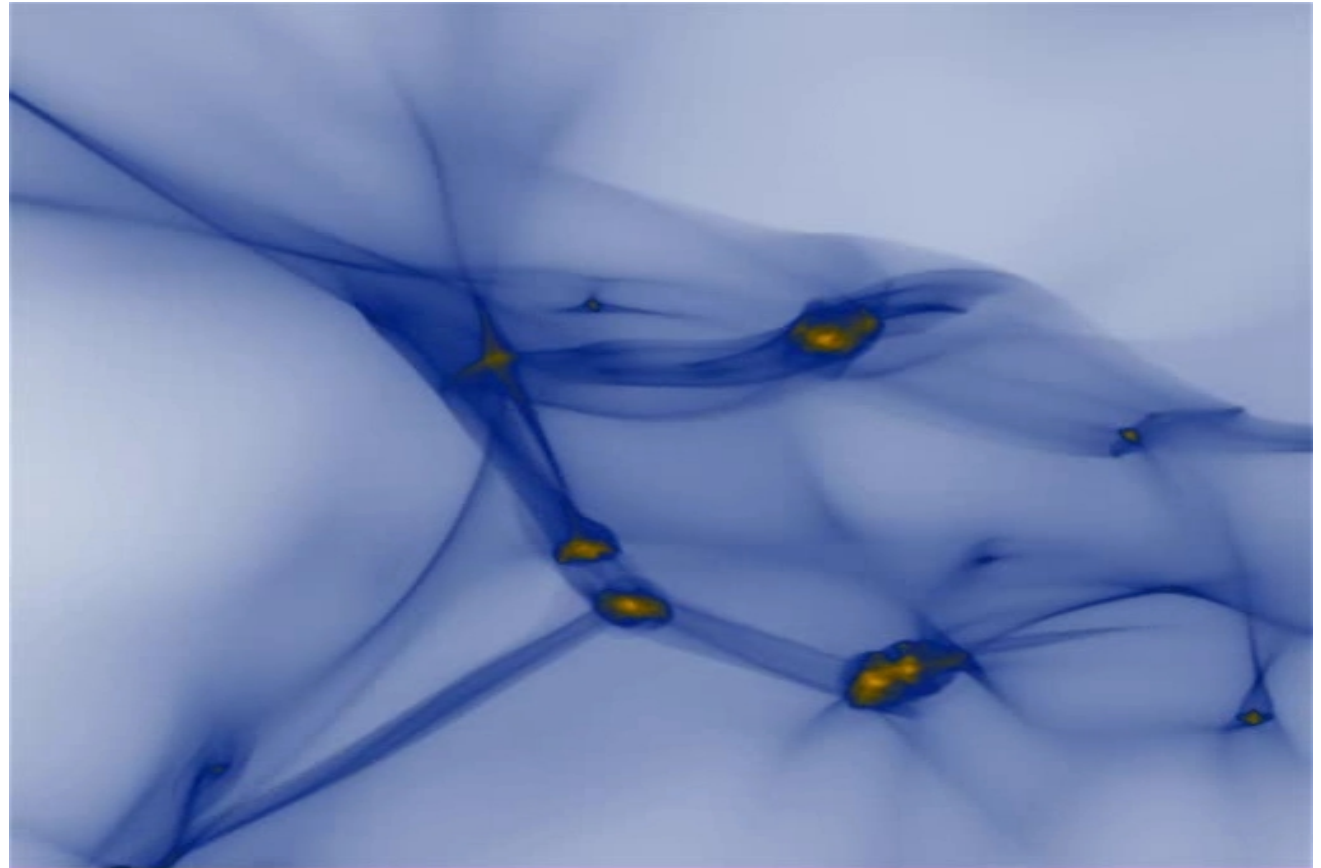


# Dark Matter Simulations for the Large-Scale Structure of the Universe

Raul E. Angulo



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Advanced Workshop on Cosmological Structures  
ICTP, Trieste  
Mayo 2015

# Simulating structure formation in the Universe

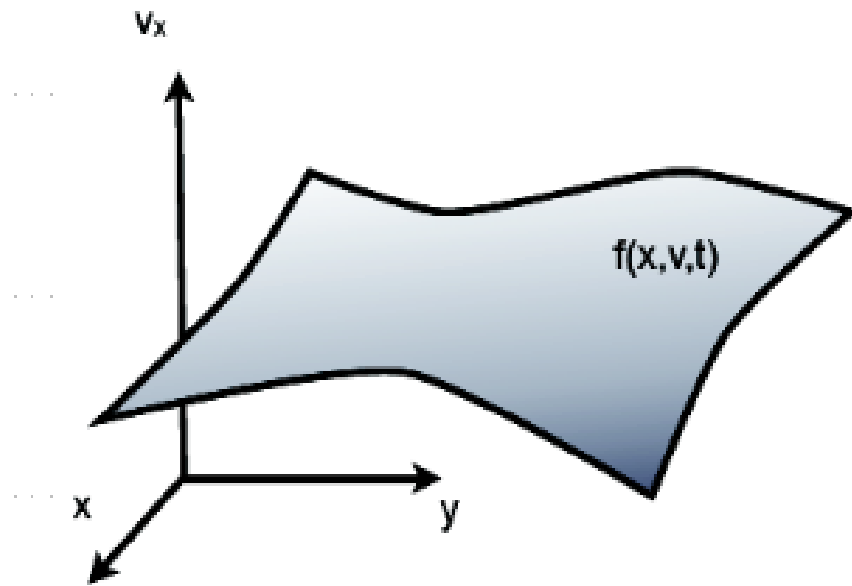
Most of the mass in the Universe is in the form of an unknown elementary particle: the Cold Dark Matter

## Properties of CDM

- No thermal velocity
- Only Gravity
- Small primordial fluctuations

*...but simulating trillions of micro-physical CDM particles is impossible*

**CDM forms a "sheet": A continuous 3D surface embedded in a 6D space**



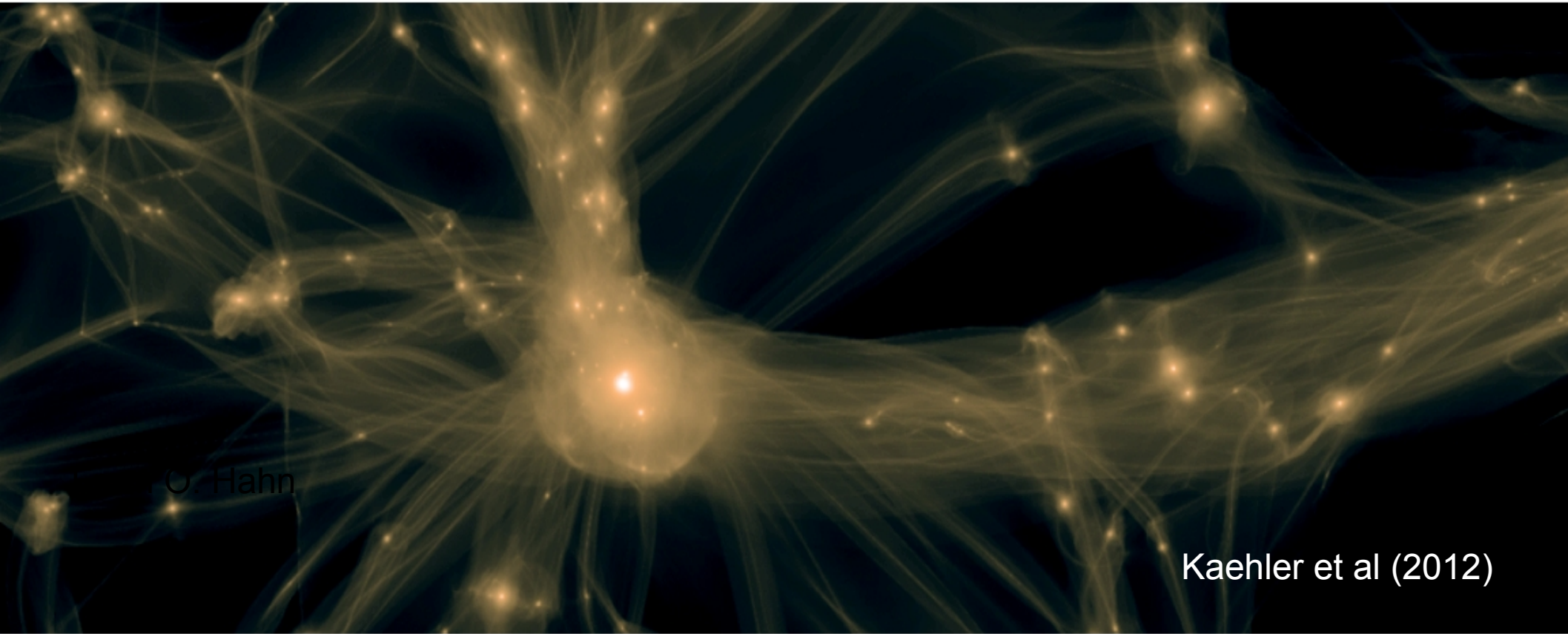
# The Vlasov-Poisson Equation

$$0 = \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \frac{\partial f}{\partial \mathbf{v}} - \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \Phi}{\partial \mathbf{x}}$$

