N-body & Hydrodynamic Simulations

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I. N-body methods
II. Hydrodynamical Methods
   i. Lagrangian methods (SPH)
   ii. Eulerian Methods
III. Applications to formation of cosmic structures
   i. Including astrophysics of galaxy formation
   ii. Cosmology with galaxy clusters and simulations

Lecture @ Advanced School of Cosmology, ICTP - Trieste, May 25TH 2015

Part I:
N-body simulations

Based on:
Springel 2005, MNRAS, 364, 1005
Dolag, SB+, 2008, Sp.Sc.Rev., 2008, 134
Springel, arXiv:1412:5187

What is an N-body code?

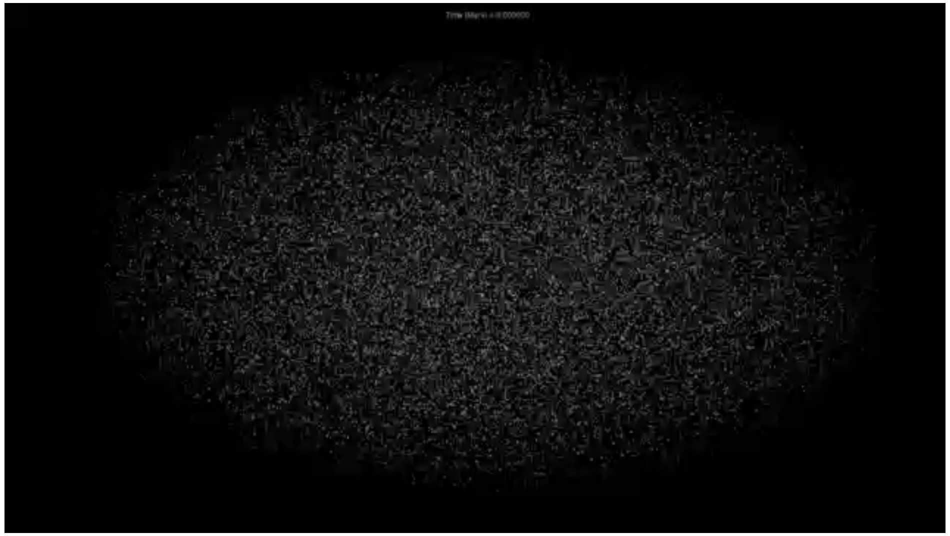
Problem: solve the dynamics of a self-gravitating collisionless system

Collisionless Boltzmann (Vlasov) + Poisson equations:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2}\nabla f - m\nabla\Phi\frac{\partial f}{\partial \mathbf{p}} = 0 \qquad f(\mathbf{x},\mathbf{p},t)$$: Phase-space distrib. function

$$\nabla^2\Phi(\mathbf{x},t) = 4\pi \mathrm{G}a^2\left[\rho(\mathbf{x},t) - \bar{\rho}(t)\right]$$

$$\rho(\mathbf{x},t) = \int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3p$$



➔ Tough to deal with: it's in 6D!

N-body approximation:

- sample the initial phase space with N discrete fluid elements
- integrate their eqs. of motion in the collective gravity field

➔ equivalent to solving the characteristic eqs., describing curves in phase space where f(**x**,**p**,t) is constant

What is an N-body code?

- Integrate the equations of motions of the N particles:

$$\frac{d\mathbf{p}}{dt} = -m\nabla\Phi \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{ma^2}$$

Direct N-body code: For each particle compute the contribution to the potential from all the other N-1 particles:

$$\Phi(\mathbf{r}) = -G\sum_j \frac{m_j}{\left(|\mathbf{r}-\mathbf{r}_j|^2+\epsilon^2\right)^{\frac{1}{2}}}$$

➔N(N-1)/2 operations!!!

ε: softening parameter (~1/20-1/50 MIS)

➔To reduce spurious two-body relaxation when a finite particle number is used to describe a collisionless fluid

<u>Solutions:</u>

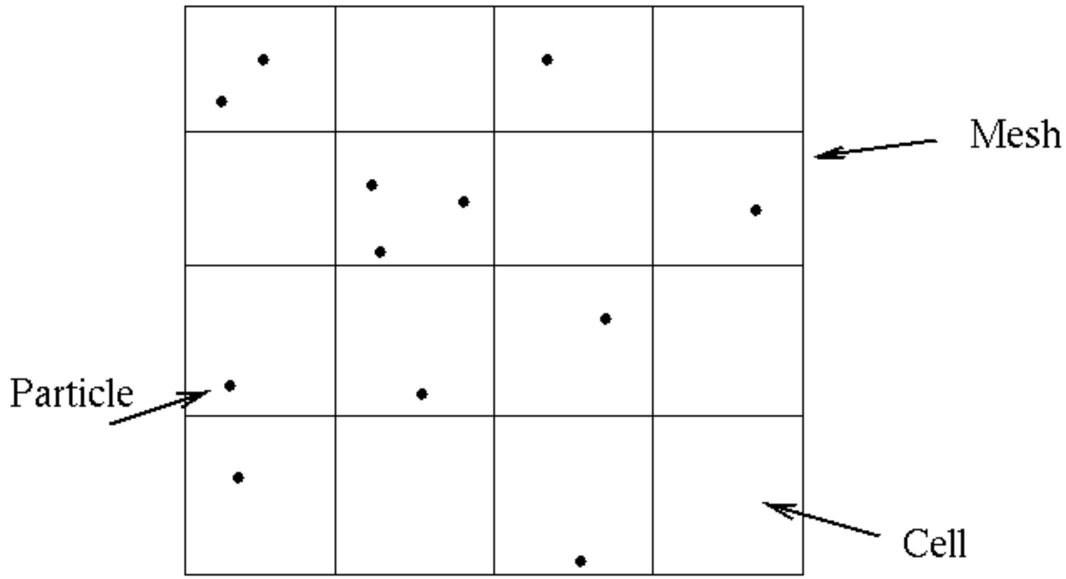
- Resort to special purpose hardware solutions (e.g. GPUs)
- Resort to faster integration methods, always implying a lower accuracy

Particle-Mesh (PM)

1st STEP: assign densities to the mesh from particle positions

$$\rho_m = \frac{1}{h^3} \sum_i m_i W(\mathbf{x}_i - \mathbf{x}_m)$$

$W(\mathbf{x}_m - \mathbf{x}_i)$: weighting function



2nd STEP: solve the Poisson equation in Fourier space

$$\Phi(\mathbf{x}) = \int g(\mathbf{x} - \mathbf{x}')\rho(\mathbf{x}')\mathrm{d}\mathbf{x}'$$

Solution of the Poisson eq. with

$g(\mathbf{x}) = -\mathrm{G}/|\mathbf{x}|$: Green's function of the Laplacian

➔ Use FFT to compute $\hat{\Phi}(\mathbf{k}) = \hat{g}(\mathbf{k})\,\hat{\rho}(\mathbf{k})$

➔ Transform back to compute $\Phi(\mathbf{x})$

Particle-Mesh (PM)

3rd STEP: compute the force on the grid: $\mathbf{f}(\mathbf{x}) = -\nabla\Phi(\mathbf{x})$

Use a finite differentiation: $f^{(x)}_{i,j,k} = -\dfrac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h}$

4th STEP: interpolate back forces to particles positions, using the same weighting scheme:

$$\mathbf{f}(\mathbf{x}_i) = \sum_m W(\mathbf{x}_i - \mathbf{x}_m)\mathbf{f}_m$$

5th STEP: update particle positions and velocities
E.g. using the "leapfrog" scheme to integrate $\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$

<u>Kick-Drift-Kick</u>

$$\mathbf{v}_{n+1/2} = \mathbf{v}_n + \mathbf{f}(\mathbf{x}_n)\Delta t/2$$
$$\mathbf{x}_{n+1} = \mathbf{x}_n + \mathbf{v}_{n+1/2}\Delta t$$
$$\mathbf{v}_{n+1} = \mathbf{v}_{n+1/2} + \mathbf{f}(\mathbf{x}_{n+1})\Delta t/2$$

<u>Drift-Kick-Drift</u>

$$\mathbf{x}_{n+1/2} = \mathbf{x}_n + \mathbf{v}_n\Delta t/2$$
$$\mathbf{v}_{n+1} = \mathbf{v}_n + \mathbf{f}(\mathbf{x}_{n+1/2})\Delta t$$
$$\mathbf{x}_{n+1} = \mathbf{x}_{n+1/2} + \mathbf{v}_{n+1}\Delta t/2.$$

$$\boxed{\Delta t = \alpha\sqrt{\epsilon/|\mathbf{a}|}}$$

$\alpha \approx 0.1$

N + $N_g \log N_g$ operations now required !

Tree codes

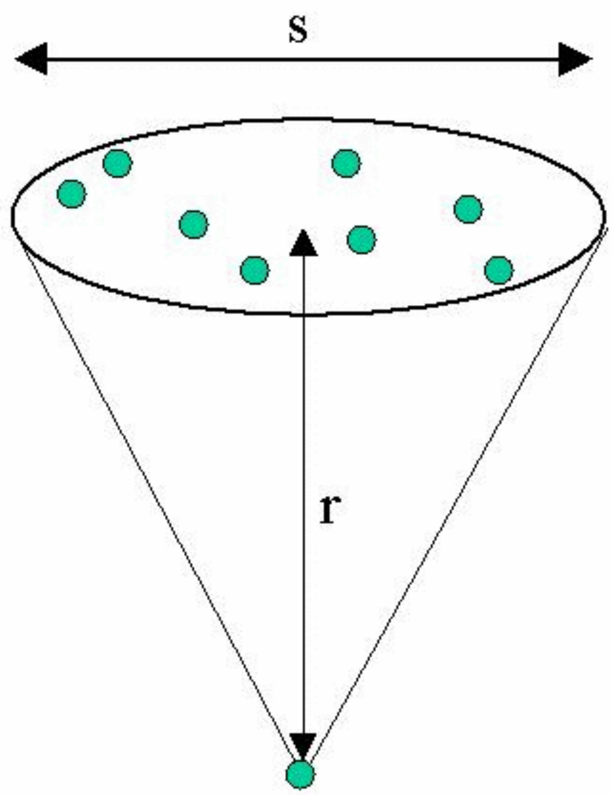
Barnes & Hut 86

Basic idea: treat a distant group of particles as a single "macro-particle"

Precision regulated by the value of the "critical opening angle": $s > r/\theta_c$ $(\theta_c \sim 0.5)$

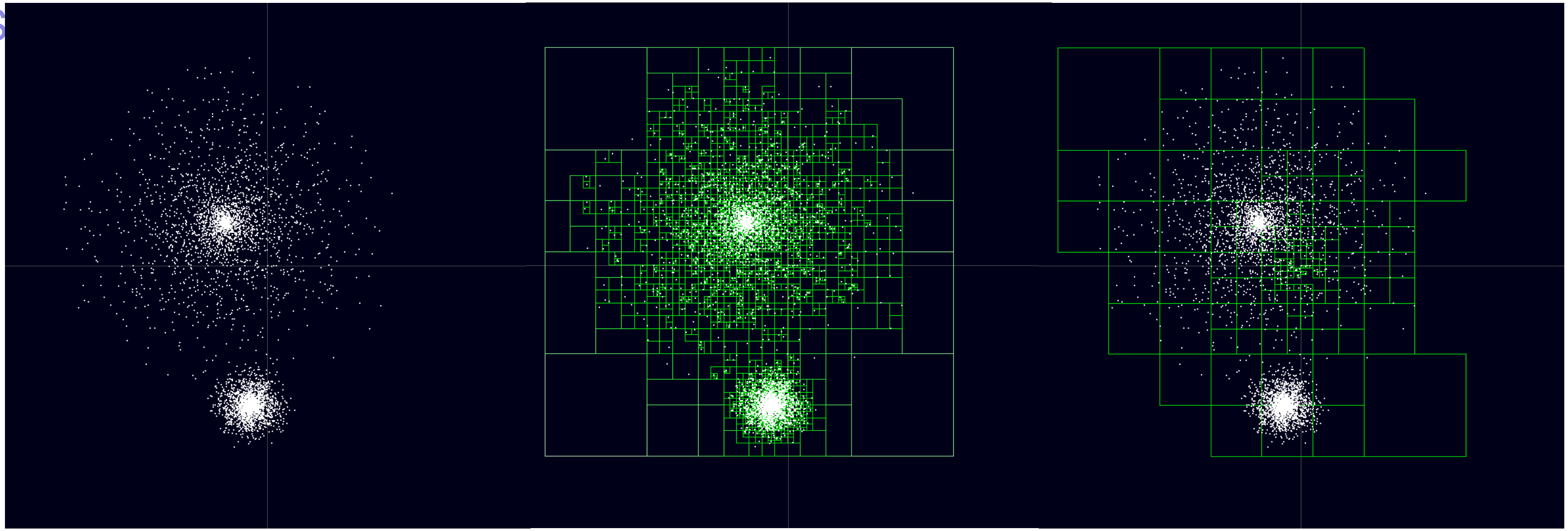
➔ *N log N* operations required, but:

- Pre-factor depending on how clustered is the particle distribution
- Need to construct and store the structure a hierarchical binary tree.



Tree codes

S



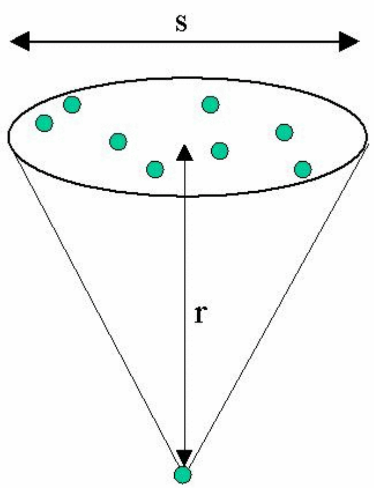
- ➔ Recursively divide the simulation box in sub-domains until each “leave” of the “tree” contains either one or zero particles
- ➔“Walk” the tree for each particle, starting from the top-node.

  r: spatial extent of the node.

  s: distance of the center of mass of the node from the partic

  $\theta_c < r/s$ ➔ open the node and iterate

  $\theta_c > r/s$ ➔ compute the force from the node on the particle



Hybrids

TreePM (e.g. Bagla 02):

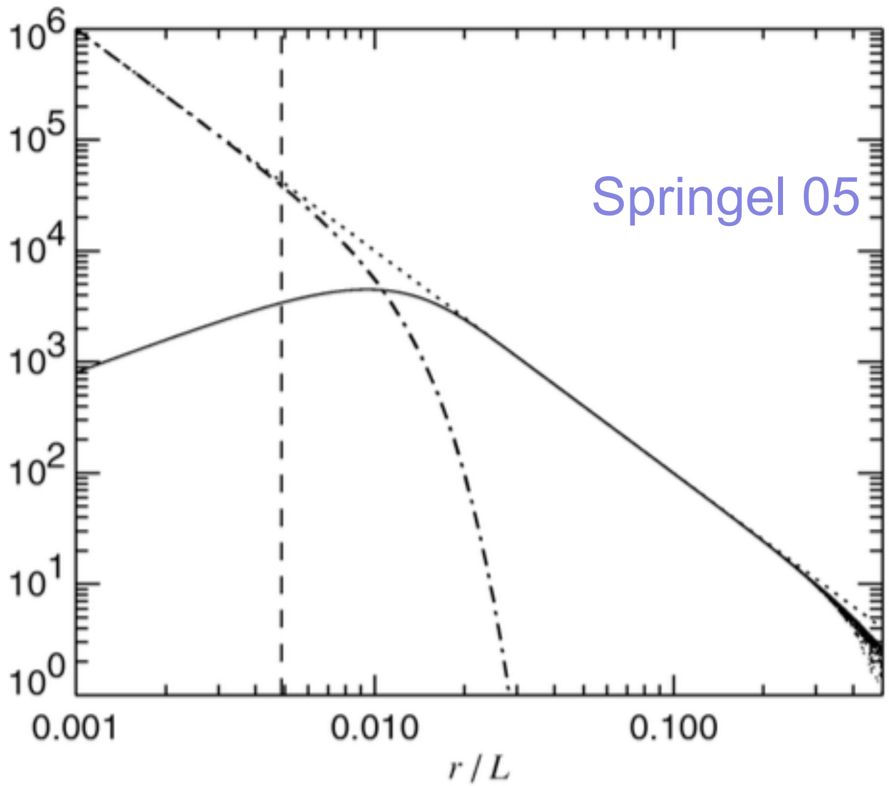
$$\Phi_{\mathbf{k}} = \Phi_{\mathbf{k}}^{\rm long} + \Phi_{\mathbf{k}}^{\rm short}$$

$$\Phi_{\mathbf{k}}^{\rm long} = \Phi_{\mathbf{k}} \exp(-\mathbf{k}^2 r_{\rm s}^2)$$

➔ from the PM method
r_s: splitting scale

$$\Phi^{\rm short}(\mathbf{x}) = -G \sum_i \frac{m_i}{\mathbf{r}_i} \operatorname{erfc}\left(\frac{\mathbf{r}_i}{2 r_{\rm s}}\right)$$

➔ from the Tree part



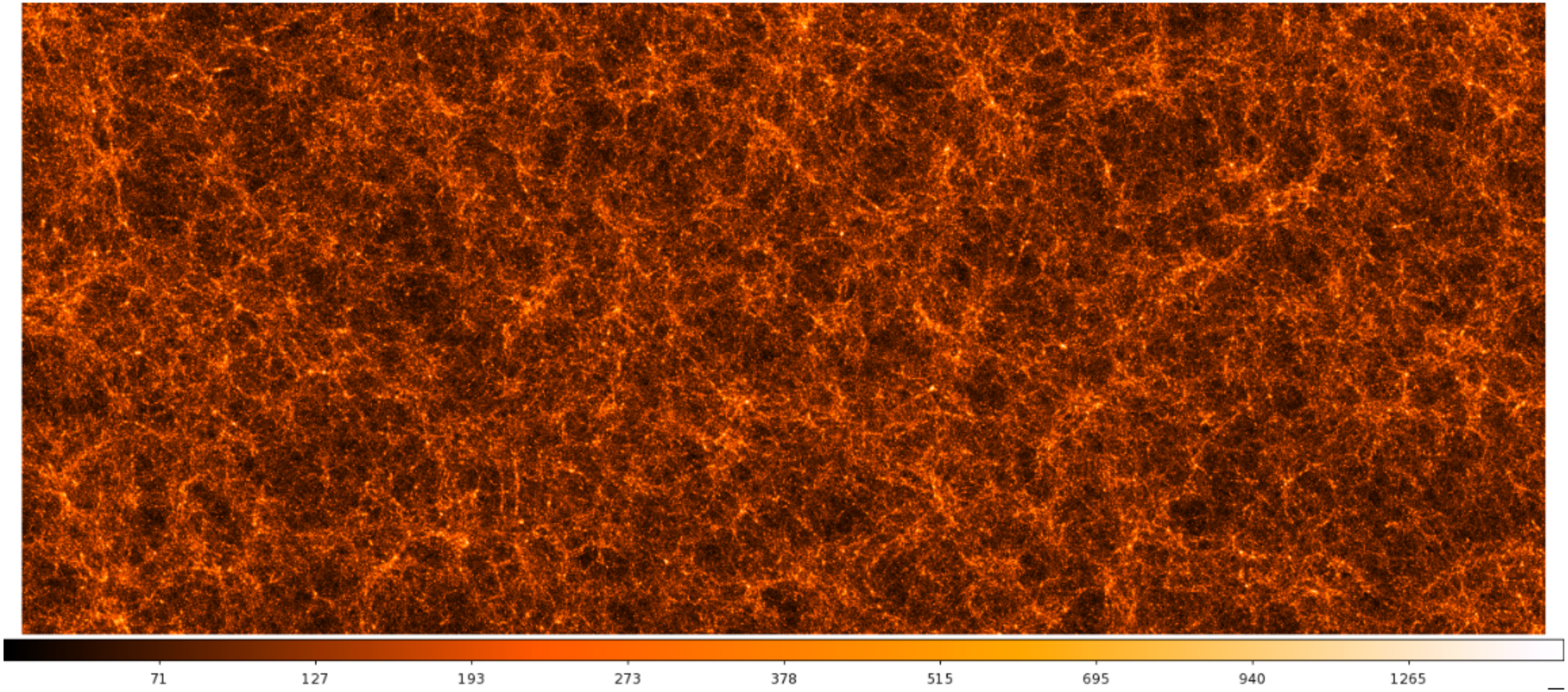
P3M (e.g. Efstathiou+85): same as TreePM with $\Phi_{short}(x)$ computed from direct integration

AP3M/ATreePM (e.g. Couchman 91, Springel 05): same as above, with Adaptive PM

The record

Dark Sky Simulations; Skillman+14

10240^3 particles; Box size = 8 h^{-1} Gpc



➔ Past-light cone between z=0.9 and 1.0

Part II:
Hydrodynamical Methods

Based on:
Monaghan, Rep. Prog. Phys.,2005, 68, 1793
Rosswog, NewA, 2009, 53, 78
Dolag, SB+, 2008, Sp.Sc.Rev., 2008, 134
Springel, arXiv:1412:5187

Numerical Hydrodynamics

- To follow the formation and evolution of *baryonic structures* inside the potential wells of Dark Matter in non-linear regime
- Two large classes of numerical methods: **Lagrangian** and **Eulerian**
- **Eulerian** methods: follow the *fluxes* of gas end energies in space. Derivatives evaluated at fixed points in space
- **Lagrangian** methods: follow the evolution of *fluid elements*. Derivatives in a coordinate system following the fluid element

Basics of fluido-dynamics

ρ : fluid density
v : fluid velocity
P : pressure;
u : internal energy;

Euler equations for a non-viscous fluid:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\vec{v}.$$

Continuity equation; mass conservation

$$\frac{d\vec{v}}{dt} = -\frac{\nabla P}{\rho} + \vec{f},\ \ \vec{f} = -\vec{\nabla}\Phi$$

Euler equation; momentum conservation

$$\frac{du}{dt} = \frac{P}{\rho^2}\frac{d\rho}{dt} = -\frac{P}{\rho}\nabla\cdot\vec{v}.$$

Energy conservation; 1st law of thermodynamics

$$dU = dQ - PdV,$$

$$P = (\gamma - 1)\rho u,\ \ \gamma = c_p/c_v$$

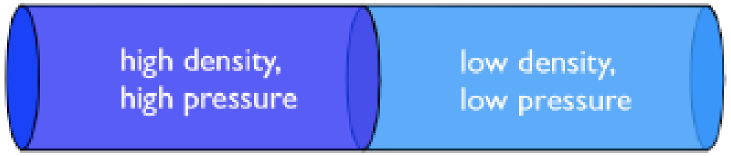
Equation of state

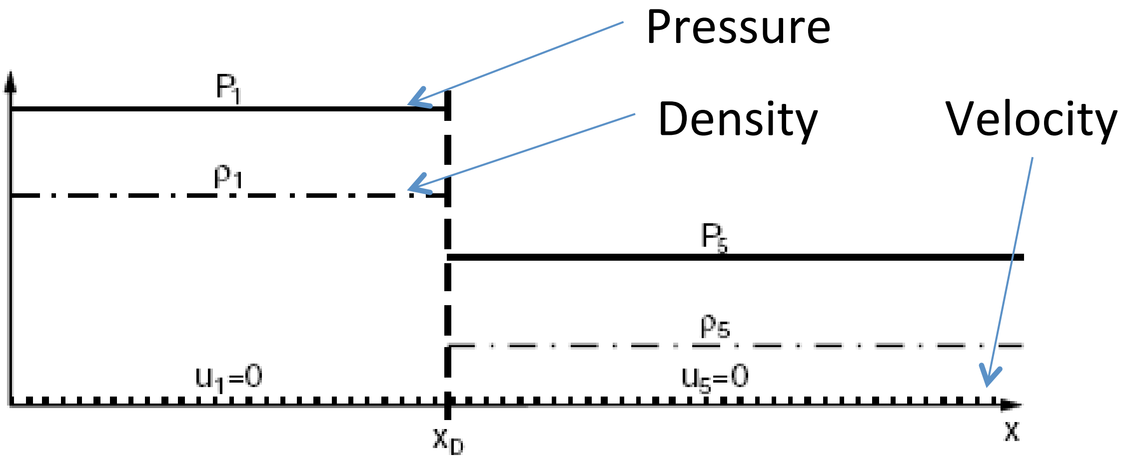
γ=5/3 for an ideal monoatomic gas

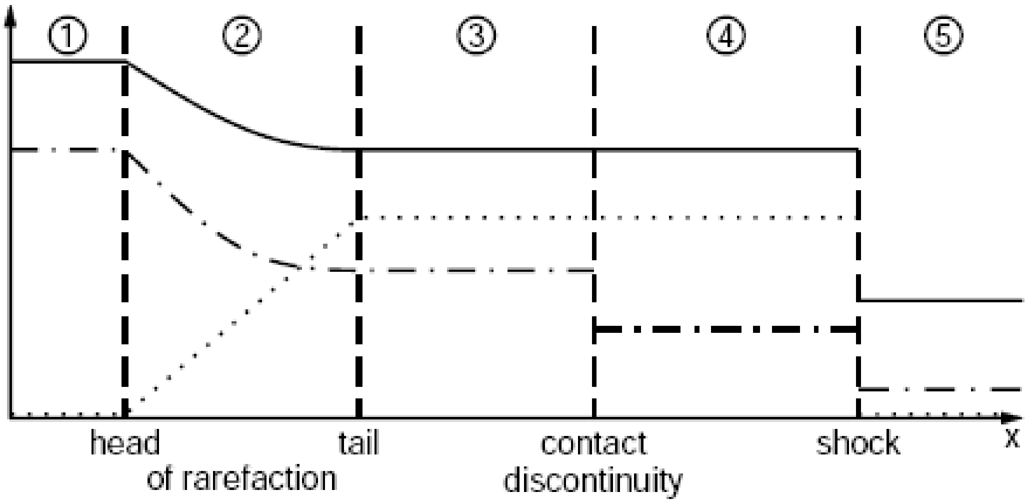
$$\frac{d}{dt} = \frac{dx^i}{dt}\frac{\partial}{\partial x^i} + \frac{\partial}{\partial t} = \vec{v}\cdot\nabla + \frac{\partial}{\partial t}$$

Lagrangian derivative

Eulerian methods – The Riemann problem







<u>What is the Riemann problem:</u>

- Initial value problem for a hyperbolic system

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F} = 0$$

- Two piece-wise constant states with an interface at $t=0$

$$U_L = \begin{pmatrix} \rho_L \\ P_L \\ \mathbf{v}_L \end{pmatrix}, \quad U_R = \begin{pmatrix} \rho_R \\ P_R \\ \mathbf{v}_R \end{pmatrix}$$

$\mathbf{v}_L = \mathbf{v}_R = 0$: Sod shock tube

<u>Shock:</u> irreversible conversion of mechanical into thermal energy

<u>Contact discontinuity:</u> original separation plane

<u>Rarefaction wave:</u> smoothly connects two states

Eulerian methods - Discretization

Equations of fluido-dynamics:

$$\frac{\partial U}{\partial t} + \nabla \cdot \boldsymbol{F} = 0$$

State variable　　　Fluxes

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho \mathbf{v} \\ \rho e \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho \mathbf{v} \\ \rho \mathbf{v}\mathbf{v}^T + P \\ (\rho e + P)\mathbf{v} \end{pmatrix}, \quad e = u + \mathbf{v}^2/2$$

$$P = (\gamma - 1)\rho u$$

	P_i, V_i, u_i, ρ_i	P_j, V_j, u_j, ρ_j

$$\mathbf{U}_i = \frac{1}{V_i}\int_{\mathrm{cell}\ i} \mathbf{U}(\mathbf{x})\,\mathrm{d}V.$$: average state within a cell

➔ Integration of conservation law within a cell in a given time interval

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathrm{d}x \int_{t_n}^{t_{n+1}} \mathrm{d}t \left(\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} \right) = 0.$$

Eulerian methods - Discretization

➔ $$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathrm{d}x \left[\mathbf{U}(x,t_{n+1}) - \mathbf{U}(x,t_n)\right] + \int_{t_n}^{t_{n+1}} \mathrm{d}t \left[\mathbf{F}(x_{i+\frac{1}{2}},t) - \mathbf{F}(x_{i-\frac{1}{2}},t)\right] = 0.$$

Cell-average at the *n-th* time-step: $$\mathbf{U}_i^{(n)} \equiv \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{U}(x,t_n)\mathrm{d}x.$$

➔ $$\Delta x \left[\mathbf{U}_i^{(n+1)} - \mathbf{U}_i^{(n)}\right] + \int_{t_n}^{t_{n+1}} \mathrm{d}t \left[\mathbf{F}(x_{i+\frac{1}{2}},t) - \mathbf{F}(x_{i-\frac{1}{2}},t)\right] = 0.$$

Godunov scheme:

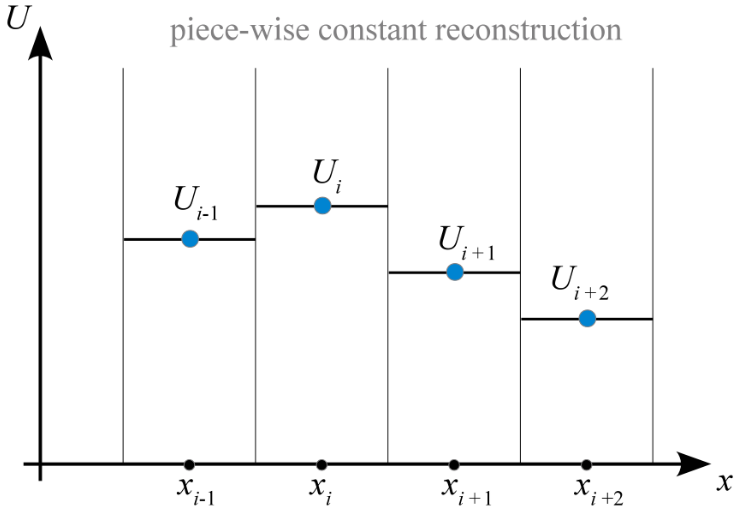
$\mathbf{F}(x_{i+\frac{1}{2}},t)$ ➔ Solution of the Riemann problem with $\mathbf{U}_i^{(n)}$ as left state and $\mathbf{U}_{i+1}^{(n)}$ as right state:

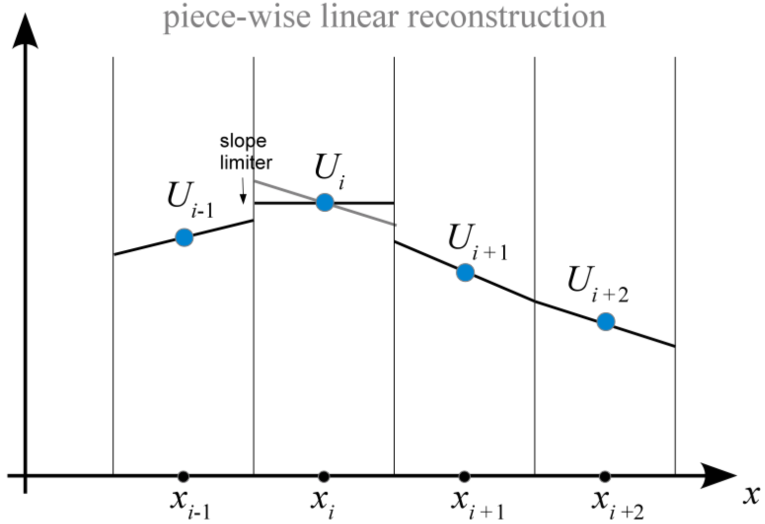
$$\mathbf{F}^{\star}_{i+\frac{1}{2}} = \mathbf{F}_{\text{Riemann}}(\mathbf{U}_i^{(n)}, \mathbf{U}_{i+1}^{(n)})$$

➔ Advancement of the state:

$$\boxed{\mathbf{U}_i^{(n+1)} = \mathbf{U}_i^{(n)} + \frac{\Delta t}{\Delta x}\left[\mathbf{F}^{\star}_{i-\frac{1}{2}} - \mathbf{F}^{\star}_{i+\frac{1}{2}}\right]}$$

Reconstruct-Evolve-Average





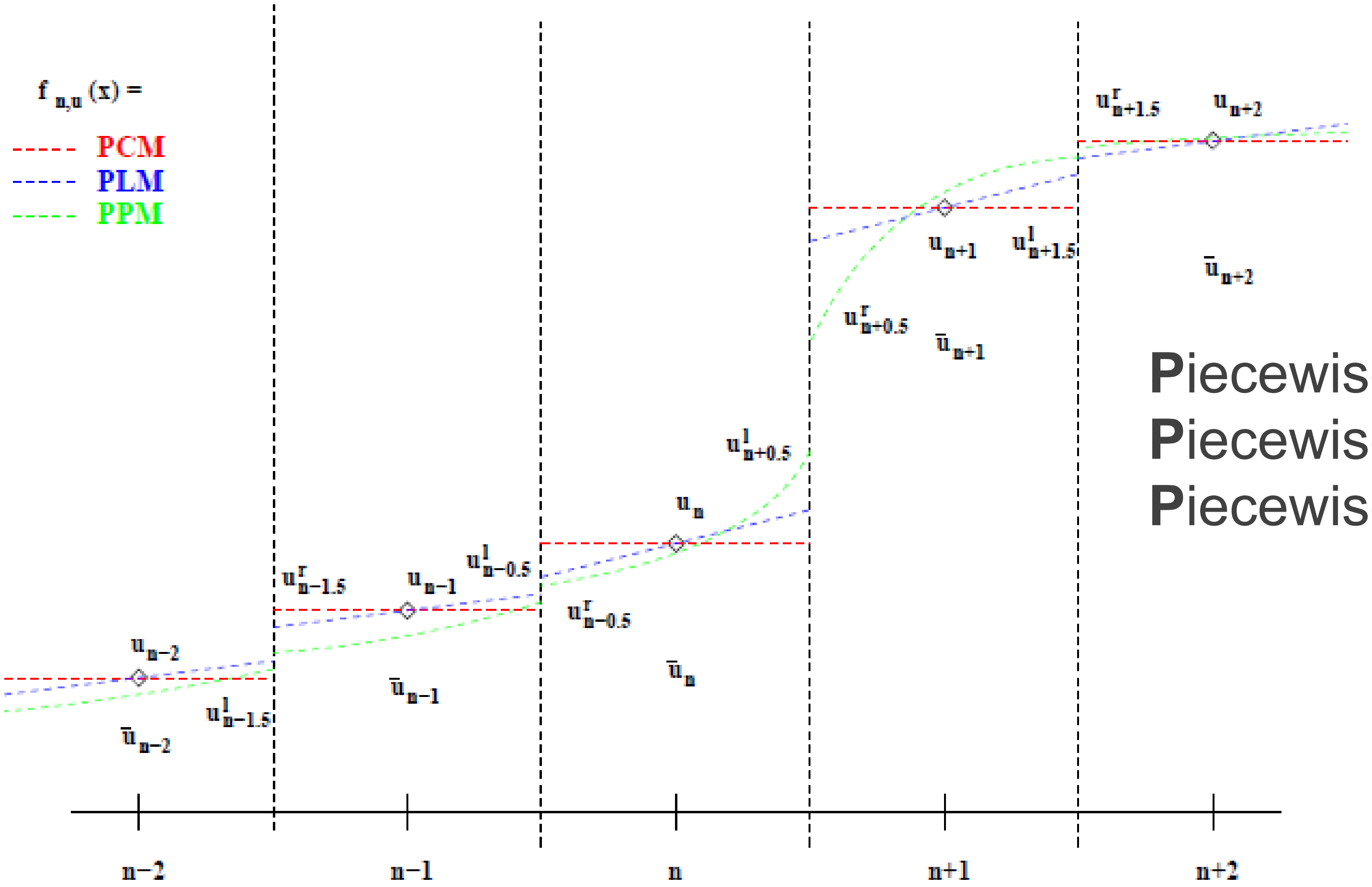
Reconstruct: define the run of a quantity U^n in a cell to compute the cell-averaged value

Evolve the reconstructed state by solving the Riemann problem at the cell interface and use this to evolve by Δt

Average: account for the fluxes entering and leaving the cell volume to compute the evolved cell-averaged state U^{n+1}

Slope Limiters: additional constraint on reconstruction, to prevent new extrema and unphysical oscillations in the solution

Reconstruction accuracy



Piecewise **C**onstant **M**ethod
Piecewise **L**inear **M**ethod
Piecewise **P**arabolic **M**ethod

- **Newer schemes:** (W)ENO: (Weighted) Essentially Non Oscillatory,
  MP: Monotonicity Preserving;
  Use more grid points with higher-order reconstruction in smooth part of the fluid, and sharp discontinuities at the shocks.

