\ell^2}{\dot{\ell}} \Rightarrow \dot{\ell}^2 = \tfrac{8\pi}{3}G_{\rm N}\rho\ell^2 + K$$

To obtain a *static* solution (Einstein's "greatest blunder") we have to set:

$$\rho + 3p = 0 \text{ i.e. } p = -\tfrac{\rho}{3} \text{ (!)} \Rightarrow \text{ universe of radius: } \mathcal{R}^2 = -\frac{\ell^2}{k} = [\tfrac{8\pi}{3}G_{\rm N}\rho]^{-1}$$

The static solution is in fact *unstable* (metric perturbations grow exponentially fast) but we do *not* have the freedom, as Einstein said, to "do away with the cosmological constant" … it is a *necessary* consequence of **general coordinate invariance** which allows an *arbitrary* constant multiplying the metric tensor to be added to the l.h.s.

So must modify the field equations to: $R_{ij} + \frac{1}{2}g_{ij}R_{\rm c} - \Lambda g_{ij} = 8\pi G_{\rm N}T_{ij}$

… which can be *interpreted* (when moved to r.h.s.) as a fluid with: $\rho_\Lambda = -p_\Lambda = \Lambda/8\pi G_N$

FLRW Dynamics

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\rm N}}{3}(\rho + 3p) \pm \frac{1}{a^2\mathcal{R}^2} \rightarrow -\frac{4\pi G_{\rm N}}{3}(\rho_{\rm b} + 3p_{\rm b}) \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}$$

b ⇒ ‘background’ (i.e. “ordinary” matter/radiation)

Conservation of energy-momentum: $\dot{\rho_{\rm b}} = -3(\rho_{\rm b} + p_{\rm b})\frac{\dot{a}}{a}$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G_{\rm N}}{3}\rho_{\rm b} \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}, \quad \text{where + is open/- is closed universe}$$

Two interesting solutions describing an expanding universe:

Einstein-de Sitter:

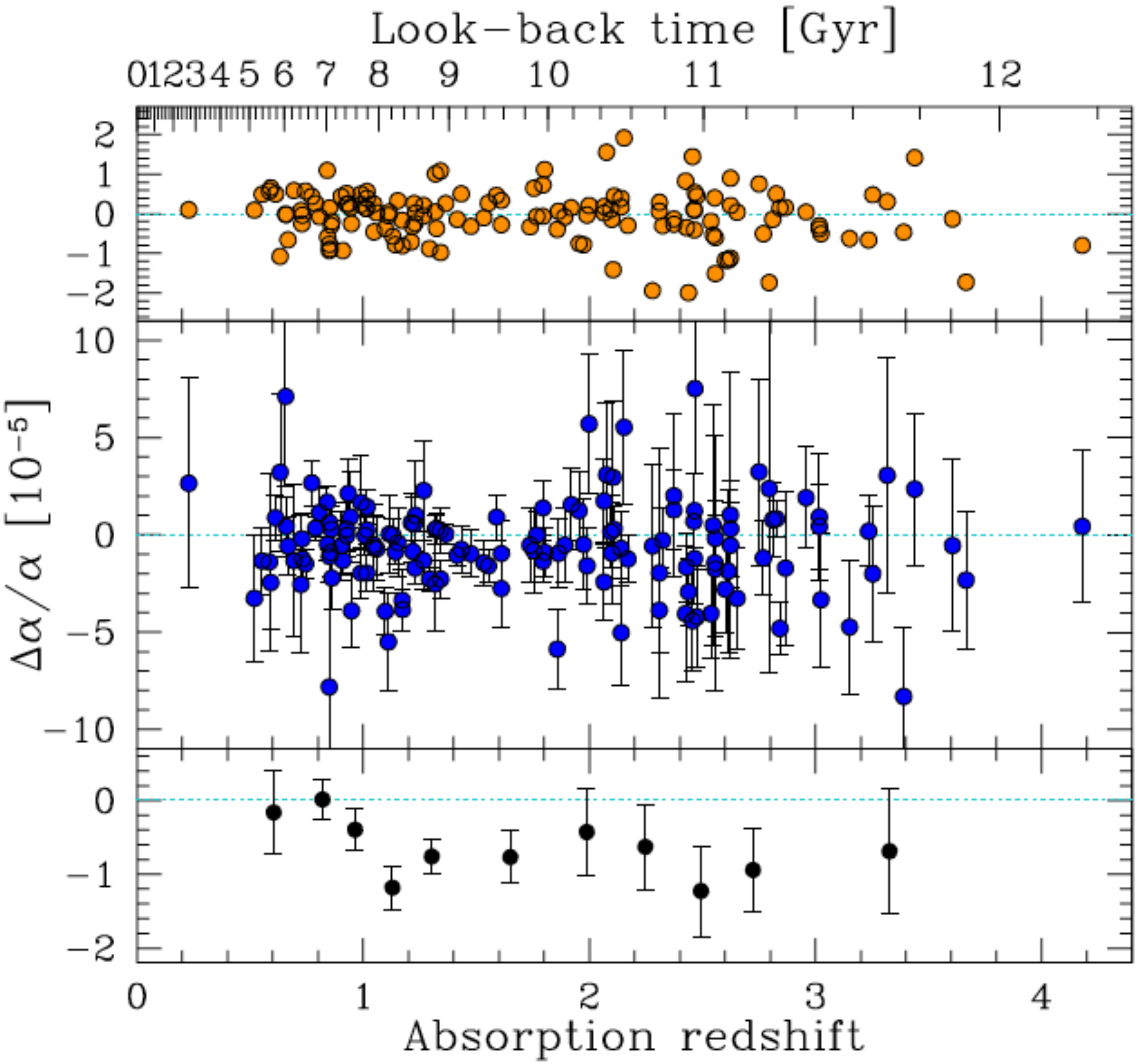
$$p_{\rm b} \ll \rho_{\rm b}, \Lambda = \frac{1}{a^2\mathcal{R}^2} = 0 \Rightarrow a(t) \propto t^{2/3}, t = \frac{2}{3H} = \frac{1}{\sqrt{6\pi G_{\rm N}\rho}}$$

