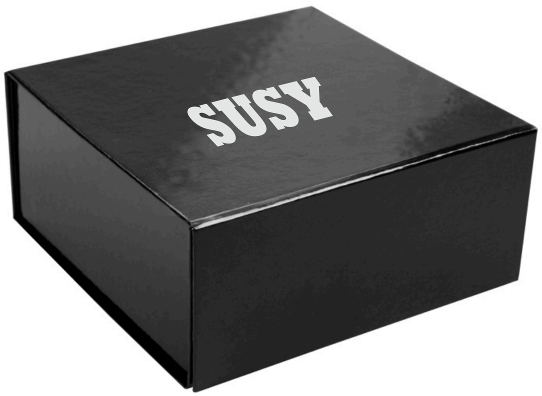
SUSY in a Black Box



Can understand the rest of the lectures treating the superfield formalism as a black box.

Chiral superfields: $\Phi = (\phi, \psi_\alpha, F)$

$U(1)$ gauge superfields: $V = (\lambda_\alpha, A_\mu, D)$ (WZ gauge)

$$\int d^2\theta\, W(\Phi) = \frac{\partial W(\phi)}{\partial \phi^a} F_a + \frac{\partial^2 W(\phi)}{\partial \phi^a \phi^b} \psi^a \psi^b$$

$$\int d^4\theta\, \Phi^\dagger e^{2gV} \Phi = D^\mu \phi^\dagger D_\mu \phi + \psi^\dagger i\bar{\sigma}^\mu D_\mu \psi + F^\dagger F + \sqrt{2} g (\phi^\dagger T_A \lambda_A \psi + \text{h.c.}) + D_A \phi^\dagger T_A \phi$$

$$W_\alpha = -\tfrac{1}{4} \bar{D}^2 D_\alpha V$$

$$\int d^2\theta\, \tfrac{1}{4} W^\alpha W_\alpha = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} + \lambda^\dagger i\bar{\sigma}^\mu D_\mu \lambda + \tfrac{1}{2} D^2$$

Supersymmetry Breaking

If SUSY exists in nature, it must be broken.

SUSY is a spacetime symmetry, so it is gauged when we include gravity (supergravity).

Because of this, SUSY must be broken spontaneously.

This means we must look for a SUSY invariant Lagrangian whose ground state breaks SUSY.

From the SUSY algebra, we know that SUSY is broken if and only if the vacuum energy is nonzero.

$$\not{\text{SUSY}} \iff \langle 0 | H | 0 \rangle > 0$$

Classical potential:

$$V = F_a^\dagger F_a + \tfrac{1}{2} D_A D_A$$

$$F_a^\dagger = \frac{\partial W(\phi)}{\partial \phi^a} \qquad D_A = \frac{g_A^2}{2} \phi_a^\dagger (T_A)^a{}_b \phi^a$$

$T_A =$ gauge generators/charges

g_A depends on A for product gauge group

We see that SUSY breaking requires nonzero VEVs for some of the axiliary fields F^a and D_A.

Polonyi Model:

Simplest model of SUSY breaking.

$$\mathcal{L} = \int d^4\theta \, \Phi^\dagger \Phi + \left(\int d^2\theta \, \kappa \Phi + \text{h.c.} \right)$$

$$F^\dagger = \frac{\partial W}{\partial \phi} = \kappa \neq 0 \qquad \text{“}F\text{-type breaking”}$$

Nonzero vacuum energy, but is SUSY really broken?

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + \psi^\dagger i \bar{\sigma}^\mu \partial_\mu \psi$$

$= $ free field theory with SUSY spectrum

$\langle \phi \rangle =$ undetermined...

Can turn this into a theory with real SUSY breaking by generalizing Kahler potential:

$$\mathcal{L} = \int d^4\theta \left[\Phi^\dagger \Phi - \frac{1}{4M^2} (\Phi^\dagger \Phi)^2 \right] + \left(\int d^2\theta \, \kappa \Phi + \text{h.c.} \right)$$

Potential terms (no spacetime derivatives):

$$\mathcal{L} = F^\dagger F - \frac{1}{M^2} \phi^\dagger \phi F^\dagger F + (\kappa F + \text{h.c.})$$

Integrate out $F \Rightarrow$

$$V = \frac{|\kappa|^2}{1 - \phi^\dagger \phi / M^2}$$

Minimizing potential fixes $\langle \phi \rangle = 0$

Can show:

$$m_\phi^2 = \frac{|\kappa|^2}{M^2} \qquad m_\psi = 0$$

Massless fermion is the Goldstino = Goldstone fermion

The Goldstino

When a global symmetry is spontaneously broken, the theory contains a massless Nambu-Goldstone excitation with the quantum numbers of the broken symmetry generator. So it is not surprising that spontaneously broken SUSY leads to a massless neutral Weyl fermion, the Goldstino.

$$Q_\alpha \text{ broken} \Rightarrow \text{ massless Goldstino } \chi_\alpha$$

This can be proven in great generality.