$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f d^3 v$$

## CDM Sheet Properties

- phase-space is conserved along characteristics
- It can never tear
- It can never intersect

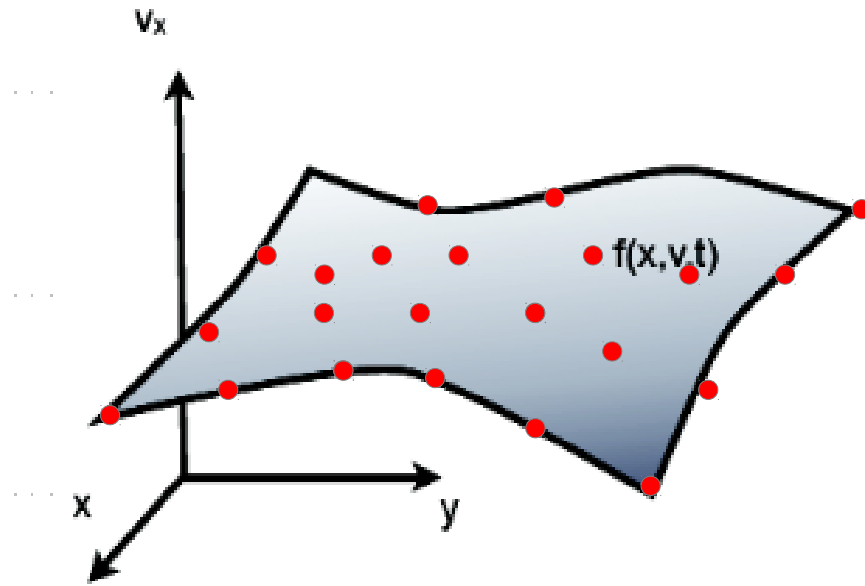


© Hahn

Kaehler et al (2012)

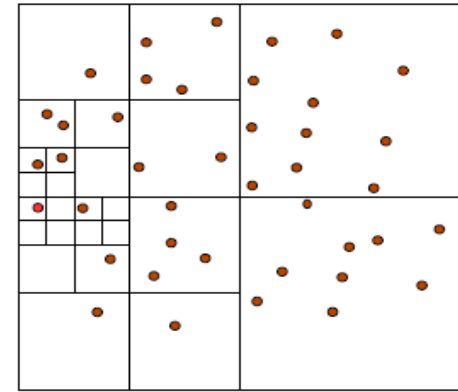
**Standard approach to solving the VP equation:**

# Montecarlo Sampling and coarse graining the CDM distribution function



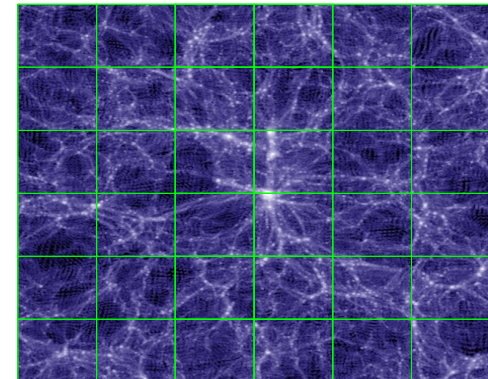
## Tree Algorithms

Multipole decomposition



## Particle-Mesh

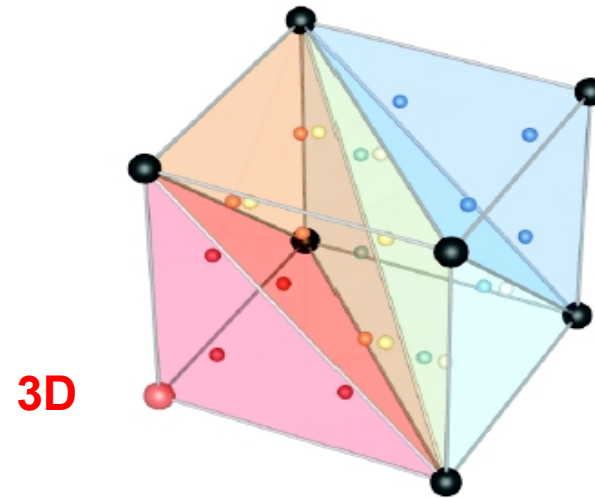
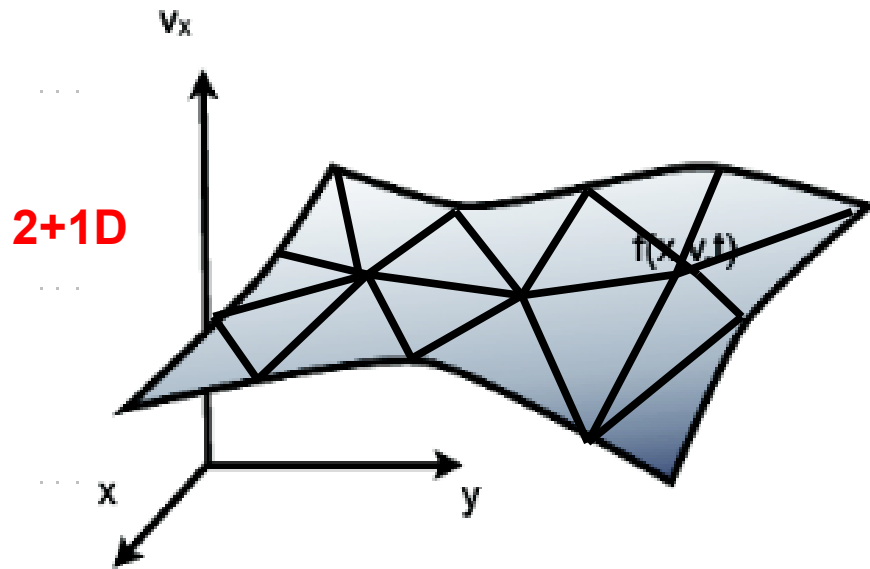
Poisson equation



$$\frac{d^2 \mathbf{x}_i}{dt^2} = \nabla_i \Phi(\mathbf{x}_i),$$
$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \epsilon^2]}$$

*An alternative approach:*

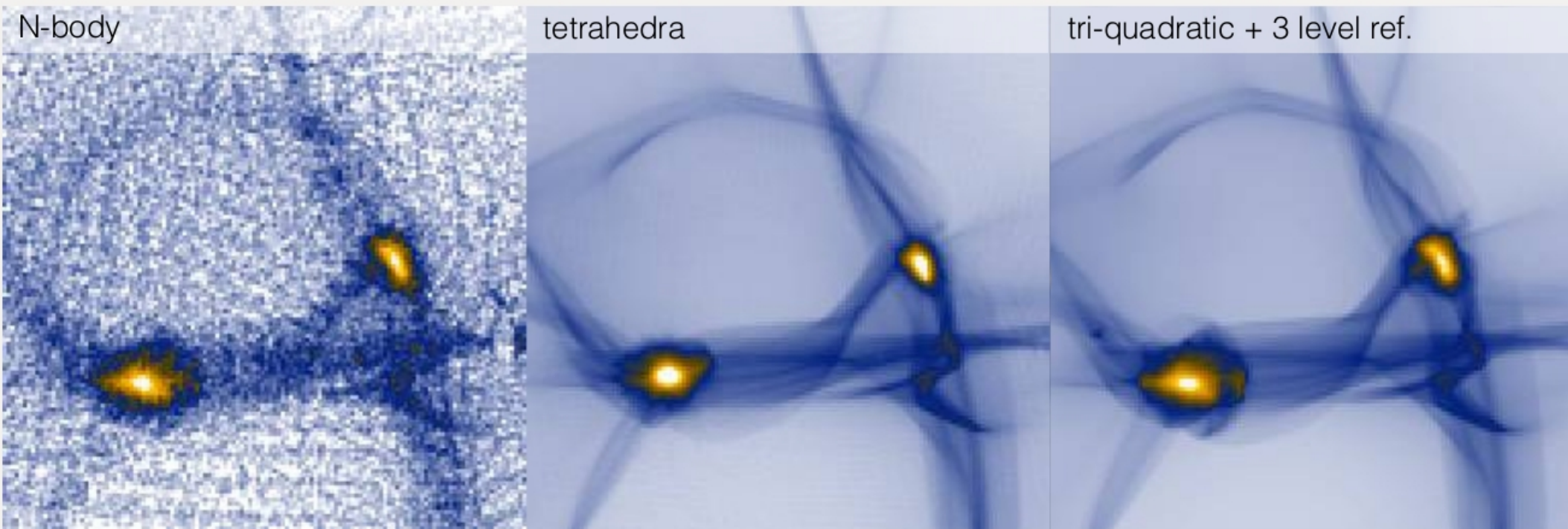
## Discretization of the DM fluid using phase-space element methods



**A tessellation of a finite number of mesh-generating points in Lagrangian space allows to continuously map the deformation of the dark matter sheet**

