Axions: Past, Present and Future

ICTP Summer School, 2015

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Axions

Axions and axion-like-particles are the goldstone bosons of symmetries broken at some high scale f_a

These particles acquire a mass because the broken global symmetry is not exact - when the symmetry is broken by QCD, the particle is called an axion (***a***). If it is due to another source, we call it an axion-like-particle ($\boldsymbol{\phi}$)

In the last class, we were talking about the effects of surface terms on non-perturbative phenomena in quantum mechanics and gauge theories. The connection to axions will become apparent in this lecture

$$\mathcal{L} \supset -G_{\mu\nu}G^{\mu\nu} + \theta G_{\mu\nu}\tilde{G}^{\mu\nu}$$

We found that for pure QCD (without any fermions), the θ term is physical and contributes to the vacuum energy of the theory. Of course, in our world, we have fermions - what does θ term do?

Instantons and Fermions

Let us consider QCD and add a massless quark (Dirac) to the theory

$$\mathcal{L} \supset -G_{\mu\nu}G^{\mu\nu} + \theta G_{\mu\nu}\tilde{G}^{\mu\nu} + \bar{Q}\gamma^{\mu}D_{\mu}Q$$

Naively, we would think that this should not do anything to the instanton effects. After all, the instanton solutions will still exist - they were simply maps from the boundary S^3 of space-time to the gauge group SU(3). The map simply picks a pure gauge configuration of SU(3). This still exists in the presence of fermions

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However, note that a massless fermion brings with it an additional symmetry - in addition to the usual vector current, classically there is a chiral rotation symmetry as well

$$Q \to e^{i\gamma_5 \alpha} Q \implies \delta L = 0$$

Is this a symmetry of the quantum theory as well?

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Classically, massless quarks have a chiral symmetry

$$Q \to e^{i\gamma_5 \alpha} Q \implies \delta L = 0$$

Famously, when the quarks are coupled to a gauge theory, this symmetry is anomalous. There is no way to pick a regulator that preserves this chiral symmetry while also preserving the gauge symmetry

The anomaly contributes to the divergence of the chiral current

$$\partial_{\mu} J^{\mu 5} = \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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Hence, if we do a chiral rotation on the massless quark by an angle α that will shift the θ term by θ -> θ + α. For a massless quark, we are free to pick α to be whatever we want it to be - and hence all the effects of θ must disappear!

But, how is that possible? We just argued that the instanton solutions do not care about the existence of fermions!

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For a massless quark, we can do a chiral rotation to eliminate all the effects of θ. How is this possible?

$$E\left(\theta\right) \sim K\cos\theta e^{-S_0}$$

The point is that the pre-factor K is some functional determinant. One can show that this operator has a zero eigenvalue when there are massless quarks (see Coleman for derivation). When there is a zero eigenvalue, the determinant is zero and the pre-factor vanishes

So when we add quarks to the theory, even though we don't affect the instanton solutions, we do change their contribution to physical parameters through the pre-factor. Importantly, this pre-factor vanishes when the quark masses go to zero as we had argued by our ability to pick an appropriate chiral rotation

Instantons in Real QCD

Of course, in real QCD, the quarks have a small mass

$$\mathcal{L} \supset -G_{\mu\nu}G^{\mu\nu} + \theta G_{\mu\nu}\tilde{G}^{\mu\nu} + \bar{Q}\gamma^{\mu}D_{\mu}Q + m_Q\bar{Q}Q$$

The mass term for the quarks breaks the chiral symmetry explicitly.
The divergence of the chiral current now reads:

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Using these chiral rotations, we can move the phases in the quark mass matrix to θ and vice-versa. What ultimately matters is the linear combination:

$$\bar{\theta} = \theta + \arg\left(\det\left(m_Q\right)\right)$$

This term is physical and cannot be removed away by any chiral rotation!

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$$\bar{\theta} = \theta + \arg\left(\det\left(m_Q\right)\right)$$

This term is physical - what effects does it have?

Term breaks CP. G and $\tilde{G}$ transform oppositely under CP.

This means there must be CP violation in QCD. One can show that this leads to an electric dipole moment for particles like the proton and the neutron

$$d_n \approx \bar{\theta} \times 10^{-16} \text{ e-cm}$$

$$\text{Experimentally } \bar{\theta} \lessapprox 10^{-9}$$

The quark masses and the bare θ term come from completely different sources - why should they cancel so exactly?

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This is the strong CP problem - the QCD axion (*a*) was invented to solve this!

Axion Solution to the Strong CP Problem

Assume that there is some global symmetry somewhere (called the Peccei-Quinn or PQ symmetry)

Suppose this symmetry is spontaneously broken at some scale f_a

As a result of this spontaneous symmetry breaking, there should be a goldstone boson (axion) that should be exactly massless

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But, assume that this global symmetry actually has a mixed anomaly under QCD. This is actually a generic expectation - one has to usually pick charges carefully (anomaly cancellation) in order to get an anomaly free theory. So a generic assignment of global charges will in general produce mixed anomalies

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What does QCD do to the goldstone boson (axion) of this anomalous symmetry? What does the axion do to QCD?

Axion Solution to the Strong CP Problem

Global PQ symmetry spontaneously broken at scale f_a. Symmetry assumed to have a mixed anomaly under QCD

At low energies, there is a goldstone boson (axion ***a***) that is a remnant of the original global symmetry.

How does ***a*** couple?

$$\mathcal{L} \supset (\partial_\mu a)^2 + \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

This coupling to QCD is generated by the anomaly - for example, this is evident from the fact that the anomalous PQ current satisfies:

$$\partial_\mu J^\mu_{PQ} = G_{\mu\nu} \tilde{G}^{\mu\nu}$$

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Notice that the axion couples a lot like the θ term

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Let us look at the full Lagrangian including both the axion and QCD

$$\mathcal{L} \supset -G_{\mu\nu}G^{\mu\nu} + (\partial_\mu a)^2 + \left(\bar{\theta} + \frac{a}{f_a}\right) G_{\mu\nu}\tilde{G}^{\mu\nu} + \bar{Q}\gamma^\mu D_\mu Q + m_Q \bar{Q}Q$$

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In particular, the coefficient $\left(\bar{\theta} + \frac{a}{f_a}\right)$ of $G_{\mu\nu}\tilde{G}^{\mu\nu}$ contributes to the energy density

$$E\left(\bar{\theta} + \frac{a}{f_a}\right) \sim K e^{-S_0} \cos\left(\bar{\theta} + \frac{a}{f_a}\right)$$

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$$E\left(\bar{\theta} + \frac{a}{f_a}\right) \sim K e^{-S_0} \cos\left(\bar{\theta} + \frac{a}{f_a}\right)$$

$$\text{Energy density minimized when } \bar{\theta} + \frac{a}{f_a} = 0$$

Axion dynamically cancels θ - its minimum is precisely set by this condition

Axion Solution to the Strong CP Problem

$$E\left(\bar{\theta}+\frac{a}{f_a}\right)\sim Ke^{-S_0}\cos\left(\bar{\theta}+\frac{a}{f_a}\right)$$

QCD instantons thus give a potential to the axion - the minimum of this potential corresponds to a point where the effective θ is zero

Thus, the axion acquires a mass (the leading term in the potential) from QCD while curing QCD of the strong CP problem!

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The fact that the axion, a goldstone boson, gets a mass from QCD is not surprising - the symmetry is anomalous and instantons precisely make those anomalous terms matter

What does the axion potential (or mass) depend upon?

Axion Potential

$$E\left(\bar{\theta}+\frac{a}{f_a}\right) \sim K e^{-S_0} \cos\left(\bar{\theta}+\frac{a}{f_a}\right)$$

The calculation of the axion potential involves calculating this ugly functional determinant. This has been done - but we can make order of magnitude guesses for what it should be

First, this potential must vanish in the limit that the quark masses are zero. This is because in that limit, as we said earlier, the pre-factor K vanishes. So in the limit of a massless quark, the axion has no mass so it cannot solve the strong CP problem. But, in this limit, there is no strong CP problem since all the effects of θ disappear

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The only other dimensionful scale in the problem is Λ_{QCD}

$$\text{After some grunge } V(a) \approx \Lambda_{\mathrm{QCD}}^4 \cos\left(\bar{\theta}+\frac{a}{f_a}\right) \implies m_a \approx \frac{\Lambda_{\mathrm{QCD}^2}}{f_a}$$

Summary of the Story So Far

We wanted to know why axions were introduced first - in other words, why are they so well motivated?

To that end, we observed that the Lagrangian of gauge theories could contain a surface term (θ). While such terms do not contribute to the classical equations of motion, they can nevertheless have very important non-perturbative effects in quantum mechanics

The ground state energy of a quantum system can be calculated using the Euclidean Path integral - this integral is dominated by finite energy solutions of the Euclidean equations of motion. We found that depending upon the geometry of the problem (the pendulum under gravity, QCD), there can be distinct classes of solutions to these equations. These solutions are called instantons

The θ term becomes physical because it tells us how to add different classes of instantons together

Summary of the Story So Far

In QCD, the θ term is also affected by contributions from the quark masses. Together, these contribute to CP violation in QCD that would imply a nucleon electric dipole moment

Experimentally, the effective value of θ must be very small - this raises a question of tuning. Why are two different parameters of the Lagrangian cancelled to such high accuracy? (strong CP problem)

Enter the axion - simply just the Goldstone boson of some anomalous global (PQ) symmetry. This axion couples to QCD the same way as the θ term. Instantons of QCD generate a potential for the axion

This minimum lies exactly at the point where the effective value of θ is completely cancelled - thus solving the strong CP problem. This strongly motivates the axion!

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$$\text{We are thus left with a light boson of mass } \approx \frac{\Lambda_{\rm QCD}^2}{f_a}$$

What does it do? How can we find it?

The Saga of Light Bosons

The axion is a well motivated light boson

Even though it emerges from physics at a high scale f_a, its mass ~ Λ^2/f_a is nevertheless much smaller than f_a since it is a goldstone boson

If we find it, we can peek into some very high scale f_a that is normally not accessible through any foreseeable collider

While the axion is particularly well motivated by the strong CP problem, there may well be other goldstone bosons like the axion out there. Such particles don't receive a mass from QCD - their mass may come from other sources that we don’t know about (and so less constrained). These are called axion-like-particles ($\boldsymbol{\phi}$)

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Given that we know very little about UV physics, good idea to search for such particles

So let us figure out how to search and constrain such light bosonic fields!

Axions and Axion-like-Particles

Global symmetry broken at high scale f_a

Light Goldstone boson

Gauge Fields

Fermions

$$\frac{a}{f_a} G\tilde{G}$$

$$\frac{\phi}{f_\phi} F\tilde{F}$$

$$\frac{\partial_\mu \phi}{f_\phi} \bar{\psi}\gamma^\mu\gamma_5\psi$$

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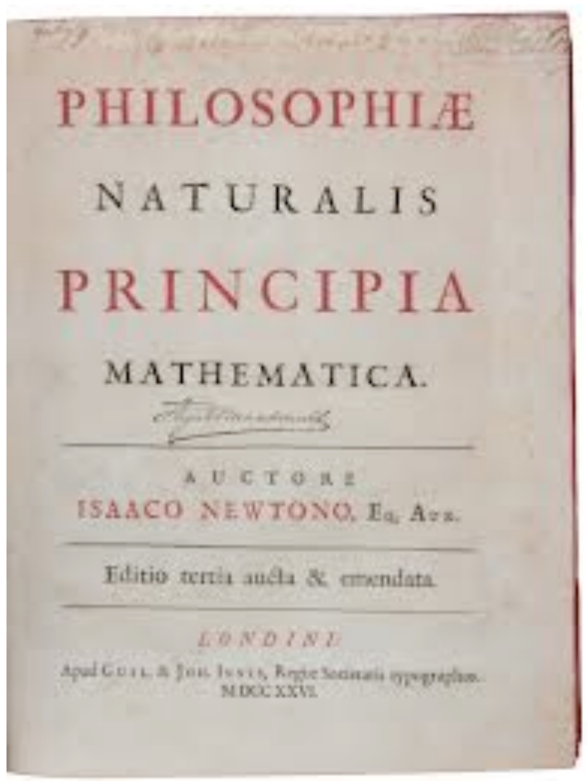
QCD axion

Axion-like Particles

Generic way to couple to UV symmetries

Constraints

Low energy, measured parameters


PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

AUCTORE

ISAACO NEWTONO, Eq. Aur.