de Sitter:

$$\rho_{\rm b} = p_{\rm b} = 0 \Rightarrow a(t) = \exp\left(H_\Lambda t\right), \text{ where } H_\Lambda = \sqrt{\frac{\Lambda}{3}}$$

The general solution is for an universe expanding under the influence of both matter (including radiation) and a cosmological constant

We can check *experimentally* that physical 'constants' such as α have been sensibly constant for the past ~12 billion years …



Webb *et al*, PRL 82:884,1999, arXiv:0911.4512

So we are entitled to extrapolate known physical laws back in time with confidence

Knowing the equation of state, we can solve the Friedman equation ...

For matter: $\frac{d}{dt}(\rho a^3) = 0 \quad \Rightarrow \quad \rho = \rho_0/a^3 = \rho_0(1+z)^3$

Hence $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_0}{3a^3} \quad \Rightarrow \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$

For radiation: $\frac{d}{dt}(\rho a^4) = 0 \quad \Rightarrow \quad \rho = \rho_0/a^4 = \rho_0(1+z)^4$

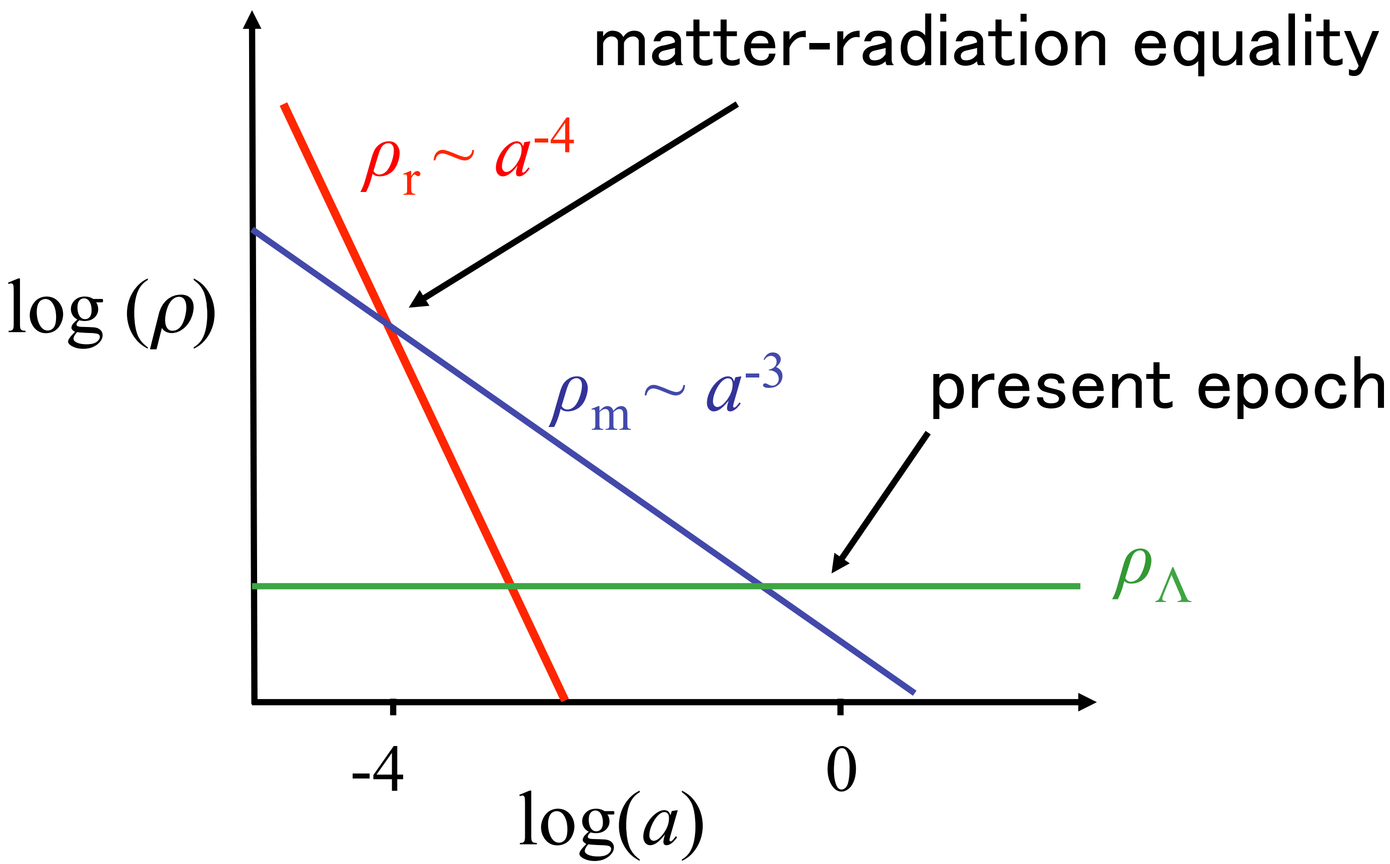
So radiation will dominate over other components as we go to early times

$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \Rightarrow \rho_\mathrm{r} \propto t^{-2}$ Radiation-dominated era

But at $a_\mathrm{eq} = \rho_{\mathrm{r},0}/\rho_{\mathrm{m},0}$ the matter density will come to dominate

Note that $\rho_\mathrm{m} \propto t^{-2}$ during the **Matter-dominated era as well**

Evolution of different energy components



The early universe was therefore radiation-dominated

Very recently (at $z \sim 1$), the expansion has supposedly become dominated by a 'cosmological constant': $\Lambda \sim 2\, H_0^2 \Rightarrow \rho_\Lambda \sim 2\, H_0^2\, M_P^2$

(This creates a severe **'why now?'** problem as $\rho_\Lambda \ll \rho_{m,\,r}$ at earlier epochs)

On the basis of known physics, the evolution of the universe can be extrapolated into our past, quite reliably up to the nucleosyntheis era and (with some caveats) back through the QCD phase transition up to the electroweak unification scale

New physics is required to account for the observed asymmetry between matter and antimatter, to explain dark matter, and also generate the density fluctuations which seeded the formation of structure