(Abel+ 2012, Shandarin+ 2012, Kaehler+ 2013, Hahn+ 2013, Angulo+ 2013, Hahn & Angulo 2015)

## Simulations of the same region of the Universe



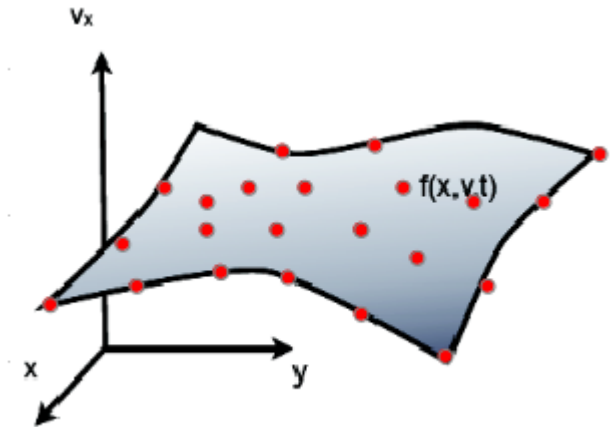
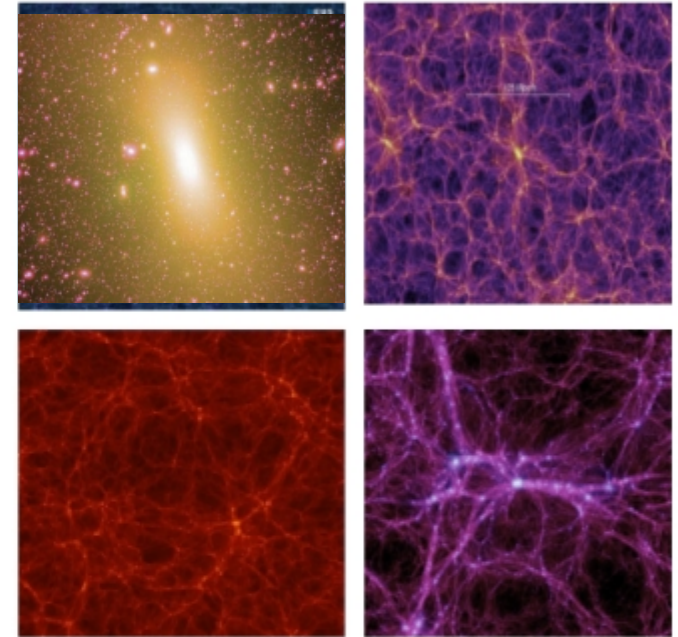
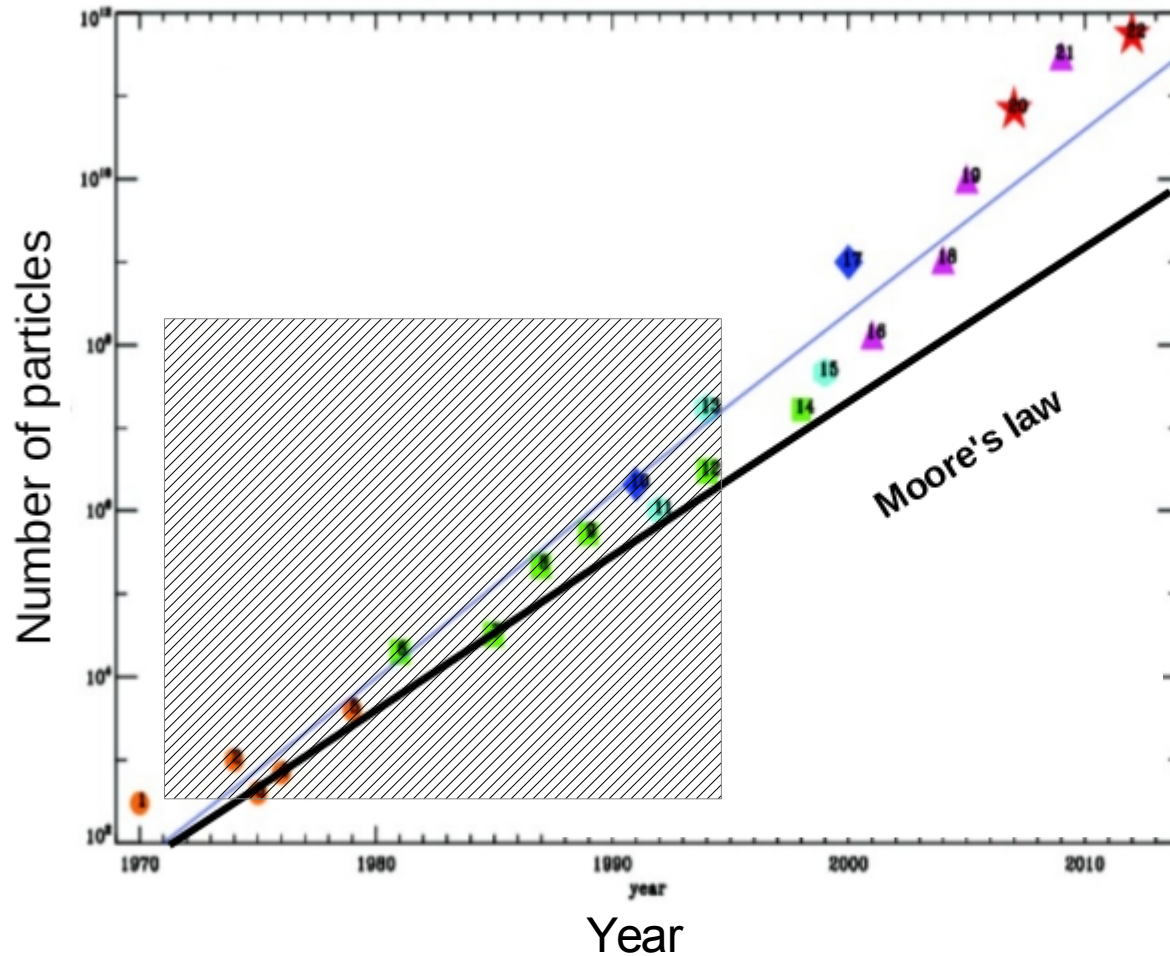
Hahn & Angulo 2015

See O. Hahn's talk

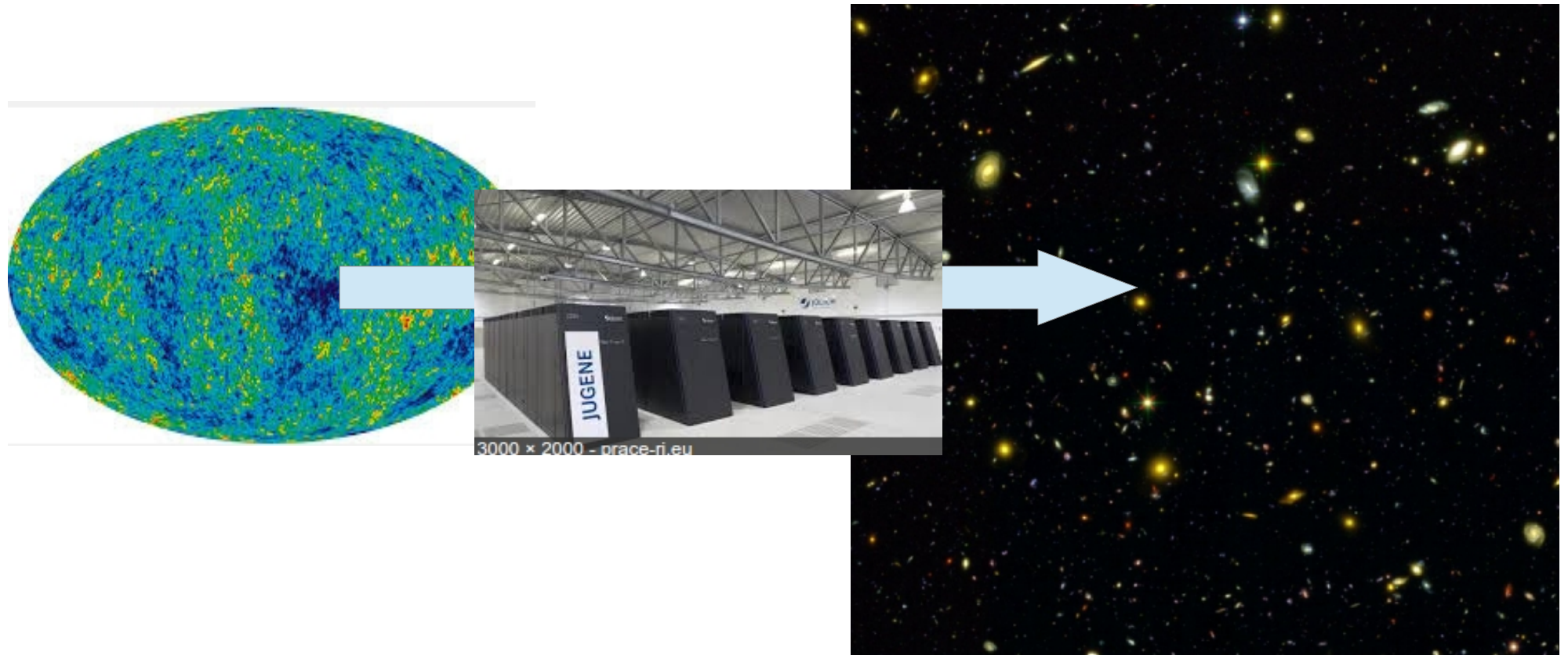
# Exponential growth of computing power over 40 years

