VERY EARLY UNIVERSE COSMOLOGY

UNIQUE PROBE: COSMOLOGICAL CORRELATIONS

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle, \quad \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle, \quad \ldots$$

EARLY UNIVERSE PHYSICS ENCODED IN SYMMETRIES OF THESE CORRELATION FCNS.

SYMMETRIES $\Longrightarrow$ WARD IDENTITIES

$\Longrightarrow$ CONSTRAIN CORRELATION FCNS.

OUTLINE

IN THESE LECTURES,

1. MULTI-FIELD INFLATION : $SO(4,1)$

2. SINGLE-FIELD INFLATION :

   $SO(4,1) \longrightarrow ISO(3)$

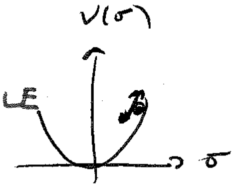
3. CONFORMAL MECHANISM :

   $SO(4,2) \longrightarrow SO(4,1)$.

1. MULTI-FIELD INFLATION

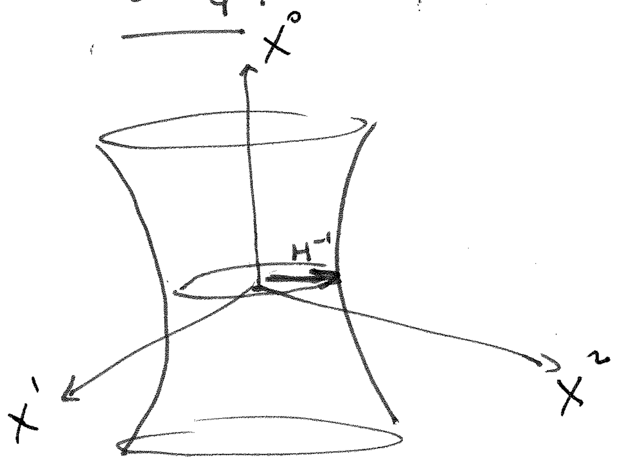
INFLATION = PHASE OF APPROXIMATE dS EXPANSION.

BY "MULTI-FIELD", WE MEAN SPECTATOR FIELD PRESENT DURING INF'N BUT WITH NEGLIGIBLE EFFECT ON BACKGROUND.



FOR SUCH FIELDS, CAN APPROXIMATE BACKGROUND AS EXACT dS.

dS_4: 4d HYPERBOLOID IN 5d MINKOWSKI.



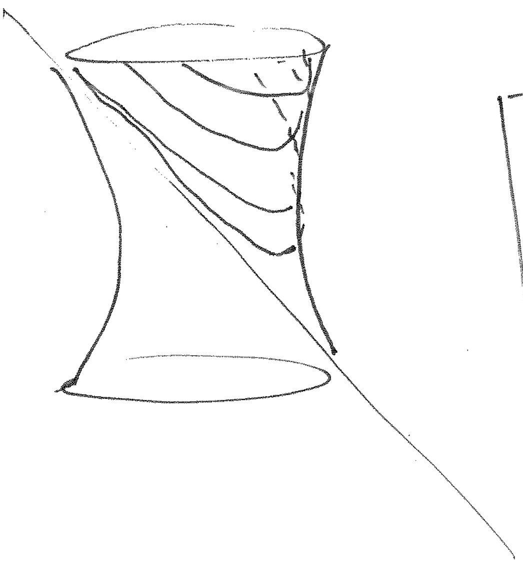
$$\boxed{ds_5^2 = \eta_{AB}\, dX^A dX^B}$$

$A, B = 0, 1, \ldots, 4$

$$\boxed{\eta_{AB} X^A X^B = \frac{1}{H^2}} \quad (*)$$

CAN ~~...~~ CHOOSE DIFFERENT COORDINATES ON THE HYPERBOLOID. FOR INFLATION, RELEVANT SLICING IS SPATIALLY-FLAT SLICING.

INTRODUCE τ TO DECOMPOSE ~~AS FOLLOWS~~ $(*)$ INTO 2 CONSTRAINTS:



$$\boxed{\begin{aligned} -(X^0)^2 + (X^4)^2 &= \frac{1}{H^2}\left(1 - \frac{\vec{x}^2}{\tau^2}\right) \\ (X^1)^2 + (X^2)^2 + (X^3)^2 &= \frac{1}{H^2}\,\frac{\vec{x}^2}{\tau^2} \end{aligned}}$$

$$\boxed{-\infty < \tau < 0}$$

A NICE COORDINATE SYSTEM :

$$X^0 = \frac{1}{2(-\tau)}\left(\frac{1}{H^2} - \tau^2 + \vec{x}^2\right)$$

$$X^i = \frac{1}{H}\,\frac{x^i}{(-\tau)}$$

$$X^4 = \frac{1}{2(-\tau)}\left(\frac{1}{H^2} + \tau^2 - \vec{x}^2\right)$$

NOTE : THIS ONLY COVERS UPPER HALF OF PARABOLOID :

$$X^0 + X^4 = \frac{1}{H^2(-\tau)} > 0 .$$

INDUCED METRIC :

$$\boxed{ds^2 = \frac{1}{H^2\tau^2}\left(-d\tau^2 + d\vec{x}^2\right)}$$

HYPERBOLOID PRESERVED BY $SO(4,1)$ LORENTZ GROUP OF 5d MINKOWSKI SPACE :