Extension to multiple dimensions

Euler equations in 3D:

$$\partial_t \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e \end{pmatrix} + \partial_x \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uw \\ \rho u(\rho e + P) \end{pmatrix} + \partial_y \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho vw \\ \rho v(\rho e + P) \end{pmatrix} + \partial_z \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + P \\ \rho w(\rho e + P) \end{pmatrix} = 0,$$

$e = e_{\mathrm{therm}} + (u^2 + v^2 + w^2)/2$: total specific energy

$\partial_t \mathbf{U} + \partial_x \mathbf{F} + \partial_y \mathbf{G} + \partial_z \mathbf{H} = 0$ $\quad \mathbf{F}, \mathbf{G}, \mathbf{H}$: fluxes along *x*, *y*, *z*

Dimensional splitting:

$$\partial_t \mathbf{U} + \partial_x \mathbf{F} = 0,$$
$$\partial_t \mathbf{U} + \partial_y \mathbf{G} = 0,$$
$$\partial_t \mathbf{U} + \partial_z \mathbf{H} = 0.$$

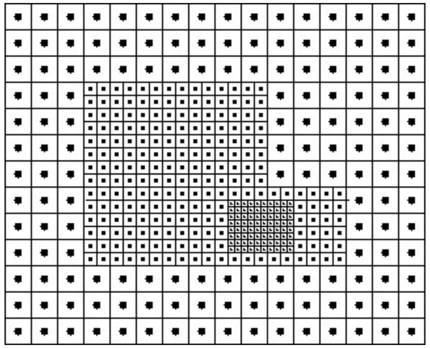
- Differentiation in one dimension each
- Flux updates made sequentially

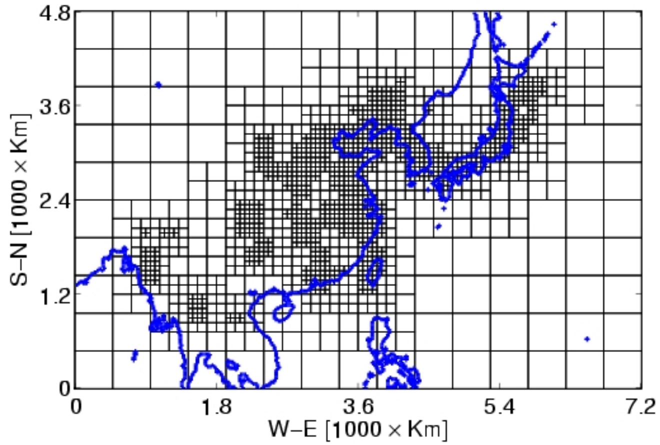
Unsplit scheme: flux updates applied simultaneously
For the 2D Cartesian case:

$$U_{i,j}^{n+1} = U_{i,j}^{n} + \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i-\frac{1}{2},j} - \mathbf{F}_{i+\frac{1}{2},j}\right) + \frac{\Delta t}{\Delta y}\left(\mathbf{G}_{i,j-\frac{1}{2}} - \mathbf{G}_{i,j+\frac{1}{2}}\right)$$

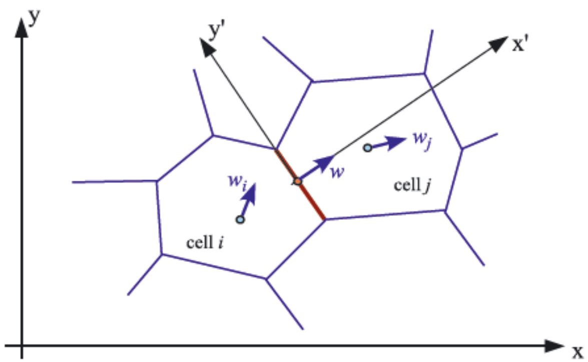
Different Eulerian codes

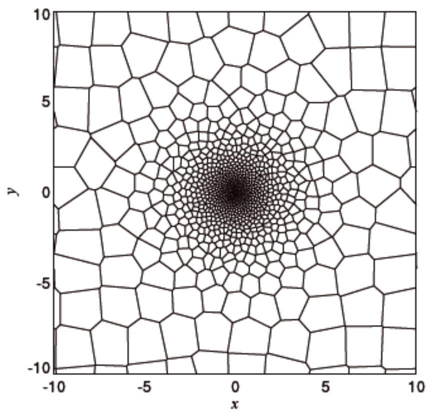
- Uniform grid: fixed resolution.
- AMR, Adaptive Mesh Refinement: hierarchy of nested grids to increase resolution where needed

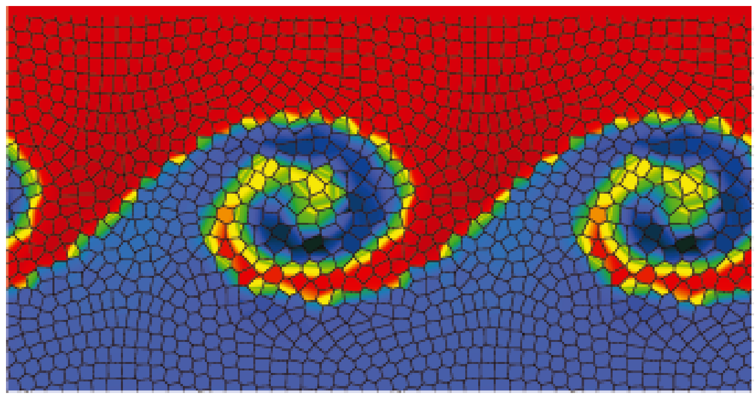




- Moving mesh (e.g., AREPO)

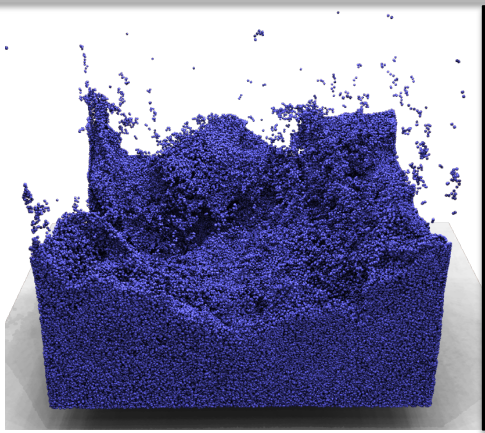
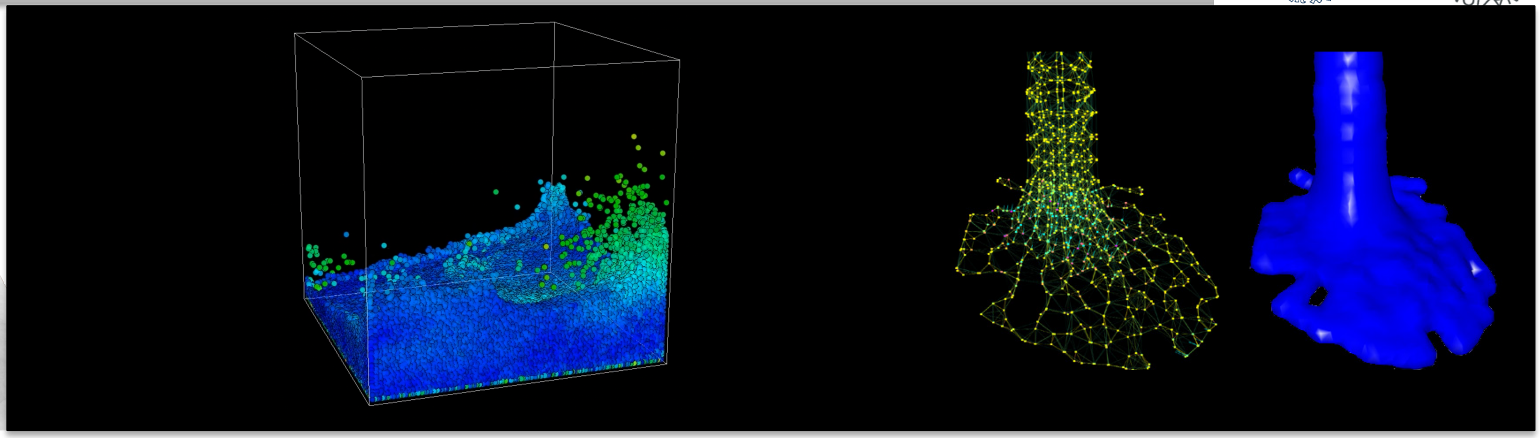






- …meshless (e.g. GSPH).

Smoothed Particle Hydrodynamics (SPH)

Basic principles:

- Fluid sampled with points (*particles*)
- Hydrodynamic quantities are carried by each particle, but their values are *smoothed* over a given number of neighbouring particles
- Particles move under the Euler equations making use of the smoothed quantities
- After each *time-step*, quantities are re-evaluated

Smoothing kernel & interpolating function

➔ Interpolating function: $\tilde{f}_h(\vec{r}) = \int f(\vec{r'}) W(\vec{r} - \vec{r'}, h)\, d^3r'$

$W(r)$: interpolating kernel

h : *smoothing lenght*

➔ <u>Properties of the kernel:</u>

$$\lim_{h \to 0} \tilde{f}_h(\vec{r}) = f(\vec{r}) \;\text{ and }\; \int W(\vec{r} - \vec{r'}, h)\, d^3r' = 1$$

To recover the original function in the limit of small h

➔ <u>Discretization:</u>

$$\tilde{f}_h(\vec{r}) = \int \frac{f(\vec{r'})}{\rho(\vec{r'})} W(\vec{r} - \vec{r'}, h)\, \rho(\vec{r'}) d^3r' \;\rightarrow\; f(\vec{r}) = \sum_b \frac{m_b}{\rho_b} f_b W(\vec{r} - \vec{r}_b, h)$$

$$f(\vec{r}) = \rho(\vec{r}) \;\rightarrow\; \boxed{\rho(\vec{r}) = \sum_b m_b W(\vec{r} - \vec{r}_b, h)}$$

Kernel function: additional properties

- Radial kernel for conservation of angular momentum

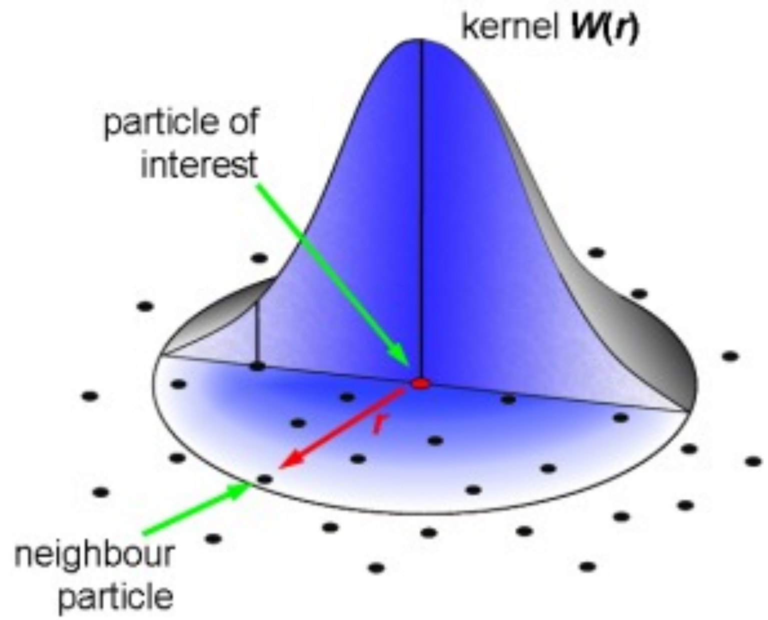
$$W(\vec{r}-\vec{r}', h) = W(|\vec{r}-\vec{r}'|, h)$$

- Compact support to avoid n^2 interactions per particle

➔ Widely used **cubic spline kernel**:

$$W(q) = \frac{1}{\pi h^3}\begin{cases} 1-\frac{3}{2}q^2+\frac{3}{4}q^3 & \text{for } 0 \le q \le 1 \\ \frac{1}{4}(2-q)^3 & \text{for } 1 < q \le 2 \\ 0 & \text{for } q > 2 \end{cases} \qquad q = |\vec{r}-\vec{r}'|/h$$

➔ Gaussian kernel: useful for analytic computations but no compact support



Derivatives in SPH

- Differentiate the discretized equation: $\nabla f(\vec{r}) = \sum_b \frac{m_b}{\rho_b} f_b \nabla W(\vec{r} - \vec{r}_b, h)$

  It doesn't vanish for $f(r)=cost$

- To enforce it: $\frac{\partial A}{\partial x} = \frac{1}{\Phi}\left(\frac{\partial(\Phi A)}{\partial x} - A\frac{\partial \Phi}{\partial x}\right)$

- In SPH this is: $\left(\frac{\partial A}{\partial x}\right)_a = \frac{1}{\Phi_a}\sum_b m_b \frac{\Phi_b}{\rho_b}(A_b - A_a)\frac{\partial W_{ab}}{\partial x_a}$

$$W_{ab} = W(\vec{r}_a - \vec{r}_b, h)$$

$\Phi = 1$ $\boxed{\frac{\partial A_a}{\partial x_a} = \sum_b \frac{m_b}{\rho_b}(A_b - A_a)\frac{\partial W_{ab}}{\partial x_a}}$ $\Phi = \rho$ $\boxed{\frac{\partial A_a}{\partial x_a} = \frac{1}{\rho_a}\sum_b m_b(A_b - A_a)\frac{\partial W_{ab}}{\partial x_a}}$

- The continuity equation becomes:

$\frac{d\rho}{dt} = -\rho\nabla\cdot\vec{v}.$ $\boxed{\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = \rho_a \sum_b \frac{m_b}{\rho_b} v_{ab}\cdot\nabla_a W_{ab}}$ $\mathrm{v}_{ab} = \mathrm{v}_a - \mathrm{v}_b$

Euler equation in SPH

$$\frac{d\vec{v}}{dt} = -\frac{\nabla P}{\rho}$$

➔ Brute force differentiation of pressure: $\dfrac{d\vec{v}_a}{dt} = -\dfrac{1}{\rho_a}\sum_b \dfrac{m_b}{\rho_b} P_b \nabla_a W_{ab}$

$$\vec{F}_{ba} = \left(m_a \frac{d\vec{v}_a}{dt}\right)_b = -\frac{m_a}{\rho_a}\frac{m_b}{\rho_b} P_b \nabla_a W_{ab}$$

$$\vec{F}_{ab} = \left(m_b \frac{d\vec{v}_b}{dt}\right)_a = -\frac{m_b}{\rho_b}\frac{m_a}{\rho_a} P_a \nabla_b W_{ba} = \frac{m_a}{\rho_a}\frac{m_b}{\rho_b} P_a \nabla_a W_{ab}$$

If $P_a \neq P_b \quad \Rightarrow \vec{F}_{ba} \neq -\vec{F}_{ab}$

➔ Momentum not conserved

➔ Use instead: $\nabla\left(\dfrac{P}{\rho}\right) = \dfrac{\nabla P}{\rho} - P\dfrac{\nabla \rho}{\rho^2}$

$$\boxed{\begin{aligned}\frac{d\vec{v}_a}{dt} &= -\frac{P_a}{\rho_a^2}\sum_b m_b \nabla_a W_{ab} - \sum_b \frac{m_b}{\rho_b}\frac{P_b}{\rho_b}\nabla_a W_{ab} \\ &= -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right)\nabla_a W_{ab}.\end{aligned}}$$

➔ Pressure part manifestly symmetric

➔ Momentum now conserved

Energy equation in SPH

$$\frac{du}{dt} = \frac{P}{\rho^2}\frac{d\rho}{dt} = -\frac{P}{\rho}\nabla\cdot\vec{v}.$$

$$\boxed{\frac{du_a}{dt} = \frac{P_a}{\rho_a^2}\frac{d\rho_a}{dt} = \frac{P_a}{\rho_a^2}\frac{d}{dt}\left(\sum_b m_b W_{ab}\right) = \frac{P_a}{\rho_a^2}\sum_b m_b \vec{v}_{ab}\cdot\nabla_a W_{ab}}$$

Based on using:

$$\frac{dW_{ab}}{dt} = \frac{\partial W_{ab}}{\partial r_{ab}}\frac{dr_{ab}}{dt} = \frac{\partial W_{ab}}{\partial r_{ab}}\frac{(\vec{r}_a - \vec{r}_b)\cdot(\vec{v}_a - \vec{v}_b)}{r_{ab}} = \frac{\partial W_{ab}}{\partial r_{ab}}\hat{e}_{ab}\cdot\vec{v}_{ab} = \vec{v}_{ab}\cdot\nabla_a W_{ab}.$$

<u>Equation of state:</u> $\boxed{P_a = (\gamma - 1)\rho_a u_a}$

...these make a full set of SPH equations!

Additional features

➔ Entropy-conserving formulation:

$P = A(s)\rho^{\gamma}$, s: specific entropy of the fluid element
$A(s)$: entropic function

In the absence of any cooling or viscosity terms:

$$\frac{dA(s)}{dt} = 0$$

➔ Entropy conservation enforced at the particle level, then internal energy computed from

$$u = \frac{A(s)}{\gamma - 1}\rho^{\gamma - 1}$$

➔ Adaptive softening: $\rho_i h_i^3 = const$: enforce constant mass within the kernel

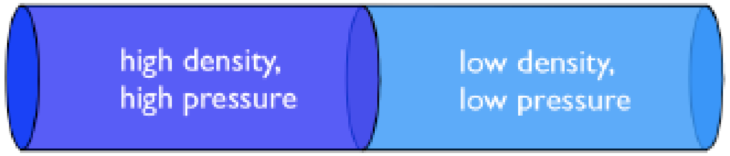
$$\frac{d\mathbf{v}_i}{dt} = -\sum_{j=1}^{N} m_j \left[f_i \frac{P_i}{\rho_i^2} \nabla_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2} \nabla_i W_{ij}(h_j) \right]$$

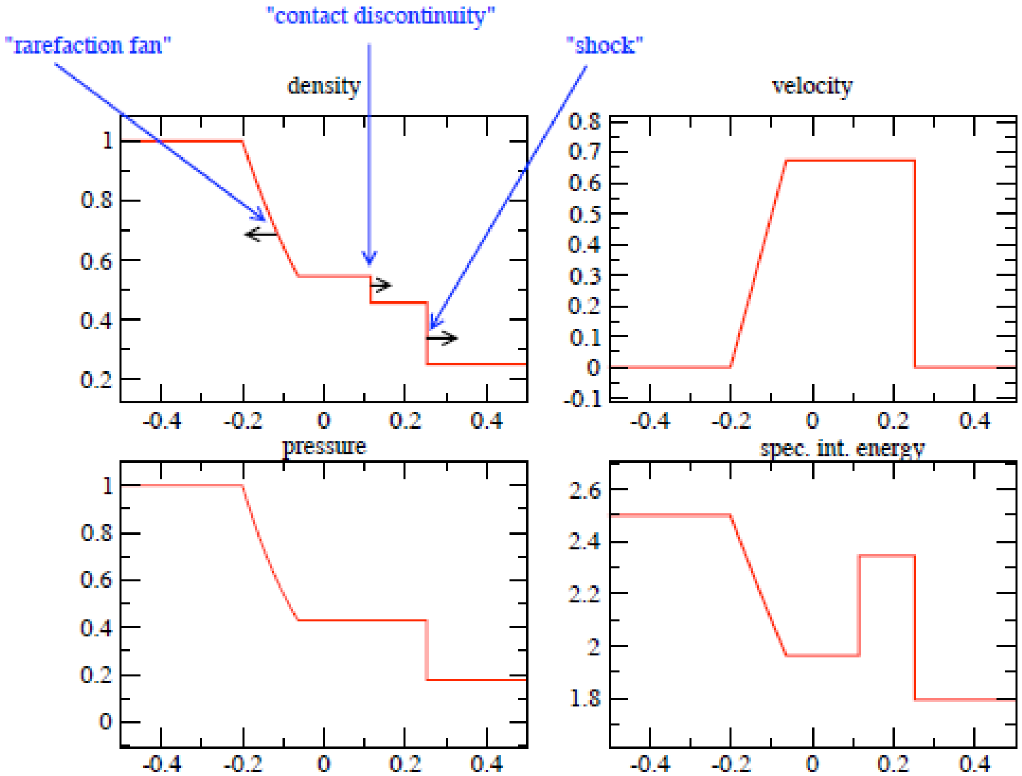
$$f_i = \left[1 + \frac{h_i}{3\rho_i} \frac{\partial \rho_i}{\partial h_i} \right]^{-1}$$

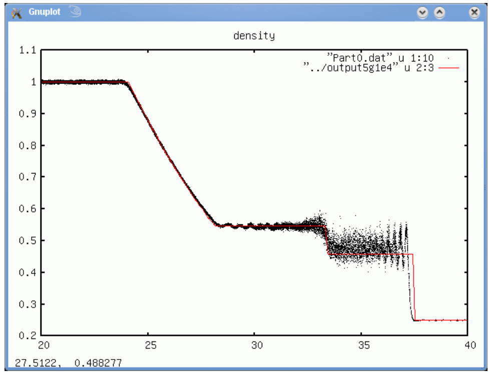
$$\frac{du_i}{dt} = f_i \frac{P_i}{\rho_i^2} \sum_j m_j (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla W_{ij}(h_i)$$

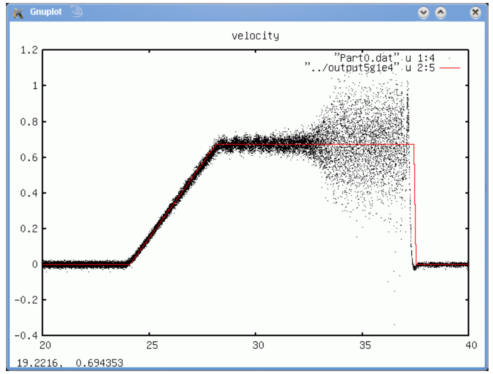
➔ To account for local softening

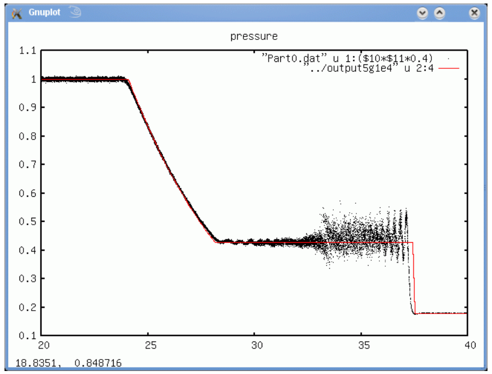
Sode shock tube: SPH solution











- ➔ Velocity noise in the pre-shock region
- ➔ Lack of diffusion of momentum
- ➔ Need to convert mechanical energy into thermal energy

Artificial viscosity

➔Add an "artificial" viscous contribution to pressure in the Euler eq.:

$$q_{\rm visc} = \underbrace{-c_1 \rho c_{\rm s} l (\nabla \cdot \vec{v})}_{\text{Bulk viscosity}} + \underbrace{c_2 \rho l^2 (\nabla \cdot \vec{v})^2}_{\text{Von Neumann-Richtmyer viscosity}}$$

$l\,(\nabla \cdot \vec{v})$ ~ velocity jump between two fluid elements; c_s: adiabatic sound speed

➔SPH translation: $\left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right) \rightarrow \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab}\right)$

$$\Pi_{ab} = \Pi_{ab,\rm bulk} + \Pi_{ab,\rm NR} = \begin{cases} \frac{-\alpha \bar{c}_{ab} \mu_{ab} + \beta \mu_{ab}^2}{\bar{\rho}_{ab}} & \text{for } \vec{r}_{ab} \cdot \vec{v}_{ab} < 0 \\ 0 & \text{otherwise} \end{cases} \qquad \mu_{ab} = \frac{\bar{h}_{ab} \vec{r}_{ab} \cdot \vec{v}_{ab}}{r_{ab}^2 + \epsilon \bar{h}_{ab}^2}$$

$\alpha \approx 1$, $\beta \approx 2$ and $\epsilon \approx 0.01$ from numerical experiments

➔ SPH equations with artificial viscosity:

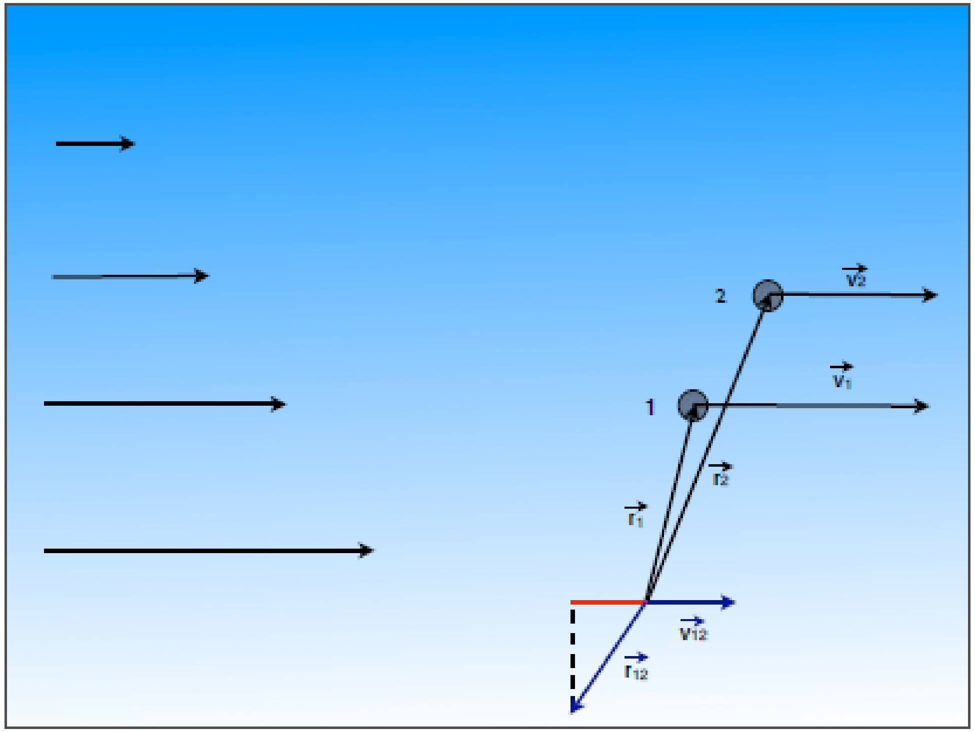
$$\boxed{\frac{d\vec{v}_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab}\right) \nabla_a W_{ab}}$$

$$\boxed{\frac{du_a}{dt} = \frac{P_a}{\rho_a^2} \sum_b m_b \vec{v}_{ab} \cdot \nabla_a W_{ab} + \frac{1}{2} \sum_b m_b \Pi_{ab} \vec{v}_{ab} \cdot \nabla_a W_{ab}}$$

Artificial viscosity

➔ Spurious viscous force in the absence of shocks (e.g. in shear flows)

$$\mu_{ab} = \frac{\bar{h}_{ab}\vec{r}_{ab}\cdot\vec{v}_{ab}}{r_{ab}^2 + \epsilon\bar{h}_{ab}^2}$$



<u>"Balsara switch":</u>

$$f_a = \frac{|\langle\nabla\cdot\vec{v}\rangle_a|}{|\langle\nabla\cdot\vec{v}\rangle_a| + |\langle\nabla\times\vec{v}\rangle_a| + 0.0001c_{s,a}/h_a}$$

$$\Pi'_{ab} = \Pi_{ab}\,\bar{f}_{ab} \qquad \bar{f}_{ab} = (f_a + f_b)/2$$

=0 in shear flows

=0 in purely compressional flows

➔ <u>Time-dependent viscosity</u>: to make viscosity decay away from shocks

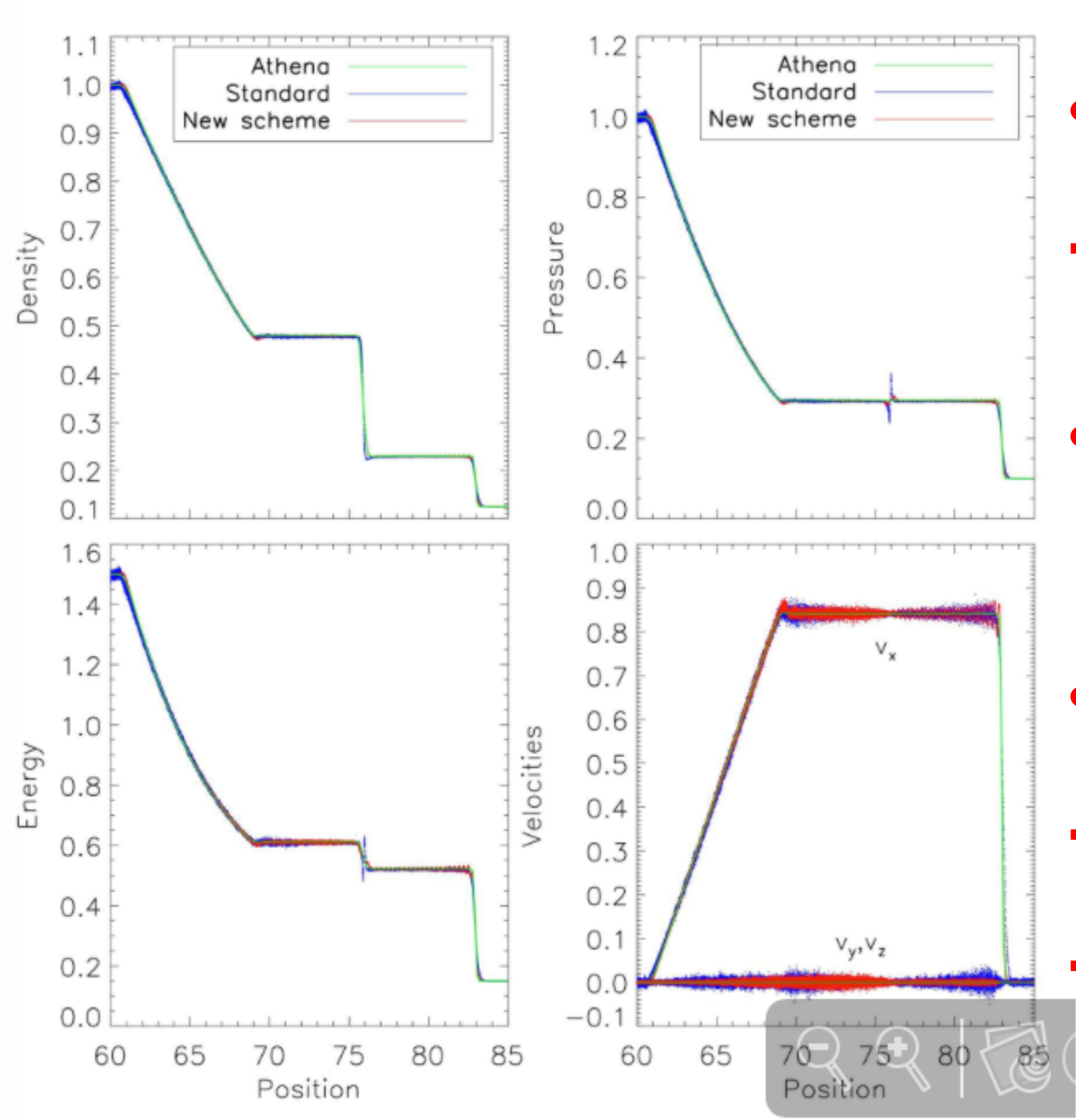
$$\frac{d\alpha_a}{dt} = -\frac{\alpha_a - \alpha_{\min}}{\tau_a} + S_a; \qquad \tau_a = \frac{h_a}{\xi c_{s,a}}$$