Editio tertia aucta & emendata.

LONDINI:

Apud Guil. & Joh. Innys, Regiæ Societatis typographos.

MDCCXXVI.

Hard to evade

Assumptions about unknown physics

May not always hold

Rigorous Constraints

Goldstone boson $\Longrightarrow$ all interactions suppressed by f_a

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Bounds depend upon mass and interactions

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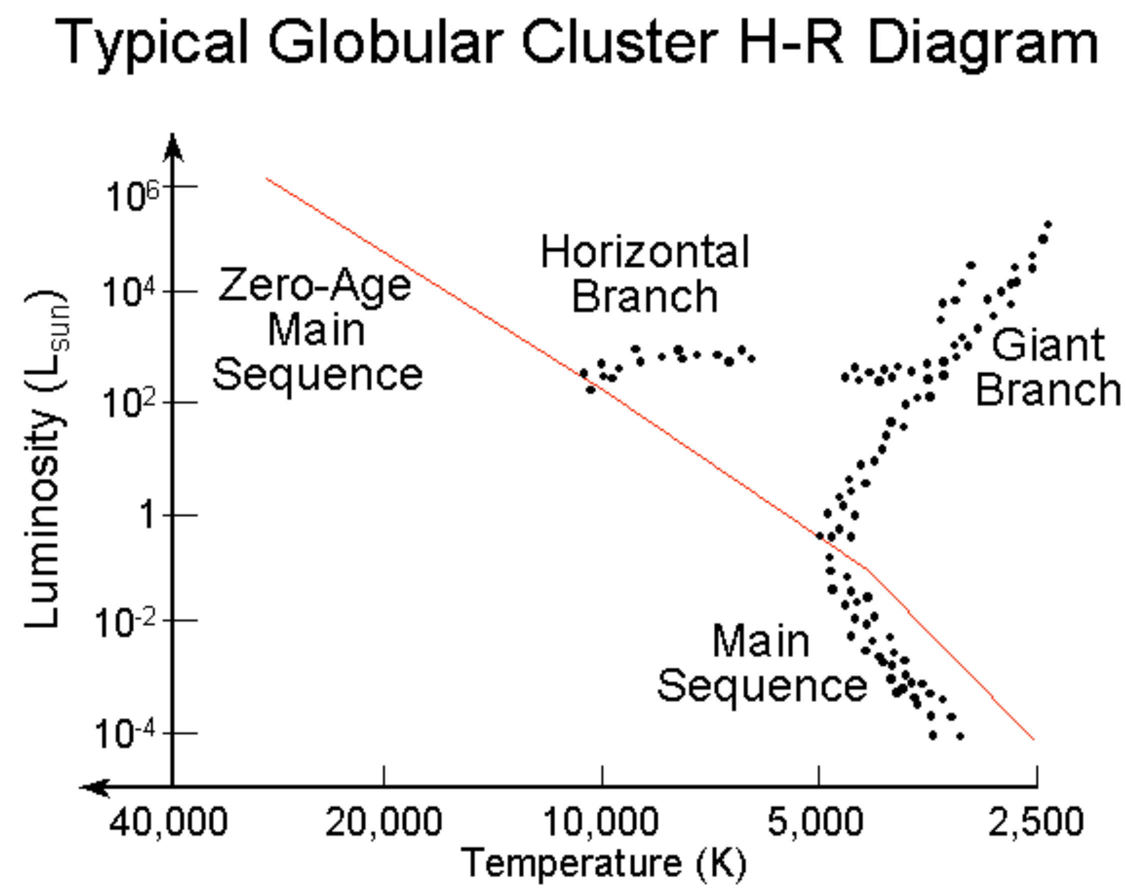
For QCD axion, mass set by QCD. f_a is the only parameter

2D parameter space (m_ϕ, f_ϕ) for axion-like particles

Stellar Constraints

Neutrinos from SN1987A measured by Kamiokande II

Burst duration and number of events



Horizontal Branch stars

Observations of globular clusters

G. Raffelt, PDG

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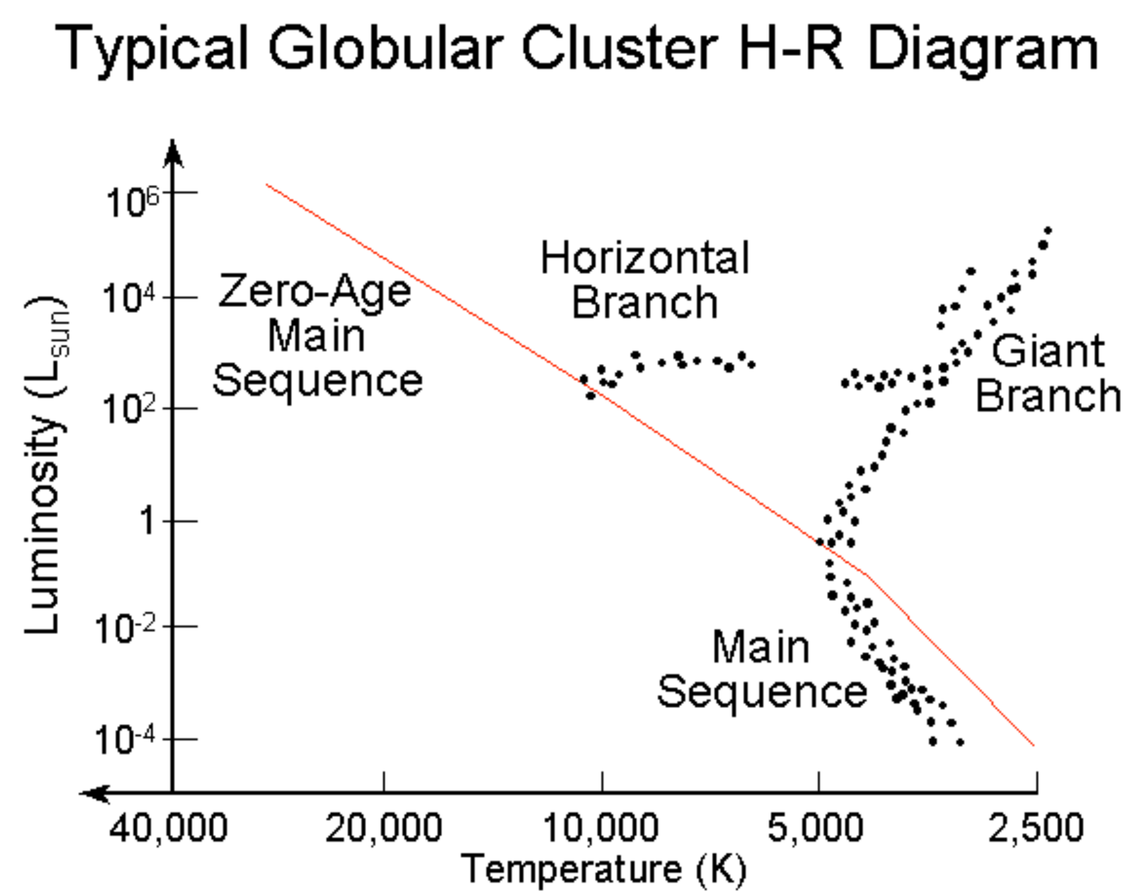

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Observations of globular clusters

Basically rules out axion-like-particles with mass < 60 MeV and f_Φ < 10^9 GeV. Tiny gap around $f_\Phi \sim 10^6$ GeV for hadronic axion

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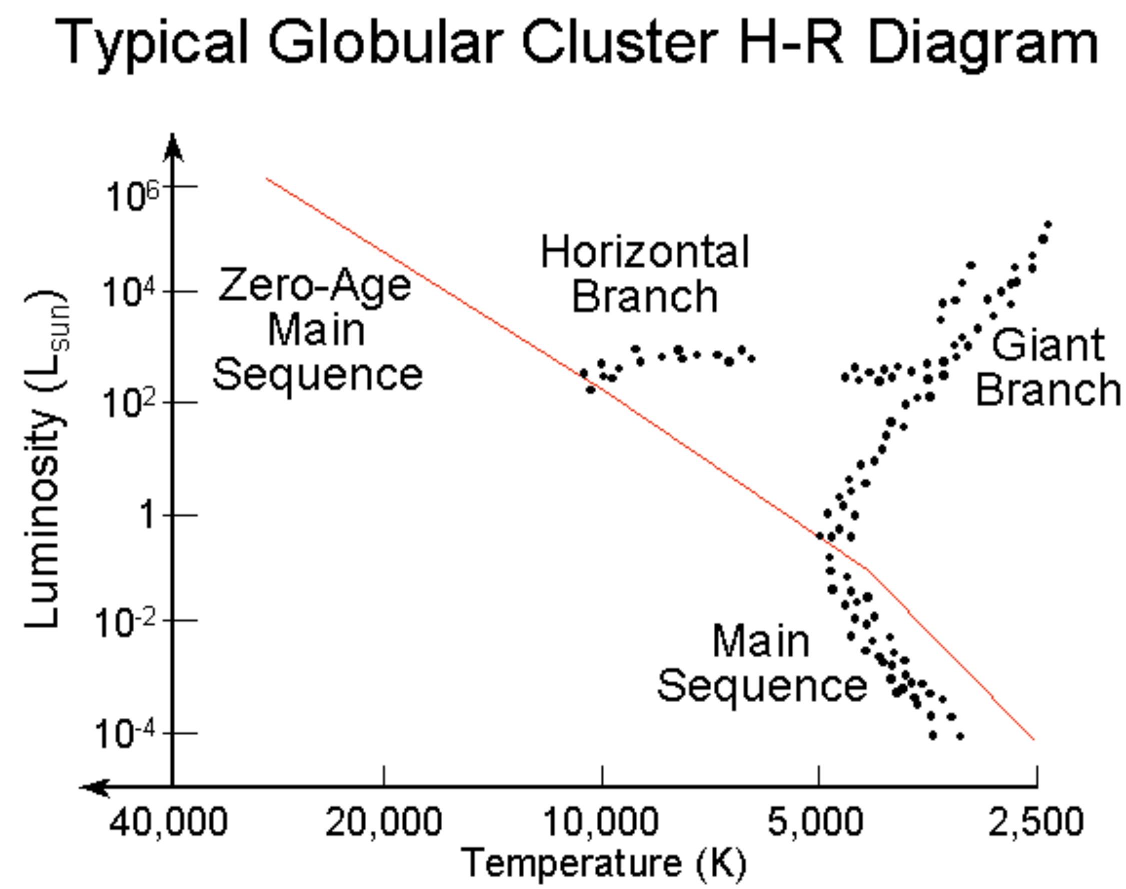

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Slightly stronger constraints on very light axions ($m_\Phi < 10^{-9}$ eV) from absence of gamma rays from SN1987A

G. Raffelt, PDG

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Astrophysical constraints basically rules out axion-like-particles with mass < 60 MeV and $f_\Phi < 10^9$ GeV. Tiny gap around $f_\Phi \sim 10^6$ GeV for hadronic axion

$$\text{For axions } m_a \sim \frac{\Lambda_{\rm QCD}^2}{f_a} \implies m_a \lessapprox 10^{-2}\ \text{eV}$$

So if the QCD axion exists, it is a very light particle!