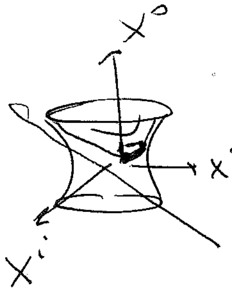
GENERATORS :

$$\boxed{J_{AB} = X_A \partial_B - X_B \partial_A}$$

WITH $SO(4,1)$ ALGEBRA :

$$\boxed{\begin{aligned}[J_{AB}, J_{CD}] &= \eta_{AD} J_{BC} - \eta_{AC} J_{BD} \\ &\quad + \eta_{BC} J_{AD} - \eta_{BD} J_{AC}\end{aligned}}$$

IN $(\tau, \vec{x})$ WORDS, THESE TAKE THE FORM:



- $J_{ij} = x_i \partial_j - x_j \partial_i$ — SPATIAL ROTATIONS
- $J_{4i} - J_{0i} = \frac{1}{H} \partial_i$ — SPATIAL TRANSLATIONS
- $J_{04} \equiv D = +\tau \partial_\tau + x^i \partial_i$ — SPACE-TIME DILATION $x^i \to \lambda x^i$, $\tau \to \lambda \tau$.
- $J_{4i} + J_{0i} \equiv K_i = H\left(2 x_i \tau \partial_\tau + 2 x^i x^j \partial_j - (-\tau^2 + \vec{x}^2)\partial_i\right)$

EXERCISE: $\boxed{\text{SHOW THESE ARE ISOMETRIES OF } ds^2 = \frac{1}{H^2\tau^2}(\cdots)}$

CONSIDER MASSIVE SCALAR FIELD:

$$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2\right)$$

$$= \int d^4x \, a^2 \left(\frac{1}{2}\phi'^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{1}{2} m^2 a^2 \phi^2\right)$$

$ds^2 = a^2(-d\tau^2 + d\vec{x}^2)$

$$= \int d^4x \, \frac{1}{H^2\tau^2}\left(\frac{1}{2}\phi'^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{1}{2}\frac{m^2}{H^2\tau^2}\phi^2\right)$$

FROM THE ABOVE, HAVE:

$$\boxed{\begin{aligned} \delta_D \phi &= \lambda(\tau \partial_\tau + \vec{x}\cdot\vec{\nabla})\phi \\ \delta_{K^i}\phi &= b^i\left(2x_i \tau\partial_\tau + 2x_i \vec{x}\cdot\vec{\nabla} - (-\tau^2 + \vec{x}^2)\partial_i\right)\phi \end{aligned}}$$

LEAVES S INVARIANT.

EOM: $$\phi'' - \frac{2}{\tau}\phi' - \vec{\nabla}^2\phi + \frac{m^2}{H^2\tau^2}\phi = 0.$$

TAKE $k \to 0$ (OR $\tau \to 0$)

HEISENBERG OP.

LET $\phi \sim \tau^\Delta \Longrightarrow \Delta^2 - 3\Delta + \frac{m^2}{H^2} = 0.$

$$\therefore \Delta_\pm = \frac{3}{2}\left(1 \pm \sqrt{1 - \frac{4m^2}{9H^2}}\right)$$

GROWING MODE:

$$\boxed{\phi \sim \tau^{\Delta}} \, , \quad \Delta \equiv \Delta_-$$

$\Longrightarrow$ AT LATE TIMES, $\tau \partial_\tau \rightarrow \Delta$.

$$\therefore \quad \boxed{\begin{aligned} \delta_D \phi &= \lambda \left(\Delta + \vec{x} \cdot \vec{\nabla} \right) \phi \\ \delta_{K^i} \phi &= b^i \left(2 x_i \Delta + 2 x_i \vec{x} \cdot \vec{\nabla} - \vec{x}^2 \partial_i \right) \phi \end{aligned}}$$

THESE ARE SPATIAL DILATION + SPECIAL CONF TRANSF'N FOR A FIELD OF WEIGHT Δ.

i.e., $SO(4,1) \cong$ CONFORMAL GROUP ON $\mathbb{R}^3$.

isometries of dS

cf. dS/CFT CORRESPONDENCE.

IN COSMOLOGY, WE ARE PRECISELY INTERESTED IN CORRELATION FCNS AT LATE TIMES ($k|\tau| \to 0$) (i.e., END OF INFLATION)

2-PT FUNCTION: Creminelli, 1108.0874

TRANS'N + ROT'N INV. $\Longrightarrow \langle \phi(\vec{x}_1, \eta) \, \phi(\vec{x}_2, \eta) \rangle = F(|\vec{x}_1 - \vec{x}_2|, \eta)$

DILATION $\Longrightarrow$ $$\boxed{\langle \phi(\vec{x}_1, \eta) \, \phi(\vec{x}_2, \eta) \rangle = \frac{A \, \tau^{2\Delta}}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}}}$$

IN PARTICULAR, FOR $\Delta \ll 1$ (i.e., $m \ll H$),

THEN $\langle \phi_1 \, \phi_2 \rangle \sim$ CONST. (i.e., SCALE INVARIANT).