➔Decay time-scale of viscosity

$$S_a = \max\left[-(\vec{\nabla}\cdot\vec{v})_a, 0\right]$$

➔Source term

Artificial viscosity: shock tube test

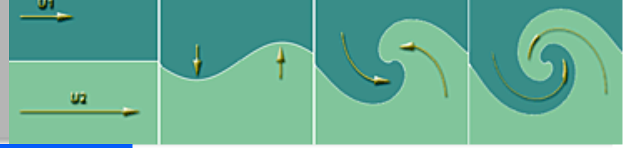


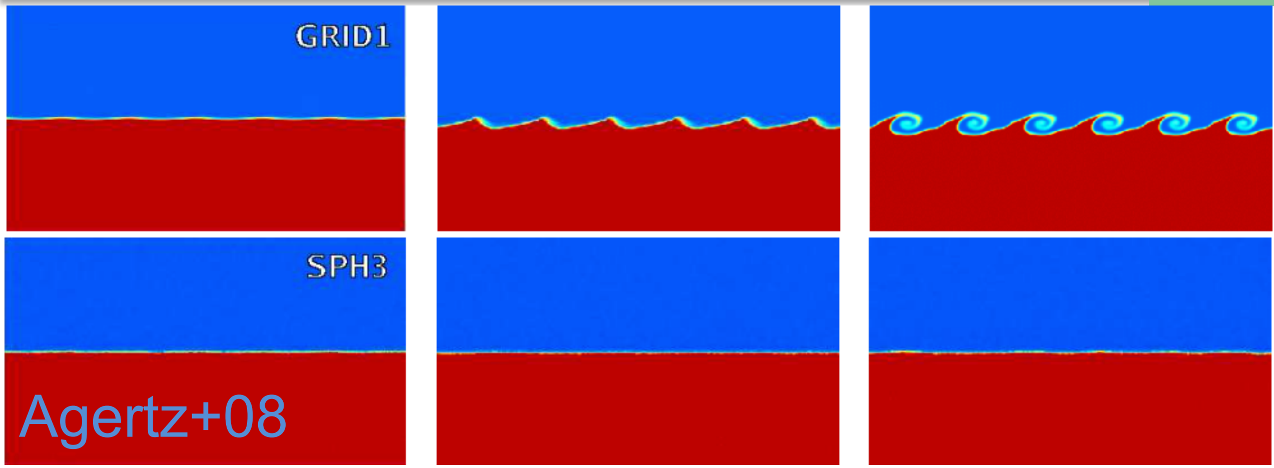
- Analytic solution better recovered
➔ Pre-shock velocity noise much reduced

- Blip in pressure and energy at the contact discontinuity

- Spurious pressure force causing "surface tension"
➔ Entropy preserved at the particle level
➔ No diffusion of energy across the discontinuity

Kelvin-Helmholtz instability

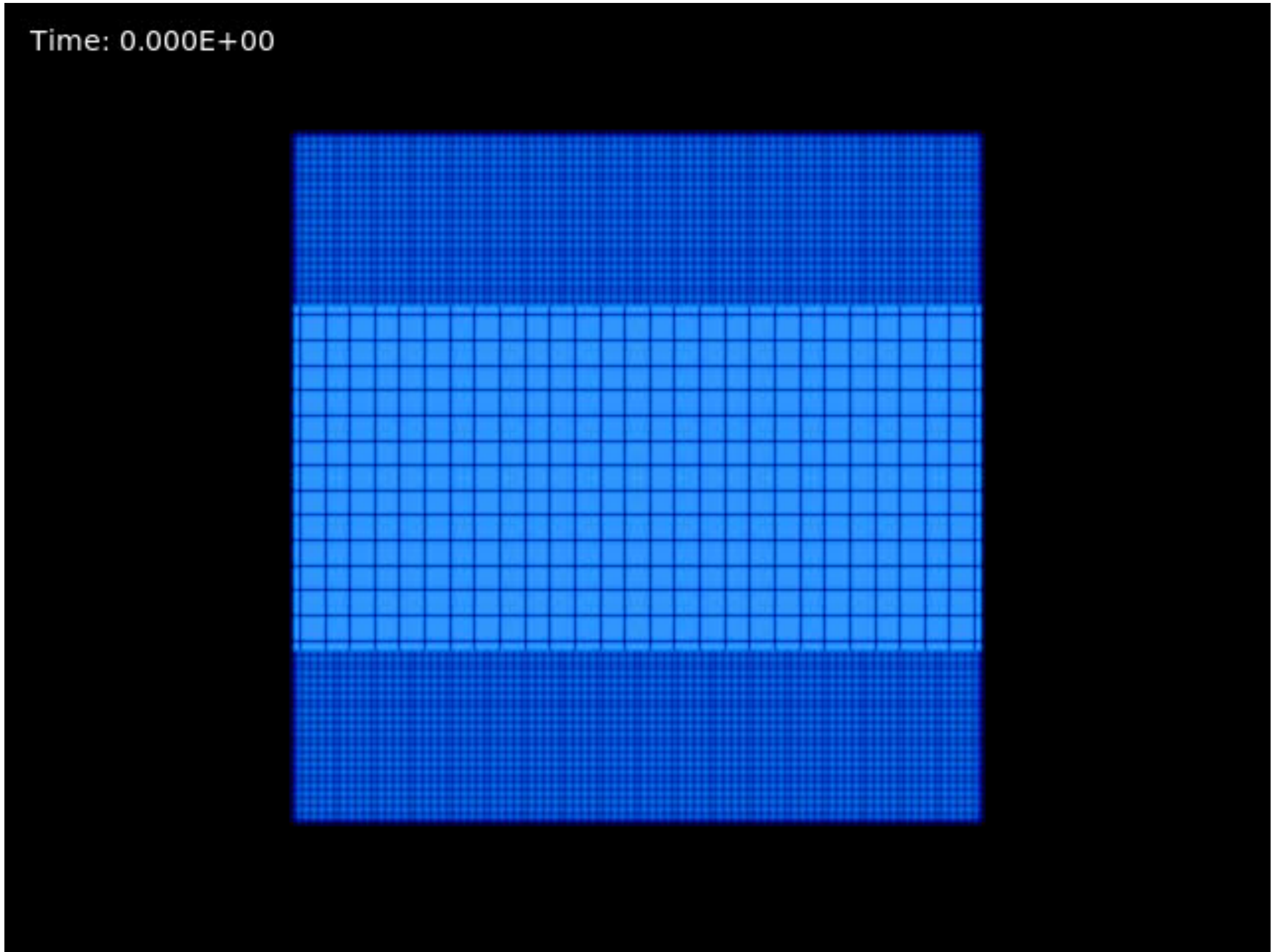




$$\chi = \rho_{\rm b}/\rho_{\rm t} = T_{\rm t}/T_{\rm b} = c_{\rm t}^2/c_{\rm b}^2$$

$$v_y(x) = \delta v_{\rm y} \sin(\lambda 2\pi x)$$

$$\tau_{KH} = \frac{\lambda(\rho_b + \rho_t)}{(v_b + v_t)(\rho_b \rho_t)^{1/2}}$$



- SPH: surface tension prevents the development of instabilities
- KH instability followed by Eulerian schemes

They happen in real life!!

Artificial thermal energy diffusion

➔ Dissipation term for a <u>conserved</u> scalar quantity A:

$$\left(\frac{dA_a}{dt}\right)_{\rm diss} = \sum_b m_b \frac{\alpha_{A,b} v_{\rm sig}}{\bar{\rho}_{ab}} (A_a - A_b)\hat{e}_{ab} \cdot \nabla_a W_{ab}$$

$\alpha_{A,b}$: amount of dissipation

$v_{\rm sig}$: maximum signal velocity between a and b

$$\left(\frac{d\vec{v}_a}{dt}\right)_{\rm diss} = \sum_b m_b \frac{\alpha v_{\rm sig} (\vec{v}_a - \vec{v}_b) \cdot \hat{e}_{ab}}{\bar{\rho}_{ab}} \nabla_a W_{ab}$$

in the momentum equation

$$\left(\frac{du_a}{dt}\right)_{\rm diss} = -\sum_b \frac{m_b}{\bar{\rho}_{ab}} \left[\alpha v_{\rm sig} \frac{1}{2}(\vec{v}_{ab} \cdot \hat{e}_{ab})^2 + \alpha_u v^u_{\rm sig}(u_a - u_b)\right] \hat{e}_{ab} \cdot \nabla_a W_{ab}$$

in the energy equation

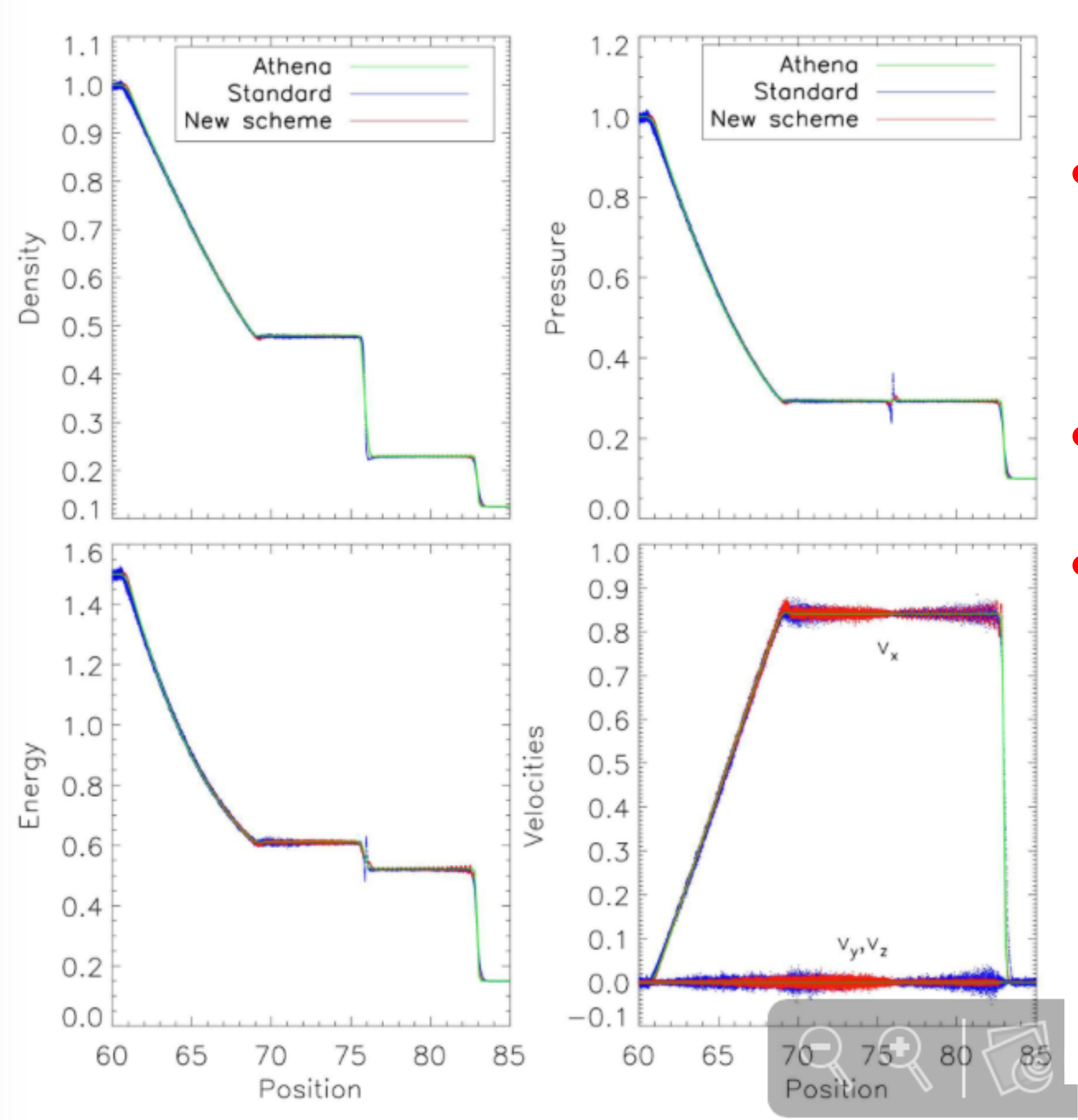
Artificial viscosity

Thermal energy diffusion

$v_{\rm sig} = c_{{\rm s},a} + c_{{\rm s},b} - \vec{v}_{ab} \cdot \hat{e}_{ab}$: signal velocity for momentum diffusion (Monaghan 97)

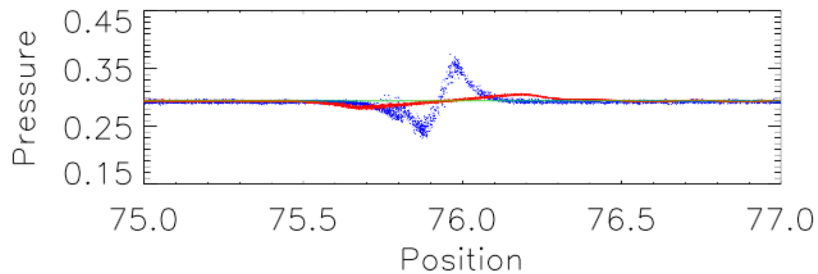
$v^u_{\rm sig} = \sqrt{\frac{|P_a - P_b|}{\bar{\rho}_{ab}}}$: signal velocity for thermal energy diffusion (Price 08)
Need to be corrected in the presence of an external (gravitational) potential

Shock tube test – Thermal diffusion in SPH

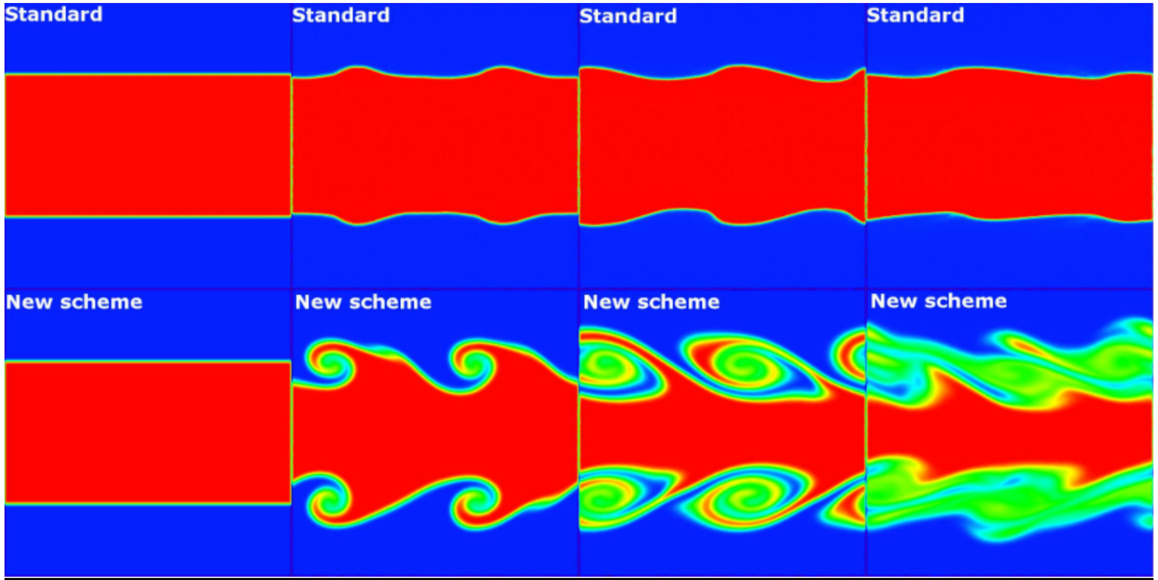


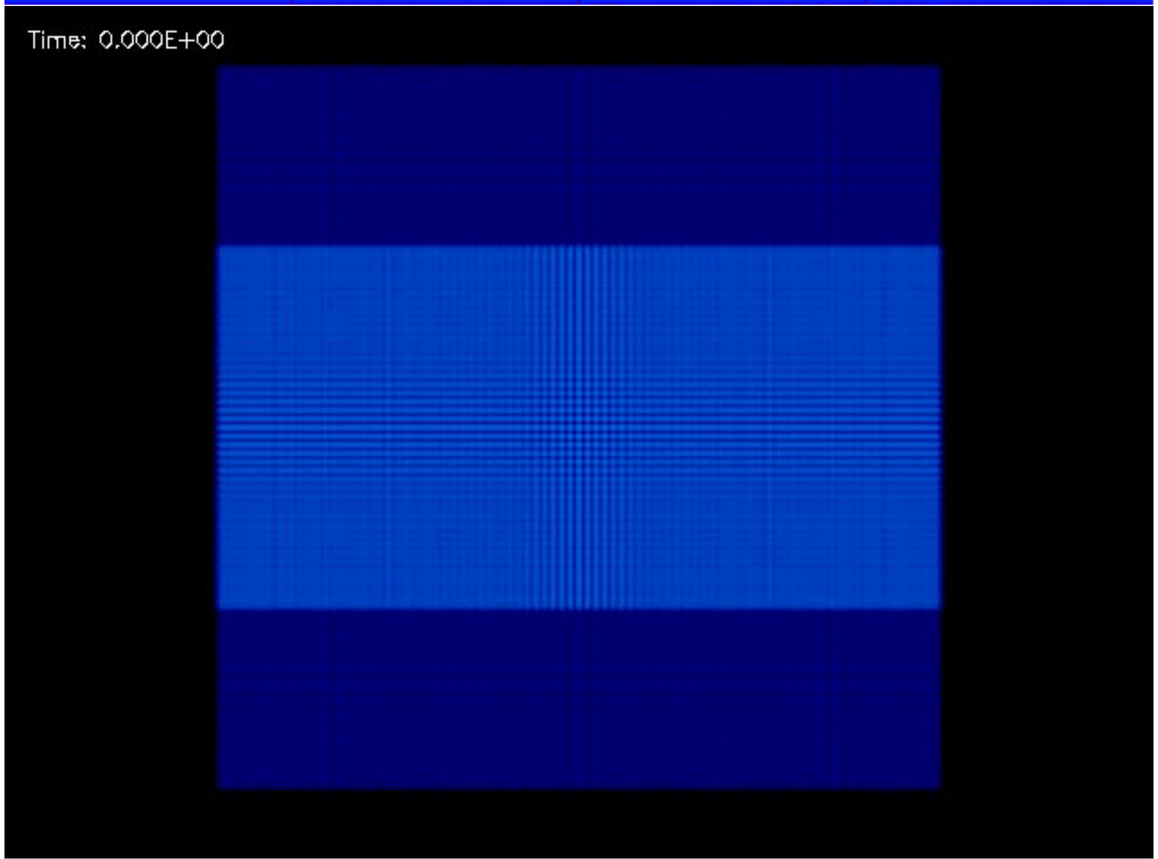
Beck+15

- Blip in pressure and energy much reduced by diffusion
- "Surface tension" reduced
- Particle noise also reduced



KH test with thermal diffusion





Beck+15

- Removal of surface tension allows development of instabilities
- Instabilities followed for several characteristic time-scales
- SPH and Eulerian results virtually indistinguishable

Eulerian vs Lagrangian

SPH pros

- Increases resolution where needed
- Easily coupled to N-Body codes
- Intrinsically Galilean-invariant

SPH cons

- Low-order accuracy for treatmen of contact discontinuities
- Sub-sonic velocity noise
- Poor shock resolution
- Difficulty in following hydro instabilities

Eulerian pros

- Sharp discontinuities and shocks accurately captured
- Hydrodynamical instabilities followed for several characteristic times

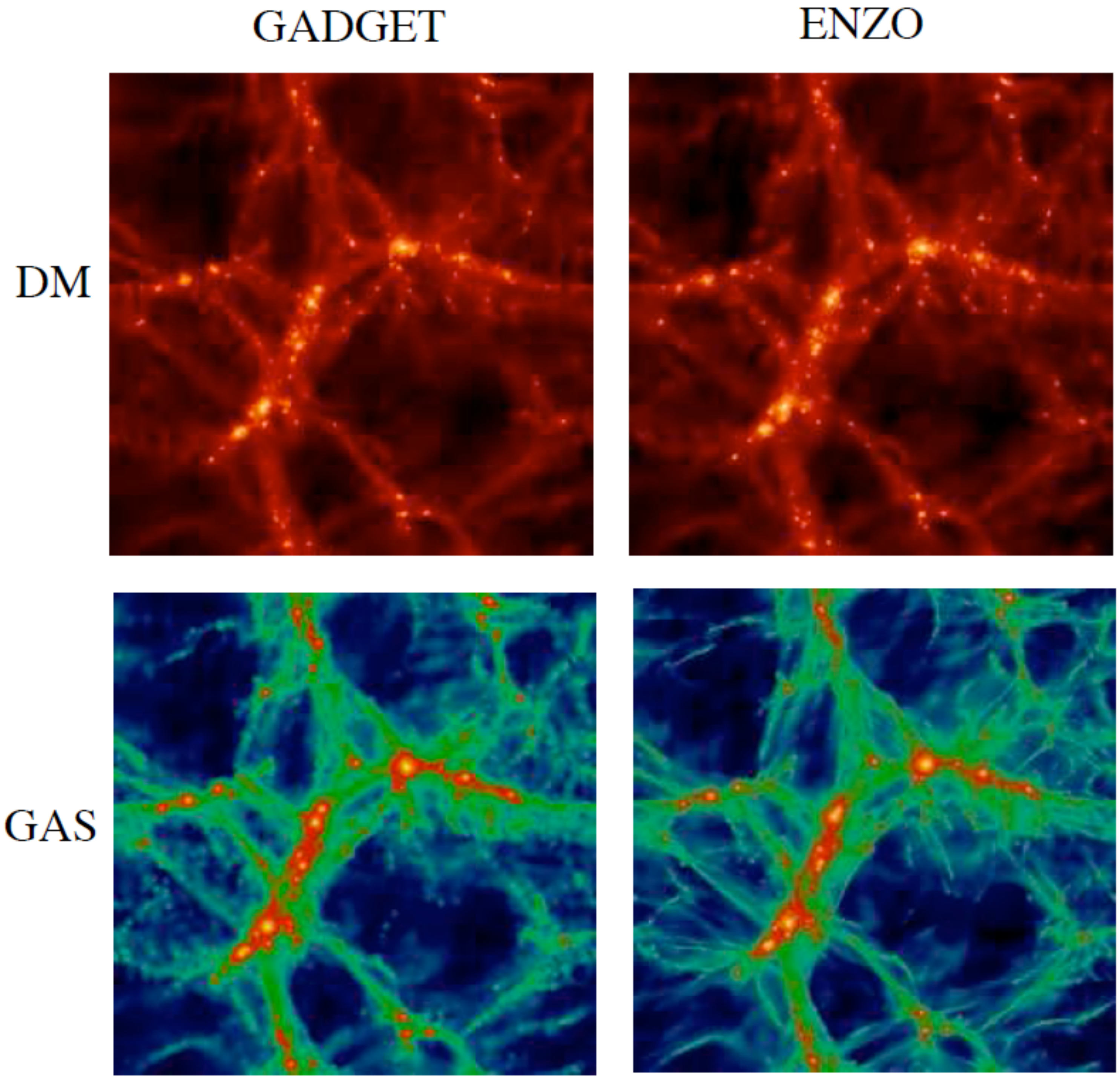
Eulerian cons

- Not manifestly Galilean invariant
- Preference of spatial directions
- Adaptive resolution not trivial
- Degree of diffusivity not easily controlled

➔ Need "ad-hoc" switches

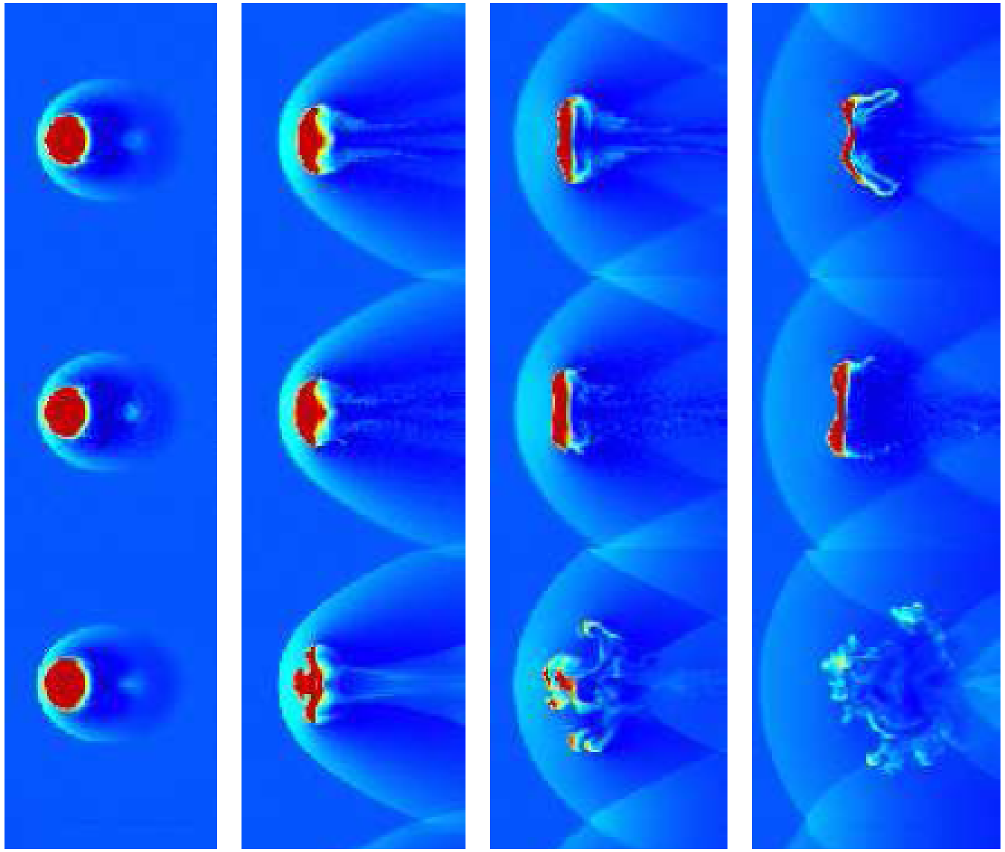
➔ Need control of diffusivity

O'Shea+05



The test of the “cold blob”

Agertz+08



Gasoline SPH

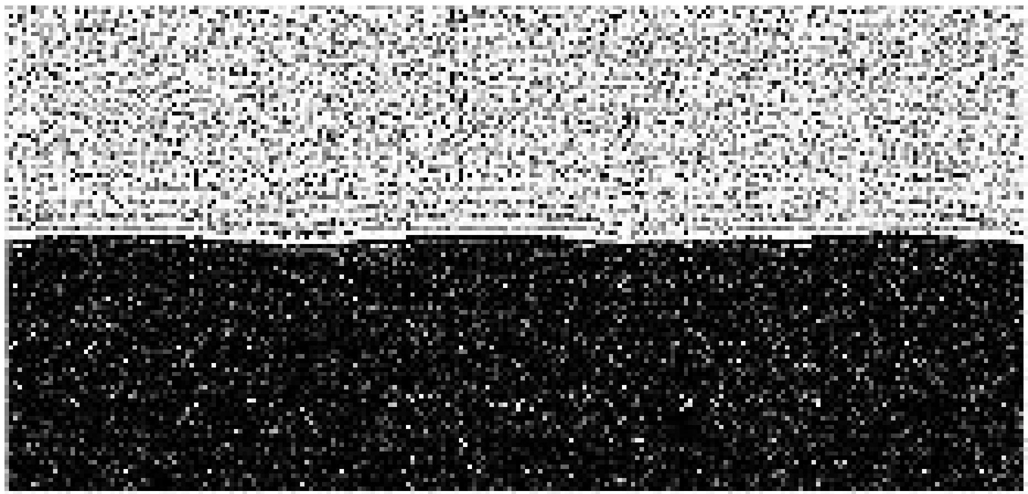
Gadget-2 SPH

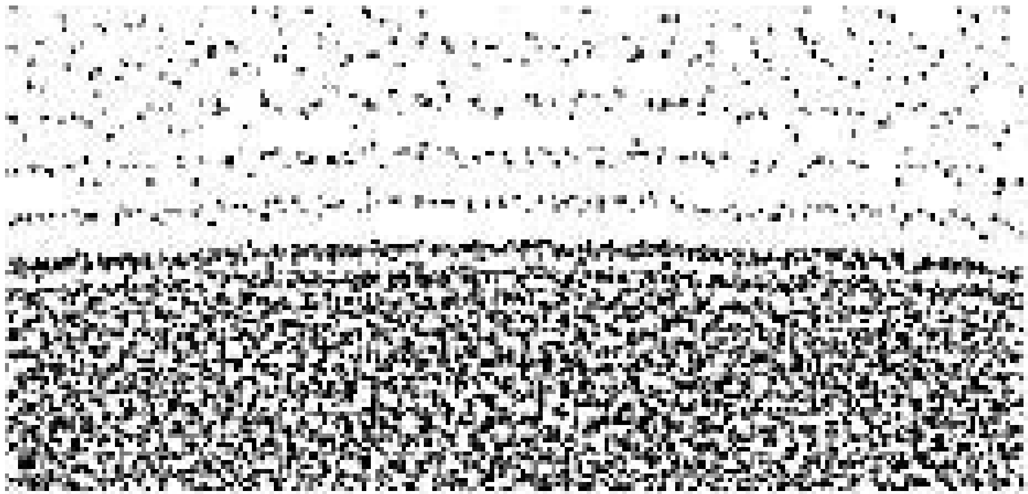
ENZO Grid

Cold dense cloud moving in a low-density hot medium in pressure equilibirum

Spurious pressure forces in SPH at density discontinuitie;

➔Suppression of KH and RT instabilities.