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We are thus looking at detecting really light particles that have really suppressed interactions

Cosmological Constraints

Not as rigorous

Depend upon assumptions about unknown physics and cosmology

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Bounds arise because axions can be dark matter

Some models/assumptions may not produce the right kind of dark matter

Too much dark matter

Too many fluctuations in dark matter density

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Too many fluctuations in dark matter density

Need to know about the production and evolution of axion dark matter density, extending to very high scales $f_\Phi > 10^9$ GeV

Axion Dark Matter

If axions exist, it is very easy for them to be the dark matter!

Photons

$$\vec{E} = E_0 \cos\left(\omega t - \omega x\right)$$

$$\text{Energy density} \sim E_0^2$$

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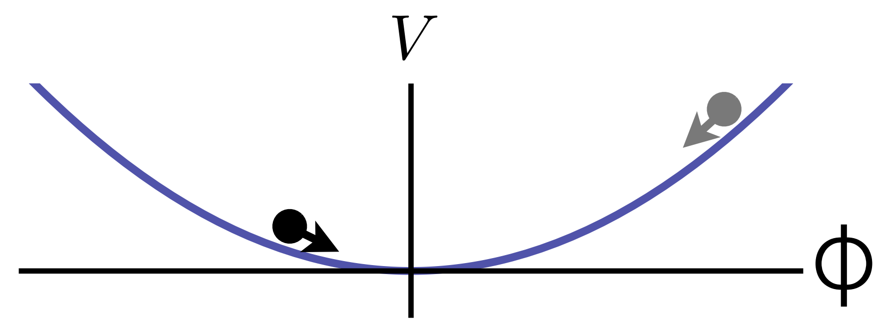
Early Universe:
Misalignment Mechanism



$$\phi\left(t\right) = \phi_0 \cos\left(m_\phi t\right)$$

Spatially uniform, oscillating field

$$m_\phi^2 \phi_0^2 \sim \rho_{DM}$$

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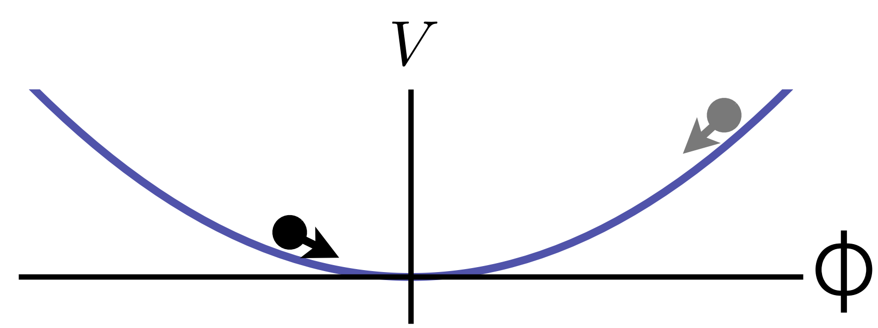

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In general, we expect the field to have a non-zero initial value. This means the field will generically have some cosmological abundance

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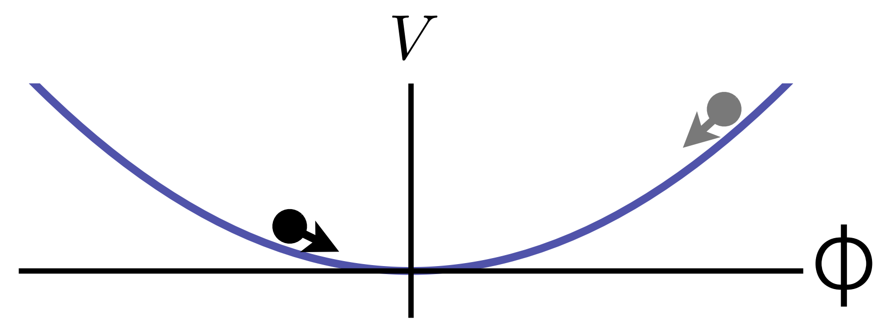
Misalignment Mechanism



$$\phi\left(t\right) = \phi_0 \cos\left(m_\phi t\right)$$

Spatially uniform, oscillating field

$$m_\phi^2 \phi_0^2 \sim \rho_{DM}$$

In general, we expect the field to have a non-zero initial value. This means the field will generically have some cosmological abundance

Key Point: Axions are so weakly coupled that this field doesn't thermalize in the early universe

Axion Dark Matter

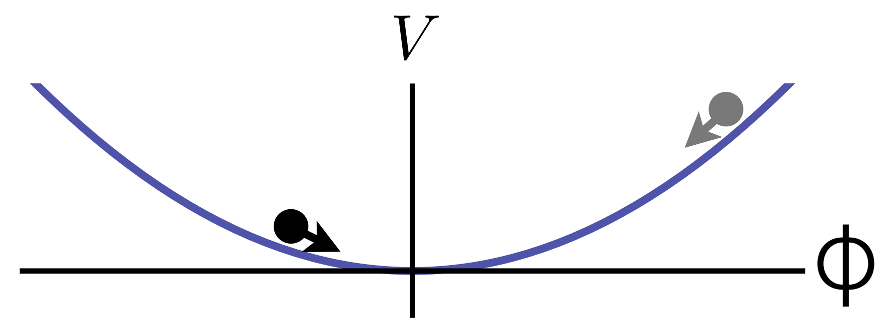
If axions exist, it is very easy for them to be the dark matter!

Photons

$$\vec{E} = E_0 \cos(\omega t - \omega x)$$

$$\text{Energy density} \sim E_0^2$$

Early Universe:
Misalignment Mechanism



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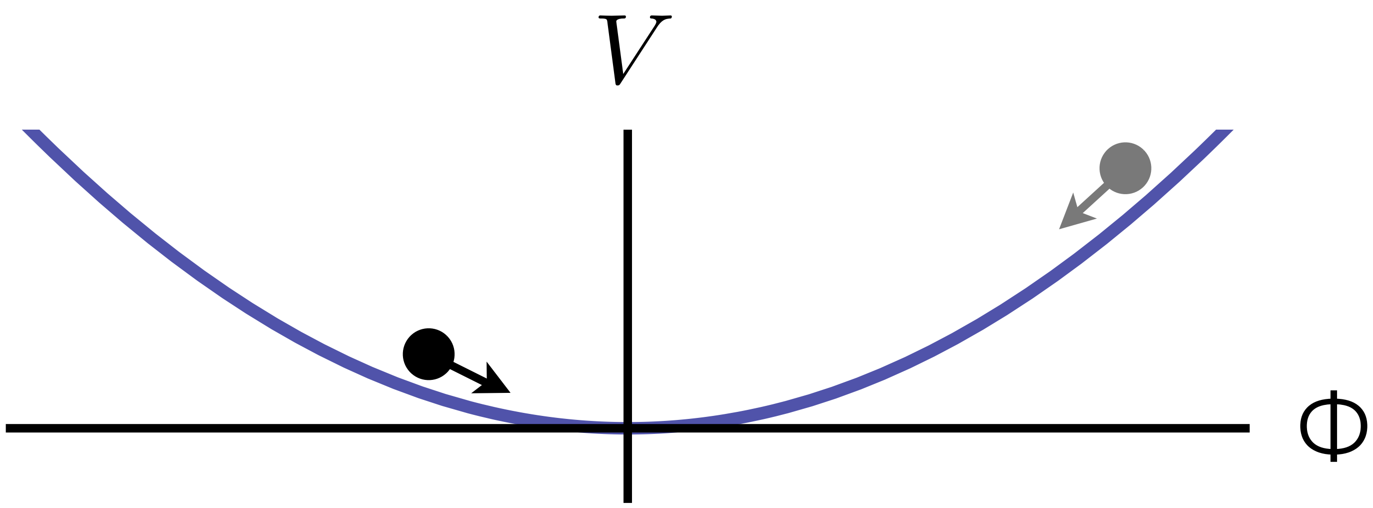
How does this energy density evolve?

Two Tenets of Axion Cosmology



The field has an initial amplitude Φ_0. How does this evolve?

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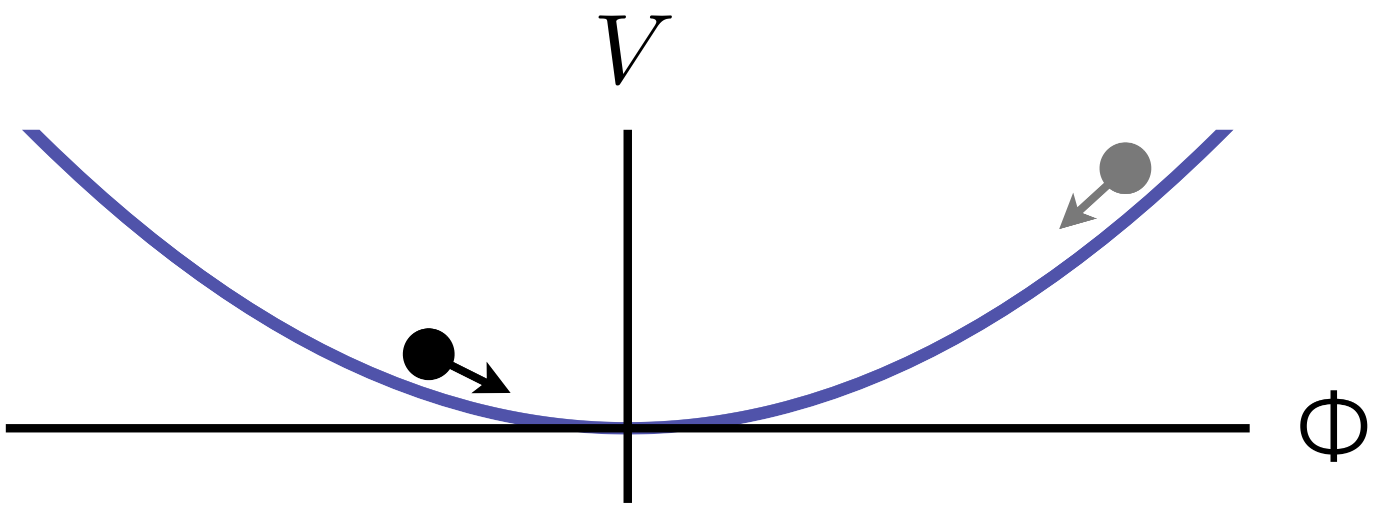

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To understand this evolution, we need the equation of motion for the scalar in an expanding universe

We will only care about the time dependence of Φ. We will assume that the field is spatially homogenous - this is consistent with our assumption that the field just starts with some initial value throughout the universe. Moreover, spatial gradients will rapidly become small as the universe expands