NO FURTHER CONSTRAINT FROM SCT.

$$\boxed{\text{3-PT FUNCTION} : \langle \phi_1 \phi_2 \phi_3 \rangle = \frac{C\, \tau^{3\Delta}}{|\vec{x}_1-\vec{x}_2|^{\Delta}\, |\vec{x}_1-\vec{x}_3|^{\Delta}\, |\vec{x}_2-\vec{x}_3|^{\Delta}}}$$

MOMENTUM SPACE REPRESENTATION :

$$\delta_D \phi = \lambda(\Delta + \vec{x}\cdot\vec{\partial})\,\phi$$

FOURIER : $\phi(\vec{x},\eta) = \int d^3k\; e^{i\vec{k}\cdot\vec{x}}\, \phi_{\vec{k}}(\tau)$

$$\Longrightarrow \quad \delta_D \phi = \lambda \int d^3k\; \phi_{\vec{k}}(\tau)\, (\Delta + \underbrace{\vec{k}\cdot\partial_{\vec{k}}}_{\text{BY PARTS}})\, e^{i\vec{k}\cdot\vec{x}}$$

$$= \lambda \int d^3k\; e^{i\vec{k}\cdot\vec{x}} \left((\Delta - 3) - \vec{k}\cdot\partial_{\vec{k}} \right) \phi_{\vec{k}}(\tau)$$

WEIGHT IS REDUCED BY 3 COMPARED TO $\phi(\vec{x})$.

$$\therefore \quad \boxed{\delta_D \phi_{\vec{k}} = \lambda\left((\Delta - 3) - \vec{k}\cdot\partial_{\vec{k}} \right) \phi_{\vec{k}}(\tau)}$$

CONSIDER GENERAL EQUAL-TIME CORRELATION $\langle \mathcal{O}(\vec{k}_1, \dots, \vec{k}_N) \rangle$.
BY TRANSLATION INV., KNOW THAT $\langle \mathcal{O} \rangle \sim \delta^3(\vec{k}_1 + \dots + \vec{k}_N)$.

DEFINE :

$$\boxed{\langle \mathcal{O}(\vec{k}_1, \dots, \vec{k}_N) \rangle = (2\pi)^3\, \delta^3(\vec{P})\, \langle \mathcal{O}(\vec{k}_1, \dots, \vec{k}_N) \rangle' \quad \text{WHERE} \quad \vec{P} \equiv \vec{k}_1 + \dots + \vec{k}_N}$$

NOW, $\delta_D \left(\delta^3(\vec{P}) \langle \mathcal{O} \rangle' \right) = \sum_{a=1}^{N} \left((\Delta_a - 3) - \vec{k}_a\cdot\partial_{\vec{k}_a} \right) \left(\delta^3(\vec{P}) \langle \mathcal{O} \rangle' \right)$

$$= -\langle \mathcal{O} \rangle' \underbrace{\sum_{a=1}^{N} \vec{k}_a}_{\vec{P}} \cdot \partial_{\vec{P}}\, \delta^3(\vec{P}) + \delta^3(\vec{P}) \sum_{a=1}^{N} \left((\Delta_a - 3) - \vec{k}_a\cdot\partial_{\vec{k}_a} \right) \langle \mathcal{O} \rangle'$$

BY PARTS

$$\stackrel{\downarrow}{=} \delta^3(\vec{P}) \left[3 + \sum_{a=1}^{N} \left((\Delta_a - 3) - \vec{k}_a\cdot\partial_{\vec{k}_a} \right) \right] \langle \mathcal{O} \rangle'$$

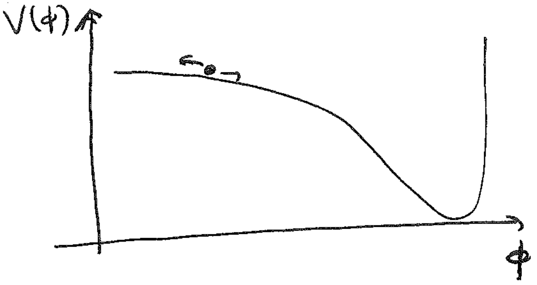
$$\Longrightarrow \quad \boxed{\delta_D \langle \mathcal{O} \rangle' = \left[\sum_{a=1}^{N} \left(\Delta_a - \vec{k}_a\cdot\partial_{\vec{k}_a} \right) - 3(N-1) \right] \langle \mathcal{O} \rangle'}$$

www.ictp.it

EXERCISE: SHOW THAT

$$\boxed{\partial_{K^i} \langle \mathcal{O} \rangle' = i \sum_{a=1}^{N} \left[2(\Delta_a - d)\frac{\partial}{\partial K_a^i} + K_a^i \vec{\partial}_{K_a}^{\,2} - 2\, \vec{K}_a \cdot \vec{\partial}_{K_a}\, \partial_{K_a^i} \right] \langle \mathcal{O} \rangle'}$$

2. SINGLE-FIELD INFLATION



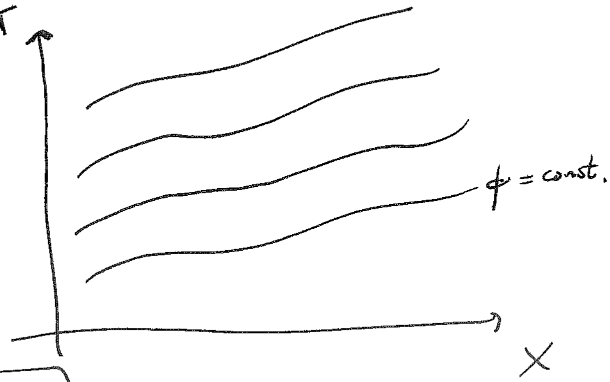
- FLUCTUATIONS $\delta\phi$ ARE SENSITIVE TO DEPARTURES FROM dS_4.