We will content ourselves with showing it in the simplest possible case of a theory with only chiral superfields.

$$V = \left(\frac{\partial W}{\partial \phi^a}\right)^\dagger \frac{\partial W}{\partial \phi^a}$$

$$0 = \frac{\partial V}{\partial \phi^b} = \left(\frac{\partial W}{\partial \phi^a}\right)^\dagger \frac{\partial^2 W}{\partial \phi^a \partial \phi^b}$$

$$\cancel{\text{SUSY}} \Rightarrow \left\langle \frac{\partial W}{\partial \phi^a} \right\rangle \neq 0$$

$$\Rightarrow \underbrace{\left\langle \frac{\partial^2 W}{\partial \phi^a \partial \phi^b} \right\rangle}_{\text{fermion mass matrix}} \text{ has a zero eigenvalue}$$

When a gauge theory is spontaneously broken, the Nambu-Goldstone excitation becomes the longitudinal polarization of a massive gauge particle. So it is not surprising that in supergravity the Goldstino becomes the longitudinal mode of the gravitino, the spin $\frac{3}{2}$ superpartner of the graviton.

$$\text{massless gravity multiplet} = \text{massless spin } 2 + \text{massless spin } \tfrac{3}{2}$$

$$m_{3/2} = \text{gravitino mass} \sim \frac{\langle F \rangle}{M_{\text{Pl}}} \qquad \langle F \rangle = \cancel{\text{SUSY}} \text{ VEV}$$

Note that for $\langle F \rangle \ll M_{\text{Pl}}^2$ the gravitino is a light particle with possible consequences for phenomenology and cosmology.

If SUSY solves the naturalness problem of the Standard Model, then the superpartners of the observed particles must have masses at the TeV scale.*

It would be very exciting if SUSY were spontaneously broken at the TeV scale by a “super Higgs” sector whose particles have masses at the TeV scale. Unfortunately, this idea does not work.

The reason is that renormalizable SUSY theories do not contain a Yukawa coupling of the form

$$\phi\lambda\lambda \qquad \begin{aligned} \phi &= \text{scalar from chiral multiplet} \\ \lambda &= \text{gaugino} \end{aligned}$$

This means there is no way to generate a mass for the gauginos at tree level.

* We’ll discuss some fine print later.

Some possibilities:

- Strong SUSY breaking at the TeV scale
- Superpartner masses from loop effects
  e.g. gauge mediation or anomaly mediation
- Hidden sector SUSY breaking

Hidden Sector SUSY Breaking

Assume SUSY is broken in a hidden sector that is coupled via higher-dimension operators to the visible sector, which contains the Standard Model fields. These higher-dimension operators can arise from new physics at high scales, for example new heavy particles or string theory.

Assume that the hidden sector breaks SUSY through the F term of a chiral superfield X:

$$\langle F_X \rangle \neq 0 \qquad X = \text{hidden sector chiral superfield}$$

Assume $\langle X \rangle = 0$ without loss of generality (shift field if necessary).

$$\langle X \rangle = F_X \theta^2 \qquad F_X \neq 0$$

Now consider higher-dimension couplings between X and the visible sector fields.