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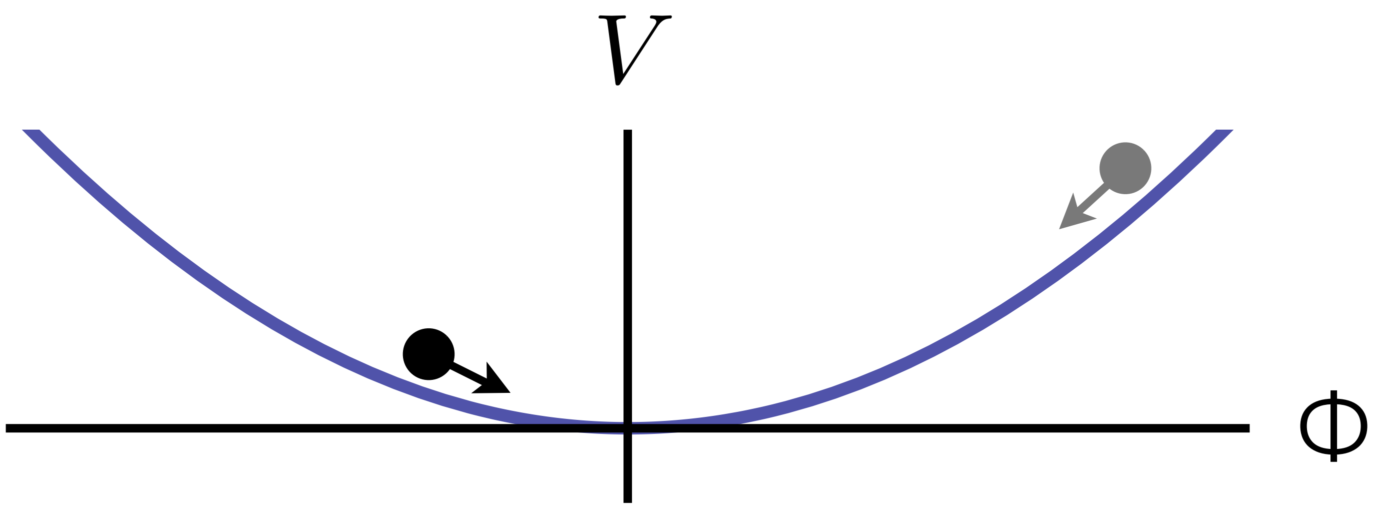

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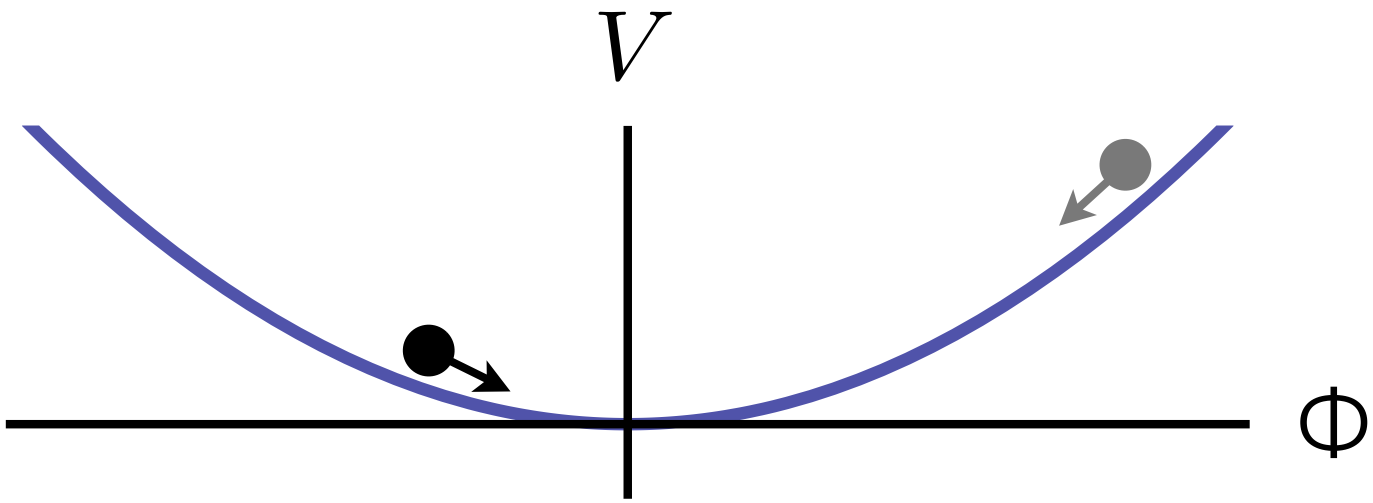
$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

This equation of course ignores interactions of the field - but this is true for axions to an excellent approximation - as a goldstone boson all interactions of the axion are suppressed by $f_\Phi > 10^9$ GeV

Two Tenets of Axion Cosmology



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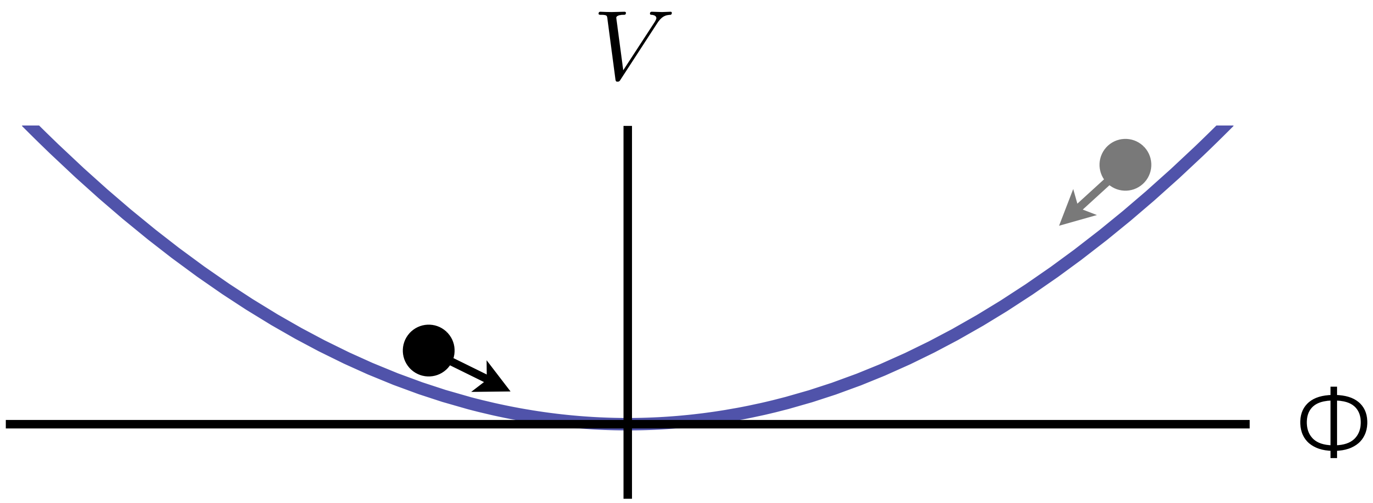
Two Different Limits, Two Different Behaviors

1. $m_\Phi << H$: Here Hubble friction dominates and the field rolls slowly.

$$\dot{\phi} = -\frac{m_\phi^2 \phi}{3H} \approxeq 0$$

Field value remains approximately constant - and so the energy density ($m_\Phi^2 \Phi_0^2$) in the field is also roughly constant

Two Tenets of Axion Cosmology



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Two Different Limits, Two Different Behaviors

2. $m_\Phi >> H$: Hubble friction can't hold the field anymore, it rolls down its potential and starts oscillating. But, as the universe expands, the amplitude of these oscillations decreases

$$\phi(t) \propto \left(\frac{1}{a(t)}\right)^{\frac{3}{2}}$$

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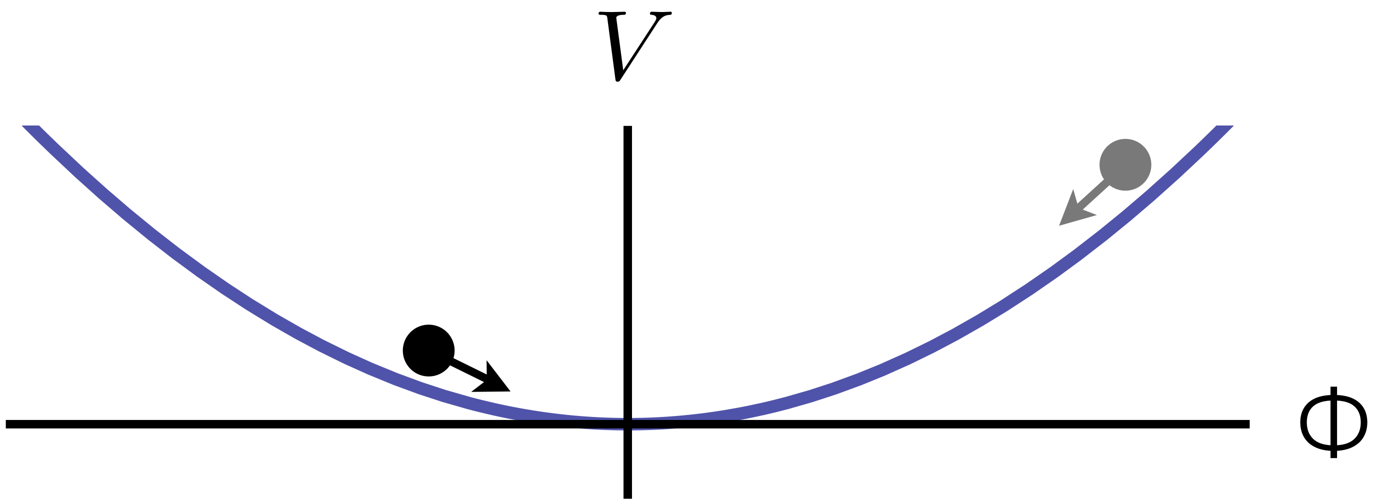

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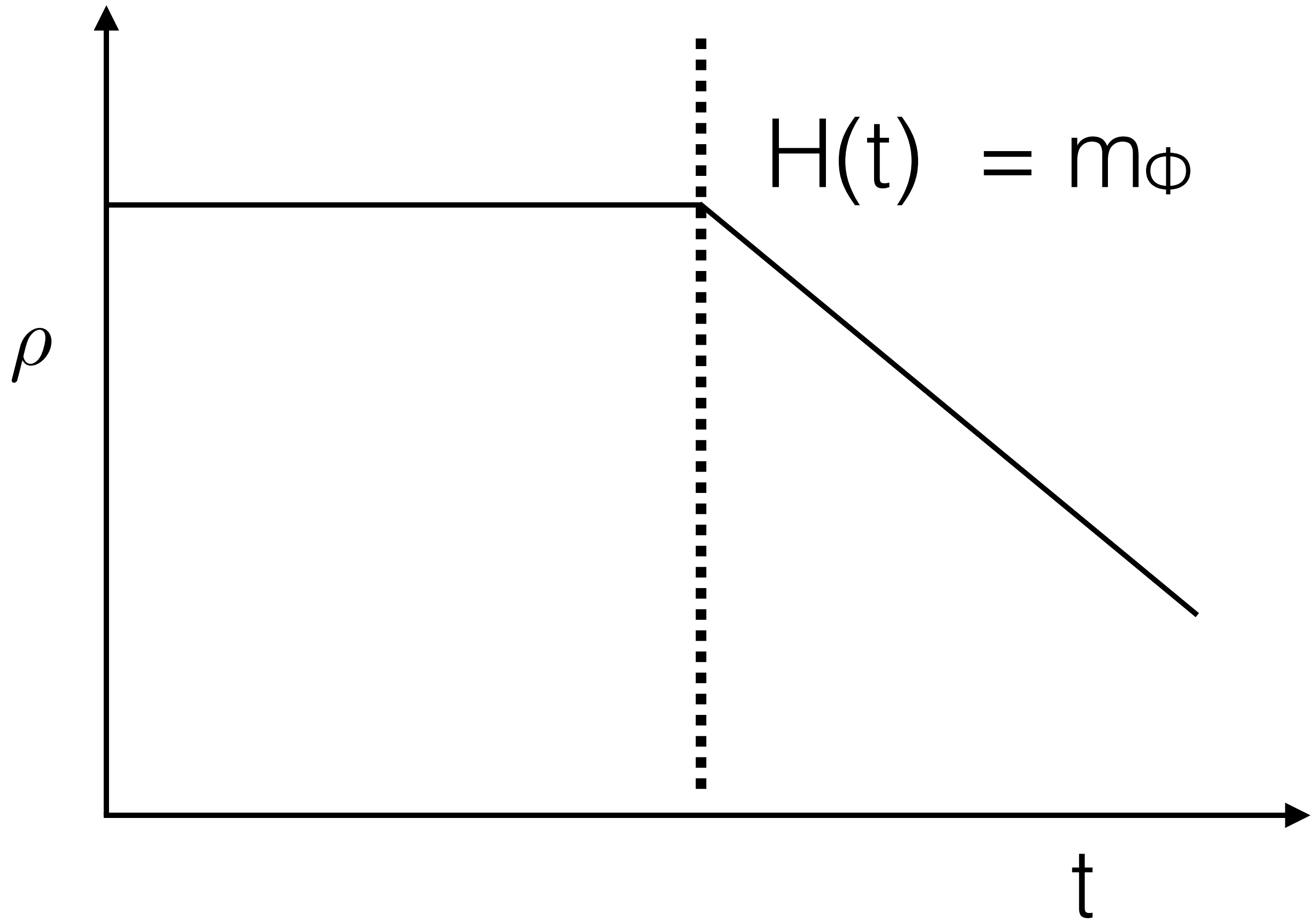
$$\phi(t) \propto \left(\frac{1}{a(t)}\right)^{\frac{3}{2}}$$