- WILL DERIVE WARD IDENTITIES THAT HOLD IRRESPECTIVE OF SLOW-ROLL, FOR <u>ANY</u> FRW BACKGROUND.

ζ-GAUGE OR UNIFORM-DENSITY GAUGE:

- USE ϕ AS CLOCK:

$$\boxed{\delta\phi = 0}$$



- USE RESIDUAL DIFF. FREEDOM TO SET:

$$\boxed{h_{ij} = a^2(t)\, e^{2\zeta(\vec{x},t)} \left(e^{\gamma(\vec{x},t)}\right)_{ij}}$$

WHERE $\zeta \equiv$ SCALAR PERT'N

$\gamma_{ij} \equiv$ TENSOR PERT'N $\quad \left(\gamma^i_{\ i} = 0\,;\ \partial_i \gamma^i_{\ j} = 0\right)$

THE LAPSE N & SHIFT N_i ARE FIXED BY THE CONSTRAINTS OF GR.

$\hookrightarrow$ THIS COMPLETELY FIXES GAUGE, FOR GAUGE TRANSF'NS THAT FALL OFF AT SPATIAL ∞. BUT THERE IS ROOM FOR SYMMETRIES THAT DON'T FALL OFF.

SYMMETRIES :

HINTERBICHLER, HUI & KHOURY, 1304.5527 .

LOOK FOR RESIDUAL DIFFS $\xi^\mu(\vec{x},t)$ THAT PRESERVE GAUGE CHOICE .

$$\boxed{\xi^i(\vec{x},t)}$$

($\xi^0 = 0$ TO LEAVE $\phi = \phi(t)$ INTACT)

GOAL : IDENTIFY $\delta\zeta$, $\delta\gamma_{ij}$ SUCH THAT

$$\boxed{\delta\left(e^{2\zeta}(e^{\gamma})_{ij}\right) = \mathcal{L}_\xi\left(e^{2\zeta}(e^{\gamma})_{ij}\right)} \quad (*)$$

WHERE $\mathcal{L}_\xi g_{ij} = \xi^k \partial_k g_{ij} + \partial_i \xi^k g_{kj} + \partial_j \xi^k g_{ik}$

THEN $\delta\zeta$, $\delta\gamma_{ij}$ WILL ACT AS GAUGE-PRESERVING DIFFS, AS DESIRED.

SOLVE $(*)$ PERTURBATIVELY IN γ_{ij} :

$$\xi_i = \xi_i^{(\gamma^0)} + \xi_i^{(\gamma^1)} + \dots$$

$$\delta\zeta = \delta\zeta^{(\gamma^0)} + \delta\zeta^{(\gamma^1)} + \dots$$

$$\delta\gamma_{ij} = \delta\gamma_{ij}^{(\gamma^0)} + \delta\gamma_{ij}^{(\gamma^1)} + \dots$$

ZEROTH-ORDER IN γ (DROP γ^0 SUPERSCRIPT)

$$\boxed{2\,\delta\zeta\,\delta_{ij} + \delta\gamma_{ij} = 2\,\xi_k \partial^k \zeta\,\delta_{ij} + \partial_i \xi_j + \partial_j \xi_i}$$

TRACE : $(\delta^{ij}\delta\gamma_{ij} = 0) \Longrightarrow$ $\boxed{\delta\zeta = \frac{1}{3}\partial^i \xi_i + \xi_i \partial^i \zeta}$

SUBSTITUTE BACK : $\frac{2}{3}\partial^k \xi_k \delta_{ij} + \cancel{2\,\xi_k \partial^k \zeta}\,\delta_{ij} + \delta\gamma_{ij} = \cancel{2\,\xi_k \partial^k \zeta}\,\delta_{ij} + \partial_i \xi_j + \partial_j \xi_i$

$$\Longrightarrow \boxed{\delta\gamma_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3}\partial^k \xi_k\,\delta_{ij}}$$

TAKE DIV: $\boxed{\partial^i \delta\gamma_{ij} = 0} \Longrightarrow \boxed{\vec{\nabla}^2 \xi_i + \frac{1}{3}\partial_i \partial^j \xi_j = 0}$ (+)

NOTE: S'POSE $\xi_i \to 0$ AS $|\vec{x}| \to \infty$.

NOW, (+) $\Longrightarrow$ $\vec{\nabla}^2(\partial^i \xi_i) + \frac{1}{3}\vec{\nabla}^2(\partial^i \xi_i) = 0$

$\Longrightarrow \partial^i \xi_i = 0$

$\Longrightarrow \vec{\nabla}^2 \xi_i = 0 \Longrightarrow \xi_i = 0.$

NO NON-TRIVIAL SOL'NS THAT FALL OFF.

SCALAR SECTOR: FOCUS ON $\delta\gamma_{ij} = 0$, i.e.

$$\boxed{\partial_i \xi_j + \partial_j \xi_i = \frac{2}{3}\partial^k \xi_k \delta_{ij}}$$

CONFORMAL KILLING EQU'N ON $\mathbb{R}^3$.

$$\Longrightarrow \xi_i^{\text{dil.}} = \lambda x_i$$

$$\xi_i^{\text{SCT}} = 2\vec{b}\cdot\vec{x}\, x_i - \vec{x}^2 b_i$$

WITH
$$\boxed{\begin{aligned} \delta_D \zeta &= \lambda\left(1 + \vec{x}\cdot\vec{\nabla}\zeta\right) \\ \delta_{K^i}\zeta &= 2\vec{b}\cdot\vec{x} + \left(2\vec{b}\cdot\vec{x}\, x^i - \vec{x}^2 b^i\right)\partial_i \zeta \end{aligned}}$$

D & K^i ARE NON-LINEARLY REALIZED.