10 trillion particle N-body simulations are expected by 2020

Springel et al 2005, Alimi et al 2012



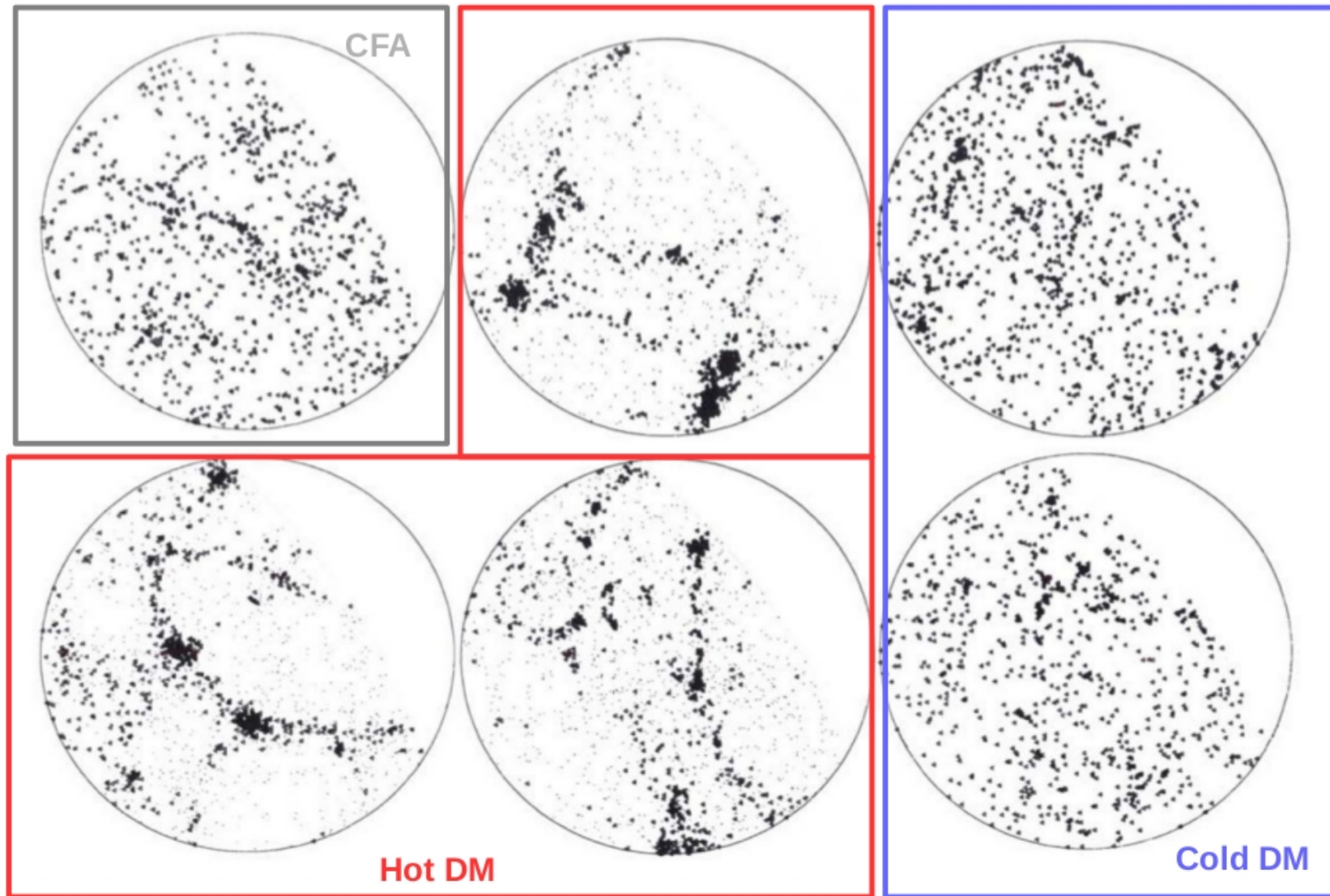
# Numerical simulations have been essential in the establishment of the “cosmology standard model”



**They aim to bridge 13.6 billion years of nonlinear evolution**

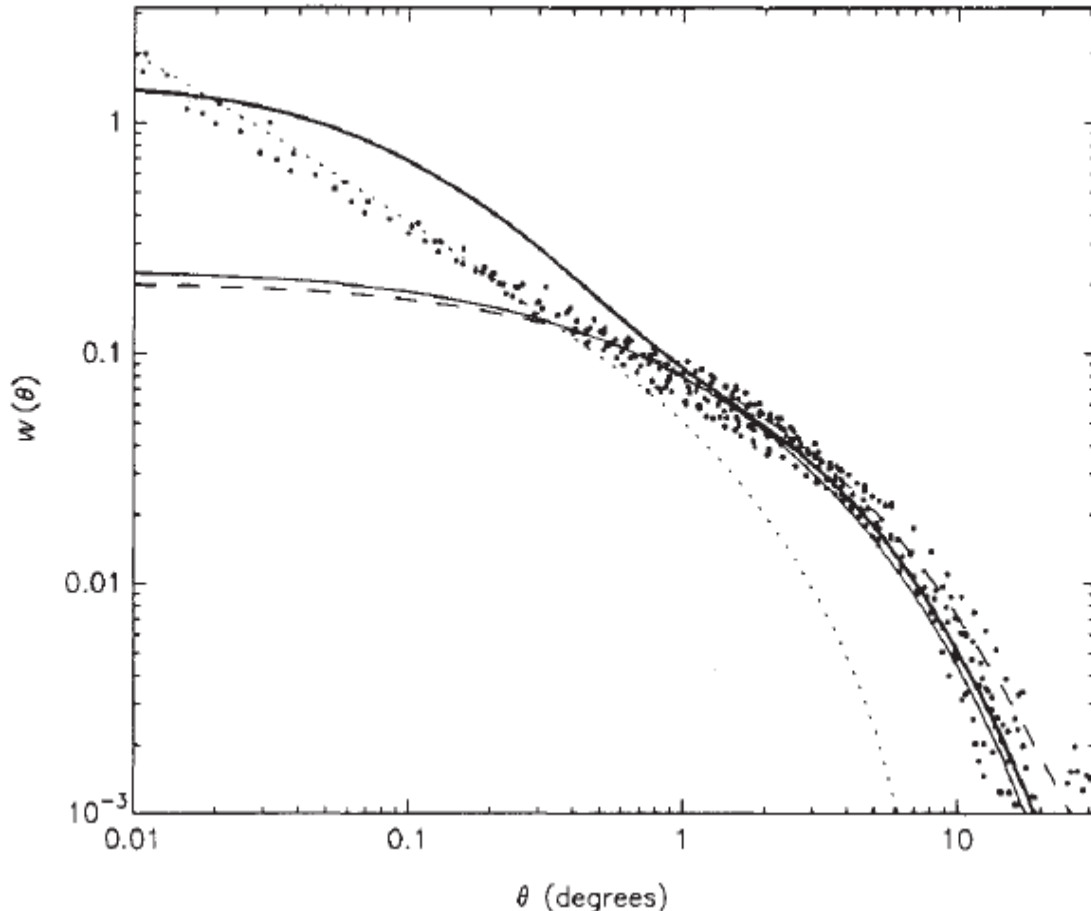


# 1985: The CDM model plus gravitational instability can explain qualitatively the observed universe



Davis, Efstathiou, Frenk & White 1985

# 1990: A cosmological constant is needed to explain the observed clustering of galaxies



**Data:** APM Survey

**Theory:** Dotted  $\Omega_m = 1$   
Solid  $\Omega_m = 0.2$   
 $\Omega_\lambda = 0.8$

**Efstathiou, Sutherland & Maddox (1990)**

*"We argue that the successes of the CDM Theory can be retained and the new Observation accommodated in a spatially Flat cosmology in which as much as 80% Of the critical density is provided by a Positive cosmological constant..."*