$$\implies \text{Energy density } m_\phi^2 \phi^2 \propto \left(\frac{1}{a(t)}\right)^3$$

Redshifts like matter

Evolution of Energy Density



Initially, the field is dominated by Hubble friction and its energy density remains constant - eventually, as the universe expands, its temperature drops and Hubble eventually becomes smaller than the mass of the particle. At this point, the energy density in the field red-shifts like dark matter

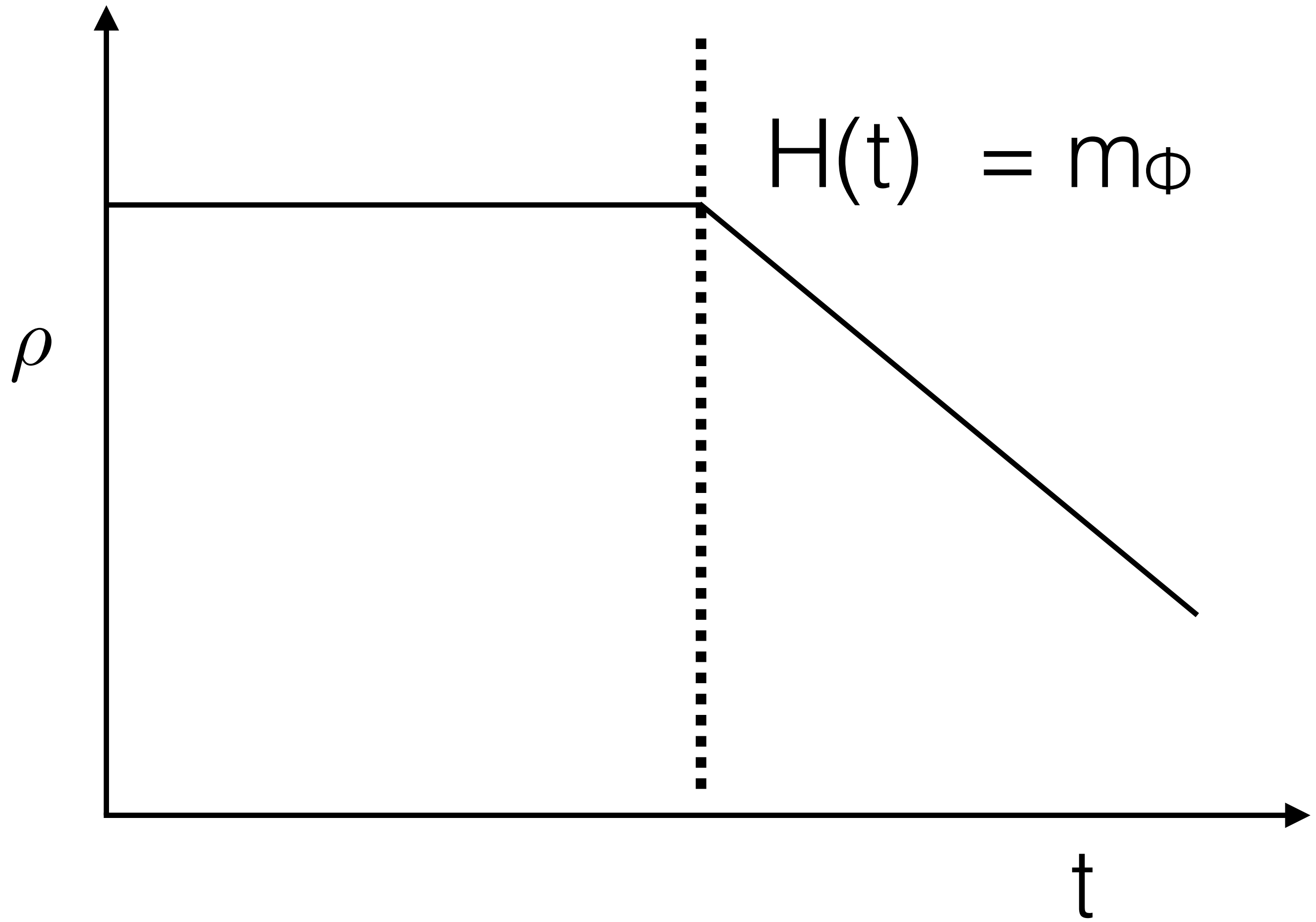
Let us fix the initial amplitude for the field to be Φ_0. As a function of the mass m for the field, what is the energy density today?

For simplicity, we just assume that the universe is always radiation dominated

Energy density is constant till H = m

$$i.e. H = \frac{T^2}{M_{pl}} = m \implies T = \sqrt{m M_{pl}}$$

Evolution of Energy Density



Let us fix the initial amplitude for the field to be Φ_0. As a function of the mass m for the field, what is the energy density today?

Energy density is constant till H = m and then it redshifts

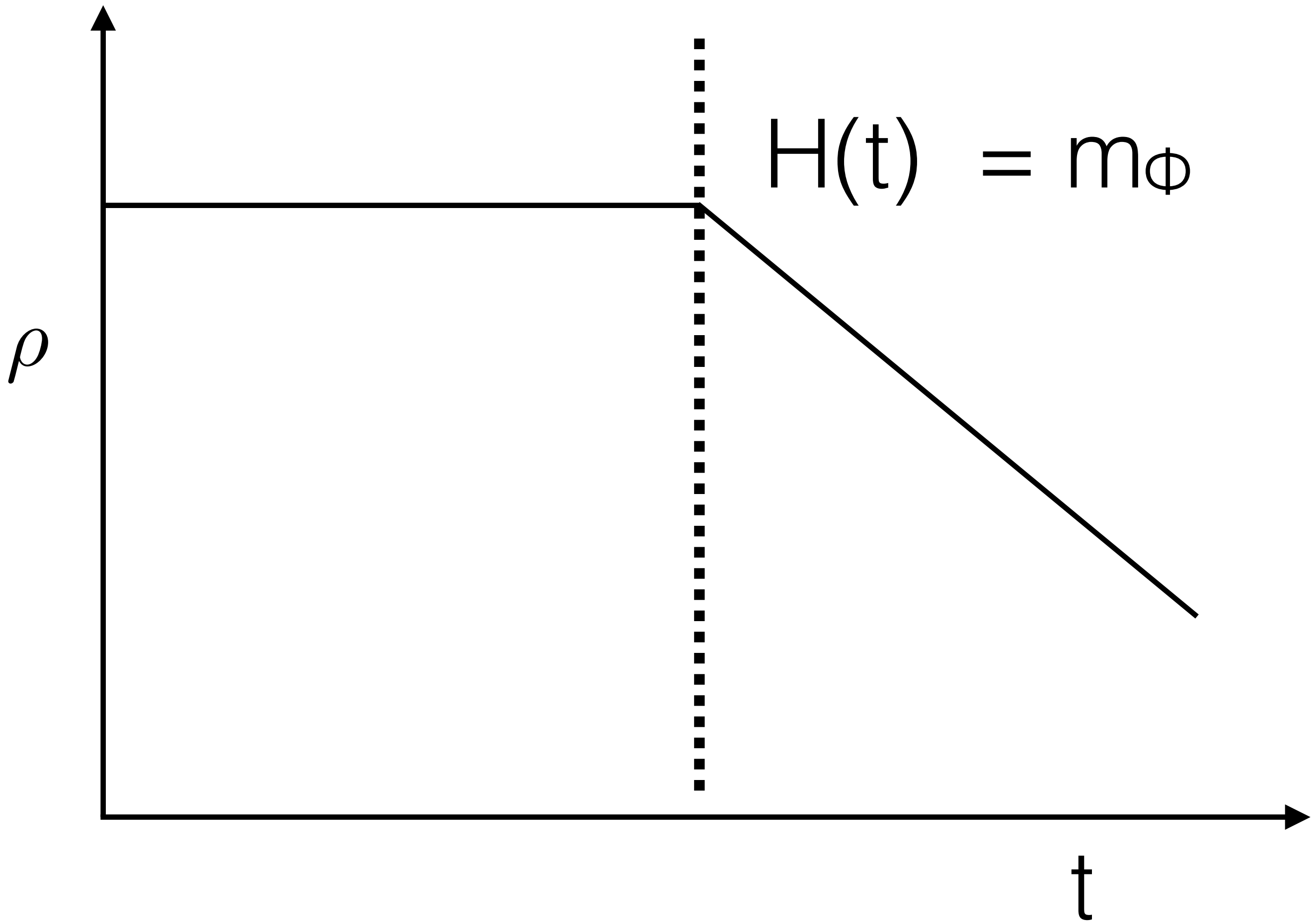
$$i.e. H = \frac{T^2}{M_{pl}} = m \implies T = \sqrt{m M_{pl}}$$

Energy density redshifts as matter afterwards: $m^2 \phi_0^2 \left(\frac{T_0}{T}\right)^3 = \sqrt{m} M_{pl} \phi_0^2 \frac{T_0^3}{M_{pl}}$

We can end up with too much dark matter for large Φ_0

This can be used to constrain parameter space - though since we don't know where Φ_0 comes from, the bound is not rigorous