$\Longrightarrow$ ζ is GOLDSTONE OR DILATON FOR

$$\boxed{SO(4,1) \longrightarrow ISO(3)}$$

WHY ONLY ONE GOLDSTONE?

MORE GENERALLY, $\vec{\nabla}^2 \xi_i + \frac{1}{3} \partial_i \partial^j \xi_j = 0.$

LET $$\boxed{\xi_i = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} M_{i l_0 \dots l_n} x^{l_0} \dots x^{l_n}}$$

$$\Longrightarrow \quad \boxed{M_{i l l l_2 \dots l_n} = -\frac{1}{3} M_{l l i l_2 \dots l_n}}$$

PHYSICALLY, A LOCAL OBSERVER SEES:

$$\boxed{\begin{array}{ccccccc} h_{ij} = & \bar{h}_{ij} & + & \partial_k \bar{h}_{ij}\, x^k & + & \frac{1}{2} \partial_k \partial_l \bar{h}_{ij}\, x^k x^l & + \dots \\ & \downarrow & & \downarrow & & \downarrow & \\ & \delta_{ij} & & 0 & & -\frac{1}{3} \bar{R}_{ijkl}\, x'^k x'^l & \end{array}}$$

$$\boxed{\#\ \text{OF COMPONENTS} - \#\ \text{OF SYMMETRIES} = \text{PHYSICAL METRIC COMPONENTS}}$$

WARD IDENTITIES : HINTERBICHLER, HUI, JK 1304.5527
(SOFT-PION THMS) GOLDBERGER, HUI, NICOLIS, 1303.1193
ASSASSI, BAUMANN, GREEN, 1204.4207
CREMINELLI, NORENA, SIMONOVIC, 1203.4595.
⋮

$$\boxed{\langle \Omega | [Q, \mathcal{O}] | \Omega \rangle = -i \langle \Omega | \delta \mathcal{O} | \Omega \rangle}$$

(IN-IN)

- FOR LINEARLY-REALIZED SYMS, $Q|\Omega\rangle = 0$

$$\Longrightarrow \langle \Omega | \delta \mathcal{O} | \Omega \rangle = 0$$

(INVARIANCE OF CORRELATOR)

- FOR NON-LINEARLY REALIZED SYMS, GET A SOFT-PION THEOREM.

ILLUSTRATE METHOD WITH DILATION:

$$\boxed{\delta_D \zeta = 1 + \vec{x}\cdot\vec{\nabla}\zeta}$$

NON-LIN PART (SYM. OF QUADRATIC THEORY)

LINEAR PART (CORRESPONDS TO $\Delta_\zeta = 0$)

RHS OF IDENTITY IS EASY: $\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N) = \zeta(\vec{k}_1, t)\cdots\zeta(\vec{k}_N,t)$

$$\Longrightarrow \boxed{\langle\Omega|\,\delta_D\mathcal{O}\,|\Omega\rangle_c' = -\left(3(N-1) + \sum_{a=1}^{N}\vec{k}_a\cdot\partial_{\vec{k}_a}\right)\langle\mathcal{O}\rangle_c'}$$

LHS: Q IS CHARGE, I.E. $[Q,\zeta] = -i\,\delta\zeta$
$= -i\,(1+\vec{x}\cdot\vec{\nabla}\zeta)$

SPLIT $\boxed{Q = Q_0 + Q_{int.}}$

GENERATES $\delta\zeta = \text{const}$.
$\Longrightarrow Q_0$ IS SYMMETRY OF FREE (QUADRATIC) THEORY.

$$\Longrightarrow \boxed{Q_0 = \int d^3x\, \Pi_\zeta(\vec{x}) = \lim_{\vec{q}\to 0}\Pi_\zeta(\vec{q})}$$

$\hookrightarrow [\zeta(x), \Pi_\zeta(\vec{y})] = i\,\delta^3(\vec{x}-\vec{y})$

NOW, $|\Omega\rangle$ IS IN-VACUUM, & RELATED TO BUNCH-DAVIES VACUUM $|0\rangle$ BY:

$$\boxed{|\Omega\rangle = \Omega(-\infty)\,|0\rangle} \qquad \boxed{\Omega(-\infty) \equiv U^\dagger(-\infty,0)\,U_0(-\infty,0)}$$

MØLLER WAVE OP.

SIMILARLY, INTERTWINING RELATION:

$$\boxed{Q\,\Omega(-\infty) = \Omega(-\infty)\,Q_0}$$

(WEAK STATEMENT)

$$\therefore \boxed{Q|\Omega\rangle = Q\,\Omega(-\infty)\,|0\rangle = \Omega(-\infty)\,Q_0\,|0\rangle}$$

CALCULATE $Q_0|0\rangle$: INSERT COMPLETE SET OF FREE-FIELD EIGENSTATES $|\zeta_0\rangle$: (SCHROEDINGER PICTURE)

$\zeta_0(\vec{x})|\zeta_0\rangle = \zeta_0(\vec{x})|\zeta_0\rangle$

$$Q_0|0\rangle = \lim_{\vec{q}\to 0}\int D\zeta_0\, |\zeta_0\rangle\langle\zeta_0|\,\pi(\vec{q})\,|0\rangle$$

$$= \lim_{\vec{q}\to 0}\int D\zeta_0\, |\zeta_0\rangle \left(-i\frac{\delta}{\delta\zeta_0(-\vec{q})}\right)\langle\zeta_0|0\rangle$$