Explore this with a toy hidden sector consisting of a single chiral superfield Φ and a gauge superfield V, with

$$W(\Phi) = \frac{\lambda}{3}\Phi^3$$

Possible couplings:

$$\int d^2\theta\, \frac{X}{M} W^\alpha W_\alpha + \text{h.c.} = \frac{F_X}{M}\lambda\lambda + \text{h.c.} \qquad \Rightarrow m_\lambda \sim \frac{F_X}{M}$$

$$\int d^4\theta\, \frac{1}{M^2} X^\dagger X \Phi^\dagger \Phi = \left|\frac{F_X}{M}\right|^2 \phi^\dagger \phi \qquad \Rightarrow\ m_\phi^2 \sim \left(\frac{F_X}{M}\right)^2$$

$\Rightarrow$ gaugino and scalar masses $\sim F_X/M$

But wait, there's more!

$$\int d^2\theta\, \frac{X}{M}\Phi^3 + \text{h.c.} = \frac{F_X}{M}\phi^3 + \text{h.c.} \qquad \Rightarrow \text{cubic scalar couplings}$$

$$\int d^4\theta \left(\frac{X}{M} + \text{h.c.}\right)\Phi^\dagger\Phi = \frac{F_X}{M} F_\phi^\dagger \phi + \text{h.c.}$$

To see what this does, integrate out F_ϕ:

$$\begin{aligned} \mathcal{L} &= F_\phi^\dagger F_\phi + \left[F_\phi \lambda \phi^2 + \text{h.c.}\right] \\ &= \left|F_\phi^\dagger + \lambda\phi^2 + \frac{F_X^\dagger}{M}\right|^2 - \left|\lambda\phi^2 + \frac{F_X^\dagger}{M}\phi^\dagger\right|^2 \end{aligned}$$

$$\Rightarrow\ \Delta V_F = \left|\frac{F_X}{M}\right|^2 \phi^\dagger\phi + \left(\frac{F_X}{M}\lambda\phi^3 + \text{h.c.}\right)$$

Note: all SUSY breaking mass parameters $\sim F_X/M$!

Higher order terms give smaller effects...

Summary: Hidden sector SUSY breaking models can naturally give gaugino masses, scalar masses, and cubic scalar couplings all of the same order of magnitude.

$$\frac{F_X}{M} \sim \text{TeV}$$

Note that if the particles in the hidden sector have masses large compared to the TeV scale, then the only observable effect of the hidden sector is the VEV F_X.

This looks just like explicit breaking...

Soft SUSY Breaking

A more phenomenological approach: break SUSY explicitly, with all mass parameters chosen to be $\sim$ TeV.

We must ensure that this explicit breaking does not give rise to quadratic sensitivity to UV physics. That is, the breaking must be *soft*.

To explore this, consider again our toy visible sector, but now allow the most general renormalizable superpotential:

$$\begin{aligned}\mathcal{L} = \int d^2\theta\, S W^\alpha W_\alpha + \text{h.c.} \\ + \int d^4\theta\, Z \Phi^\dagger \Phi \\ + \int d^2\theta \left[\kappa\Phi + \tfrac{1}{2}\mu\Phi^2 + \tfrac{1}{3}\lambda\Phi^3\right] + \text{h.c.}\end{aligned}$$

Note we have written the couplings as superfields and included an arbitrary normalization for the chiral superfield kinetic term:

$$S = \frac{1}{2g^2} + \cdots = \text{chiral superfield}$$

$$Z = 1 + \cdots = \text{real superfield}$$

$$\kappa, \mu, \lambda = \text{chiral superfield}$$

The SUSY non-renormalization theorem guarantees that this theory has only logarithmic renormalization of Z and S.

Logarithmic UV divergences $\Leftrightarrow$ logarithmic sensitivity to UV physics. We want to preserve this feature wen we include SUSY breaking.

A simple way to do this is to break SUSY by turning on θ^2 and $\theta^2\bar{\theta}^2$ components of the coupling superfields.

$$Z \to 1 + (\theta^2 B + \text{h.c.}) + \theta^2\bar{\theta}^2(-m_0^2 + |B|^2)$$

$$S \to \frac{1}{2g^2} - \theta^2 \frac{m_{1/2}}{g^2}$$

$$\kappa \to \hat{\kappa}\left(1 + \theta^2 B_\kappa\right)$$

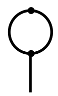
$$\mu \to \hat{\mu}\left(1 + \theta^2 B_\mu\right)$$

$$\lambda \to \hat{\lambda}\left(1 + \theta^2 B_\mu\right)$$

The UV divergent terms in the effective action can be written in terms of superfields, and the fact that the higher components of the superfield couplings are nonvanishing does not change the log divergent terms. For example,

$$\Delta\mathcal{L}_{1PI} \sim \int d^4\theta \,(Z\lambda^\dagger\lambda \times \ln\Lambda)\Phi^\dagger\Phi$$

There is one subtle point in this argument: there is an additional allowed UV divergence:



$$\Rightarrow \quad \Delta\mathcal{L}_{1PI} \sim \int d^4\theta \,(\lambda\mu^\dagger \times \ln\Lambda)\Phi + \text{h.c.}$$

This term is a total derivative in the SUSY limit, but a contribution to the linear term in the scalar potential when SUSY breaking is turned on.