Evolution of Energy Density



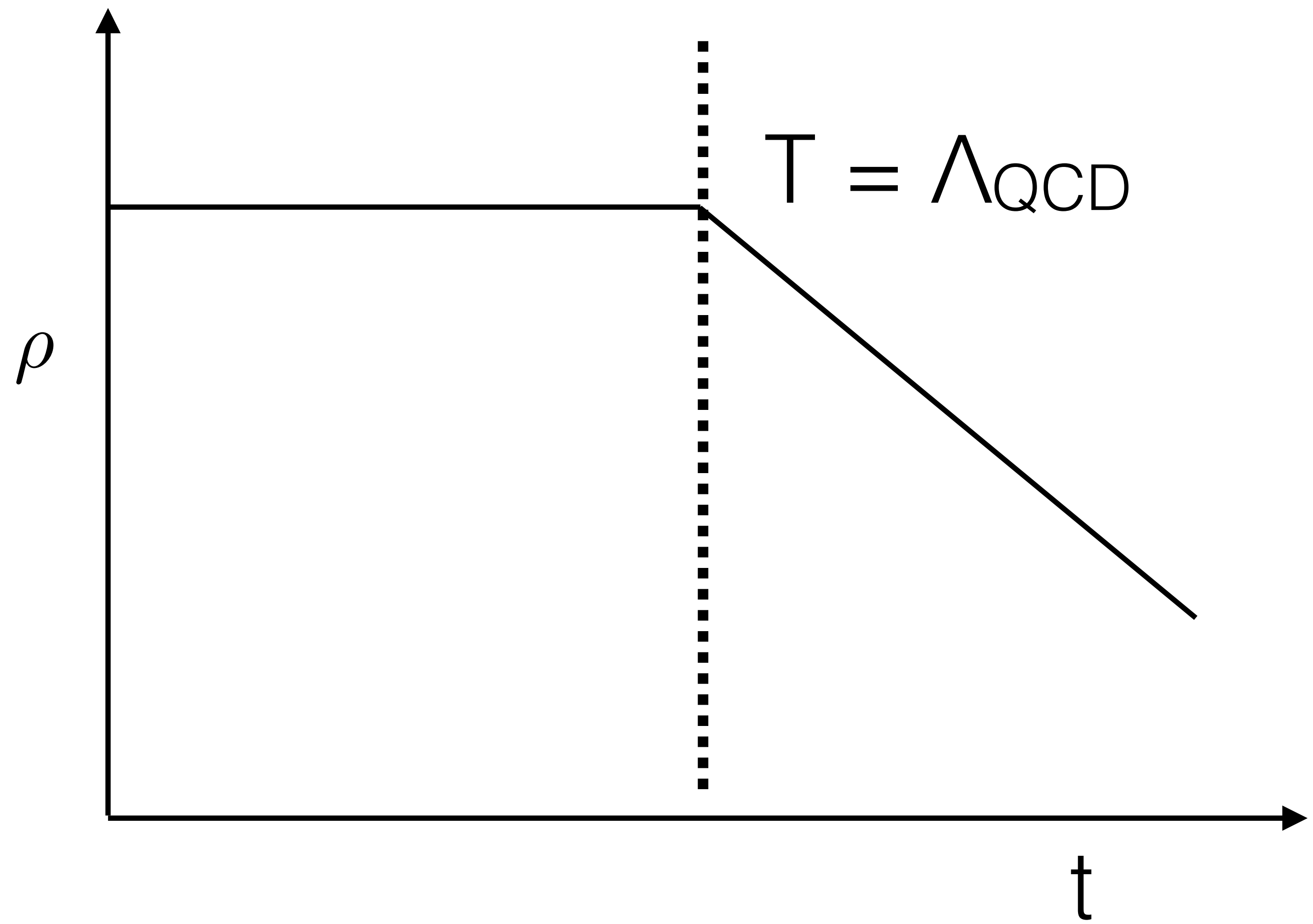
We can end up with too much dark matter for large Φ_0

The bound depends on when the mass turns on - if the mass is on earlier, the field has more time to damp

For the QCD axion, the mass comes from QCD instantons. The instantons are relevant only when the gauge coupling is large - else they are exponentially suppressed

For QCD axion, the mass becomes non-zero only at temperatures T < GeV

Cosmological Bounds on QCD Axion



With some grunge, one can show that if $a_0 \sim f_a$, QCD axions over-close the universe if $f_a > 10^{12}$ GeV

With $a_0 \sim f_a$, QCD axions will be all of dark matter if $f_a \sim 10^{12}$ GeV (conventional axion window)

People some-times say that axions with $f_a > 10^{12}$ GeV are thus “ruled out” - but this is way too quick. We don’t know the initial value of the axion field - so this is a soft bound.

Inflation and Axions

During inflation, there is a de Sitter temperature for all fields with a mass lighter than the Hubble scale H_I of inflation

All such light fields get fluctuations $\Phi \sim H_I$ during inflation

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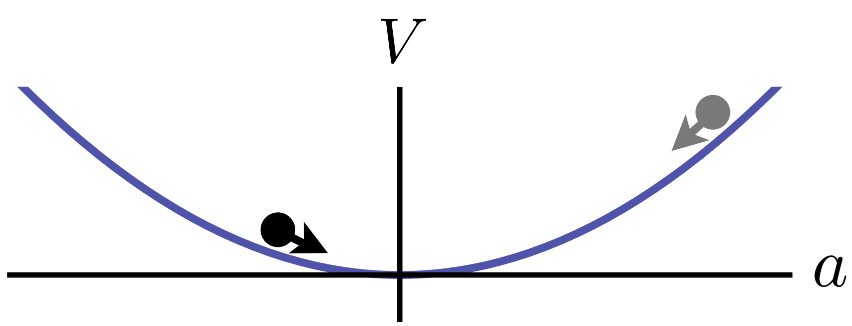
For the QCD axion, this usually constrains axions with $f_a > 10^{12}$ GeV if $H_I > 10^{13}$ GeV - though bound depends upon extrapolation of cosmology from QCD scale to inflationary scale

Features of Axion Dark Matter

What does axion dark matter look like today?

Early Universe:
Misalignment Mechanism



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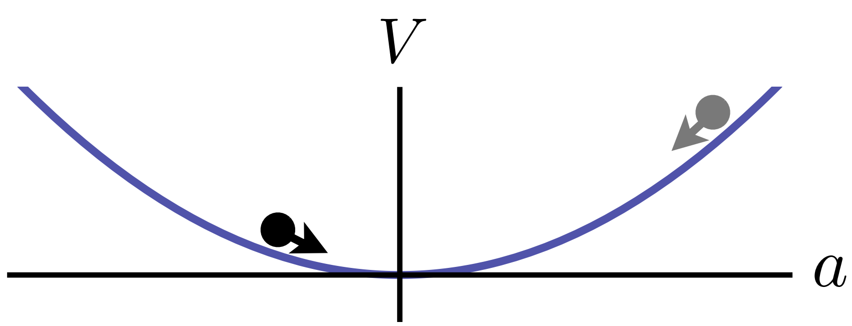

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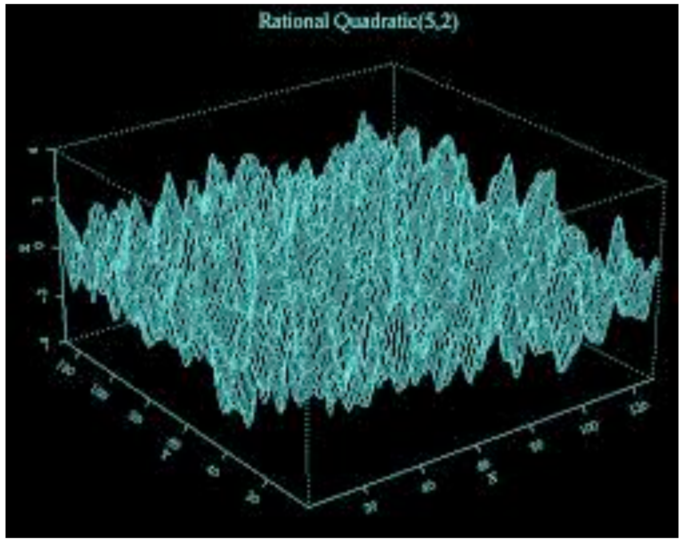
Today:
Random Field



Random field - though we can still compute two point function

That is, given $\Phi(x_1)$ how far does x_2 have to be for $\Phi(x_2)$ to be order one different?

Think in Fourier space:
Correlation length ~ $1/(m_a v)$

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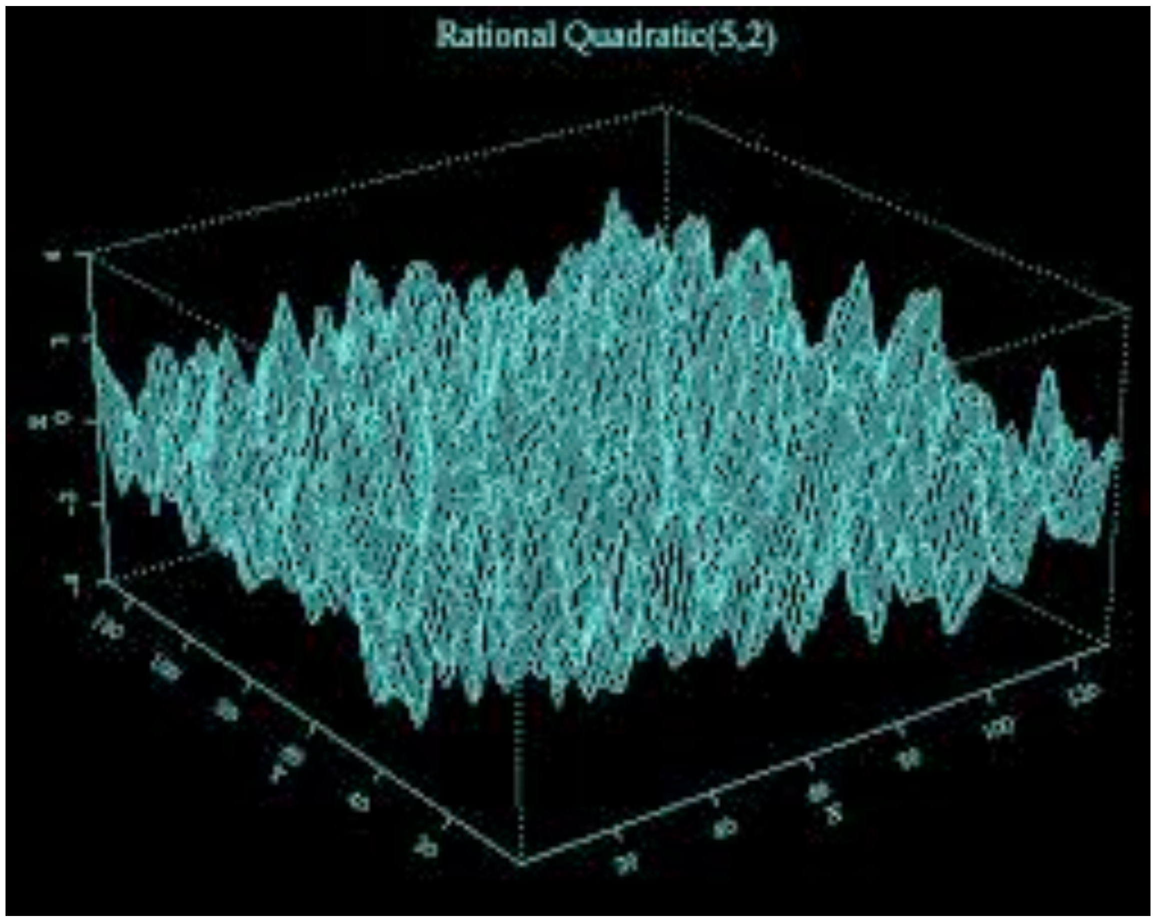

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Think in Fourier space:
Correlation length ~ $1/(m_a v)$

$$m_a \sim \frac{\Lambda_{\rm QCD}^2}{10^{12}\text{GeV}} \implies \text{Correlation length } \sim 100 \text{ m}$$

Even though this is short on galactic length scales, this is long on human scales and is thus of interest for experiments