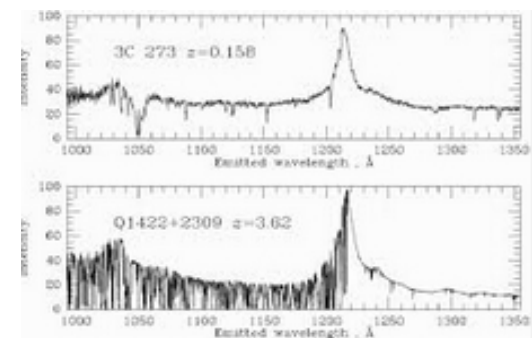
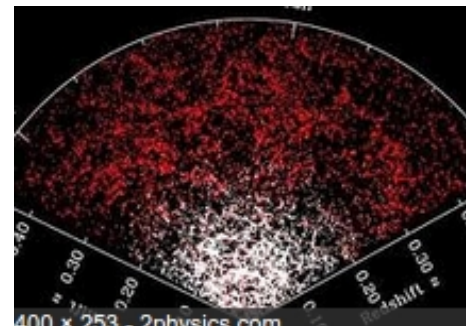
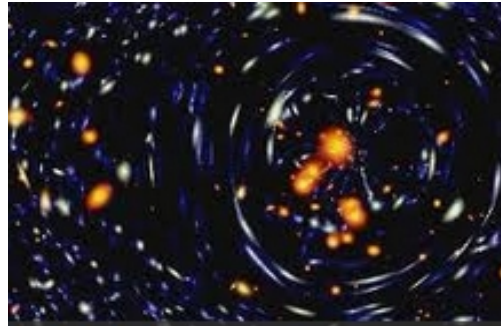
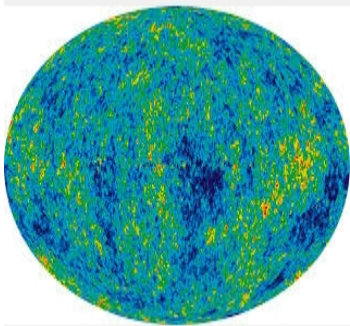
# Our current understanding of structure formation in the Universe stands on four key ideas:

**General Relativity**

**Dark Matter**

**Dark Energy**

**Inflation**



# There are fundamental open questions about each of its pillars.

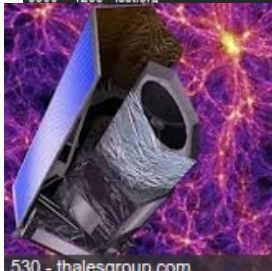
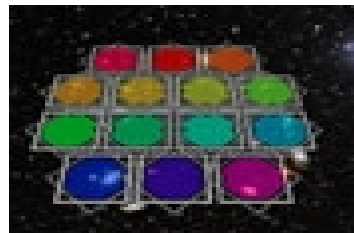
**General Relativity**  
**Galileon,  $f(R)$ ?**

**Dark Matter**  
**Warm or Cold?**

**Dark Energy**  
 **$w(z)$  or  $\Lambda$ ?**

**Inflation**  
**Single/multi field?**

These enigmas have driven multi-million dollar experiments.

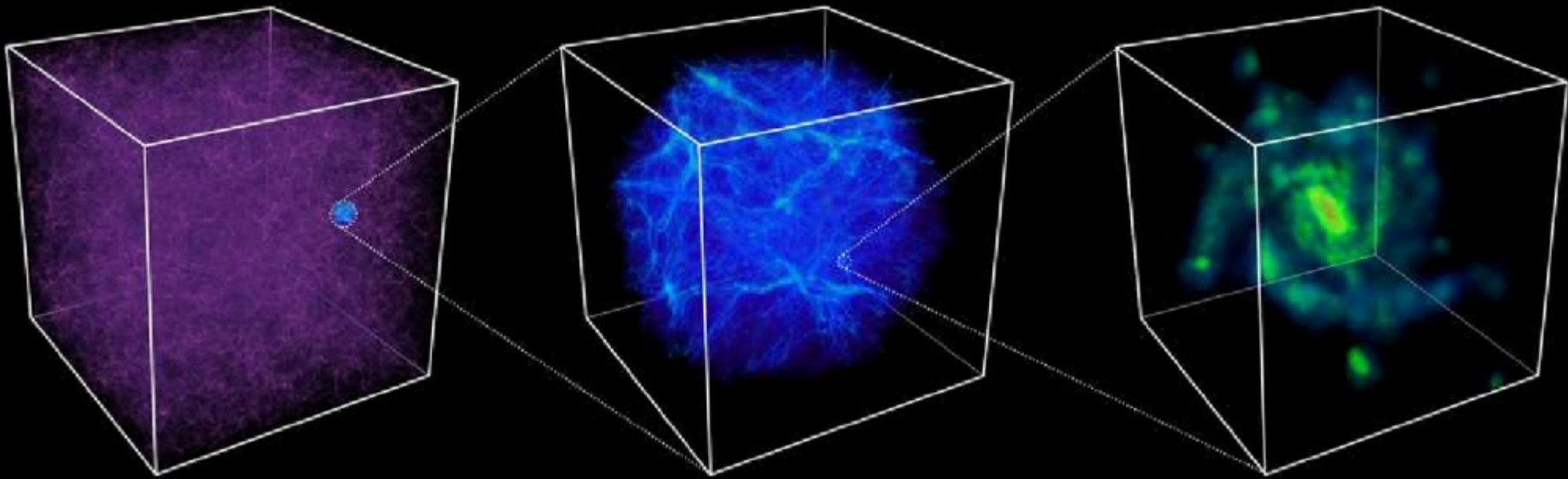


# The signature of departures from $\Lambda$ CDM depend sensitively on:

- the detailed distribution of dark matter
- the precise impact of dark energy on cosmic structure
- the physics of galaxy formation

**All this from gigaparsecs down to subgalactic scales**

GIMIC Project, Crain et al. (2009)

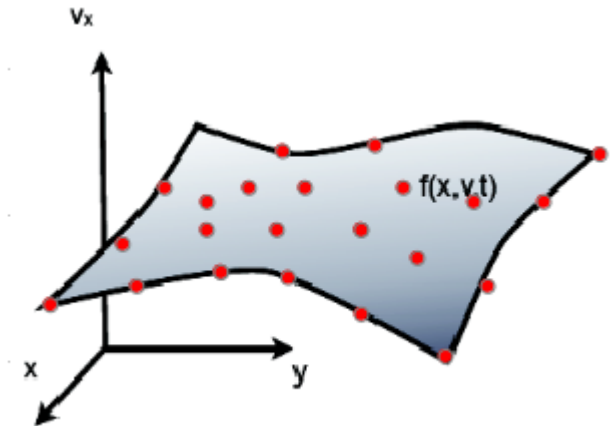
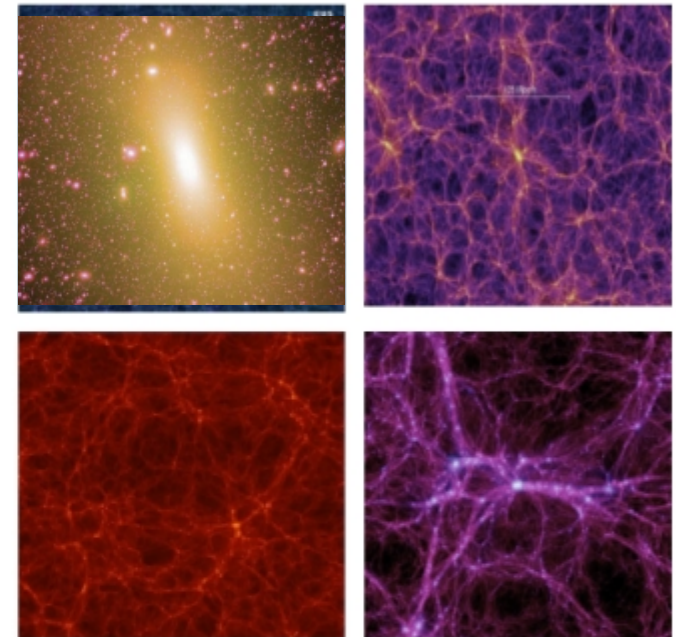
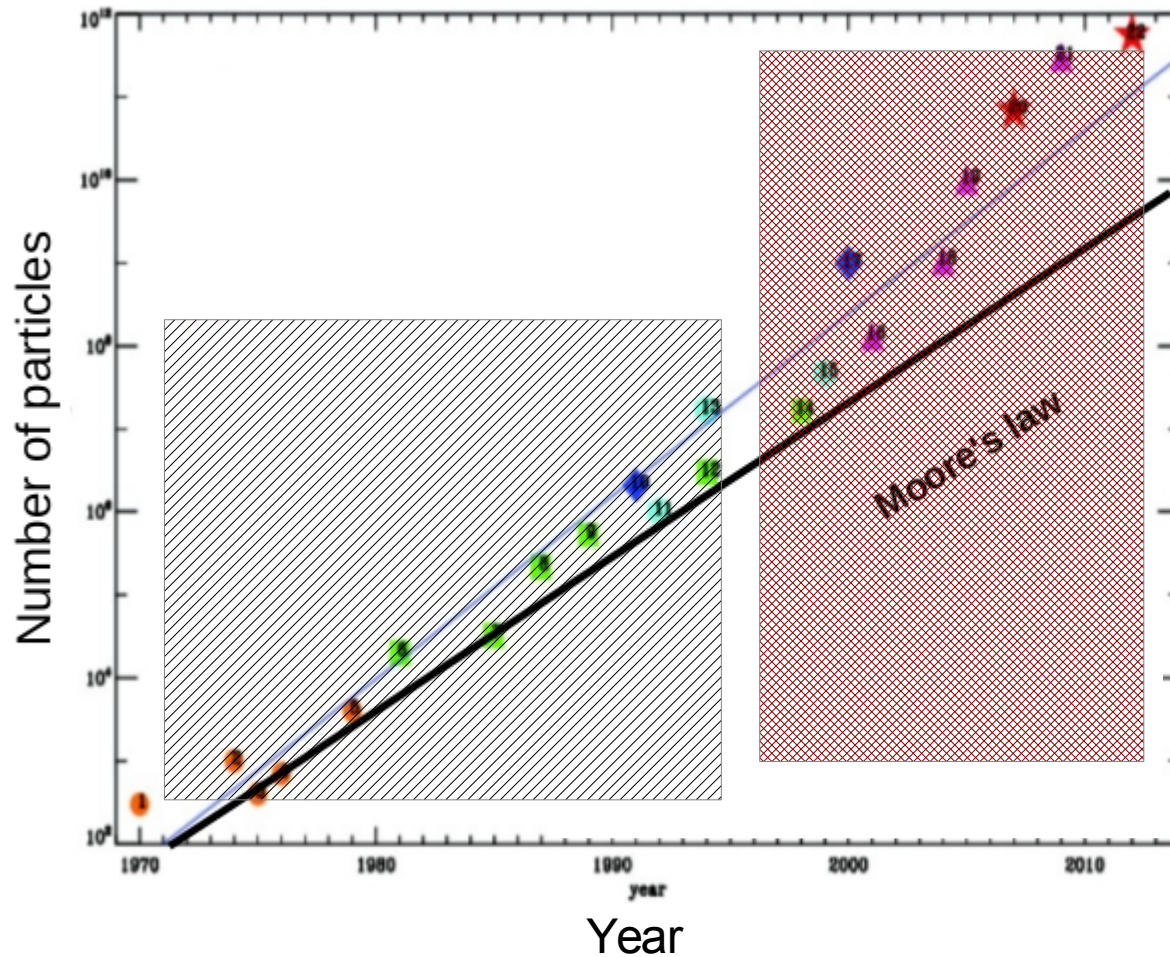