Note: allowed only if Φ is a gauge singlet...

We can write an arbitrary breaking term using superfield spurions, but all other terms are higher-dimension operators in superspace, and therefore give rise to power divergences.

For example, a non-holomorphic cubic coupling:

$$\Delta\mathcal{L} = \int d^4\theta\, X\Phi^\dagger\Phi^2 + \text{h.c.} \qquad X = \theta^2\bar{\theta}^2 h \qquad [h] = 1$$

$$= h\phi^\dagger\phi^2 + \text{h.c.}$$

Even though the coupling h has positive mass dimension, the breaking is not soft: there is a counterterm

$$\Delta\mathcal{L}_{1PI} \sim \int d^4\theta\, \Lambda^2 X\Phi + \text{h.c.} \sim \Lambda^2 h\phi + \text{h.c.}$$

Summarize: softly broken SUSY is equivalent to turning on higher components of superfield couplings.

These are also the effects that we expect if SUSY is broken in a hidden sector.

This is a good starting point for a phenomenological treatment of models with broken SUSY.

The Minimal Supersymmetric Standard Model

MSSM

MAINE SCHOOL OF SCIENCE AND MATHEMATICS

An excellent reference with many more details:

S. Martin, "A Supersymmetry Primer," `arXiv:9709356 (v6)`.

But note: uses $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$...

Start by writing the Standard Model in terms of L Weyl fermions:

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	
q	3	2	$\frac{1}{6}$	×3
u^c	$\bar{3}$	1	$-\frac{2}{3}$	×3
d^c	$\bar{3}$	1	$\frac{1}{3}$	×3
ℓ	1	2	$-\frac{1}{2}$	×3
e^c	1	1	1	×3
h	1	2	$\frac{1}{2}$	

Notation:

$$q_\alpha = q_{L\alpha} \qquad (u^c)_\alpha = \epsilon_{\alpha\beta} u_R^{\dagger\alpha} = \epsilon_{\alpha\beta} (u_R^{\dot{\alpha}})^\dagger$$

To embed this in a SUSY model, each fermion multiplet must be part of a chiral superfield. This means that we are introducing a complex scalar superpartner for each Weyl fermion.

Denote the superfield by a capital letter and the scalar partners with a tilde, *e.g.*

$$Q(x, \theta, \bar{\theta}) = \tilde{q}(x) + \sqrt{2}\theta^\alpha q_\alpha(x) + \cdots$$

It is traditional to name the scalar superpartners of fermions by adding an "s" in front of the name, *e.g.*

quark ⟷ squark

lepton ⟷ slepton

top ⟷ stop

The $SU(3) \times SU(2) \times U(1)$ gauge fields of the Standard Model must embedded in gauge superfields. This means that we are introducing a Weyl fermion superpartner for each gauge boson.

It is traditional to name the fermion superpartners of gauge by adding an "ino" to the end of the name, *e.g.*

gauge boson ⟷ gaugino

gluon ⟷ gluino

W boson ⟷ wino

photon ⟷ photino

What about the Higgs doublet?

It has the same quantum numbers as a left-handed slepton* if we complex conjugate it. We could therefore think about identifying it with a linear combination of left-handed sleptons.

But this is a really bad idea, since the Higgs VEV would then break lepton number.

We therefore embed the Higgs in its own chiral superfield.

$$H = h + \sqrt{2}\theta^\alpha \tilde{h}_\alpha + \cdots$$

* the scalar partner of ℓ

Now let us write the most general allowed couplings of these fields in the SUSY limit.

The gauge boson (and gaugino) couplings are dictated by gauge invariance and SUSY.

The most general superpotential has the form

$$W = QHU^c + U^c D^c D^c + QLD^c + LLE^c + LH$$

• The last 4 terms violate baryon number, lepton number, or both.

• There is no Yukawa coupling of the Higgs to d^c or e^c.

Note that holomorphy of the superpotential does not allow us to write a term $QH^\dagger D^c$ or $LH^\dagger E^c$.