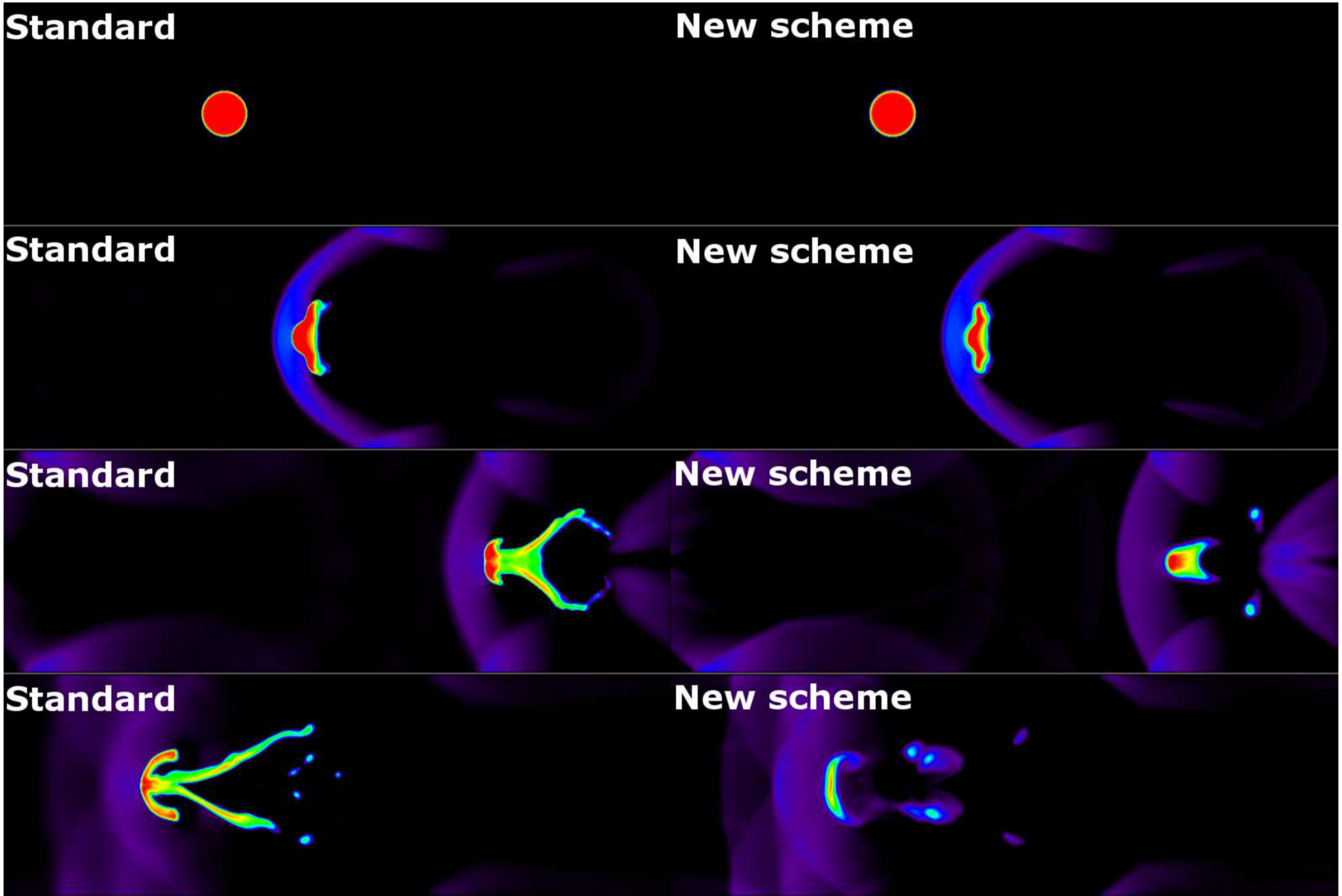




Cold blob with “diffusive” SPH

Beck+15

➔Artificial thermal diffusion promotes mixing, breaks the tension force and allows dissociation of the cloud

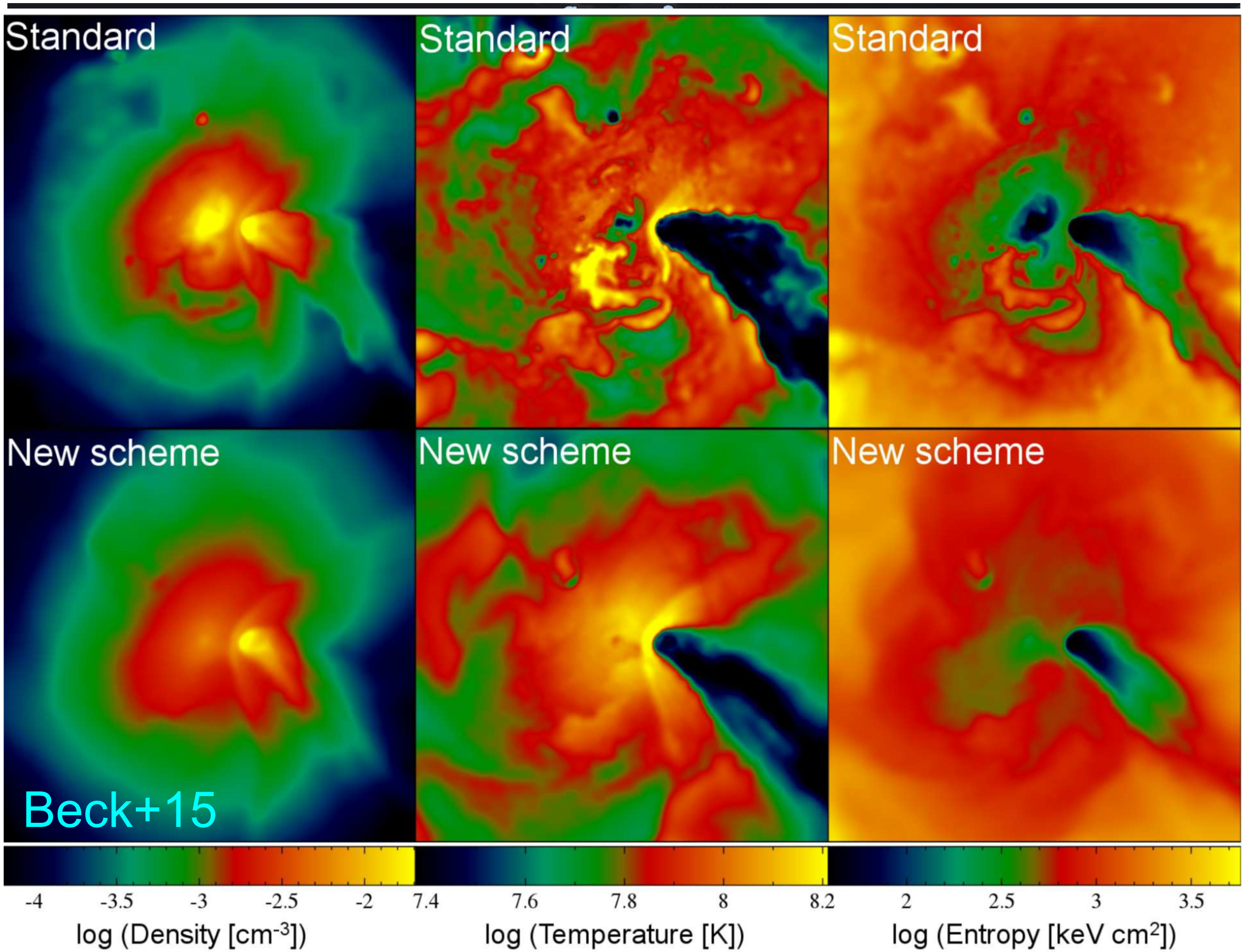


Idealised merger between two clusters

From Mitchell+09

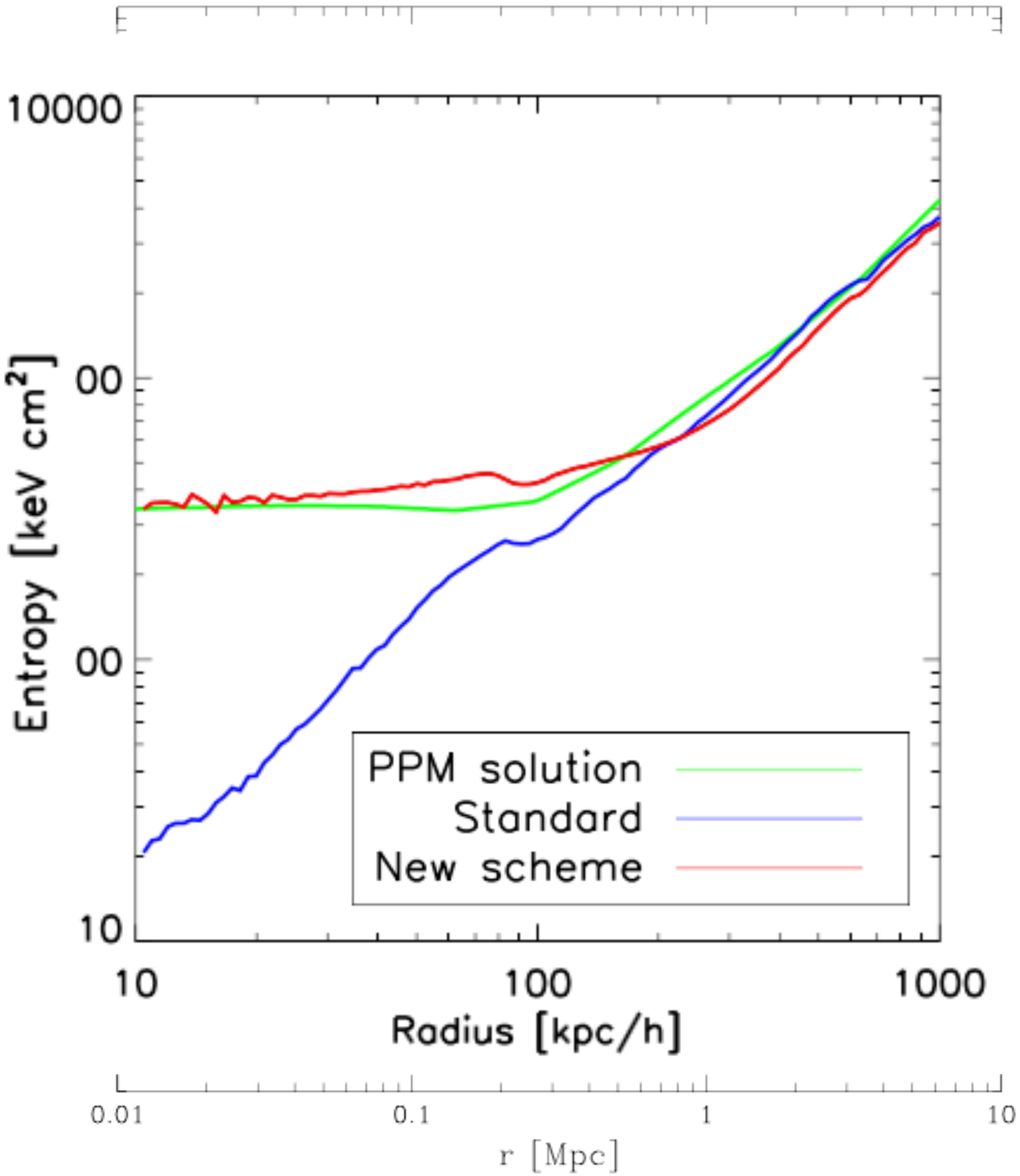
Santa Barbara cluster

➔ Cosmological build-up of a massive cluster in an EdS CDM Universe (Frenk+99)



Entropy profiles

$$K(r) = \frac{T(r)}{[\rho_{gas}(r)]^{2/3}}$$



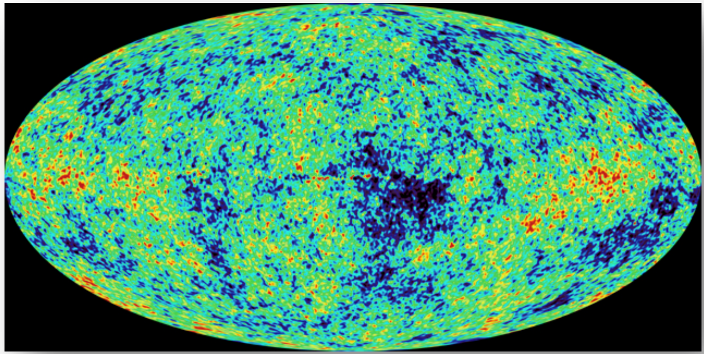
- Frenk+99: SB cluster comparison
  ➔ 1st evidence for AMR to produce entropy cores wrt SPH

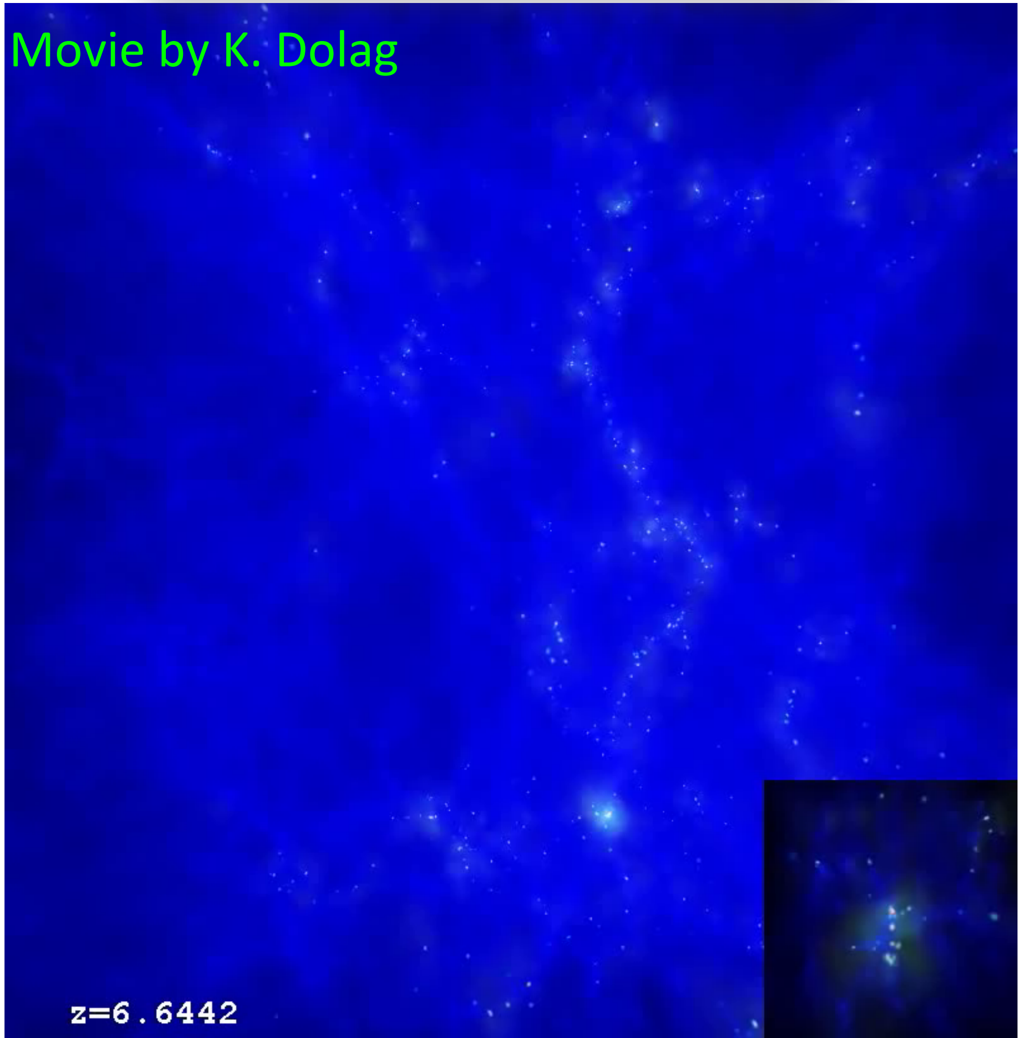
- Beck+15: effect of thermal diffusion in SPH
  ➔ TD to promote mixing and creation of entropy cores
  ➔ Now SPH and Eulerian quite close

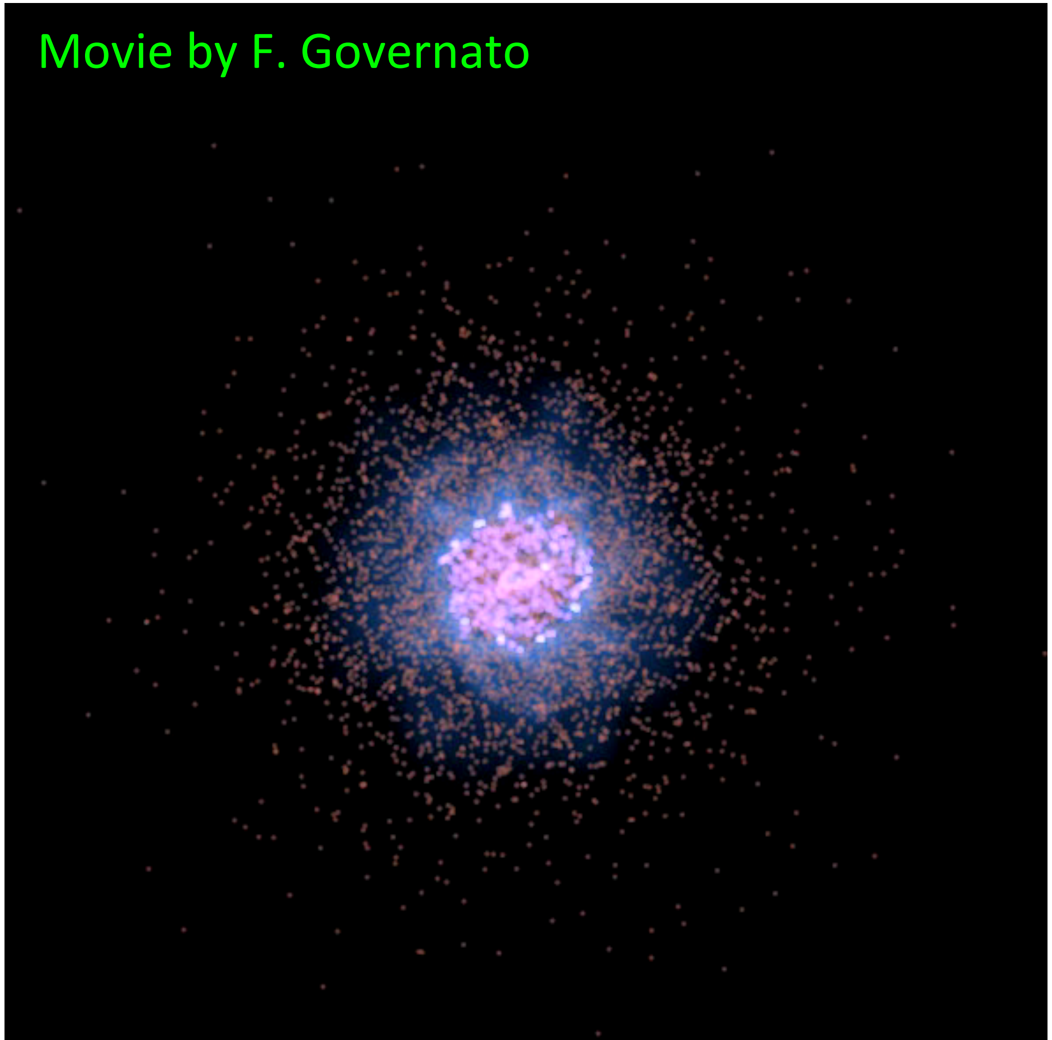
- Sembolini+15: comparison of a variety of SPH and Eulerian codes

Part III:
Application to Formation of Cosmic Structures

What’s a cosmological simulation?







Initial conditions for cosmological simulations

Problem: generate a distribution of particles whose positions and velocities are a realization of a **Gaussian** random field with a given **power spectrum**:

$$\hat{\delta}_k = \left|\hat{\delta}_k\right| e^{i\theta_k} \qquad\qquad P\left(\left|\hat{\delta}_k\right|, \vartheta_k\right) = \frac{\left|\hat{\delta}_k\right|}{P(k)} \exp\left(-\frac{\left|\hat{\delta}_k\right|^2}{P(k)}\right) d\hat{\delta}_k \, d\vartheta_k$$

➔ Rayleigh distribution for $|\delta_k|$ and uniform distribution for θ_k

Step 1: generate a Gaussian δ_k on a grid in Fourier space:

$$\hat{\delta}_k = \sqrt{-2P(k)\ln r_1}\; e^{i2\pi\, r_2} \quad ; \quad r1, r2 : \text{random in } [0,2\pi[$$

Step 2: FT to generate the potential on a grid in configuration space:

$$\Phi(\vec{q}) = \sum_{\vec{k}} \frac{\hat{\delta}_k}{k^2} e^{i\vec{k}\cdot\vec{q}}$$

Step 3: compute the linear-theory velocity field on the grid and displace particles from the grid positions using the **Zeldovich** approximation:

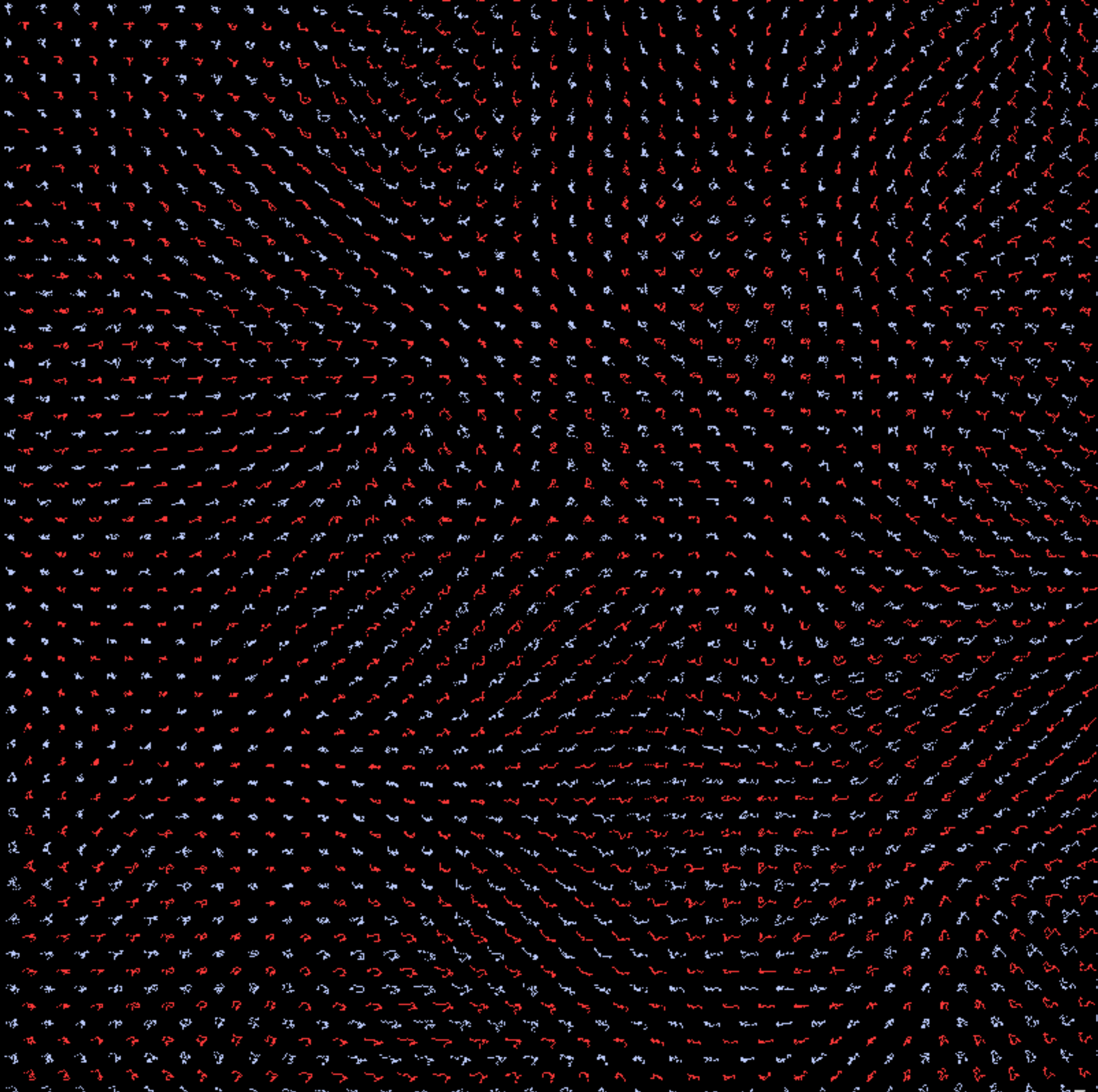
$$\vec{u} = -\dot{D}(z)\,\vec{\nabla}\Phi(\vec{q}) \qquad ; \qquad \vec{x} = \vec{q} - D(z)\,\Phi(\vec{q})$$

→ ICS generated at high enough redshift to guarantee no shell crossing on the grid scale

→ Golden rule: σ_{displ} = 0.1 x grid-spacing

Refinements:

- Generate ICS on a “glass” rather than on a regular grid (White 1993)
- Use 2LPT instead of Zeldovich approximation (1LPT)



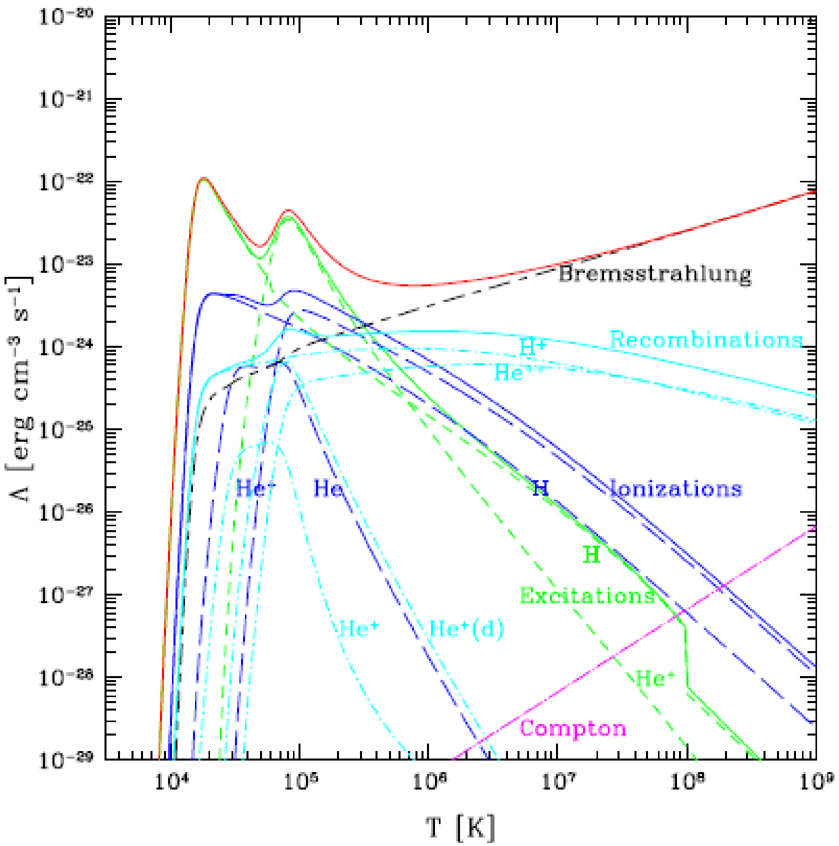
Including Astrophysics – Gas cooling

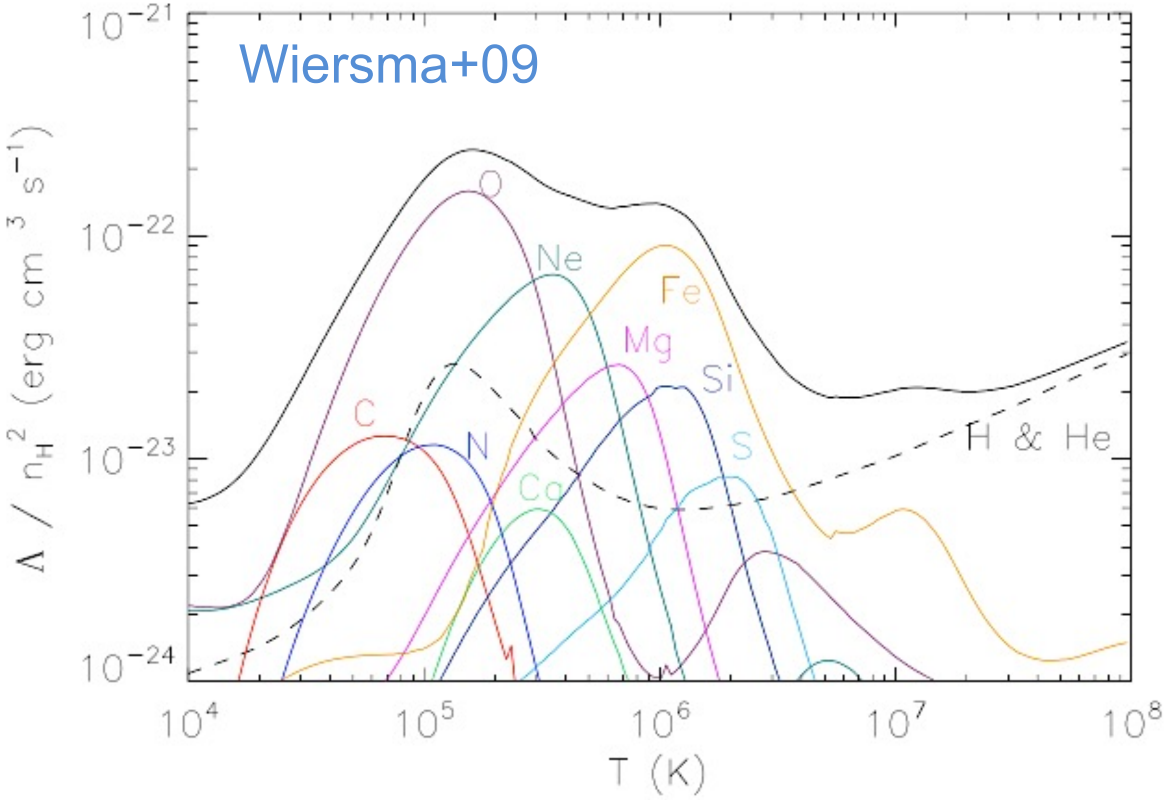
- Include an additional term to the energy equation:

$$\frac{du}{dt} = -\frac{P}{\rho}\vec{\nabla}\cdot\vec{v} + \frac{H-\Lambda}{\rho}; \quad \Lambda(u,\rho,Z)$$

Cooling function: rate of energy loss per unit volume

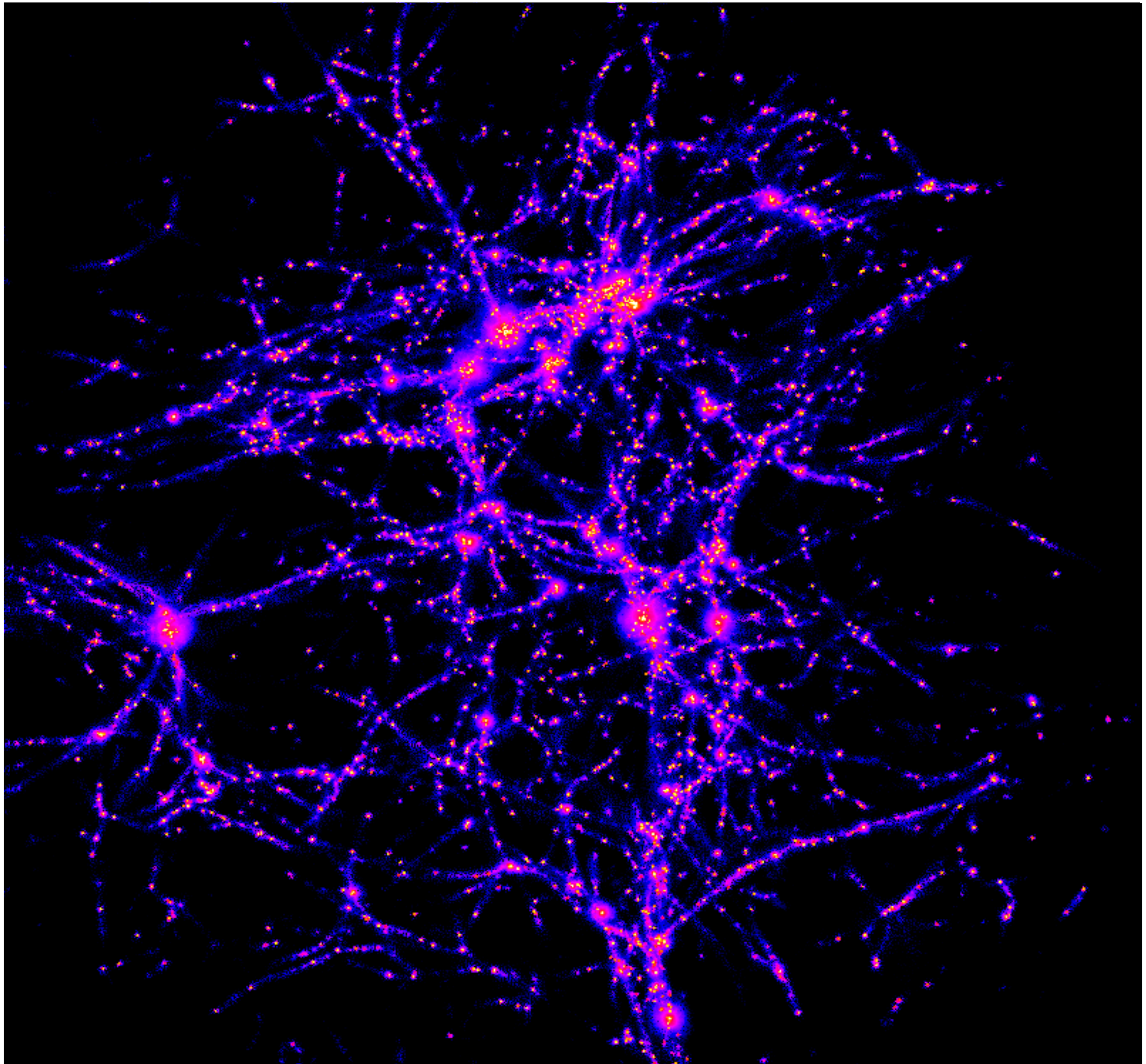
- Assuming gas optically thin, in ionization equilibrium, ignoring three-body cooling