Features of Axion Dark Matter

We are normally used to thinking about dark matter in a very “particle” way - that is, dark matter is simply a bunch of particles that are whizzing around the galaxy

This is certainly appropriate if the particles are heavy like the 100 GeV WIMP dark matter

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In this case, taking the local dark matter density ~ 0.2 GeV/cm^3, we see that the number density of a 100 GeV dark matter particle is ~ 10^{-3}/cm^3. That is obviously very tiny and so to a good approximation, the dark matter can be viewed as individual particles moving around in the galaxy

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$$m_a \approx \frac{\Lambda_{\mathrm{QCD}}^2}{10^{12}\ \mathrm{GeV}} \approx 10^{-5}\ \mathrm{eV} \implies n \approx \frac{10^{14}}{\mathrm{cm}^3}$$

Huge occupation number - better thought of as random classical fields

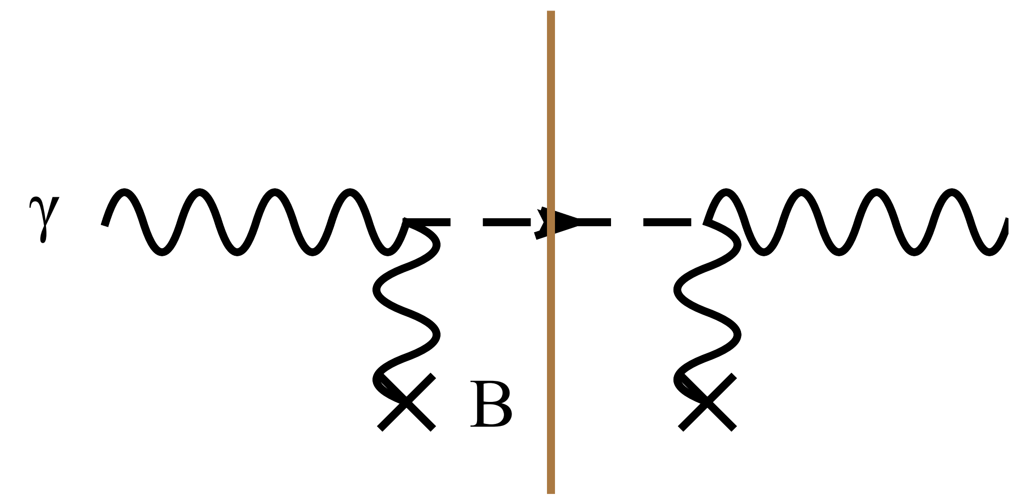
Experiments

Traditional Approaches

Lab Only

Light Shining Through a Wall

$$\frac{\phi}{g_\phi} F\tilde{F} = \frac{\phi}{g_\phi} \vec{E}.\vec{B}$$



Axion-photon oscillation

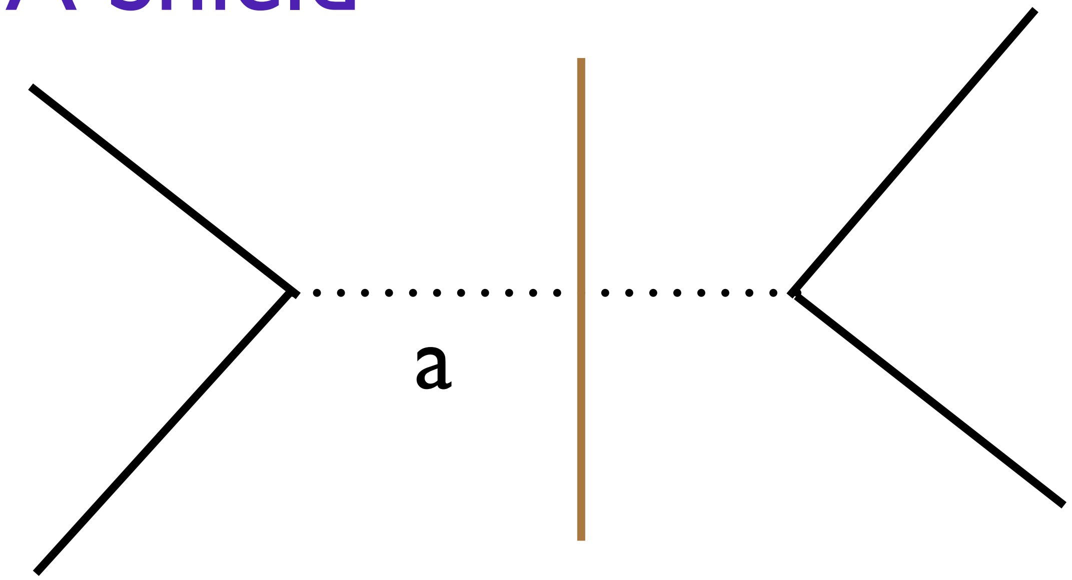
Enhanced in high Q cavities

ALPS II: $g_\Phi < 10^{11}$GeV for $m_\Phi <$ eV

Lab Only

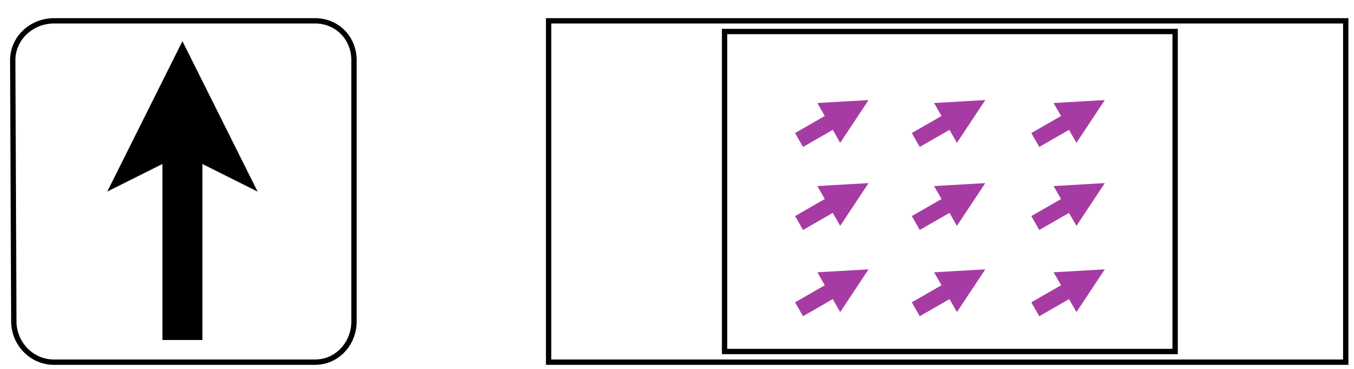
Force Through A Shield

$$\frac{\partial_\mu \phi}{f_\phi} \bar{\psi} \gamma^\mu \gamma_5 \psi \implies H_N \supset \frac{\nabla \phi}{f_\phi} . \vec{S}$$



Axion couples to
fermion spin a lot
like a magnetic field

NMR Searches

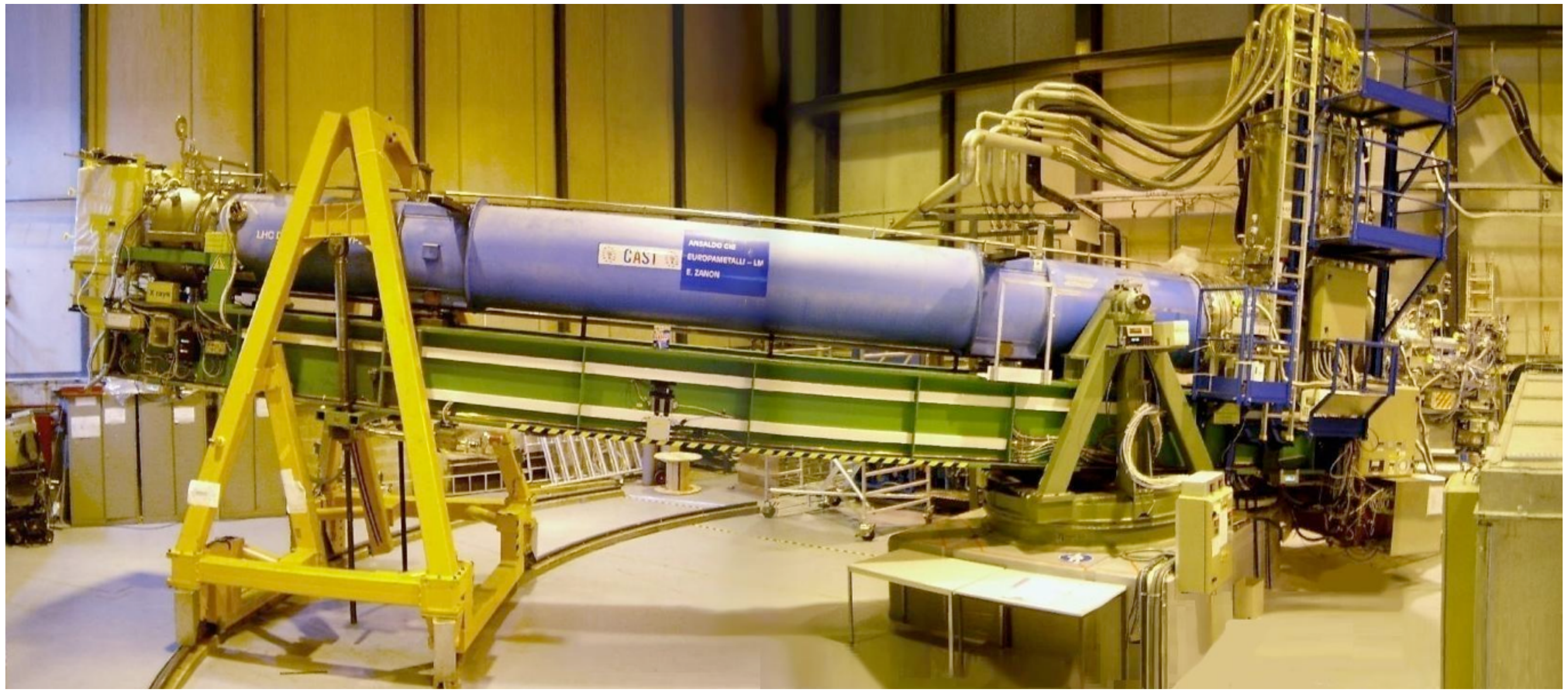


Take sample of polarized spins - place another set of spins inside a shield. See if the spins inside the shield precess as though there is a magnetic field in there. If so, you have detected the axion!

Constraints on $f_\Phi < 10^5$ GeV for $m_\Phi < 10^{-3}$ eV

Astro + Lab

Sun Shining Through A Wall



Sun produces axions.