**Modern simulations face new challenges in terms of their accuracy and predictive power.**

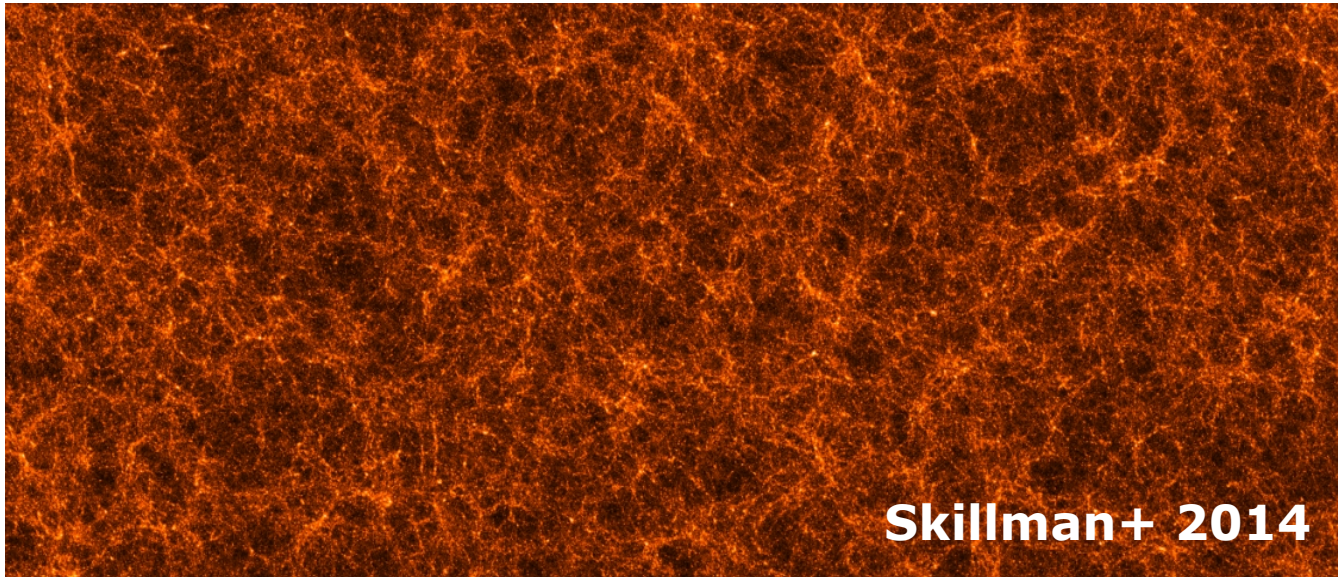
# Exponential growth of computing power over 40 years

10 trillion particle N-body simulations are expected by 2020

Springel et al 2005, Alimi et al 2012



# The record holder: DarkSky simulations

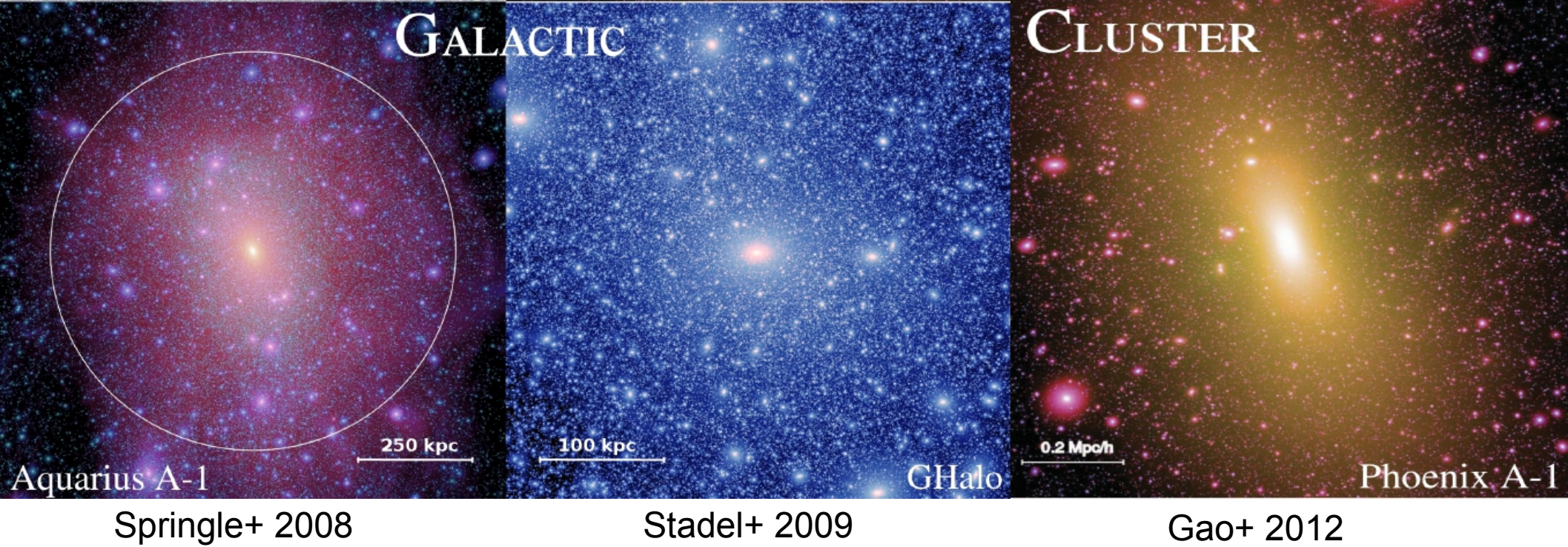


- 1 trillion particles
- 10 Gpc box
- 200,000 CPUs
- 70 Tb RAM

**Large-scale N-body simulations aim to predict:**

**BAO & Galaxy Clustering**  
**Abundance of Clusters**  
**Weak Gravitational Lensing**  
**Redshift-Space Distortions**

- The nonlinear state of mass
- The velocity field
- Abundance and properties of collapsed DM structures
- The places of galaxy formation



**Zoom-In N-body simulations  
aim to predict:**

**Direct Detection**  
**Indirect Detection**  
**Astrophysical Probes**

- Halo density and velocity profiles
- Substructure mass function
- Substructure spatial distribution



# Dark Matter simulations are robust and provide testable results

- Haloes are triaxial and rotate slowly.
- Halos density profile is described by an universal functional form

Accurate characterization of:

- Mass function
- clustering
- subhalo population
- cosmic web

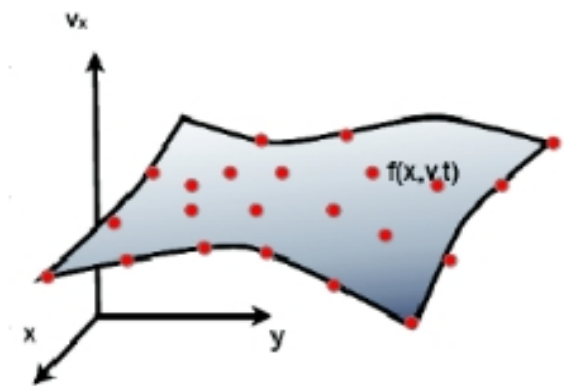
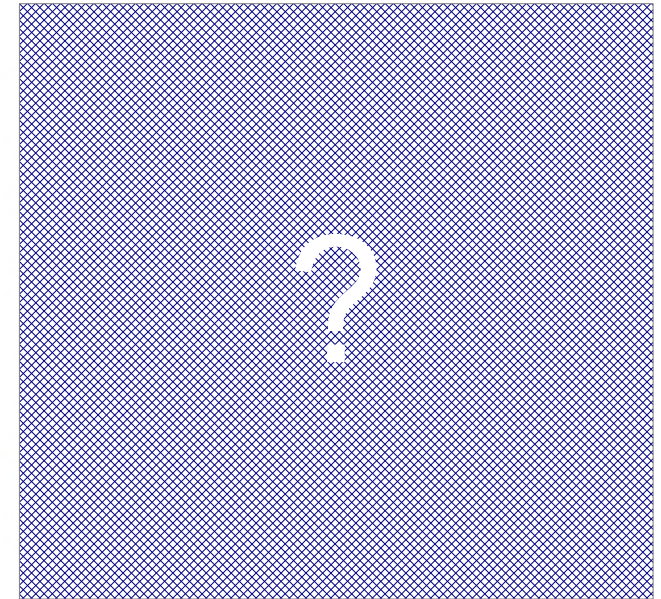
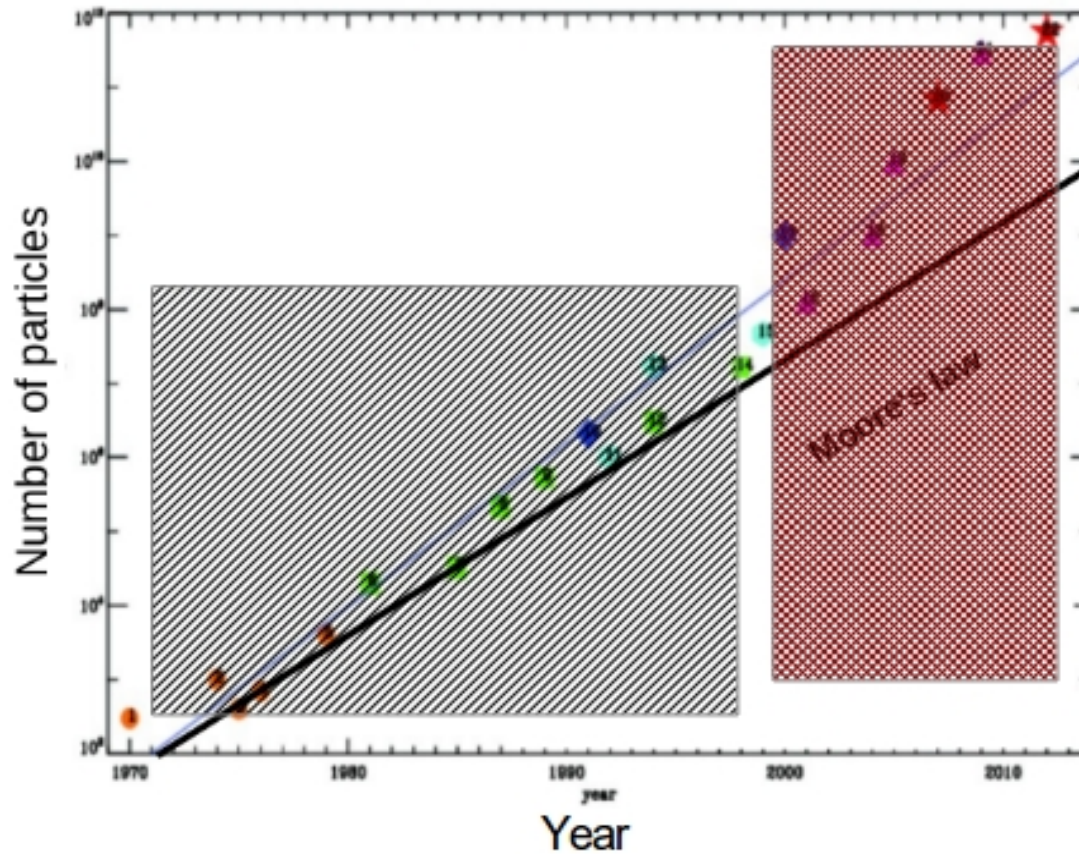
...as a function of cosmological Ingredients.

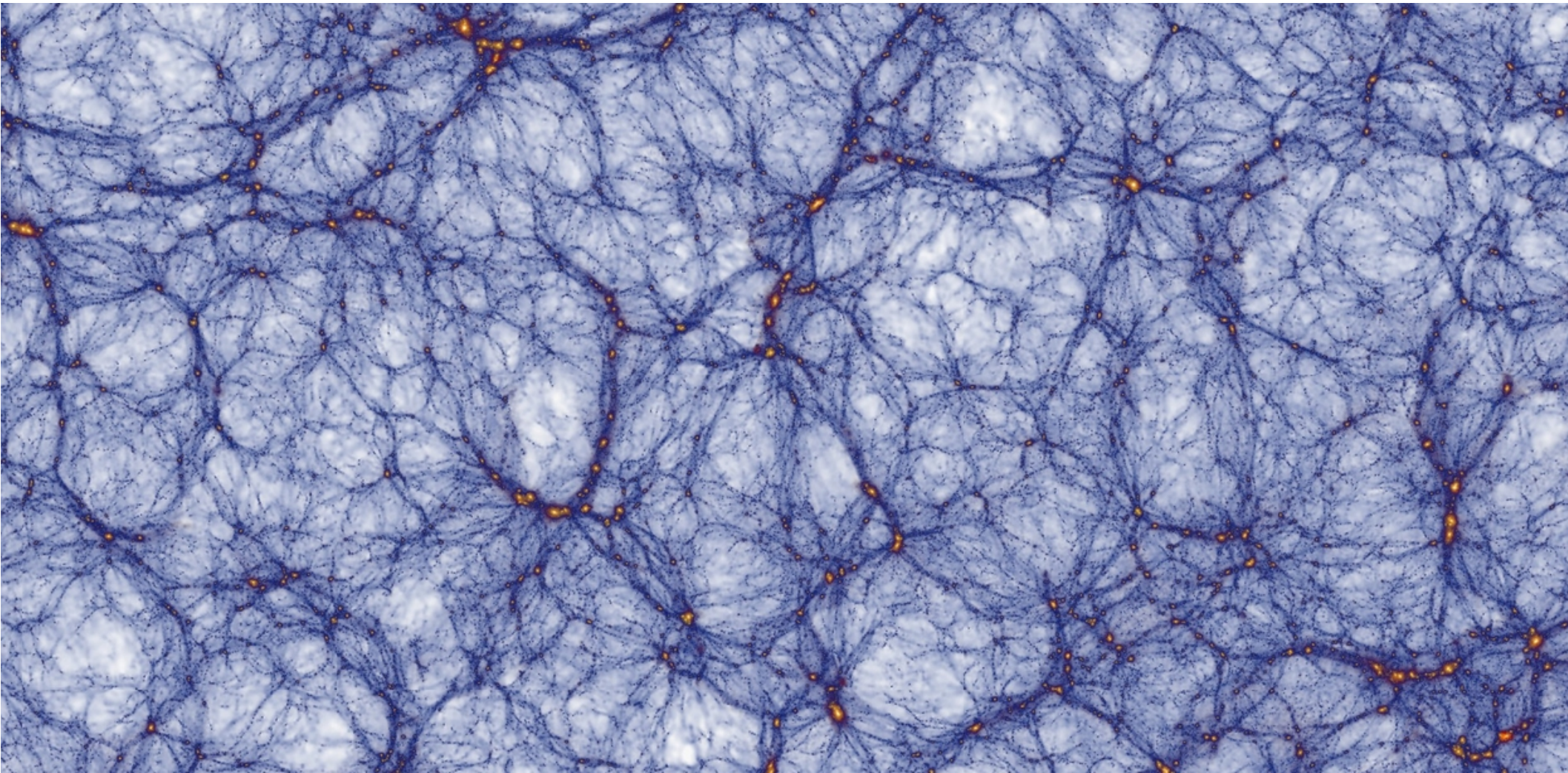


$\alpha = 0.13631347$

# Is there anything left for Dark Matter simulations after 40 years of development?

Springel et al 2005, Alimi et al 2012





MXXL, Angulo+ 2012





# How can we optimally extract all the cosmological information encoded in the clustering of galaxies?

## The challenge

- (Nonlinear) density field
- (Nonlinear) velocity field
- (Nonlinear, stochastic, non-local) Galaxy bias
- Higher order correlation functions
- Precise accounts of observational setups