BUNCH - DAVIES WAVEFUNCTIONAL (GAUSSIAN):

$$\langle\zeta_0|0\rangle \sim \exp\left[-\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{4P_\zeta(k)}\,\zeta_0(\vec{k})\,\zeta_0(-\vec{k})\right]$$

$$\Longrightarrow \boxed{Q_0|0\rangle = \lim_{\vec{q}\to 0}\frac{1}{2P_\zeta(q)}\,\zeta_0(\vec{q})\,|0\rangle}$$ (HEISENBERG)

IF $\lim_{\vec{q}\to 0}\zeta_0(\vec{q}) =$ CONSTANT & $\lim_{\vec{q}\to 0}\zeta(\vec{q}) =$ CONSTANT,

THEN $\lim_{\vec{q}\to 0}\Omega(-\infty)\,\zeta_0(\vec{q}) = \lim_{\vec{q}\to 0}\zeta(\vec{q})\,\Omega(-\infty)$

$$\Longrightarrow Q|\Omega\rangle = \Omega(-\infty)\,Q_0|0\rangle$$

$$= \lim_{\vec{q}\to 0}\frac{1}{2P_\zeta(\vec{q})}\,\Omega(-\infty)\,\zeta_0|0\rangle$$

$$= \lim_{\vec{q}\to 0}\frac{1}{2P_\zeta(\vec{q})}\,\zeta(\vec{q})\,|\Omega\rangle$$

HENCE,

$$\boxed{\langle\Omega|[Q,\mathcal{O}]|\Omega\rangle = -i\lim_{\vec{q}\to 0}\frac{1}{P_\zeta(\vec{q})}\langle\Omega|\zeta(\vec{q})\mathcal{O}|\Omega\rangle}$$

PUTTING LHS & RHS TOGETHER, OBTAIN:

$$\boxed{\lim_{\vec{q}\to 0} \frac{1}{P_\zeta(q)} \langle \zeta(\vec{q})\, \mathcal{O}(\vec{k}_1,\dots,\vec{k}_N)\rangle_c' = -\left(3(N-1) + \sum_{a=1}^{N} \vec{k}_a \cdot \frac{\partial}{\partial \vec{k}_a}\right) \langle \mathcal{O}(\vec{k}_1,\dots,\vec{k}_N)\rangle_c'}$$

MORE GENERALLY, FOR OUR GENERAL SYMMETRIES,

$$\delta\zeta,\ \delta\gamma \sim x^n$$

$$\Longrightarrow \quad Q_0 = \int d^3x\; x^n\, \Pi(\vec{x})$$

$$= \lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n} \Pi(\vec{q})$$

$$\therefore \boxed{\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{1}{P_\zeta(q)} \langle \zeta_{\vec{q}}\, \mathcal{O}(\vec{k}_1,\dots,\vec{k}_N)\rangle_c' + \frac{1}{P_\gamma(q)} \langle \gamma_{\vec{q}}\, \mathcal{O}(\vec{k}_1,\dots,\vec{k}_N)\rangle_c' \right) \sim \frac{\partial^n}{\partial k^n} \langle \mathcal{O}\rangle_c'}$$

- PHYSICAL STATEMENTS (i.e. CAN BE VIOLATED)
- HOLDS ON <u>ANY</u> FRW BACKGROUND (NO SLOW-ROLL)
- q^0 & q^1 PARTS COMPLETELY FIXED

  q^n, $n \geq 2$ <u>PARTIALLY</u> FIXED.

Craps, Hertog & Turok, 0712.4180
Rubakov, 0906.3693
Creminelli, Nicolis Trincherini, 1007.0027
Hinterbichler, Khoury, 1106.1478

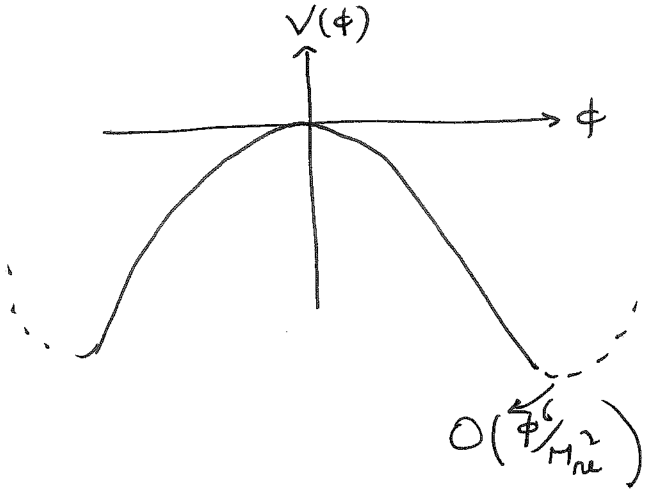
3. Conformal mechanism

- No inflation
- Gravity is unimportant, i.e. space-time $\approx$ Minkowski
- Relies on (approximate) conformal invariance. $SO(4,2)$
- Spontaneously broken:

$$SO(4,2) \longrightarrow SO(4,1).$$

- Solves flatness, horizon problems.
- Needs bounce ~~or~~ and ~~NEC~~.