The effect of radiative cooling

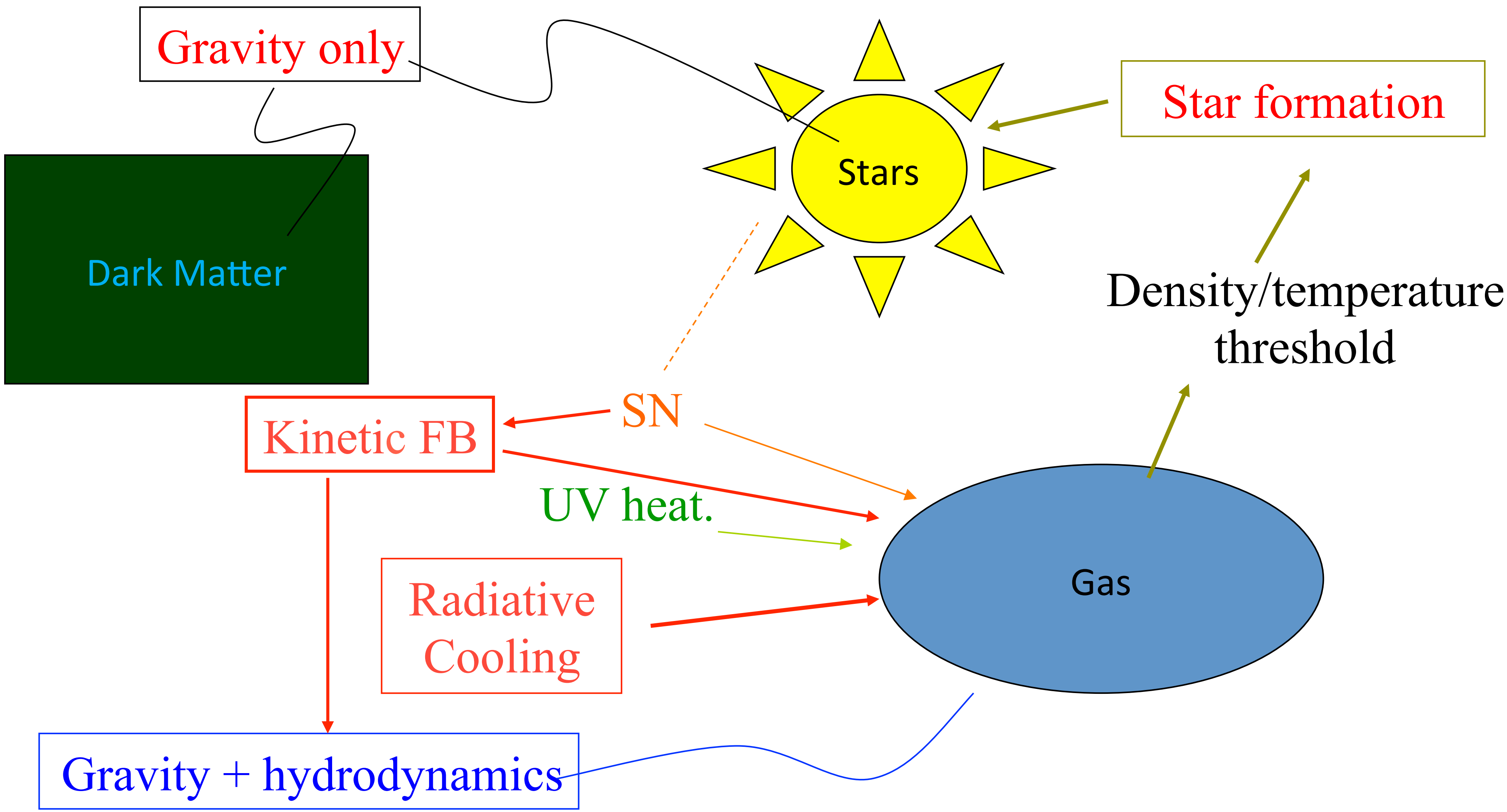
A protocluster region @ z=2



Star formation and feedback

- Everything happens well below the numerically resolved scales (<1 pc)
- Effects important at resolved scales (>10 kpc)

➔ Need to resort to phenomenological “sub-resolution” models



Star formation and feedback

A simple scheme: convert cold dense gas particles into stars (Katz+96; KWH)

- Density criterion: $n_H > 0.1$ cm^{-3}
- Jeans instability criterion: $$\frac{h_i}{c_i} > \frac{1}{\sqrt{4\pi G\rho_i}}$$
- Star formation rate: $$\frac{d\rho_\star}{dt} = -\frac{d\rho_g}{dt} = \frac{c_\star \rho_g}{t_g}$$

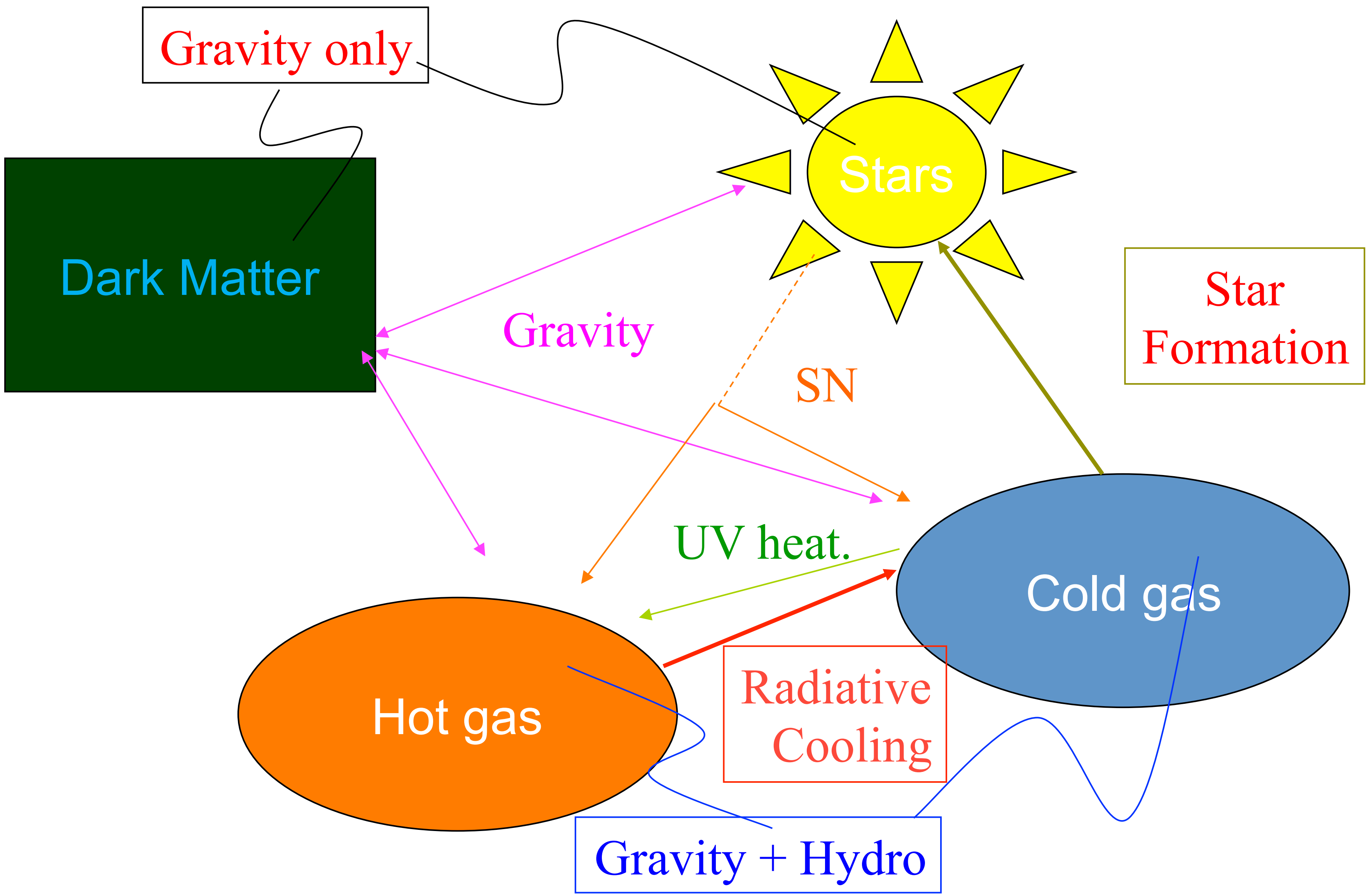
  c_* : star formation efficiency ; t_g : gas consumption time scale
- Gas elements stochastically converted into collisionless star particles within Δt with probability
  $$p = 1 - \exp\left(-c_* \Delta t / t_g\right)$$
- Number of SNe from an assumed IMF, each SN providing 10^{51} erg

Limitations:

- No description of the ISM, unless extremely high resolution is reached
- Thermal energy from SNe given to nearby high density gas particles
  - → Promptly radiated away
  - → Inefficient feedback
  - → Cooling runaway and exceedingly high star formation

Multi-phase schemes for star formation

- Hot and cold phases co-existing in pressure equilibrium within dense gas elements



A multi-phase star formation model

Springel & Hernquist 2003

$$\frac{d\rho_\star}{dt} = \frac{\rho_c}{t_\star} - \beta\frac{\rho_c}{t_\star} = (1-\beta)\frac{\rho_c}{t_\star}$$

Stars form from COLD gas
β: star mass fraction in supernovae

$$\frac{d}{dt}(\rho_h u_h)\Big|_{SN} = \epsilon_{SN}\frac{d\rho_\star}{dt} = \beta u_{SN}\frac{\rho_c}{t_\star}$$

SNe energy heats up HOT gas
ε_{SN}: average SN energy per $M_\odot$ of stars formed

$$\frac{d\rho_c}{dt}\Big|_{EV} = A\beta\frac{\rho_c}{t_\star}$$

SNe evaporate a fraction of COLD gas
A: evaporation efficiency

$$\frac{d\rho_c}{dt}\Big|_{TI} = -\frac{d\rho_h}{dt}\Big|_{TI} = \frac{1}{u_h - u_c}\Lambda_{net}(\rho_h, u_h)$$

HOT gas cools to COLD gas
u_c: specific energy of the cold clouds (T_c=1000 K assumed)

$$\frac{d\rho_c}{dt} = -\frac{\rho_c}{t_\star} - A\beta\frac{\rho_c}{t_\star} + \frac{1-f}{u_h - u_c}\Lambda_{net}(\rho_h, u_h)$$

$$\frac{d\rho_h}{dt} = \beta\frac{\rho_c}{t_\star} + A\beta\frac{\rho_c}{t_\star} - \frac{1-f}{u_h - u_c}\Lambda_{net}(\rho_h, u_h).$$

f=0 for $\rho<\rho_{th}$
f=1 for $\rho>\rho_{th}$
ρ_{th}: threshold density for the onset of star formation

A multi-phase star formation model

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_c u_c) = -\frac{\rho_c}{t_\star}u_c - A\beta\frac{\rho_c}{t_\star}u_c + \frac{(1-f)u_c}{u_h - u_c}\Lambda_{\mathrm{net}},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_h u_h) = \beta\frac{\rho_c}{t_\star}(u_{\mathrm{SN}} + u_c) + A\beta\frac{\rho_c}{t_\star}u_c - \frac{u_h - f u_c}{u_h - u_c}\Lambda_{\mathrm{net}}$$

Evolution of energy of cold and hot phases

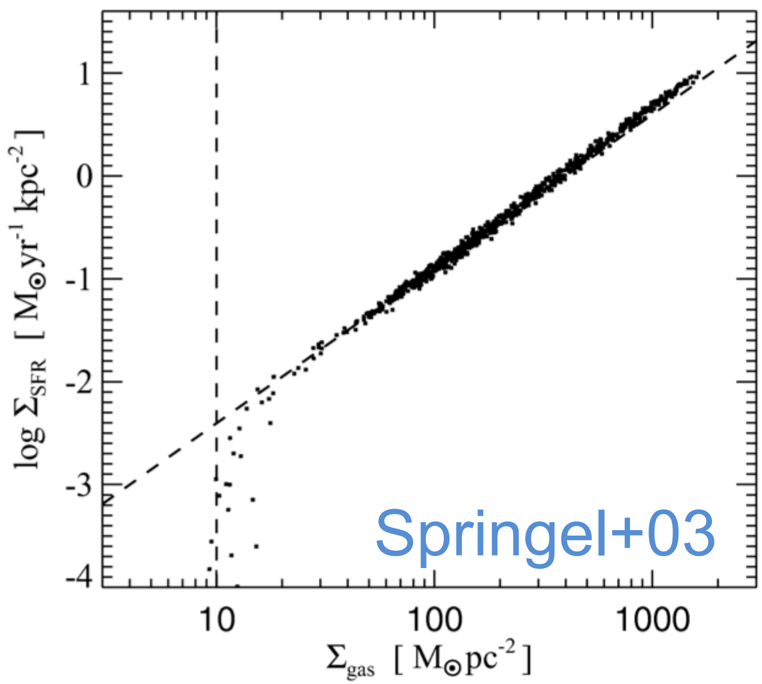
Assume:

- Self-regulated star formation
- Hot and Cold phases in pressure equilibrium
- Constant temperature of the cold phase

Solve the system of equations to calculate star formation rate:

$$\rightarrow \boxed{\frac{\rho_c}{t_\star} = \frac{\Lambda_{\mathrm{net}}(\rho_h, u_h)}{\beta u_{\mathrm{SN}} - (1-\beta)u_c}}$$

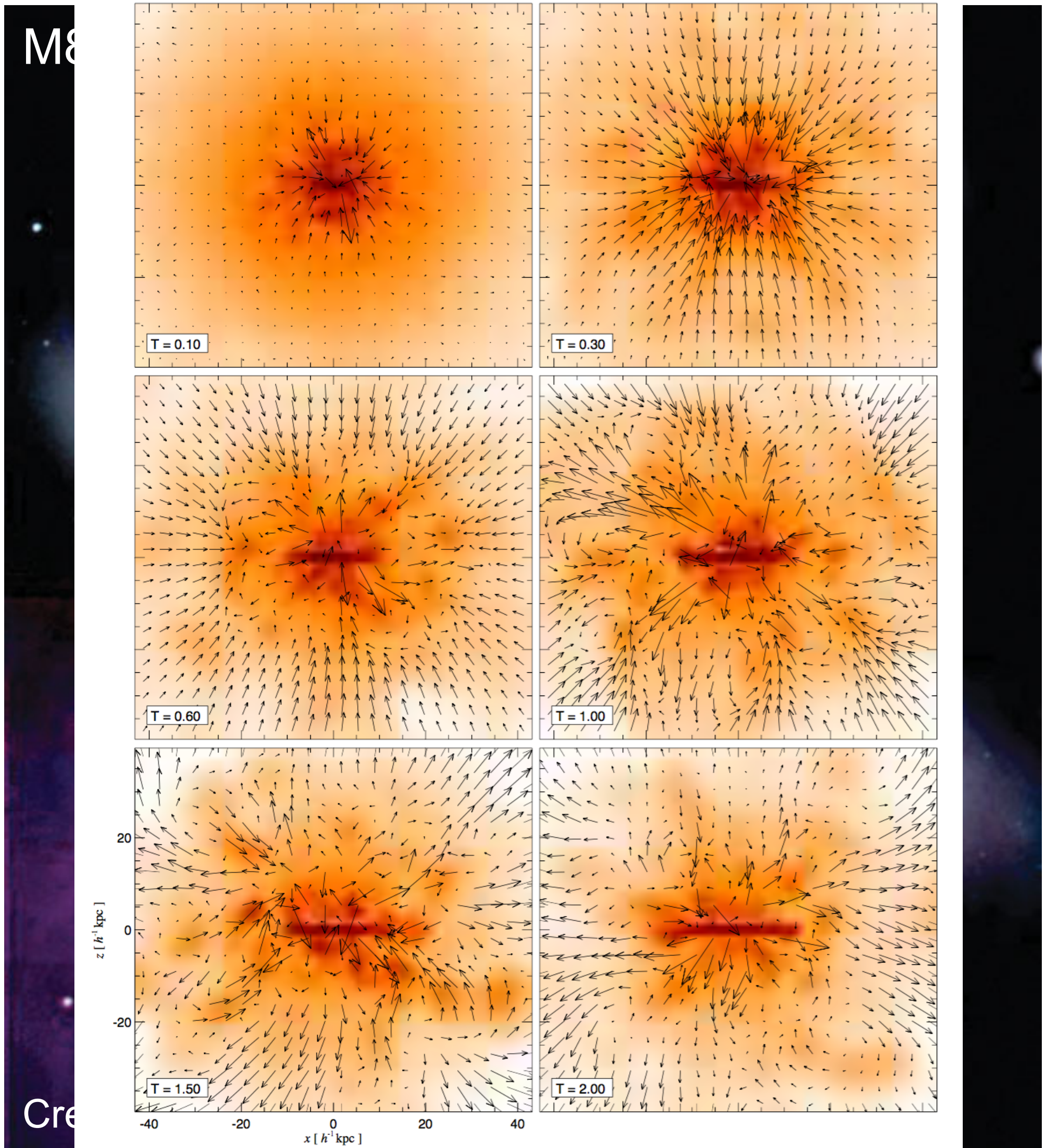
Model parameters fixed so as to reproduce the observed Σ_{SFR}-Σ_{gas} Schmidt-Kennicutt relation:



SN-driven galactic winds

➔ To deposit energy away from gas with short cooling time

➔ Gas particles «kicked» with a given velocity and from the ambient gas



Energy-driven winds:

(e.g. Springel & Hernquist 2003)

$$\dot{M}_w = \eta \dot{M}_* \quad ; \quad \frac{1}{2}\dot{M}_w v_w^2 = \chi \varepsilon_{SN} \dot{M}_*$$

➔ $$v_w = \sqrt{\frac{2\beta\chi u_{SN}}{\eta(1-\beta)}} \approx (300-600)km/s$$

η~2-3: mass-upload factor

χ~0.5-1: feedback efficiency

Momentum-driven winds:

Arising from radiation pressure (e.g. Oppenheimer & Dave' 2006)

$$v_w \approx 3\sigma \quad ; \quad \eta = \sigma / \sigma_0$$

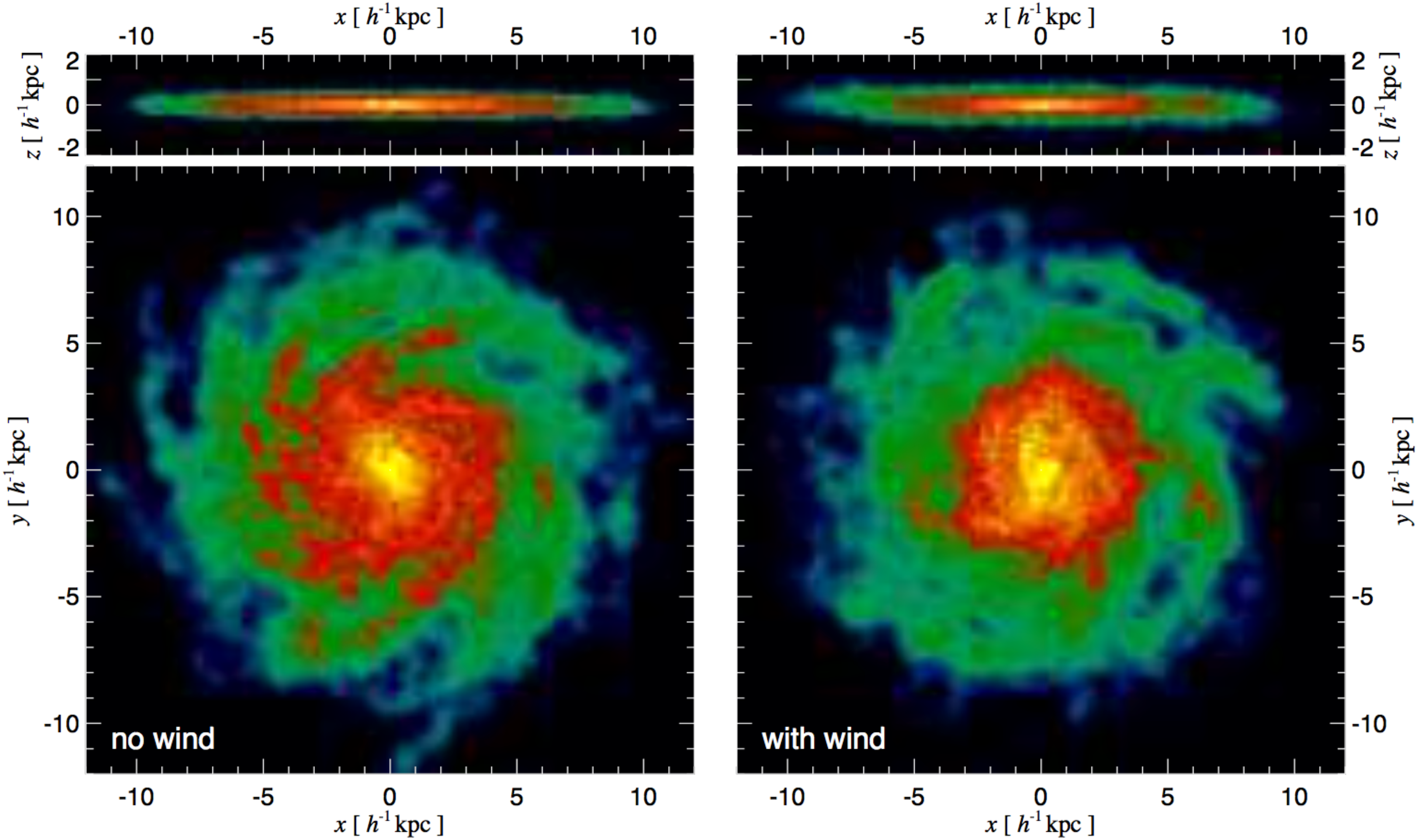
σ: galaxy velocity dispersion

σ_0~ 300 km s^{-1}

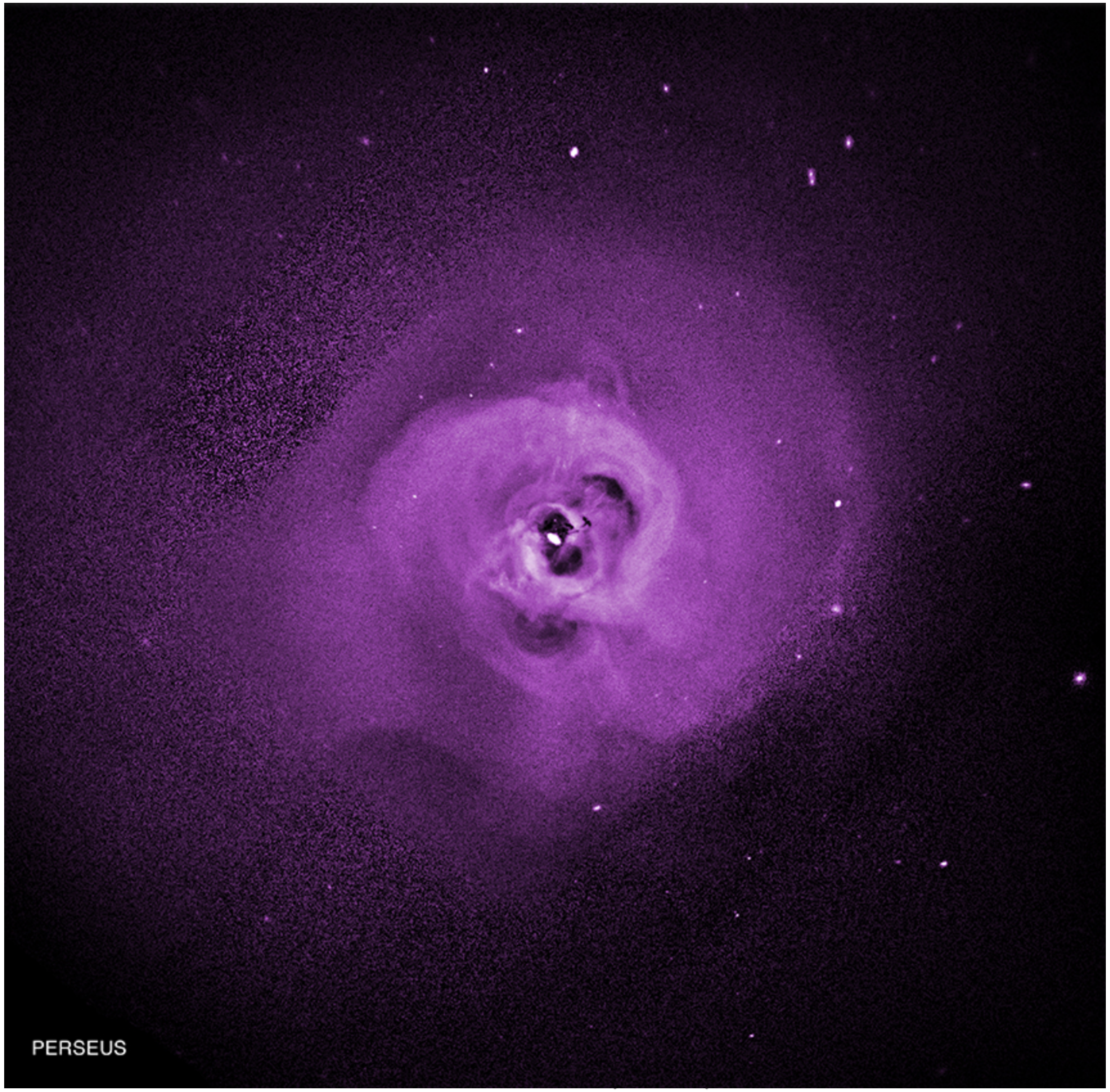
➔ Stronger feedback in more massive galaxies

SN-driven galactic winds

Springel & Hernquist 2003



AGN feedback: motivations



- Suppress the bright end of the galaxy luminosity function
- Quench star formation in massive galaxies at low z
- Steepen the L_X-T relation in galaxy clusters and groups
- Establish the cool-core structure within relaxed galaxy clusters

➔ Requires an energy feedback mechanism of non-stellar origin

AGN feedback: implementation

Original implementation: Springel+05

- Include BHs as sink particles
- Seeded in resolved DM halo
- Growing by merging and gas swallowing:

Bondi-Hoyle accretion: $$\dot{M}_B = \frac{4\pi\alpha G^2 M_{BH}^2 \rho}{\left(c_s^2 + v^2\right)^{3/2}}$$

α : fudge factor to account for unresolved density at the Bondi radius (~100 originally)
ρ : gas density at the BH position

Eddington-limited: $$\dot{M}_{Edd} \equiv \frac{4\pi G M_{BH} m_p}{\epsilon_r \sigma_T c}$$

$$\epsilon_r = \frac{L_r}{\dot{M}_{BH} c^2}$$: radiative efficiency ~0.1

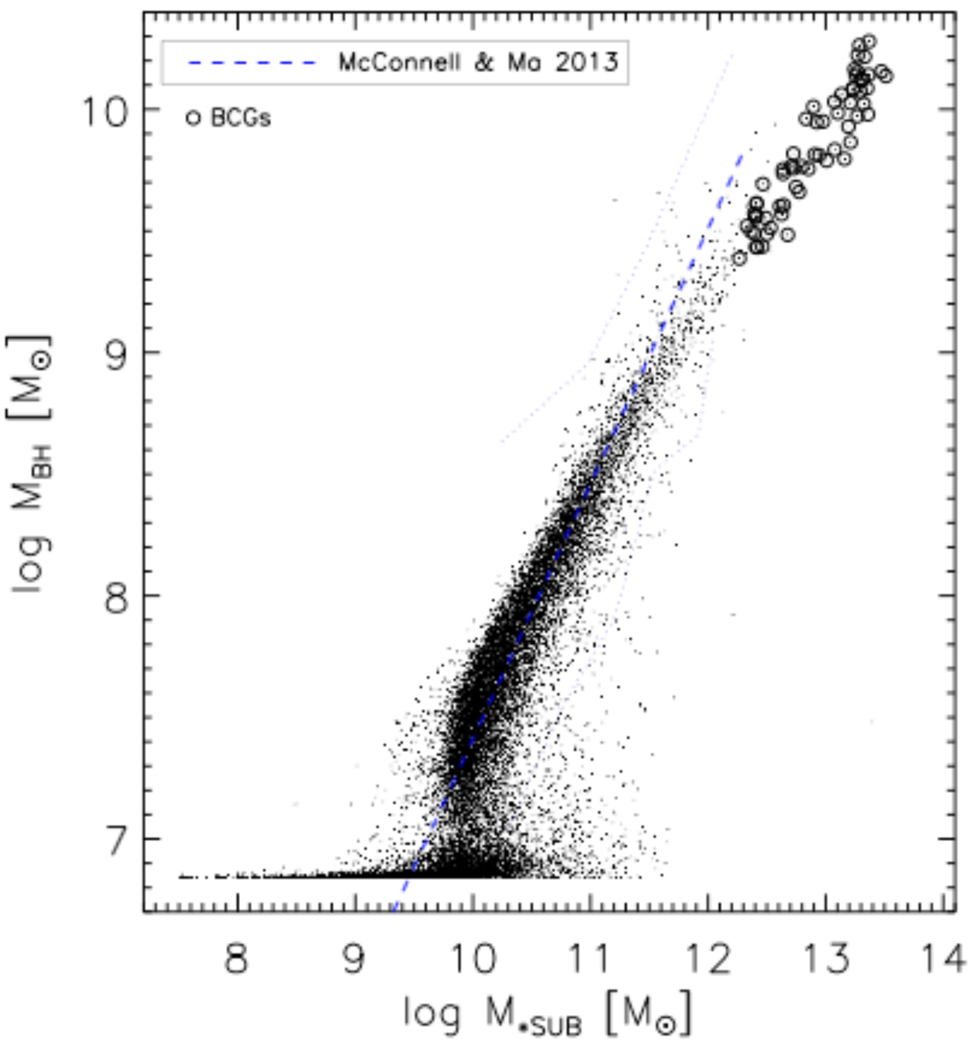
- Thermal energy to surrounding gas: $\dot{E}_{feed} = \epsilon_f L_r = \epsilon_f \epsilon_r \dot{M}_{BH} c^2$

ε_f : feedback efficiency ~0.05

- Model parameters tuned to as to reproduce observational results (e.g. M_{bulge}-M_{BH} relation; e.g. Magorrian+98; Marconi & Hunt 03)

AGN feedback - Implementation

Ragone-Figueroa+14



A number of variants:

- Purely Bondi accretion not realistic (e.g. cold & hot accretion modes; e.g. Steinborn+15)
- Make α variable to account for resolved accretion (Booth & Schaye 09)
- Different modes (i.e. QSO and radio) in different regimes (e.g. Sijacki & Springel 06; Fabjan+10)
- Energy thermalization:

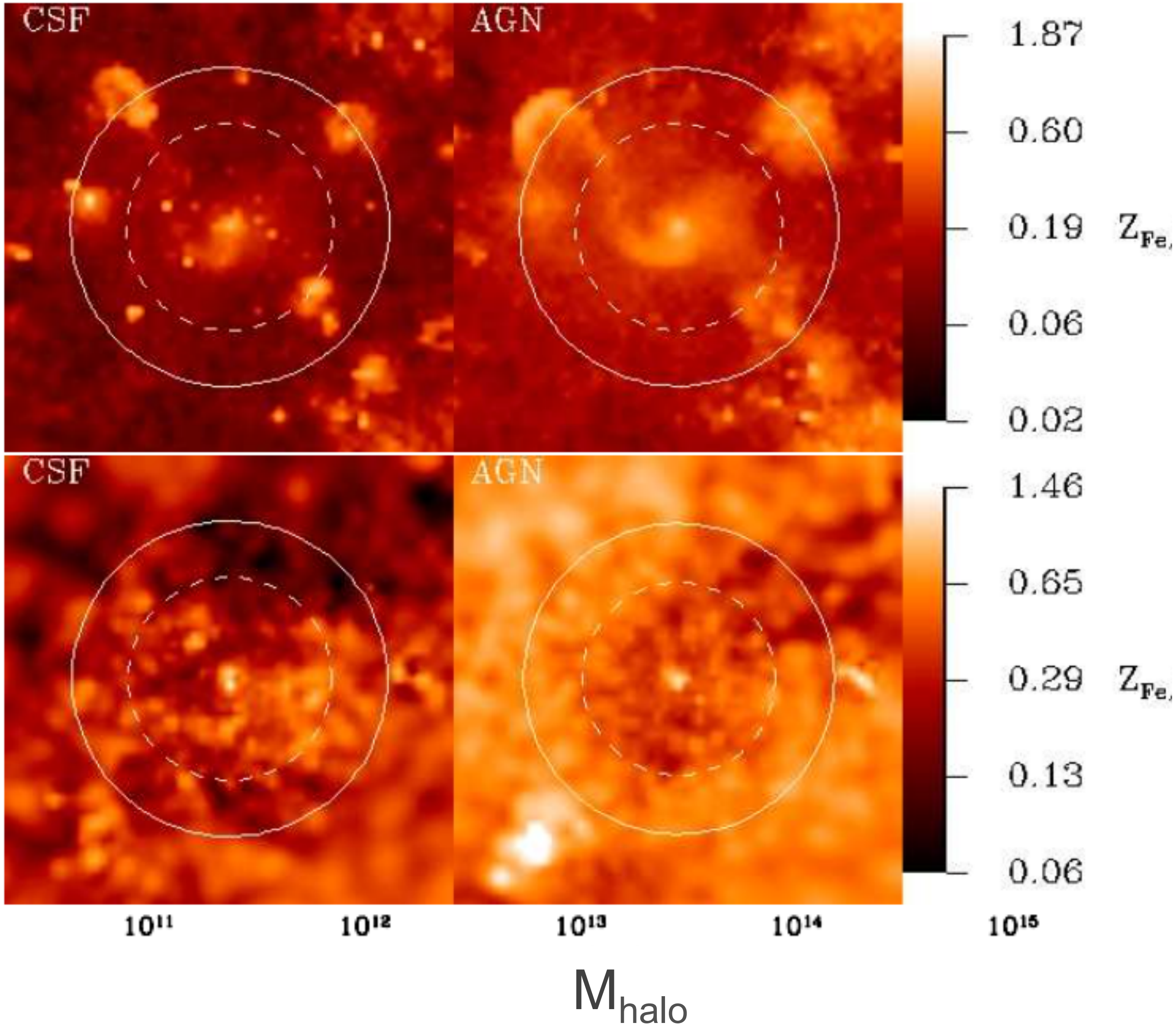
  Mimic injection of low-entropy bubbles (Sijacki & Springel 07)

  Describe explicitly sub-relativistic jets (Dubois+12; Barai+13)

....