We could choose to embed the Higgs scalar doublet in a chiral superfield $\tilde{H}$ with hypercharge $-\frac{1}{2}$. Then we could write

$$W = Q\tilde{H}D^c + L\tilde{H}E^c + U^c D^c D^c + QLD^c + LLE^c + LH$$

Now we don't have Yukawa couplings of the Higgs to u^c.

To have Yukawa couplings to all standard model fermions, we need two Higgs superfields, traditionally called H_u and H_d instead of H and $\tilde{H}$.

This is the (super)field content of the minimal supersymmetric standard model, the MSSM.

The allowed superpotential terms are now

$$W = QH_uU^c + QH_dD^c + LH_dE^c + H_uH_d + U^cD^cD^c + QLD^c + LLE^c + LH$$

This gives us the MSSM wallet card:

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	
Q	3	2	$\frac{1}{6}$	×3
U^c	$\bar{3}$	1	$-\frac{2}{3}$	×3
D^c	$\bar{3}$	1	$\frac{1}{3}$	×3
L	1	2	$-\frac{1}{2}$	×3
E^c	1	1	1	×3
H_u	1	2	$\frac{1}{2}$	
H_d	1	2	$-\frac{1}{2}$	

We will assume for now that the terms that violate baryon/lepton number are absent.

(For reasons we will explain later, these are commonly referred to as "R-parity violating terms.")

As we will see, this has far-reaching implications, and we will revisit it later.

With this assumption, the superpotential is given by

$$W = (y_u)_{ij} Q^i H_u U^{cj} + (y_d)_{ij} Q^i H_d D^{cj} + (y_e)_{ij} L^i H_d E^{cj} + \mu H_u H_d$$

$i, j = 1, 2, 3 =$ generation index

How "minimal" is the MSSM?

Although the number of degrees of freedom is (slightly more than) double that of the Standard Model, the extra degrees of freedom are related by a symmetry.

We need an additional Higgs superfield, and we need to suppress the interactions that violate baryon/lepton number.

Soft SUSY Breaking

We take a phenomenological approach to breaking SUSY in the MSSM, and break SUSY softly.

- Gaugino mass terms

$$\Delta\mathcal{L} = -M_1 \tilde{B}\tilde{B} - M_2 \tilde{W}_a \tilde{W}_a - M_3 \tilde{G}_A \tilde{G}_A + \text{h.c.}$$

$\tilde{B} =$ bino $\quad a = 1, 2, 3 = SU(2)_W$ adjoint index

$\tilde{W}_a =$ wino

$\tilde{G}_A =$ gluino $\quad A = 1, \ldots, 8 = SU(3)_C$ adjoint index

- Non-holomorphic scalar mass terms

$$\Delta V = (m^2_{\tilde{q}})^i{}_j \, \tilde{q}^\dagger_i \tilde{q}^j + (m^2_{\tilde{u}^c})^i{}_j \, \tilde{u}^{c\dagger}_i \tilde{u}^{cj} + (m^2_{\tilde{d}^c})^i{}_j \, \tilde{d}^{c\dagger}_i \tilde{d}^{cj} + (m^2_{\tilde{\ell}})^i{}_j \, \tilde{\ell}^\dagger_i \tilde{\ell}^j + (m^2_{\tilde{e}^c})^i{}_j \, \tilde{e}^{c\dagger}_i \tilde{e}^{cj} + m^2_{H_u} H^\dagger_u H_u + m^2_{H_d} H^\dagger_d H_d$$

Note I am using $H_{u,d}$ for scalar fields instead of $h_{u,d}$...

Important: scalar masses depend on flavor in general.

- Holomorphic cubic scalar couplings ("A terms")

$$\Delta V = (A_u)_{ij} \tilde{q}^i H_u \tilde{u}^{cj} + (A_d)_{ij} \tilde{q}^i H_d \tilde{d}^{cj} (A_e)_{ij} \tilde{\ell}^i H_d \tilde{e}^{cj} + \text{h.c.}$$

More flavor dependence...

- Holomorphic mass term ("$B\mu$ term")

$$\boxed{\Delta V = B H_u H_d + \text{h.c.}}$$

$$H_u H_d = \epsilon_{ab} H_u^a H_d^b$$

$a, b = 1, 2 = SU(2)_W$ fundamental index

$$\epsilon_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

There are other more exotic soft terms that do not arise from higher components of superfield couplings. These are unlikely to arise from complete models of SUSY breaking, so we will neglect them.

In any case, we have plenty to deal with already...