Simplest example



$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 \right]$$

$$\boxed{\lambda < 0}$$

- $\lambda < 0 \Longrightarrow$ asymptotically free
- (classically) conformal:

$$\delta_{P_\mu}\phi = -\partial_\mu \phi \quad ; \quad \delta_{J_{\mu\nu}}\phi = (x_\mu \partial_\nu - x_\nu \partial_\mu)\phi$$

$$\delta_D \phi = -(1 + x^\mu \partial_\mu)\phi \quad ; \quad \delta_{K_\mu}\phi = (-2x_\mu - 2x_\mu x^\nu \partial_\nu + x^2 \partial_\mu)\phi$$

Define:

$$\delta_{J_{5\mu}} = \frac{1}{2}(\delta_{P_\mu} + \delta_{K_\mu})$$

$$\delta_{J_{6\mu}} = \frac{1}{2}(\delta_{P_\mu} - \delta_{K_\mu})$$

$$\delta_{J_{56}} = \delta_D$$

$$\Longrightarrow \boxed{[\delta_{J_{AB}}, \delta_{J_{CD}}] = \eta_{AC}\,\delta_{J_{BD}} - \eta_{BC}\,\delta_{J_{AD}} + \eta_{AD}\,\delta_{J_{AC}} - \eta_{AD}\,\delta_{J_{BC}}}$$

$$\eta_{AB} \equiv \mathrm{diag}(\eta_{\mu\nu}, 1, -1) \qquad SO(4,2)$$

ASSUME $\phi = \phi(t)$ (JUSTIFY LATER),
THEN EOM ADMITS CONSERVED ENERGY:

$$\boxed{E = \frac{1}{2}\dot{\Phi}^2 + \frac{\lambda}{4}\Phi^4}$$

FOCUS ON $E = 0$ (ATTRACTOR, AS WE'LL SEE):

$$\boxed{\Phi(t) = \sqrt{\frac{2}{-\lambda}}\frac{1}{(-t)}}, \quad -\infty < t < 0 .$$

THIS PRESERVES THE FOLLOWING SYMMETRIES:

$$\{ \delta_{P_i}, \delta_{K_i}, \delta_{J_{ij}}, \delta_D \}$$

TAKE LINEAR COMB'NS:

$$\delta_{J_{5i}} = \frac{1}{2}(\delta_{P_i} + \delta_{K_i})$$
$$\delta_{J_{6i}} = \frac{1}{2}(\delta_{P_i} - \delta_{K_i})$$
$$\delta_{J_{56}} = \delta_D$$

$$\Longrightarrow \boxed{[\delta_{J_{ab}}, \delta_{J_{cd}}] = \eta_{ac}\delta_{J_{bd}} - \eta_{bc}\delta_{J_{ad}} + \eta_{bd}\delta_{J_{ac}} - \eta_{ad}\delta_{J_{bc}}}$$

$$\eta_{ab} = \mathrm{diag}(\delta_{ij}, 1, -1).$$

THIS IS $SO(4,1)$. SAME SYMMETRIES AS FOR SPECTATOR FIELDS IN INFLATION!

$$\boxed{\text{EXPECT FIELDS WITH } \Delta \ll 1 \text{ TO ACQUIRE NEARLY SCALE-INVARIANT SPECTRUM}}$$

e.g. CONSIDER SPECTATOR FIELD Θ, WITH

$$\Delta_\Theta = 0 .$$

$$S_\Theta = \int d^4x \left(- \frac{1}{2} \phi^2 (\partial\Theta)^2 \right)$$

REQUIRED BY CONFORMAL INV.

$\Longrightarrow$ AS IF Θ COUPLES TO

$$\boxed{g_{\text{eff}\,\mu\nu} = \bar{\phi}^2 \eta_{\mu\nu} = \frac{2}{|\lambda| t^2} \eta_{\mu\nu}}$$

de Sitter !

$\therefore$ $\boxed{\langle \Theta\Theta \rangle \text{ is SCALE-INVARIANT !}}$

~~GENERAL SCENARIO~~:

STABILITY : $\phi = \bar{\phi}(t) + \pi(\vec{x},t)$

$$\Longrightarrow \ddot{\pi} - \cancel{\vec{\nabla}^2 \pi} - \frac{6}{t^2}\pi = 0 .$$

$t \to 0$ LIMIT

$$\therefore \boxed{\pi \sim \frac{1}{t^2} \quad \text{OR} \quad \pi \sim t^3} .$$

GROWING MODE

JUST A TIME SHIFT OF THE BACKGROUND:

$$\bar{\phi}(t+\varepsilon) = \bar{\phi}(t) + \varepsilon \dot{\bar{\phi}}(t) \sim \frac{1}{(-t)} \left(1 + \frac{\varepsilon}{(-t)} \right)$$

$\therefore$ $\boxed{\text{SOLUTION IS AN ATTRACTOR}}$

GENERAL SCENARIO: HINTERBICHLER & JK (2011)

S'POSE HAVE CFT, SUCH THAT ~~AT LEAST~~ ~~ONE SCALAR~~ PRIMARY OPS WITH Δ_I ACQUIRE TIME-DEP BACKGROUND:

$$\boxed{\mathcal{O}_I(t) = \frac{c_I}{(-t)^{\Delta_I}}}$$

IF ~~SCALAR~~ $\Delta_I \neq 0$ FOR AT LEAST ONE OF THESE,

THEN $\boxed{SO(4,2) \longrightarrow SO(4,1)}$.