AGN feedback - Results

Planelles+13



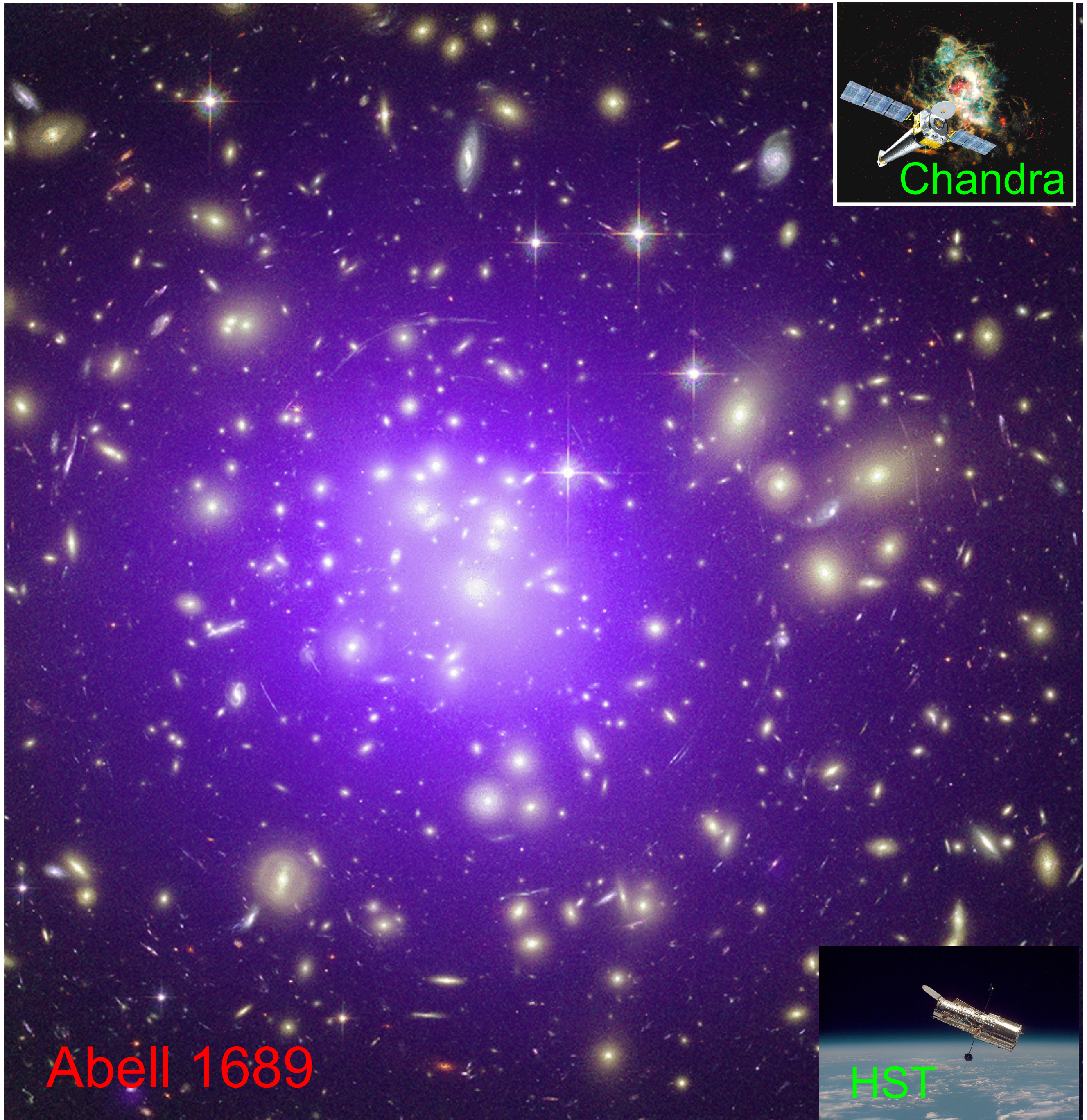
- Suppress star formation efficiency in the most massive galaxies (e.g. Martizzi+12)
- Selectively remove X-ray emitting gas from galaxy groups and steepen the L_X-T relation (e.g. Puchwein+08, Fabjan+10)
- Change the pattern of chemical enrichment in galaxy clusters (e.g. McCarthy+13, Planelles +13)

What I won't talk about?

- Stellar evolution and chemical enrichment
- Radiative transfer
- Magnetic fields
- Plasma effects:
  - Spitzer thermal conduction
  - Spitzer-Braginsky viscosity
  - Electron-ion equilibration
  - Particle acceleration at shocks and non-thermal emission

- ......

Applications to cluster cosmology



Concentrations of ~10^3 galaxies

σ_v~500-1000 km s^{-1}

Size: ~1-2 Mpc

Mass: ~10^{14}-10^{15} $M_\odot$
→ $\lambda_i \approx 10$ Mpc

<u>Baryon content:</u>

→ cosmic share in hydrostatic equilibrium

<u>ICM temperature:</u>

→ T ~ 2-10 keV

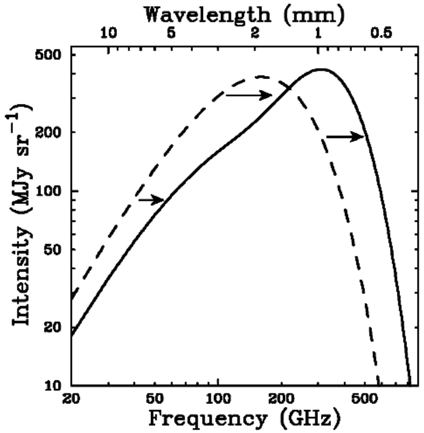
→ fully ionized plasma;

<u>Thermal bremsstrahlung</u>

→ n_e~10^{-2}-10^{-4} cm^{-3}

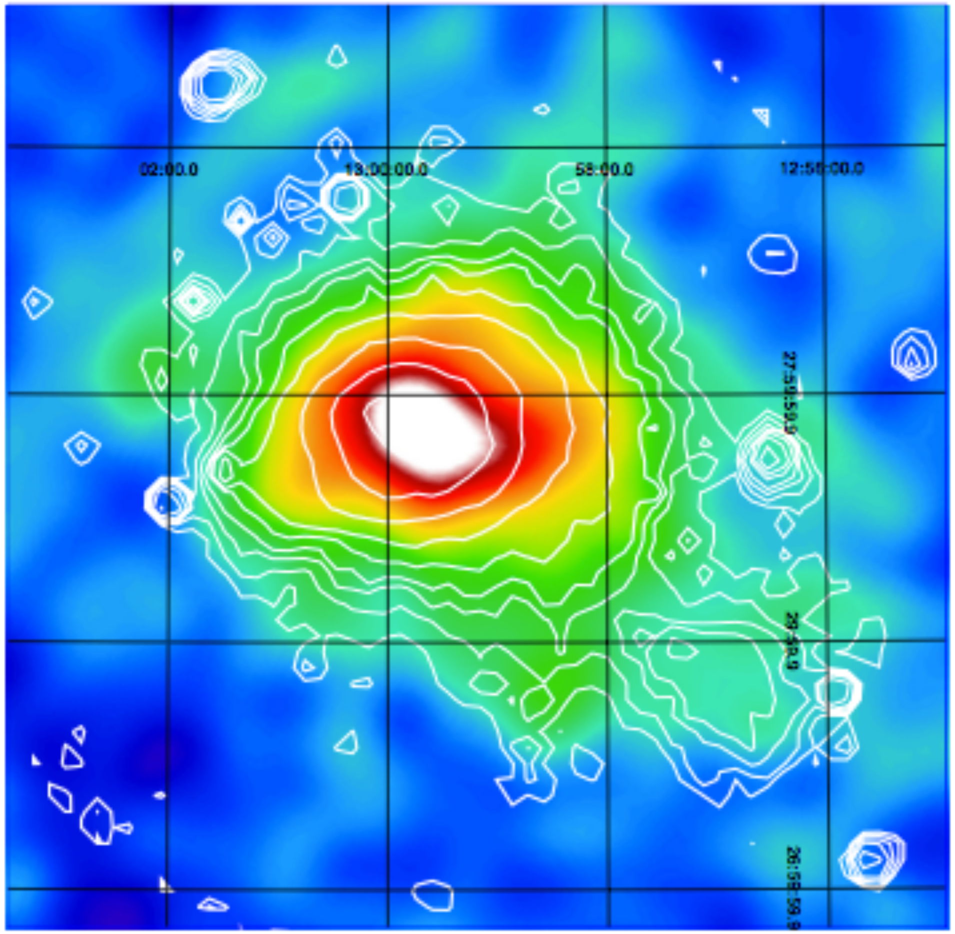
→ L_X~10^{45} erg s^{-1}

Sunyaev-Zeldovich Effect



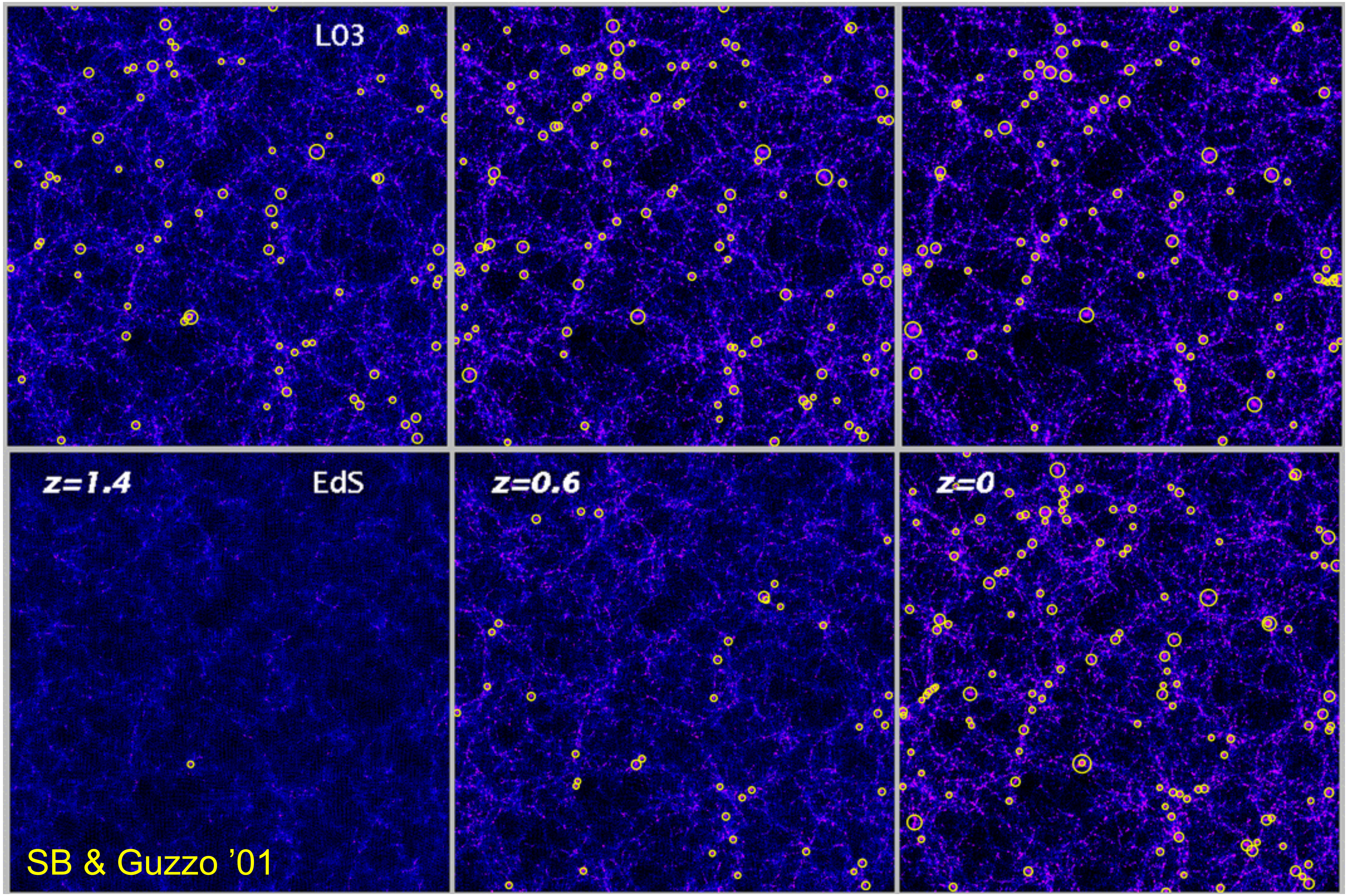
Inverse Compton scattering of CMB photons off the ICM electrons

Coma as seen by Planck



- Signal virtually independent of redshift
- Proportional to the l.o.s. integration of $n_e T_e$ ~ pressure
- Wider dynamic range accessible
- We are now in the era of SZ cluster cosmology (e.g. ACT, SPT, Planck)

Clusters & cosmic growth



Information from a cluster survey

$$\frac{dN(X;z)}{dXdz} = \frac{dV}{dz} f(X,z) \int_0^{\infty} \frac{dn(M,z)}{dM} \frac{dp(X \mid M,z)}{dX} dM$$

→ No. of clusters of given observable X and z within the survey area

$$P_{cl}(k;M,z) = b_{cl}^2(k;M,z) P_m(k,z)$$

→ Clustering of clusters of given mass

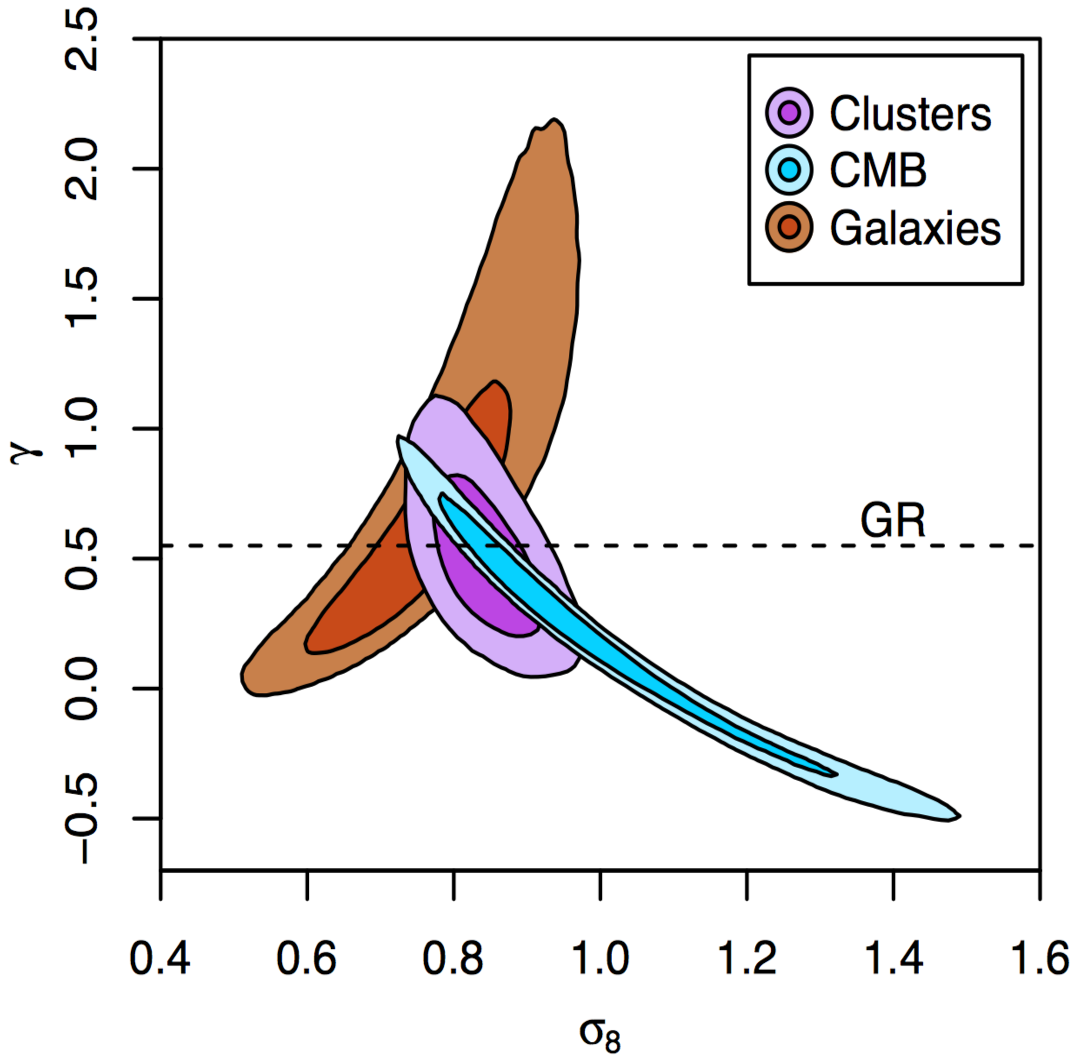
1. Friedmann background: $\frac{dV}{dz}$ → Priors from CMB, BAO, SN-Ia, ...

2. Selection function: $f(X,z)$ → Observational strategy

3. Growth history and nature of perturbations: $\frac{dn(M,z)}{dM}$ → Precisely calibrated with N-body simulations

4. Astrophysics: $p(X \mid M,z)$ → Priors on "nuisance parameters" p_j from follow-up observations and/or <u>cosmological simulations</u>

Cluster Cosmology as of today



<u>Reichardt+13</u>

100 SPT clusters at z>0.3

+ Chandra/XMM follow-up for 14 clusters

➔ Fit at the same time cosmology and mass scaling

<u>Mantz+15</u>

RASS clusters out to z~0.9

+ Chandra follow-up for M_{gas}

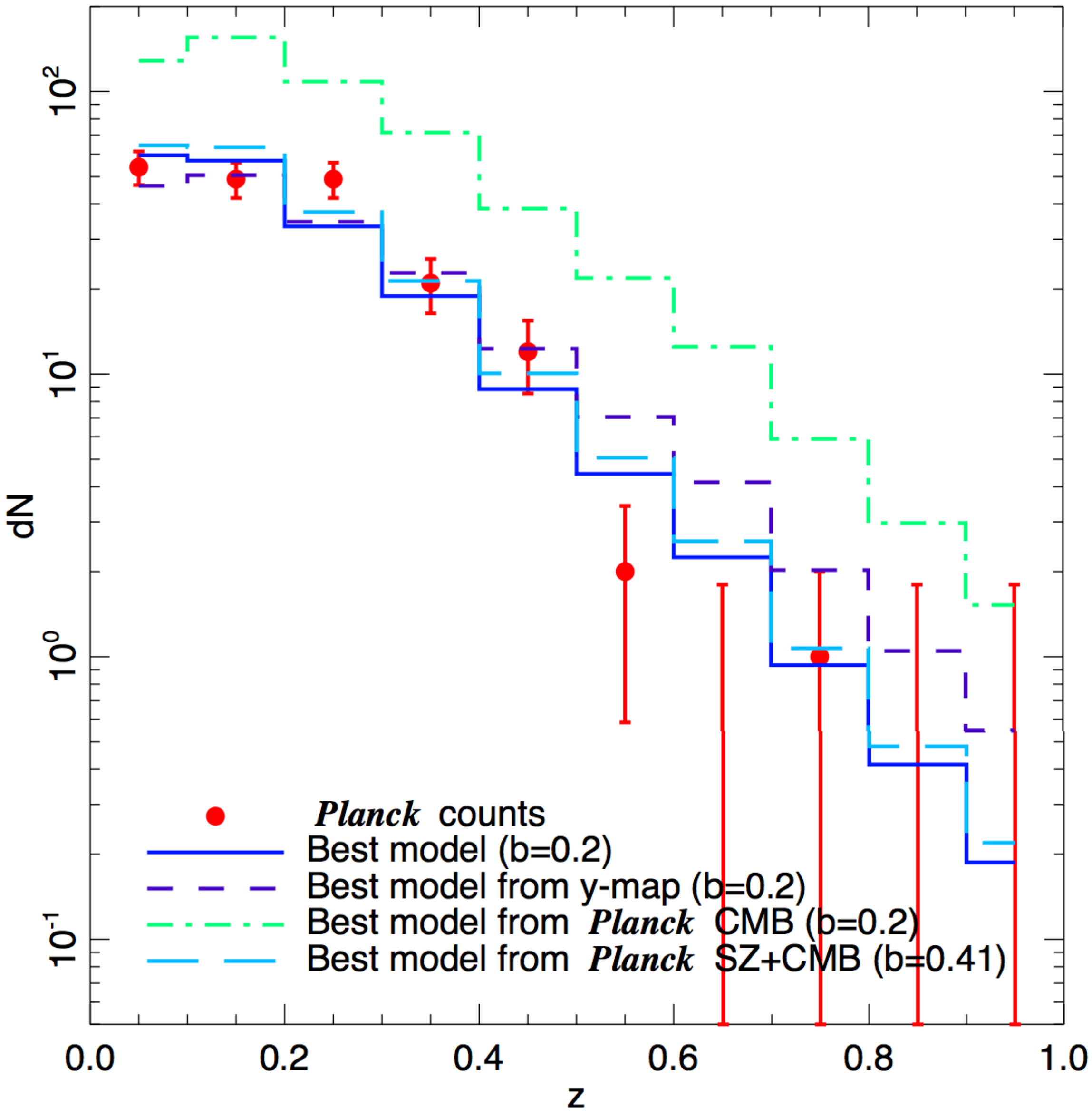
+ mass calibration from WtG WL

➔ Constraints on deviations from GR:

$$f(\Omega_m(z)) \equiv \frac{d\log D_+(a)}{d\log a} = [\Omega_m(z)]^\gamma$$

Planck CMB & clusters

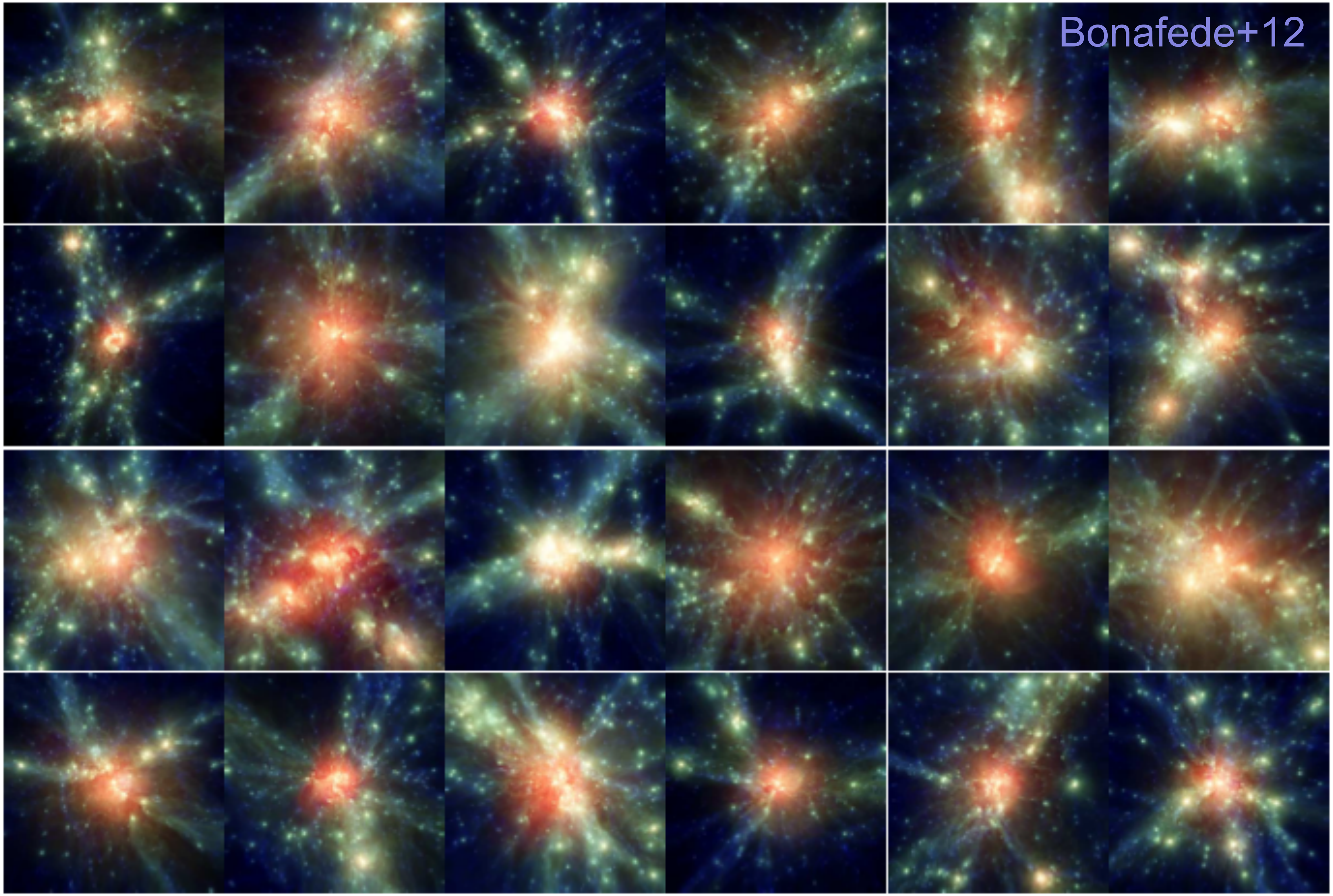
Planck collab XX 2014



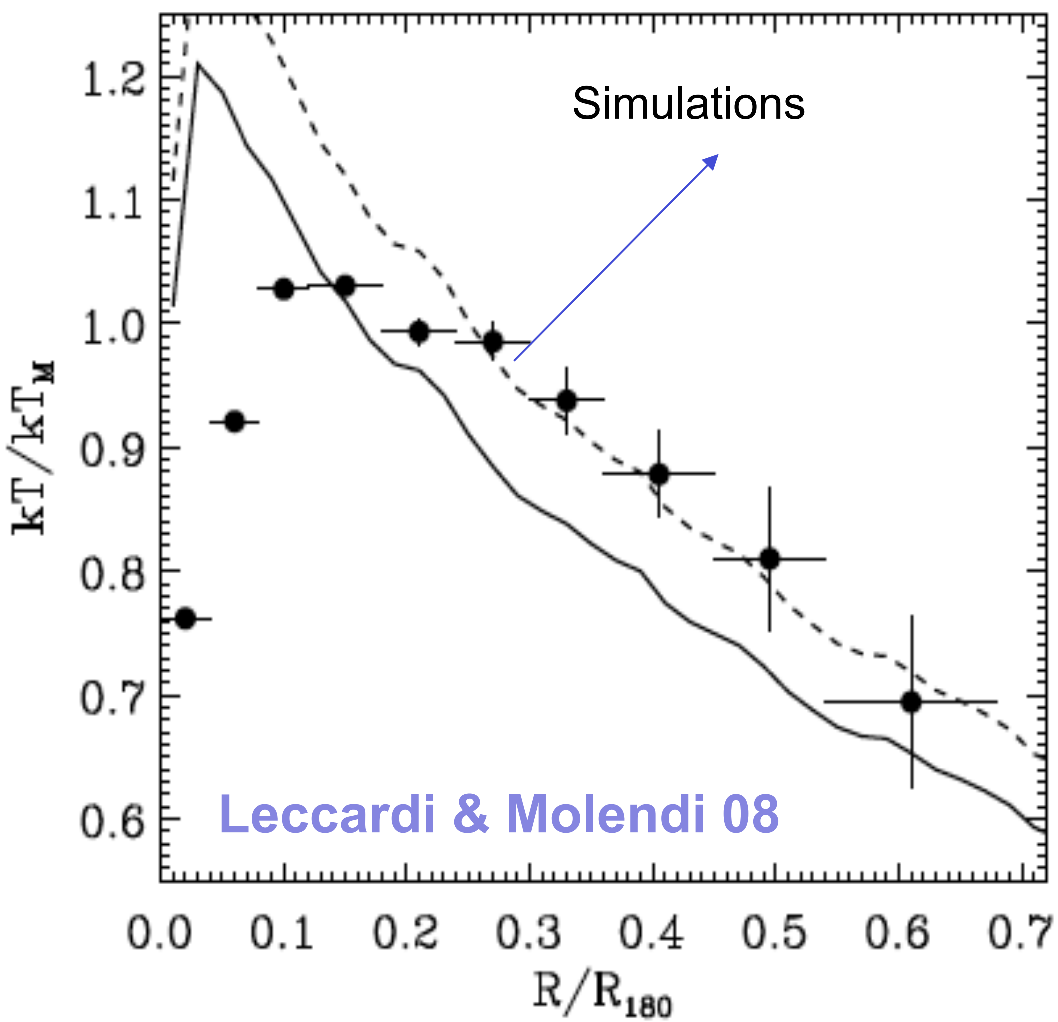
Number counts for 189 Planck-SZ clusters

- X-ray (XMM) calibrated mass scaling
- Tension with Planck primary CMB
- **b=0.2** (HE mass bias) suggested by simulations
- **b=0.4** to recover agreement with CMB cosmology
- Agreement with constraints from:
  - Planck-y map
  - Other cluster counts
  - Cosmic shear