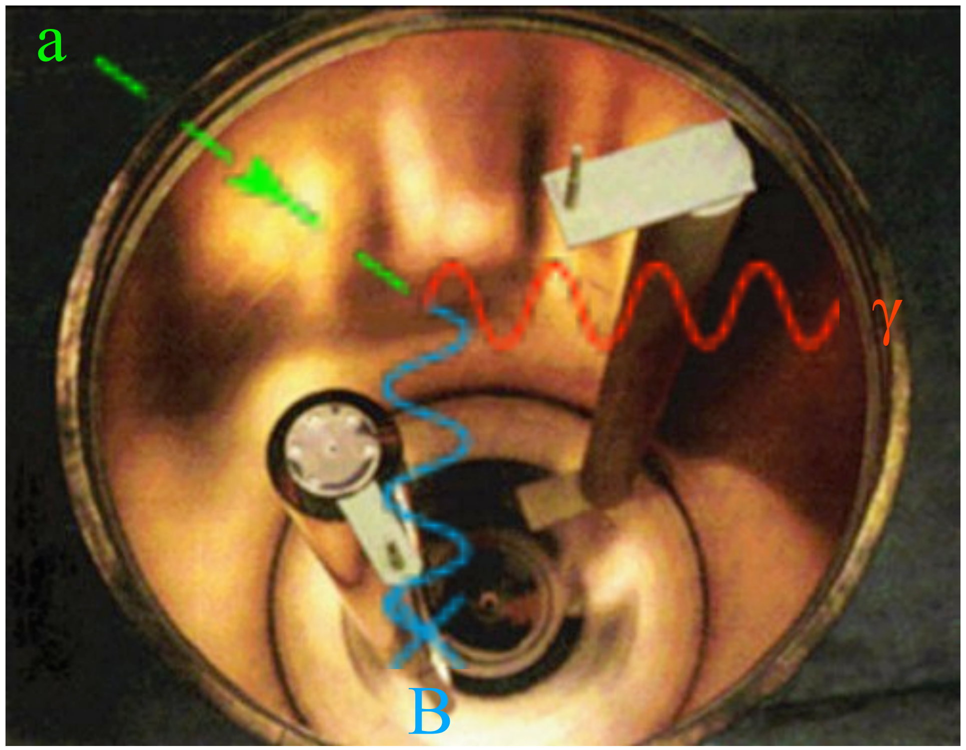
Axions converted to photons in a magnetic field

CAST: $g_\phi \sim 10^{10}$GeV for $m_\phi <$ eV

Can probe up to ~ keV axion masses. Oscillation length suppressed

Cosmo + Lab

Axion Dark Matter



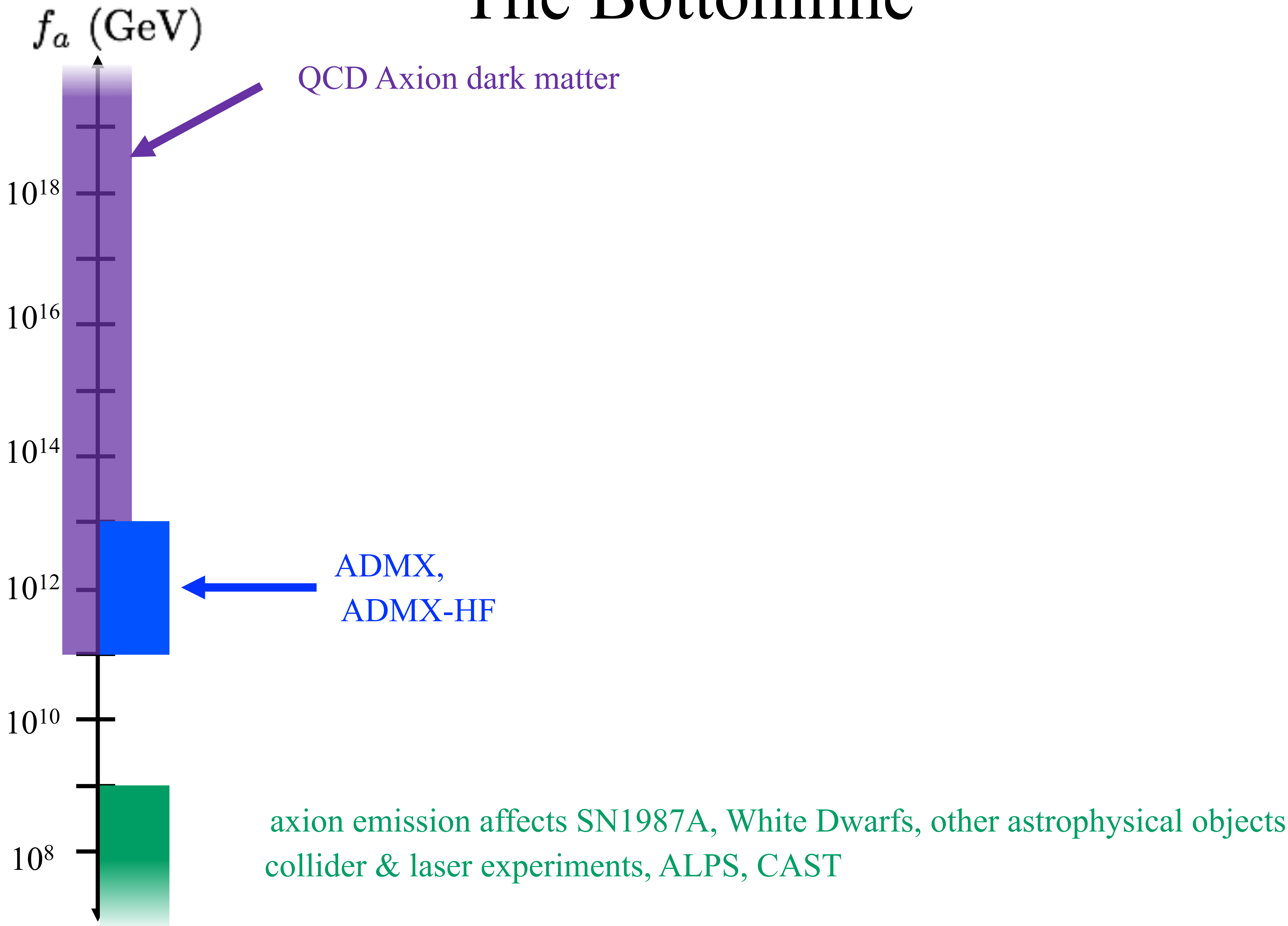
microwave cavity (ADMX)

$$\mathcal{L} \supset \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}F\tilde{F} = \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}\vec{E}.\vec{B}$$

Resonant conversion of cosmic axion to photon

Axions with mass ~ GHz,
$f_\phi \sim 10^{11} - 10^{12}$ GeV

The Bottomline



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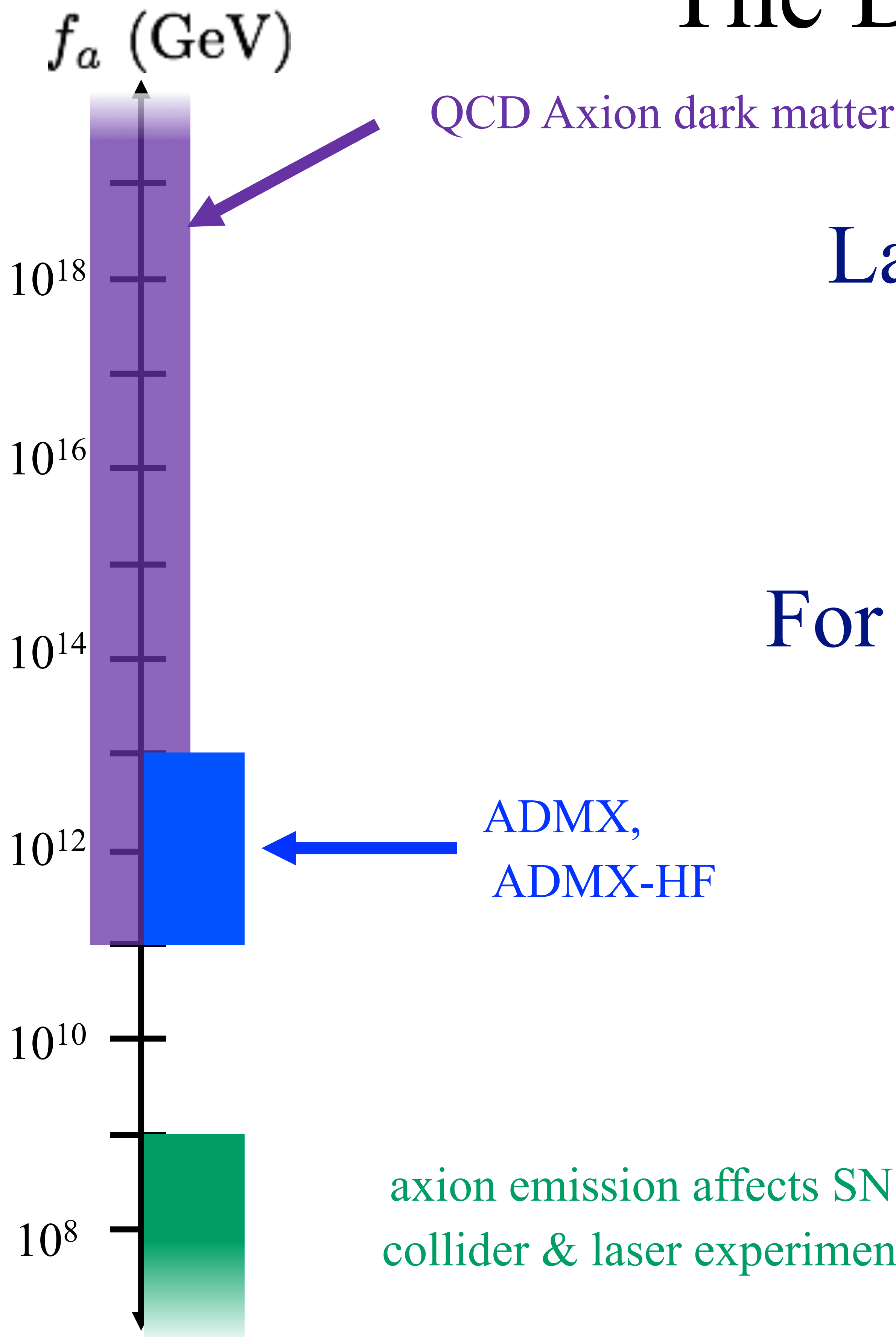
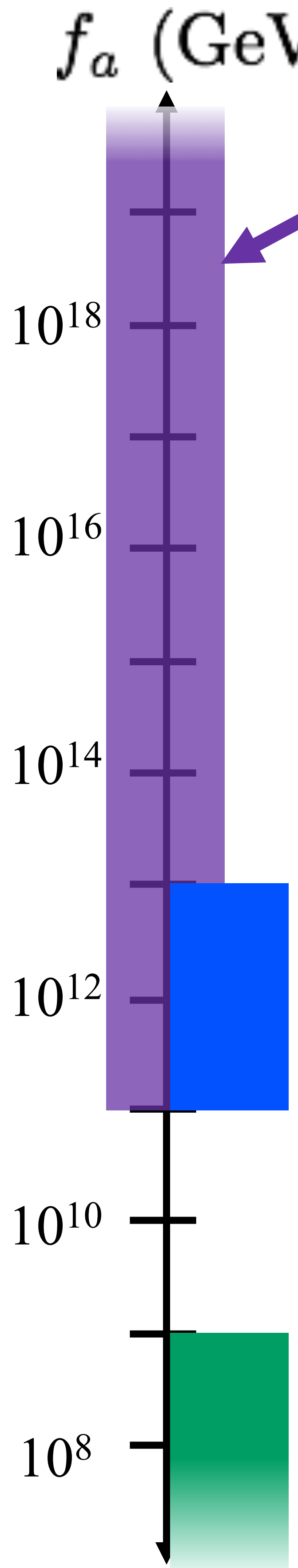


Large parameter space still unconstrained

For large f_a, need to beat small coupling

axion emission affects SN1987A, White Dwarfs, other astrophysical objects
collider & laser experiments, ALPS, CAST

The Bottomline



QCD Axion dark matter

Large parameter space still unconstrained

For large f_a, need to beat small coupling

ADMX,
ADMX-HF

How do we find these?

axion emission affects SN1987A, White Dwarfs, other astrophysical objects
collider & laser experiments, ALPS, CAST