BASED ON $SO(4,1)$ ALONE,

$$\langle \delta\mathcal{O}_I(\vec{x}_1)\, \delta\mathcal{O}_J(\vec{x}_2)\rangle = \begin{cases} \dfrac{C_{IJ}}{|\vec{x}_1 - \vec{x}_2|^{\Delta_I+\Delta_J}} & ; \ \Delta_I = \Delta_J \\ 0 & ; \ \Delta_I \neq \Delta_J . \end{cases}$$

$$\langle \delta\mathcal{O}_I(\vec{x}_1)\, \delta\mathcal{O}_J(\vec{x}_2)\, \delta\mathcal{O}_K(\vec{x}_3)\rangle = \frac{C_{IJK}}{|\vec{x}_1-\vec{x}_2|^{\Delta_I+\Delta_J-\Delta_K}|\vec{x}_2-\vec{x}_3|^{\Delta_J+\Delta_K-\Delta_I}\cdot|\vec{x}_1-\vec{x}_3|^{\Delta_I+\Delta_K-\Delta_J}}$$

BUT WE ALSO HAVE SOFT-PION THMS FOR $SO(4,2) \longrightarrow SO(4,1)$ BROKEN SYMS (MORE ON THIS LATER)

EXPLICIT REALIZATIONS:

- ϕ^4
- GALILEAN GENESIS
- DBI

$\vdots$

COUPLING TO GRAVITY

ASSUME MINIMAL COUPLING :

$$S = \int d^4x \sqrt{-g}\left(\frac{M_{pl}^2}{2} R + \mathcal{L}_{CFT}[g] \right)$$

$\downarrow$ BREAKS CONFORMAL INV. AT $\mathcal{O}(1/M_{pl})$.

THINK OF PERTURBATIVE EXPANSION IN $1/M_{pl}$:

BY SYMMETRY, $\boxed{\rho_{CFT} \simeq \frac{\alpha}{t^4} \; ; \; P_{CFT} \simeq \frac{\beta}{t^4}}$.

BUT ENERGY CONSERVATION $\Longrightarrow \boxed{\rho_{CFT} \simeq 0}$

NOW, $\dot{H} = -\frac{1}{2M_{pl}^2}\left(\cancel{\rho_{CFT}}^{\,0} + P_{CFT}\right) = -\frac{1}{2M_{pl}^2}\frac{\beta}{t^4}$.

$\therefore \boxed{H(t) \simeq \frac{\beta}{6t^3 M_{pl}^2}} \Longrightarrow$ $\beta > 0$ CONTRACTING

$\beta < 0$ EXPANDING.

INTEGRATE AGAIN :

$\boxed{a(t) \simeq 1 - \frac{\beta}{12 t^2 M_{pl}^2}}$ (MINKOWSKI + $\mathcal{O}(\frac{1}{M_{pl}^2})$ CORRECTION)

EQUATION OF STATE : $w \simeq -\frac{\dot{H}}{H^2} \sim \frac{t^2 M_{pl}^2}{\beta}$

$\gg 1 \; . \; \beta > 0$

$\ll -1 \quad \beta < 0$.

KEY POINT: COSMO BACKGROUND IS SLOWLY EXPANDING OR CONTRACTING.

COSMIC NO-HAIR

$$3M_{Pl}^2 H^2 = \frac{C_m}{a^3} + \frac{C_r}{a^4} - \frac{K}{a^2} + \frac{C_{aniso}}{a^6} + \dots + \frac{C_\phi}{a^{3(1+w)}}$$

- EXPANDING : NEED $w < -\frac{1}{3}$
  (ACCELERATED EXPANSION)
- CONTRACTING : NEED $w > 1$

$$a(t) \sim t^{\frac{2}{3(1+w)}} \Longrightarrow \text{SLOW CONTRACTION.}$$

~~SOFT-PION THMS (CREMINELLI, JOYCE, KHOURY & SIMONOVIC (2013))~~

OBSERVATIONAL CONSEQUENCES

RUBAKOV & CO.
CREMINELLI, JOYCE, JK & SIMONOVIC (2013)

SPECTRUM FOR π : $\ddot{\pi}_k + k^2\pi_k - \frac{6}{t^2}\pi_k = 0$

GOLDSTONE IS TACHYONIC!

$$\Longrightarrow \pi_k \sim \frac{1}{t^2} \quad \text{AS } t \to 0$$

$$\therefore \boxed{k^3 P_\pi(k) \sim \frac{1}{k^2 t^2}} \qquad \text{VERY RED!}$$

- OBSERVED SPECTRUM COMES FROM OTHER FIELDS
- REDNESS IS HARMLESS $\longrightarrow$ $k^3 P_\zeta(k) \sim k^3$.
  (VERY BLUE)

> BUT HAS CONSEQUENCES FOR OTHER OBSERVABLES.

SOFT-PION THMS

ON TOP OF SO(4,1), CORRELATORS ARE ~~ALSO~~ FURTHER CONSTRAINED BY 5 BROKEN SYMMETRIES :

$$\boxed{P_0 \ , \quad J_{0i} \ , \quad K_0}.$$

GOLDSTONE TRANSFORMS AS :

$$\delta_{P_0}\pi = \frac{1}{t} - \partial_t \pi$$

$$\delta_{J_{0i}}\pi = \frac{x_i}{t} + t\partial_i \pi - x_i \partial_t \pi$$

$$\delta_{K_0}\pi = -\frac{\vec{x}^2}{t} - \left(2t x^\nu \partial_\nu - x^2 \partial_t\right)\pi$$

$\Longrightarrow$ SEE SLIDES