The population of galaxy clusters



Temperature Profiles



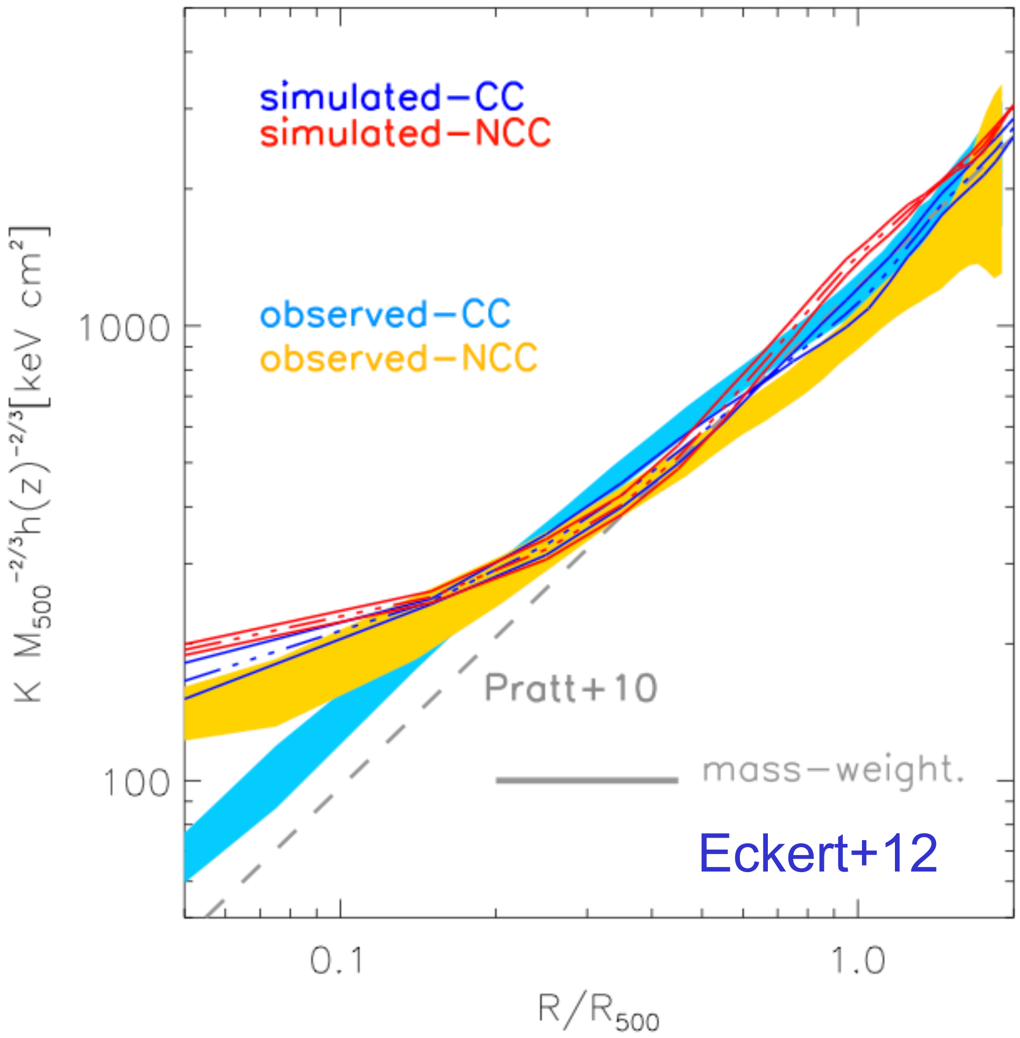
Data: X-ray (XMM) analysis of nearby clusters

Simulations: SPH simulations with star formation and SN feedback (SB+04)

➔ Central profiles in simulations steep and negative
➔ Strong disagreement with data
➔ Requires including AGN feedback

Entropy profiles

$$K = T/n_e^{2/3}$$



Eckert+12

Data: Joint analysis of X-ray (ROSAT imaging) and SZ (Planck)

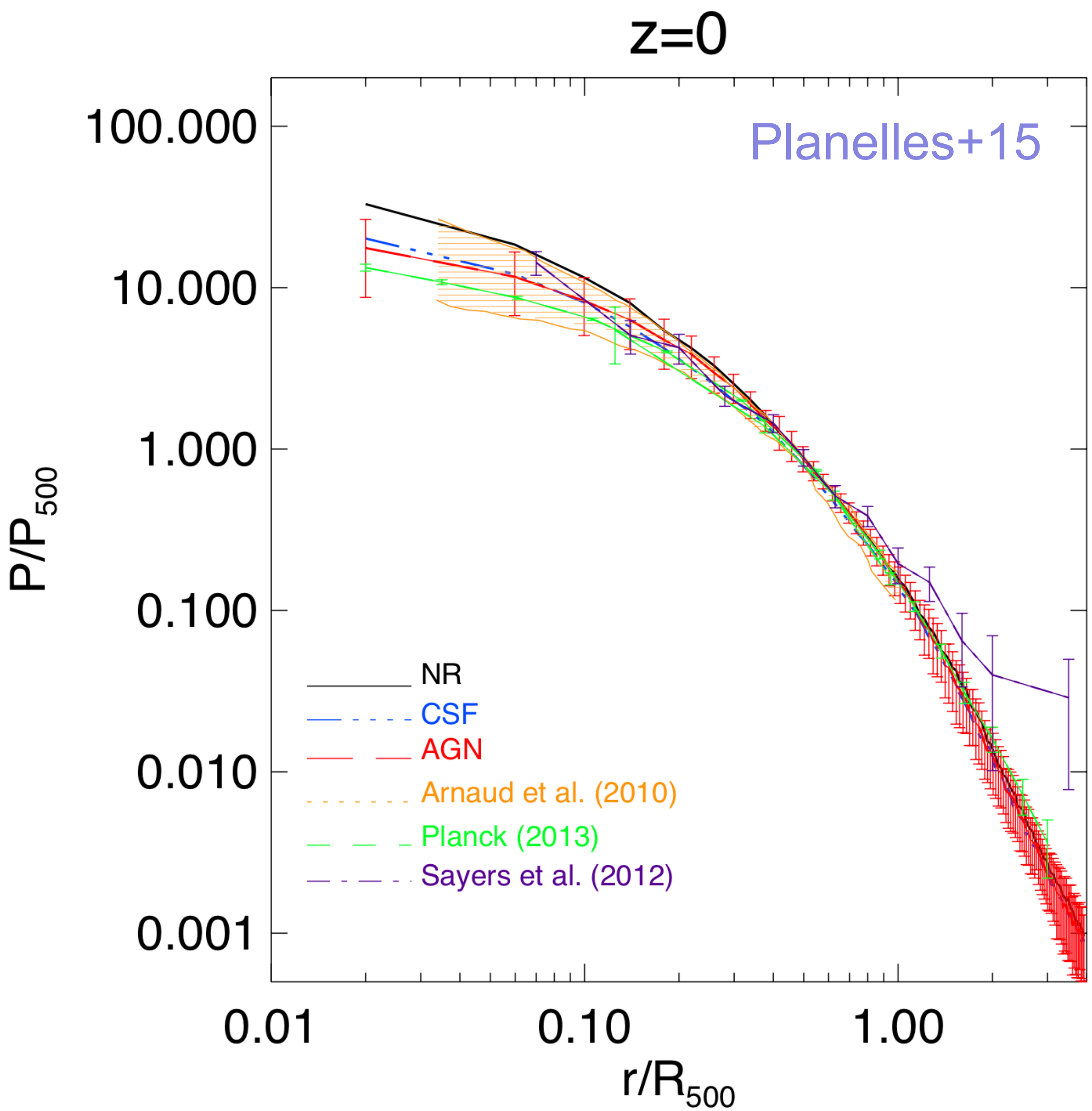
Simulations: Non-radiative with ENZO AMR (Vazza+12)

➔ Good agreement outside core regions ($r > 0.2\ r_{500}$)

➔ CC vs. non-CC dichotomy from data not reproduced in simulations

➔ Including radiative cooling not enough to account for the observed diversity in the cluster population

Pressure profiles



- Shallower profiles in radiative simulations
- Radiative P-profile consistent with observed profiles from both:
- X-ray (XMM); Arnaud+10
- SZ from Planck (Planck coll. XX), SPT (McDonald+14) & Bolocam (Sayers+13)

X-ray masses: hydrostatic bias

- Hydrostatic equilibrium (HE):

$$M_{hyd}(< r) = -\frac{rkT}{G\mu m_p}\frac{d\ln(nkT)}{d\ln(r)}$$

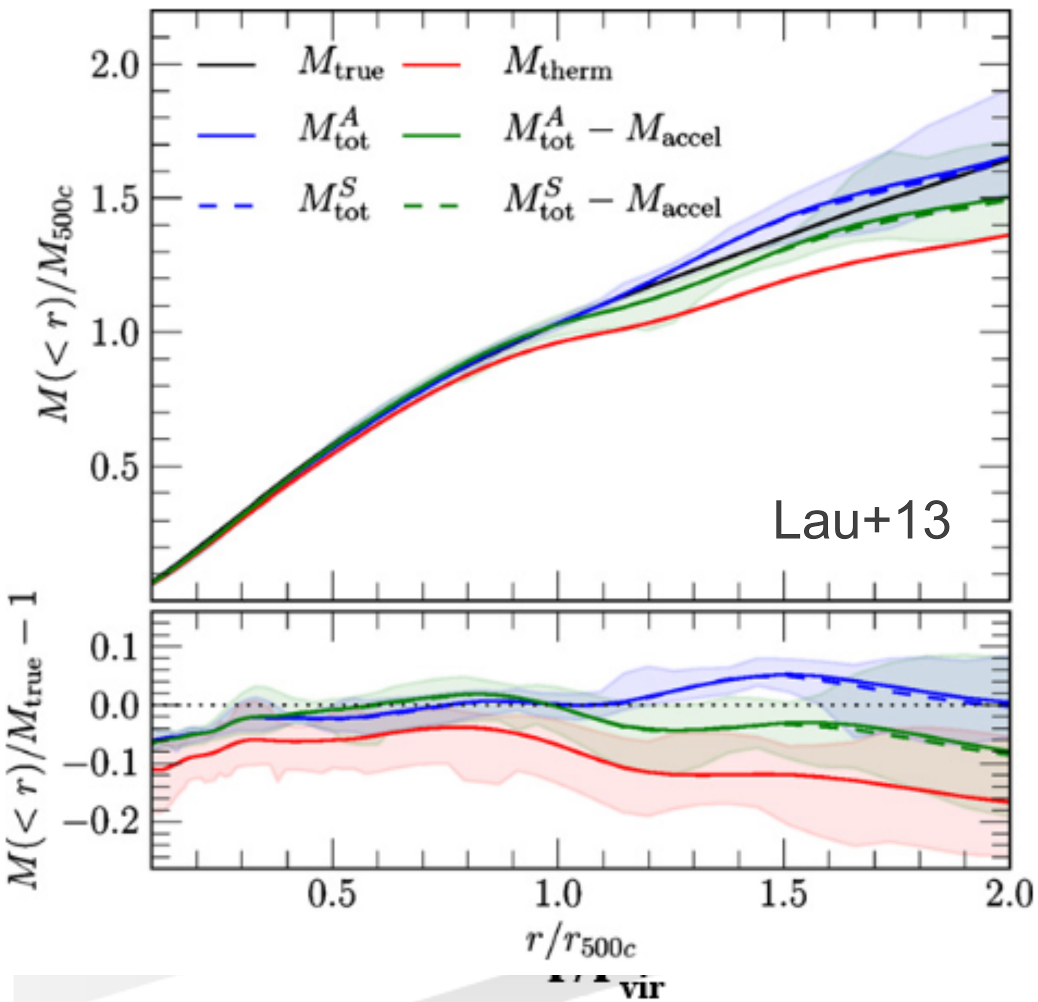
➔ HE violated at the ~10% level within r_{500}

➔ Larger deviations at larger radii ($>R_{500}$)

Also Rasia+06,12, Nagai+07, Morandi+07, Piffaretti & Valdarnini 08, Meneghetti+09, Lau+09, Kay+11, Suto+13, ...

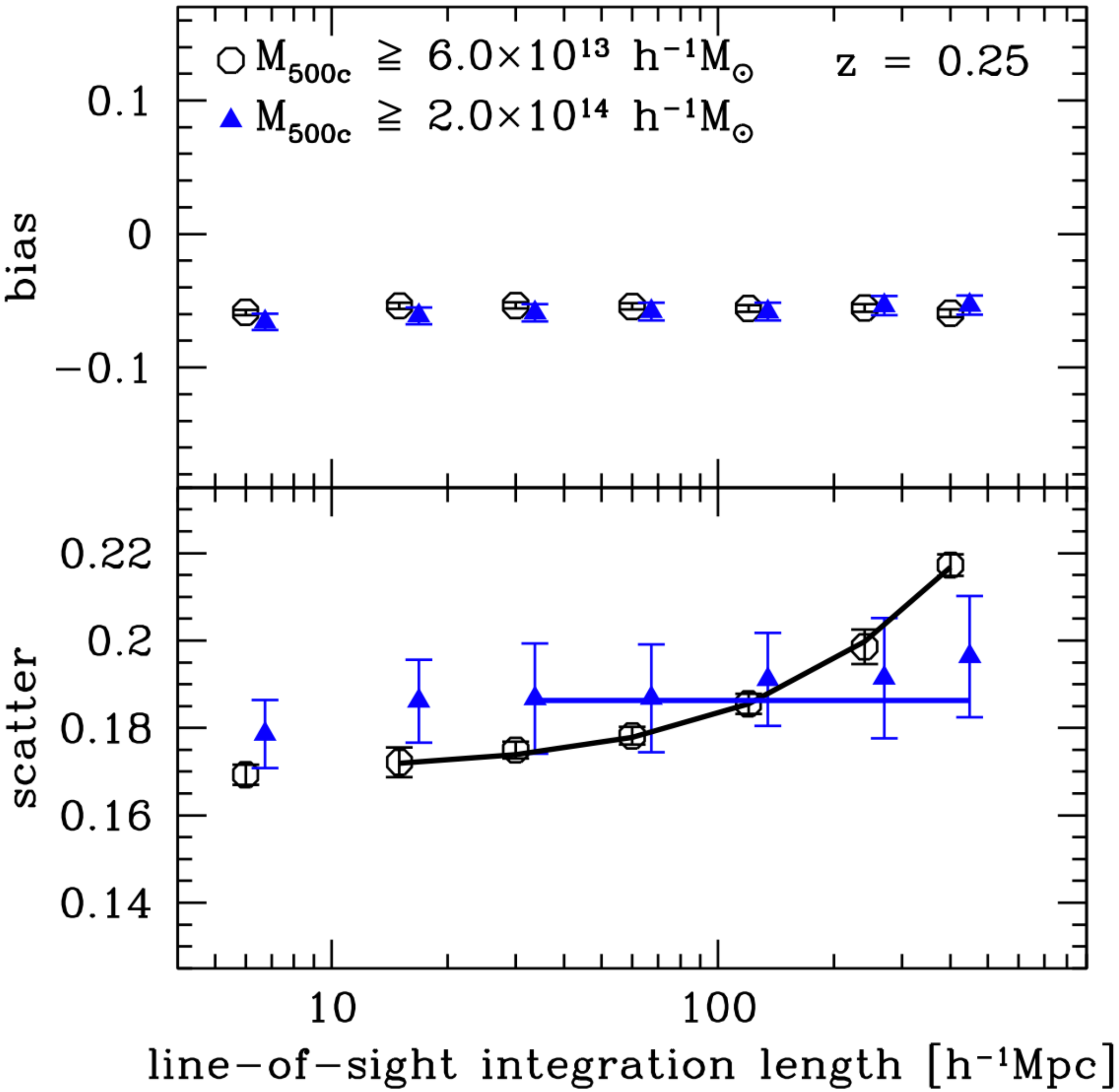
➔ Non-thermal pressure support from subsonic turbulent motions

- Acceleration term in the Euler eq. dominates the HE violation (Suto +13; Lau+13)



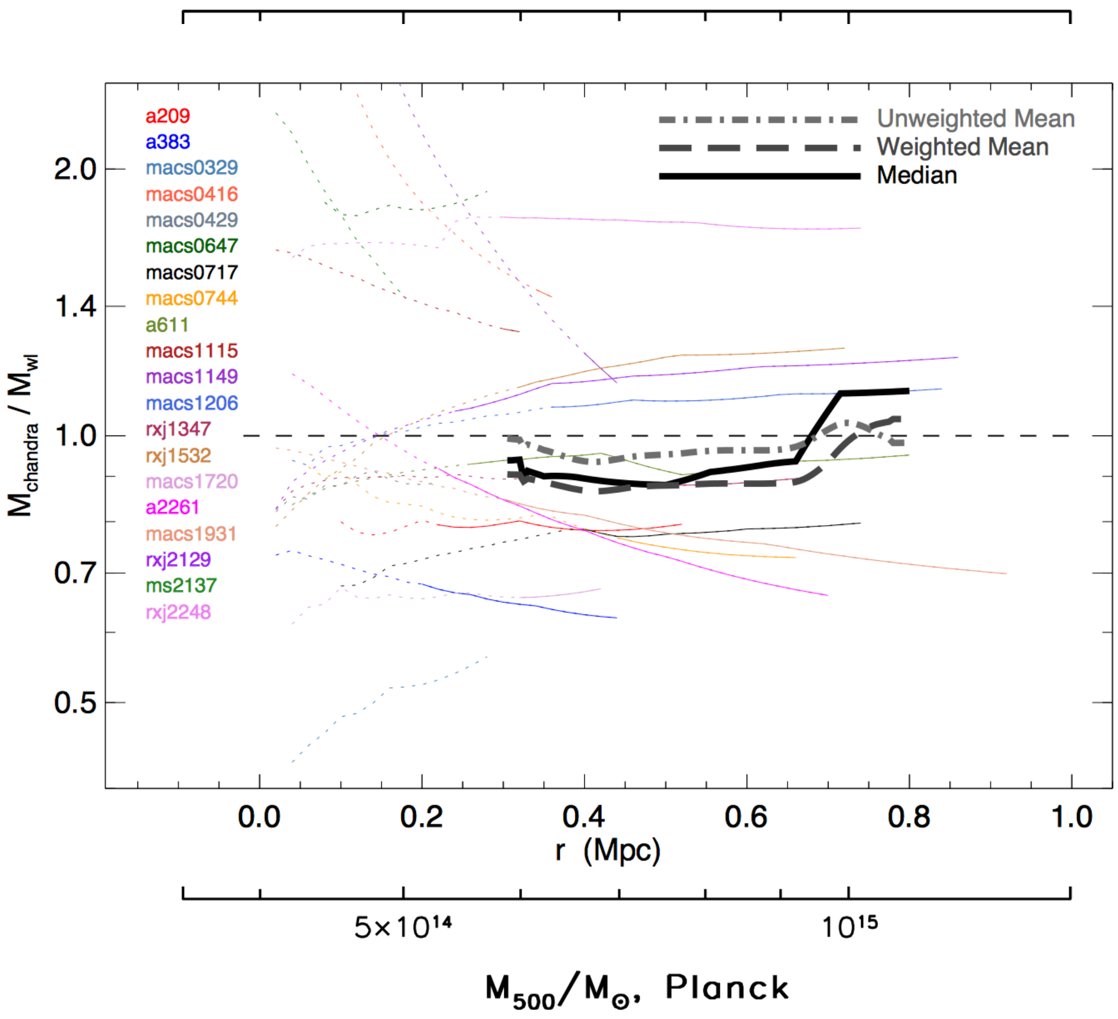
Reliability of WL masses

Becker & Kravtsov 11



- ➔ Spherical NFW fitting to tangential shear profile
- ➔ 5-10% negative bias in recovered masses
- ➔ Significant bias induced by triaxial halo shape, correlated and uncorrelated structures

Weak-Lensing and X-ray Masses



Von der Linden+14:
Planck clusters with WL (WtG) and X-ray calibrated (XMM) masses

➔ $M_X \sim 0.7\ M_{WL}$ on average

Donahue+14:
CLASH clusters with WL and X-ray calibrated (Chandra) mass profiles

➔ $M_X \sim 0.9\ M_{WL}$

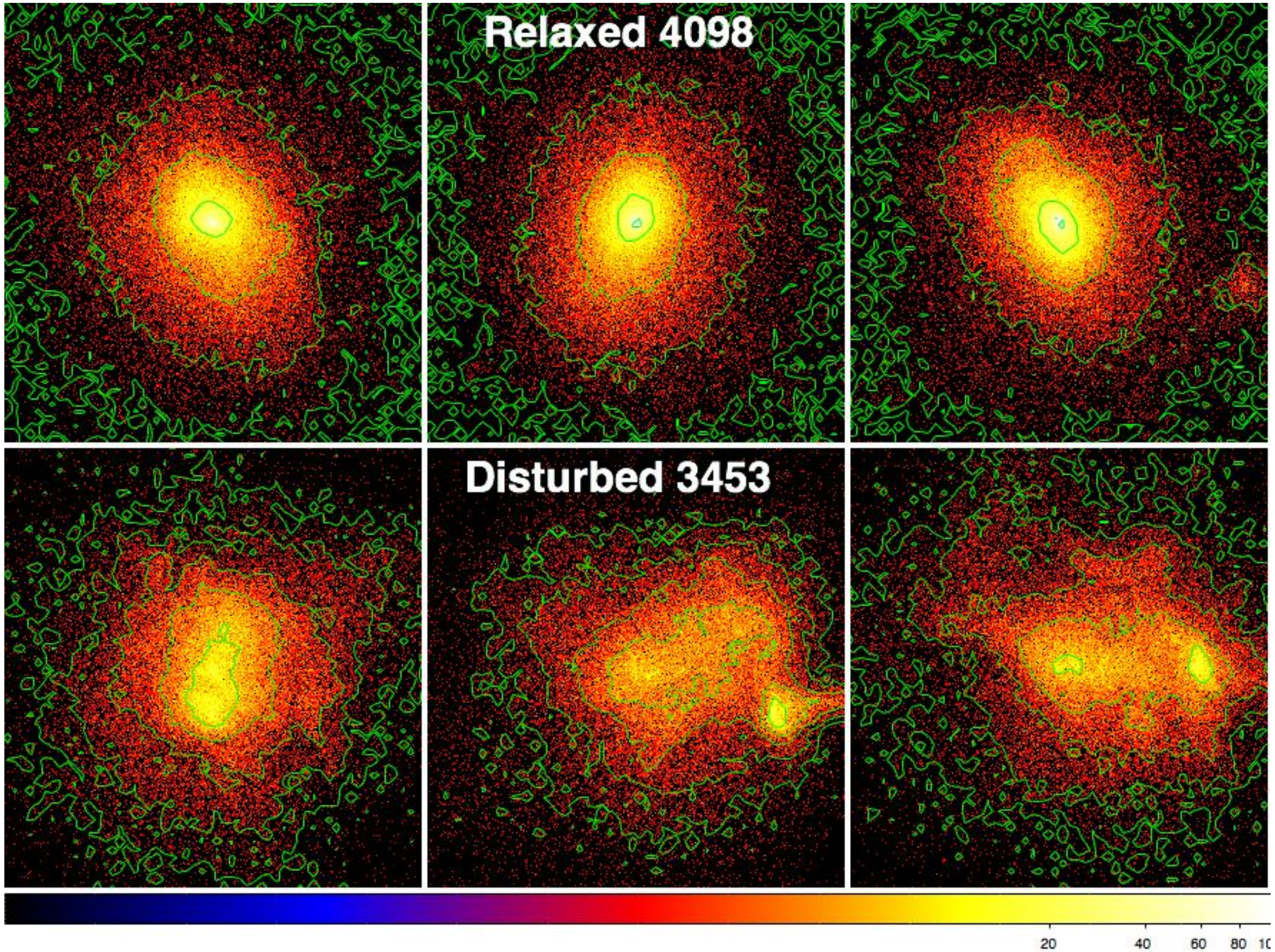
MIND: different WL mass estimators and **X-ray calibrations**!

E.g. also Zhang+10, Mahdavi+12, Israel+14

Simulating X-ray observations

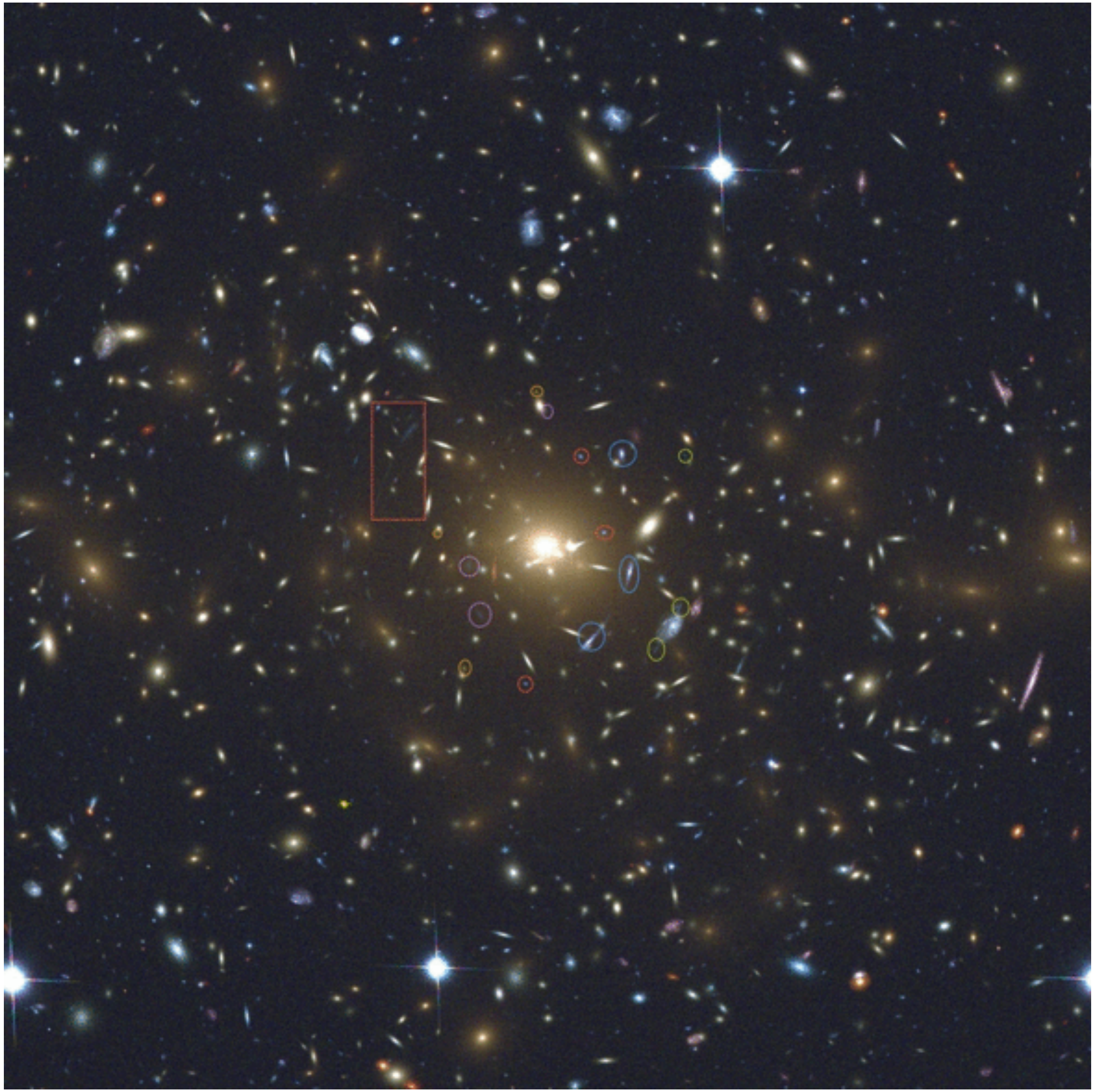
Rasia+12

- Event files from X-MAS Chandra simulator with 100 ks exp. time
- [0.7-2] keV X-ray image (16 x 16 arcmin^2)



- 20 clusters @ z=0.25 with $M_{200} > 5\times10^{14}\ M_\odot$
- 3 projections for each cluster
- Generate mock event files
- Quntitative assessment of observational biases

Simulating lensing observations

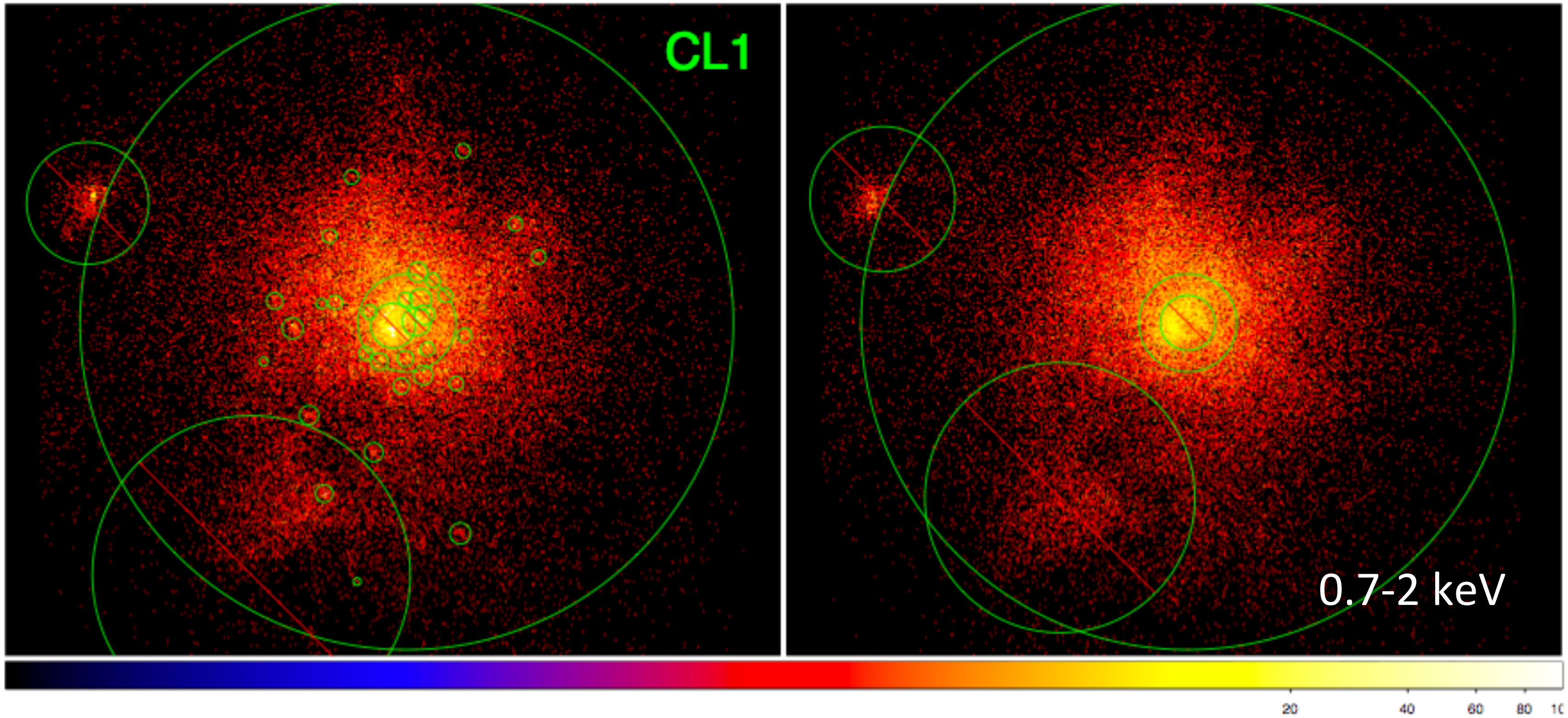


➔ HST-WFC3 lensing of a massive simulated cluster at z=0.25

Based on the SkyLens tool (Meneghetti+08)

Origin of X-ray mass bias (in SPH simulations)

Black: M_X/M_{true} Red: M_X/M_{true} using T_{mw} Green: M_X/M_{WL}



Bias in WL masses: ~10% underestimate at R_{500} (also Becker & Kravtsov 11)

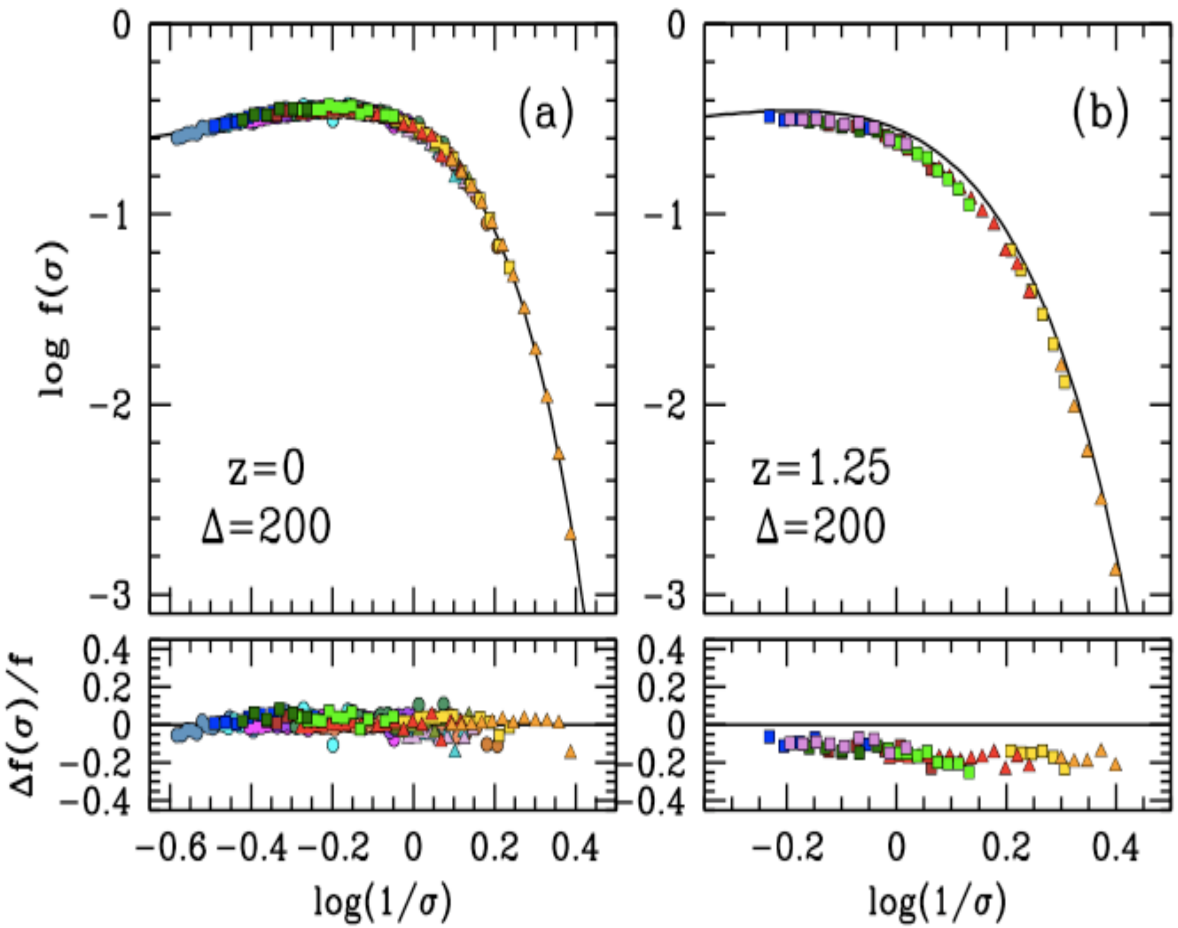
Bias in X-ray masses:

- ➔ 10-15% from violation of hydrostatic equilibrium
- ➔ ~15-20% from bias in X-ray temperature estimate (but see Nagai+07)

Calibration of the halo mass function

E.g. for ΛCDM: Sheth & Tormen 01, Jenkins+01, Evrard+02, Springel+05, Warren+07, Reed+08, Tinker+09, Crocce+10, Courtin+11, Bhattacharya+11, Angulo+12, Watson+13, ...

Tinker et al. 09



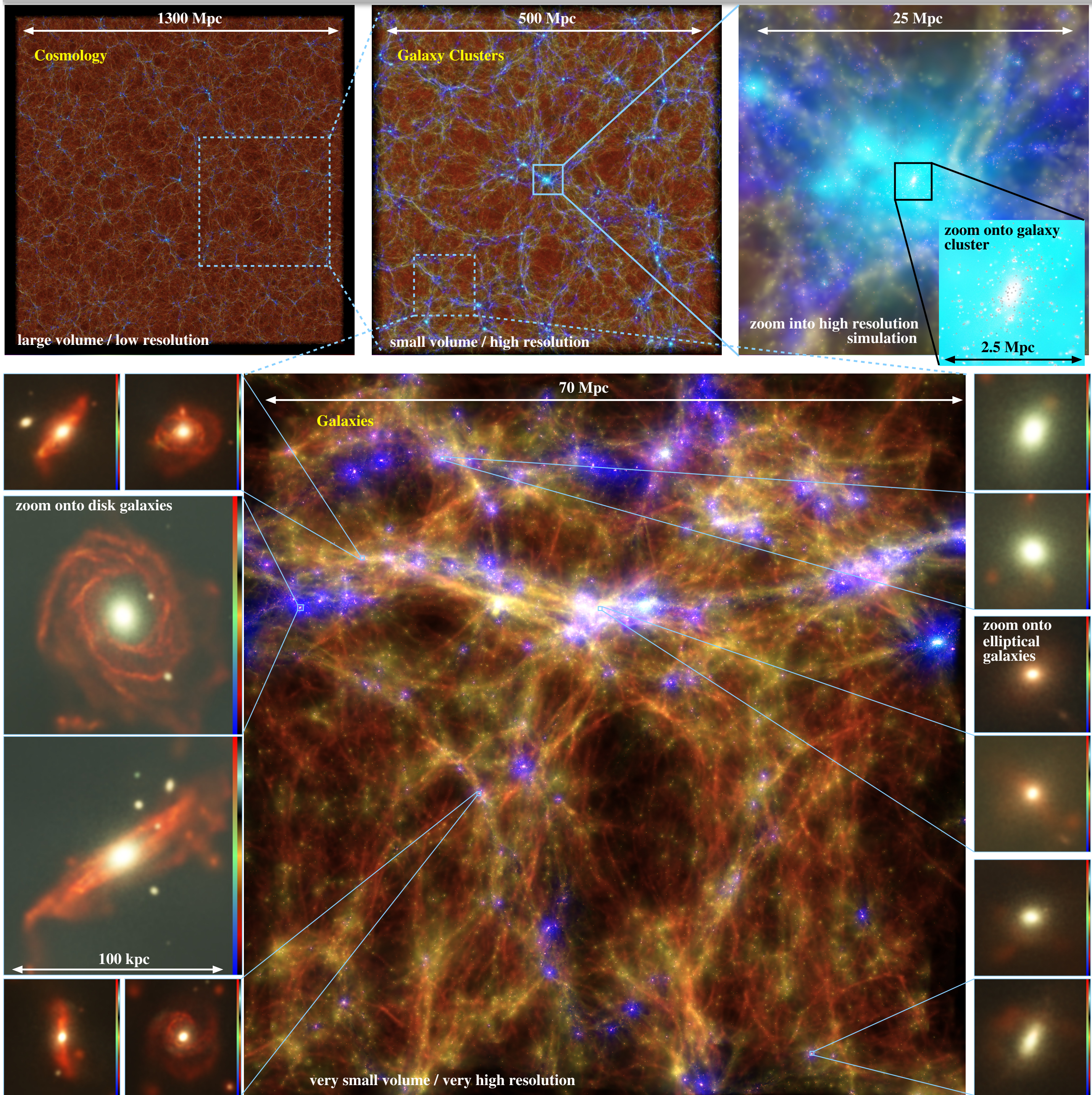
0-th order statements:

(a) N-body simulations provide the DM halo MF to any required precision

(b) Corrections to the Press-Schechter HMF are still almost universal (i.e. independent of cosmology and reshift)

(c) Residuals <10% at the cluster mass-scale

Effects of baryons on the HMF



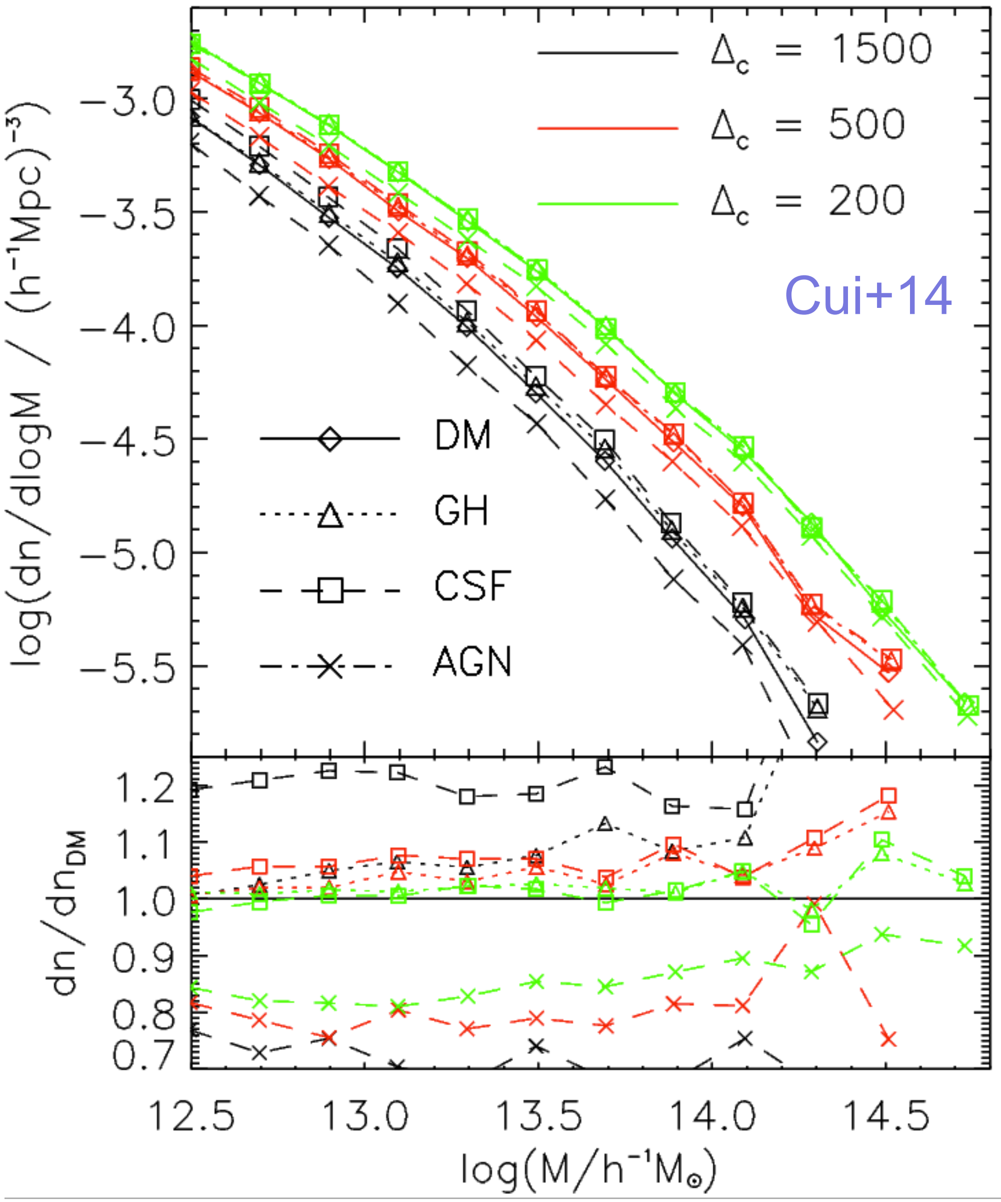
"Magneticum" simulations

Dolag+14

www.magneticum.org

Effects of baryons on the HMF

Rudd+08, Stanek+08, Cui+12,14, Martizzi+14, Velliscig+14, Vogelsberger+14, Schaller+14, Bocquet+15



Effect of non-radiative gas

➔ Slight increase of the HMF

Effect of radiative hydro

➔ Stronger increase of the HMF

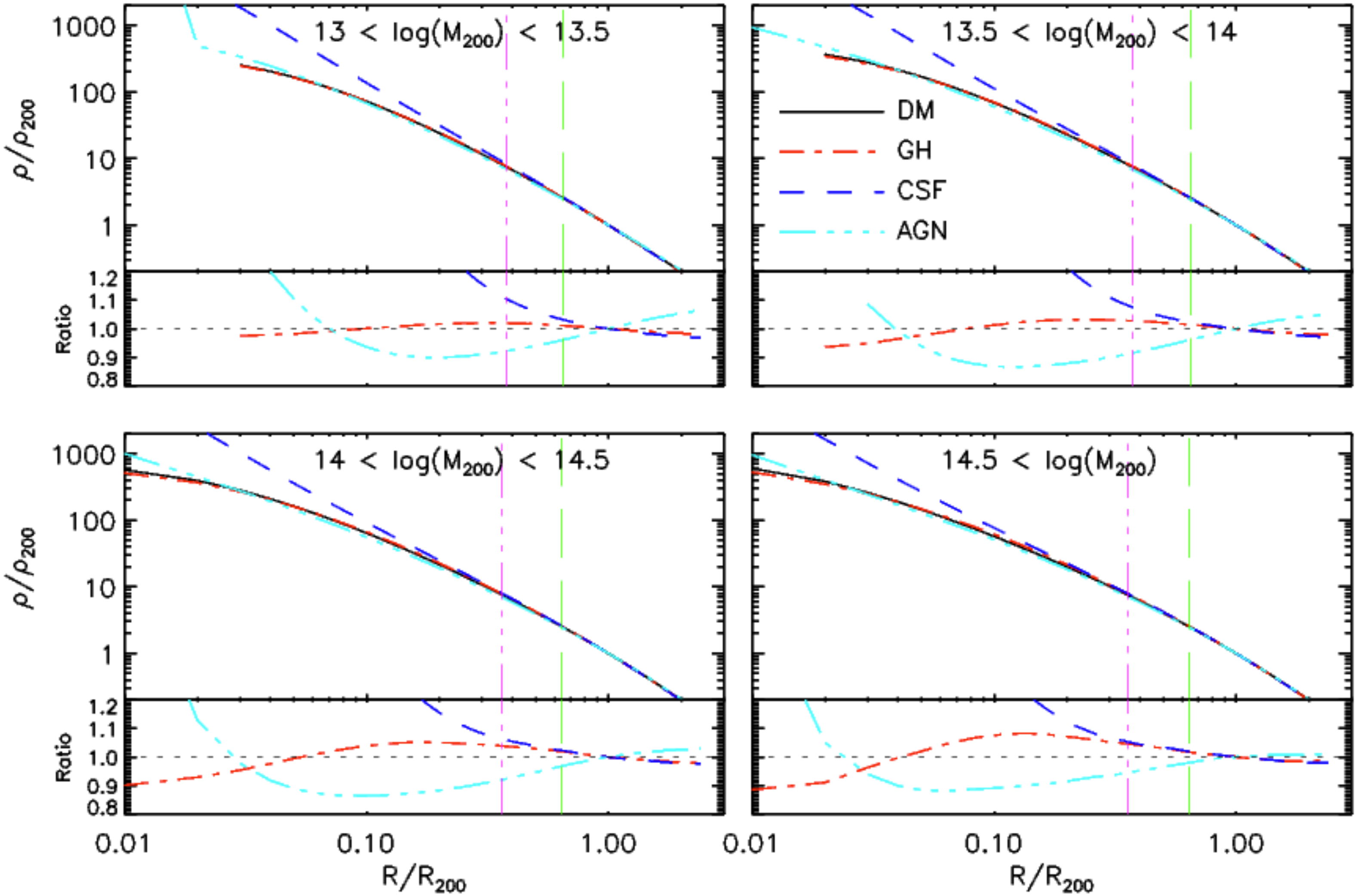
Effect of AGN feedback

➔ Decrease of the HMF

Effect of changing Δ_c

➔ Deviations increase at higher overdensity

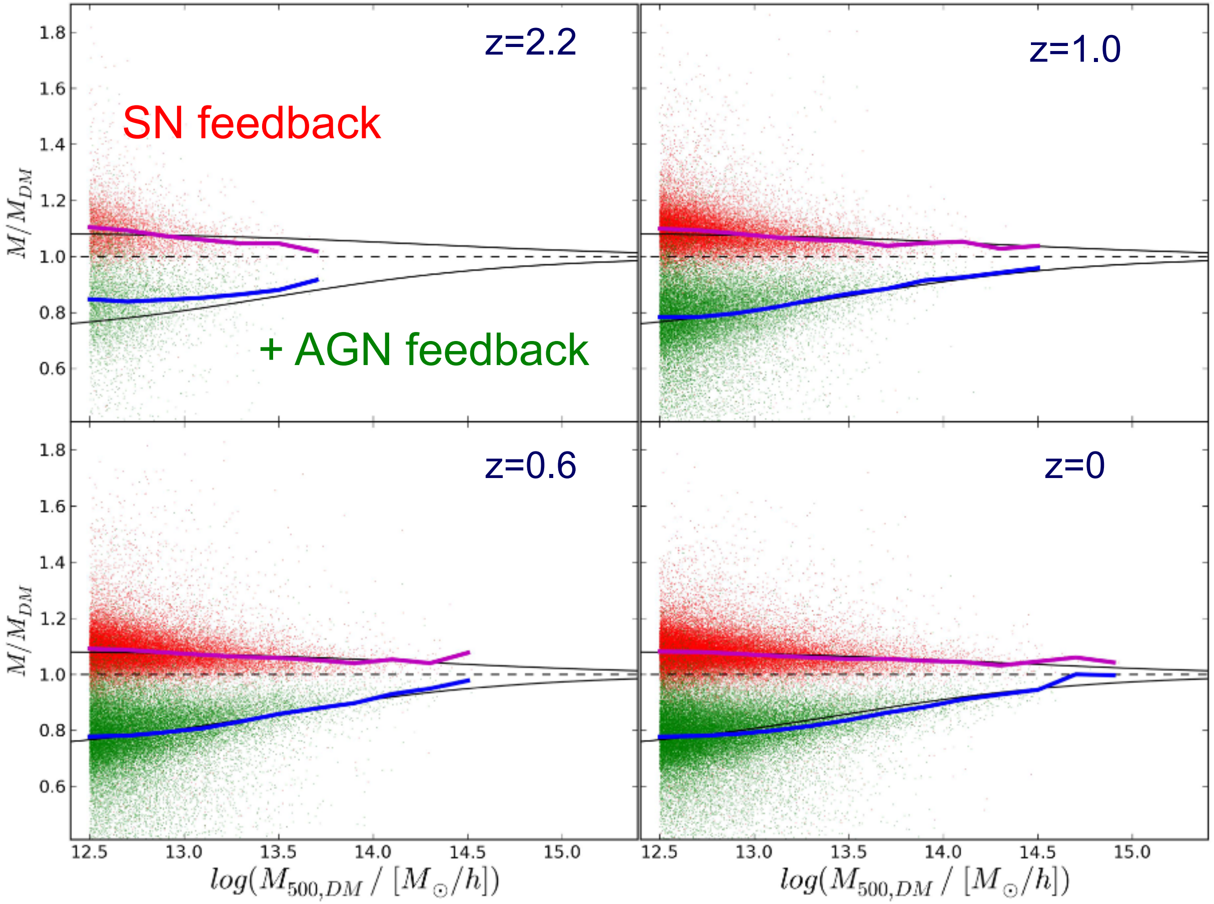
Effects of baryons on density profiles



<u>Simulations with AGN</u>: shallower density profiles (e.g., Cui+14, Martizzi +11; Duffy+10)

➔ Adiabatic expansion of the halo in reaction to sudden gas expulsion at z~2-3.

Effects of baryons on the HMF



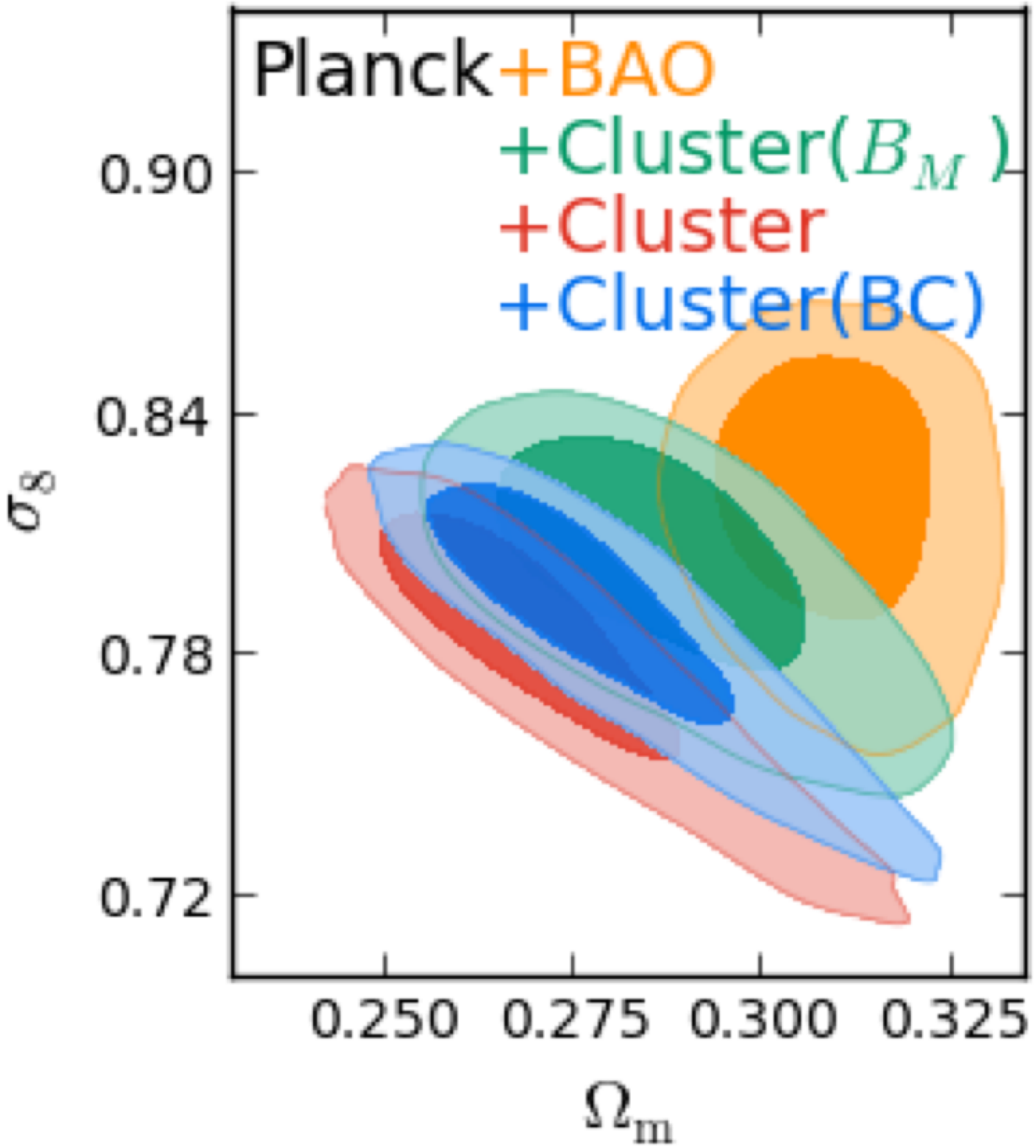
- ➔ Opposite effects for CSF and AGN simulations
- ➔ AGN: ~20% decrease at M_{500}=dex(13.5) h^{-1} $M_{\odot}$
- ➔ Independent of redshift

Q1: what's the impact on cosmological constraints?

Q2: how robust is the calibration of the baryon effects on halo masses?

Impact on cosmological constraints

Costanzi+14



- Planck CMB
- BAO from SDSS-DR11 (Anderson+14)
- CCCP clusters (Vikhlinin+09)
- Massive neutrinos included

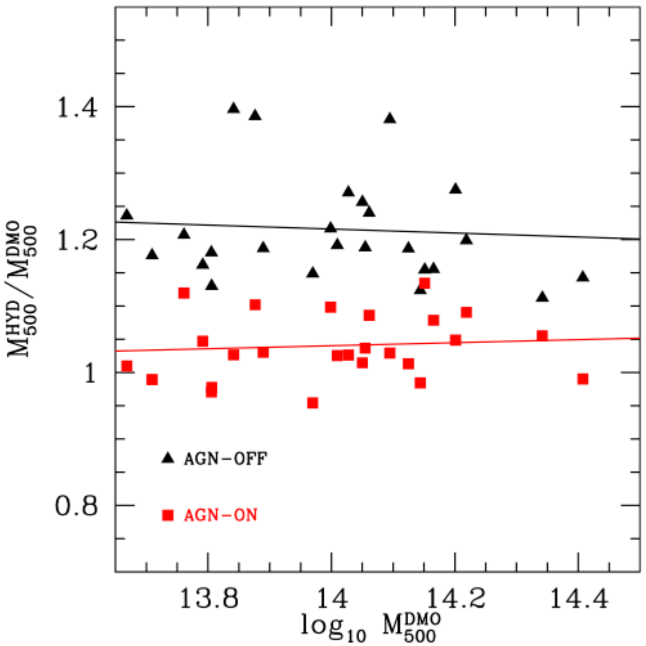
B_M: mass bias = [0.8-1]

BC: HMF baryonic correction

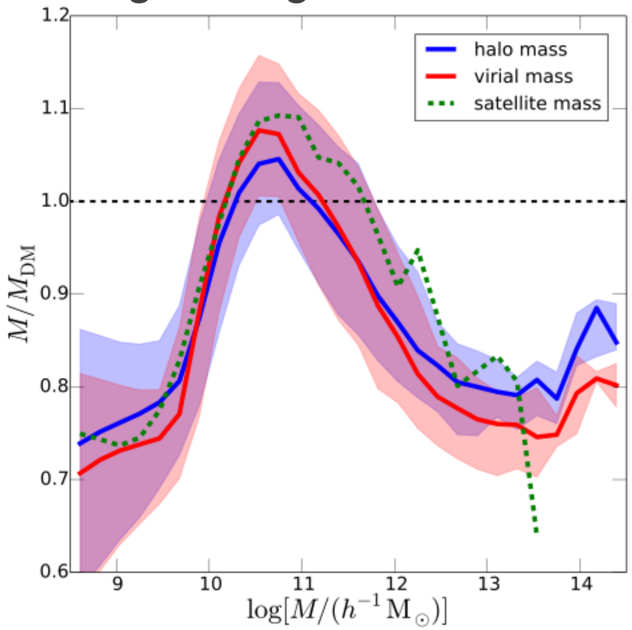
- Alleviate tension with Planck CMB
- Crucial to calibrate for future surveys

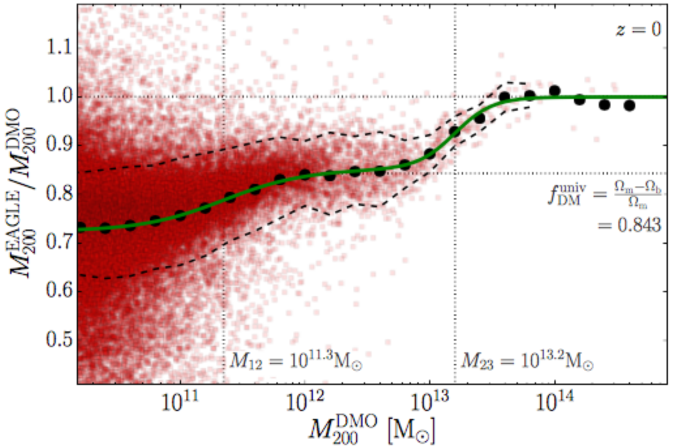
Robustness of calibration

Martizzi+14 : RAMSES

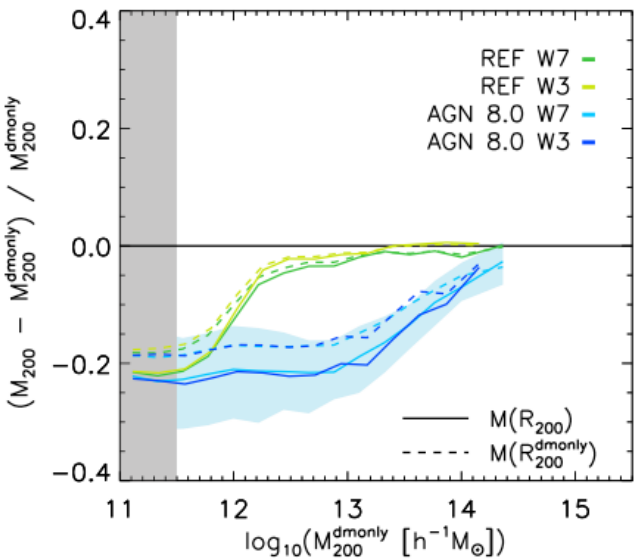


Vogelsberger+14 : AREPO



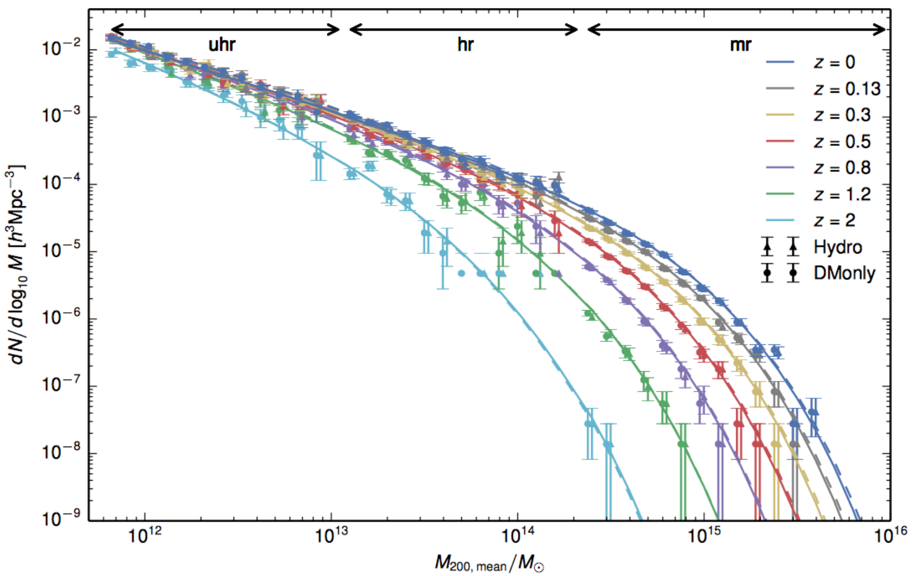


Schaller+14: GADGET-3 (Eagle)

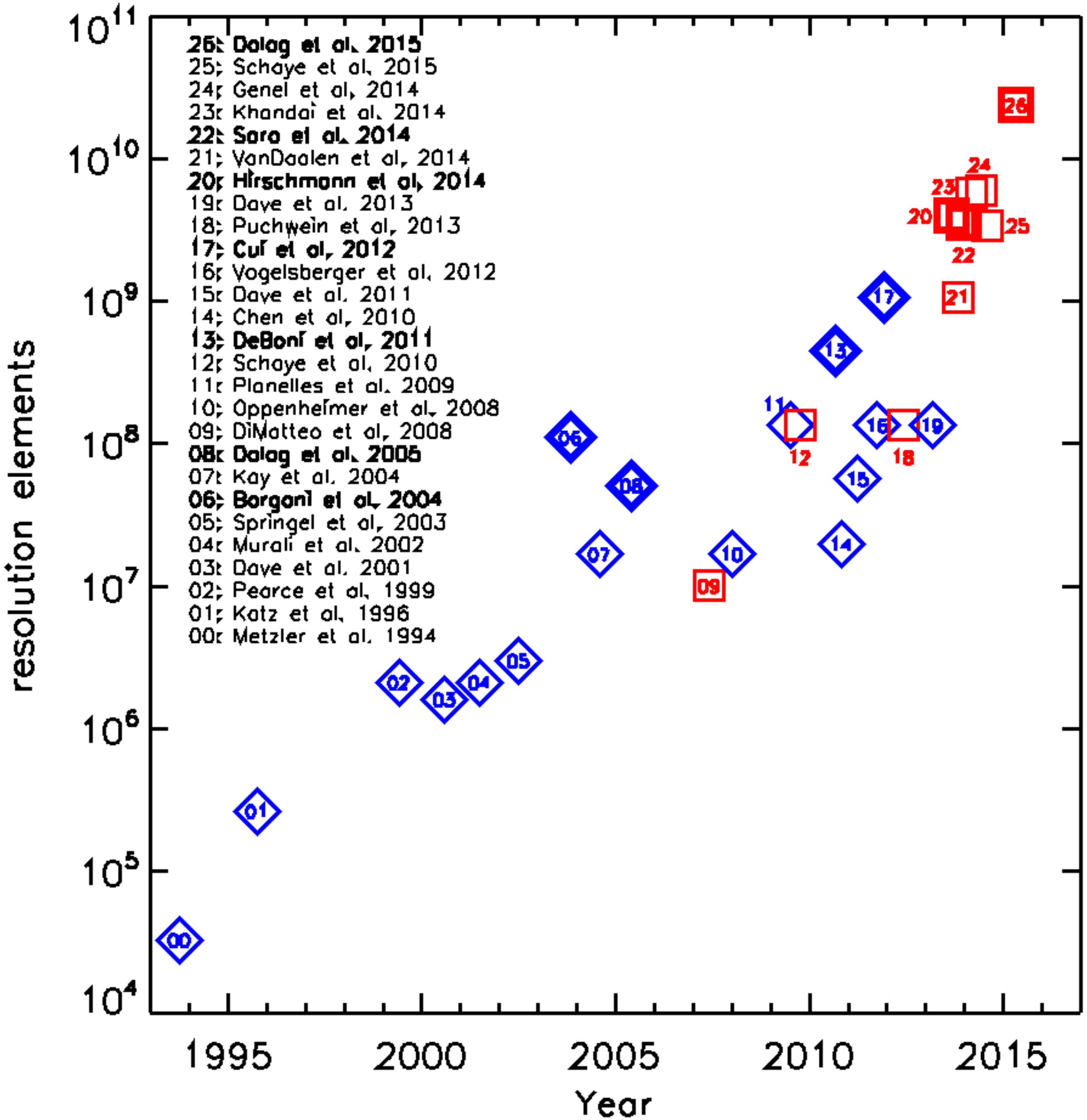


Velliscig+14 : GADGET-3 (OWLS)

Bocquet+15 : GADGET-3 (Magneticum)



Moore's law for hydro simulations



To be kept in mind:

- Different resolutions
- Different physics included
- Trend contributed both by improvement of hardware and code design

Challenge for the future:

➔ Code re-engineering for exa-scale HPC facilities

To bring home

- **Numerical N-body + hydro simulations**:
  → Ideal framework to capture the complexity of cosmic structure formation

- An exact numerical hydrodynamical method?
  → There is not such a thing.....

- Always test and compare different methods to understand range of validity and limitations

- Astrophysical processes: not self-consistently described
  → Phenomenological sub-resolution models

- Galaxy clusters: simulations help to calibrate as cosmological tools
  → Use simulations "cum grano salis"

To bring home

INCLUDE IN SIMULATIONS ALL THE (ASTRO)PHSICAL PROCESSES?

AS DIFFICULT TO INTERPRET AS OBSERVATIONS ....