## The reward

- More accurate and robust constraints on
  - Inflation, Gravity, Dark Energy, Dark Matter
  - Galaxy Formation physics

→ (*Higher order, Tree loop, Renormalized, Lagrangian, Eulerian, Effective Field Theory of LSS, augmented, integrated*) Perturbation theories; Halo Model; Halo Fit

# **N-body simulations can and should be used to directly to constraint cosmological parameters**

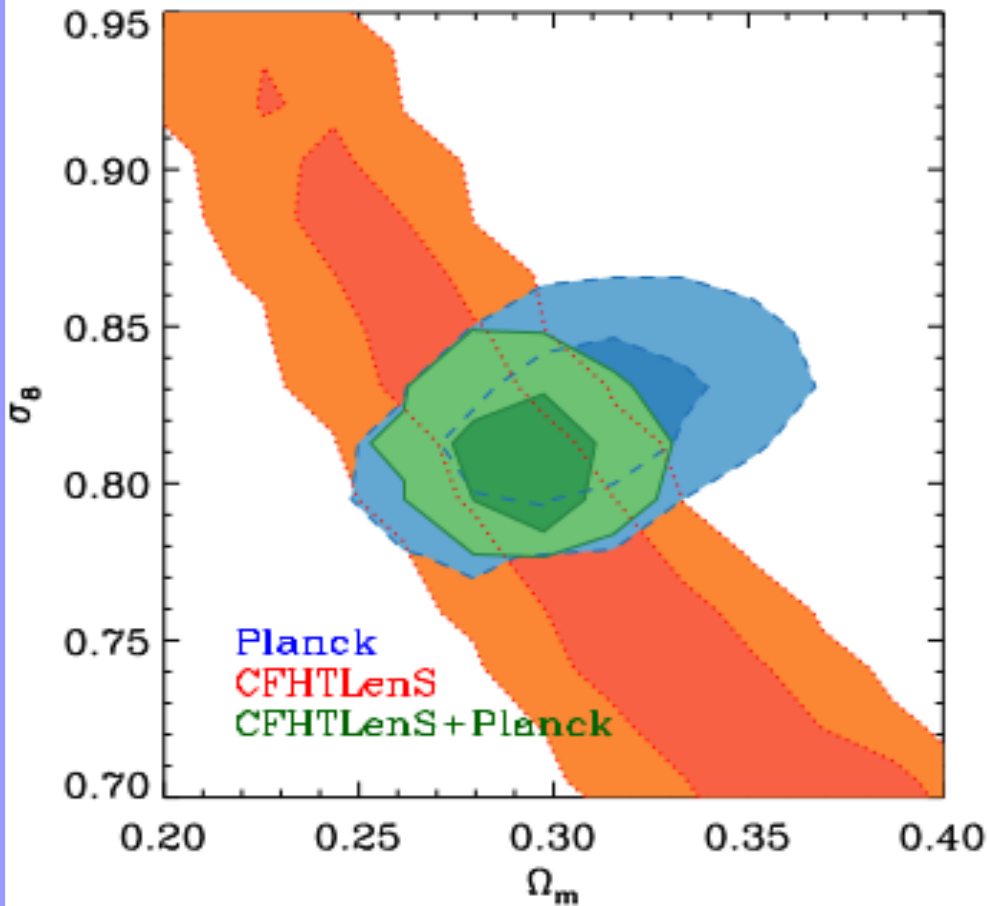
## **The dark matter as a function of cosmology**

- A grid of DMO simulations
- Emulators
- **Cosmology scaling**

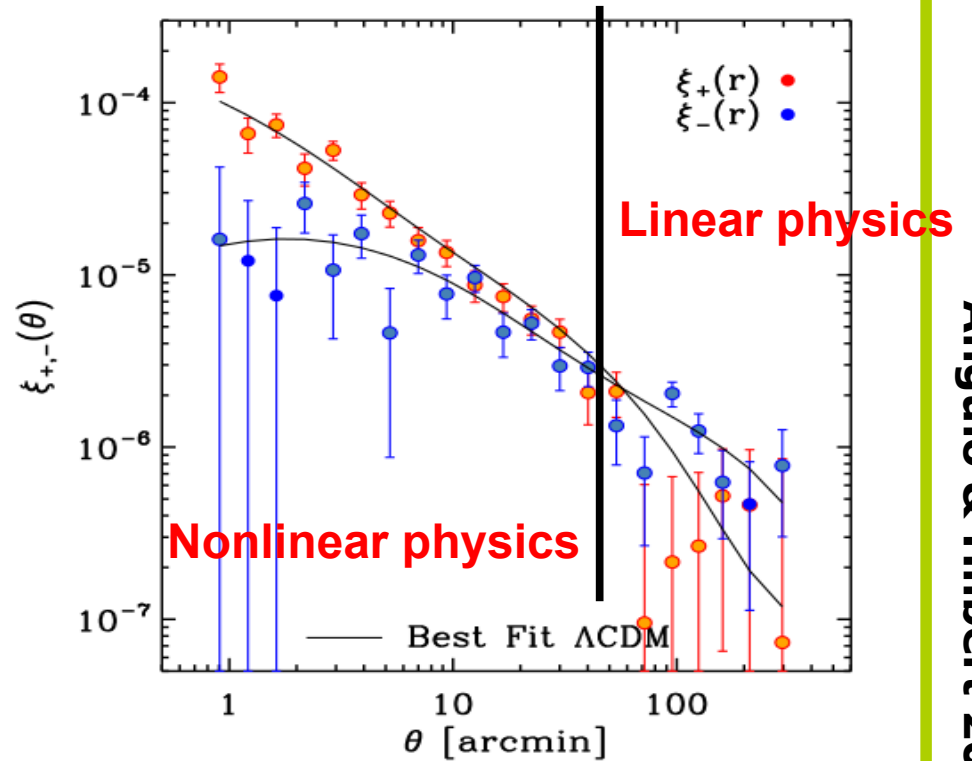
## **The galaxy population**

# N-body simulations can nowadays be used to directly constraint cosmological parameters

$$\Omega_m = 0.29 \pm 0.01$$
$$\sigma_8 = 0.81 \pm 0.01$$



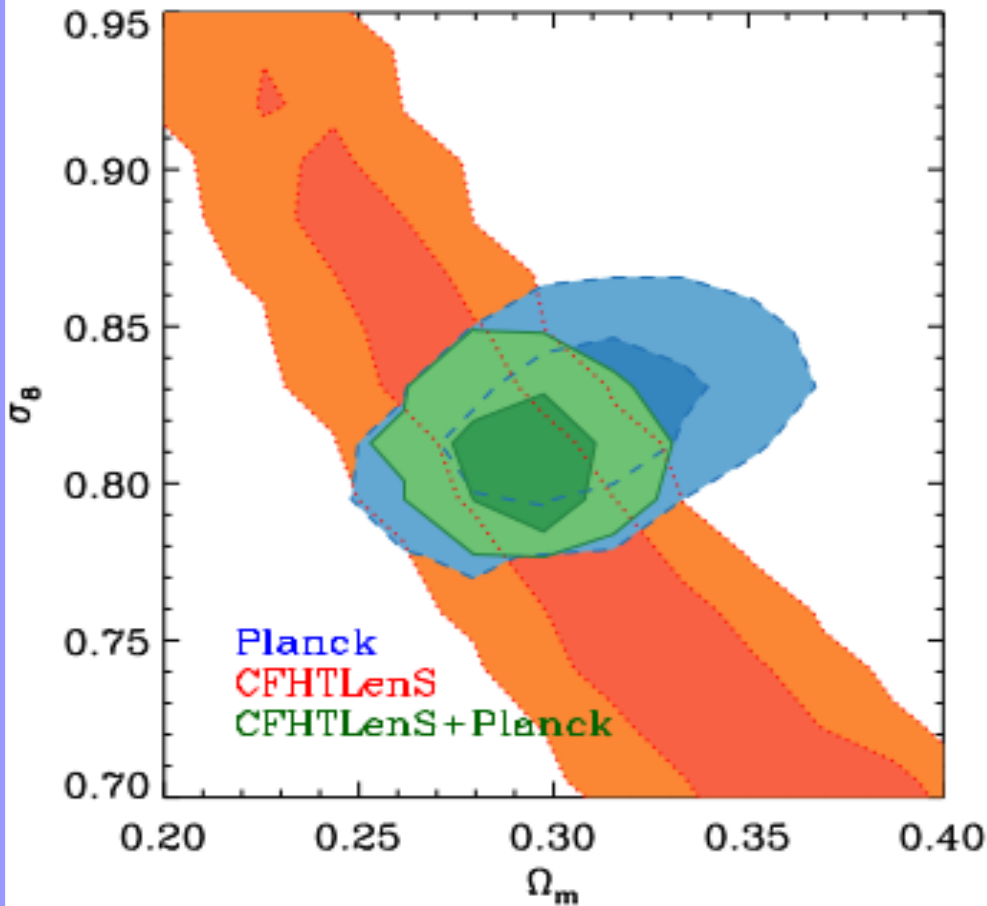
## Shear Correlation measurements



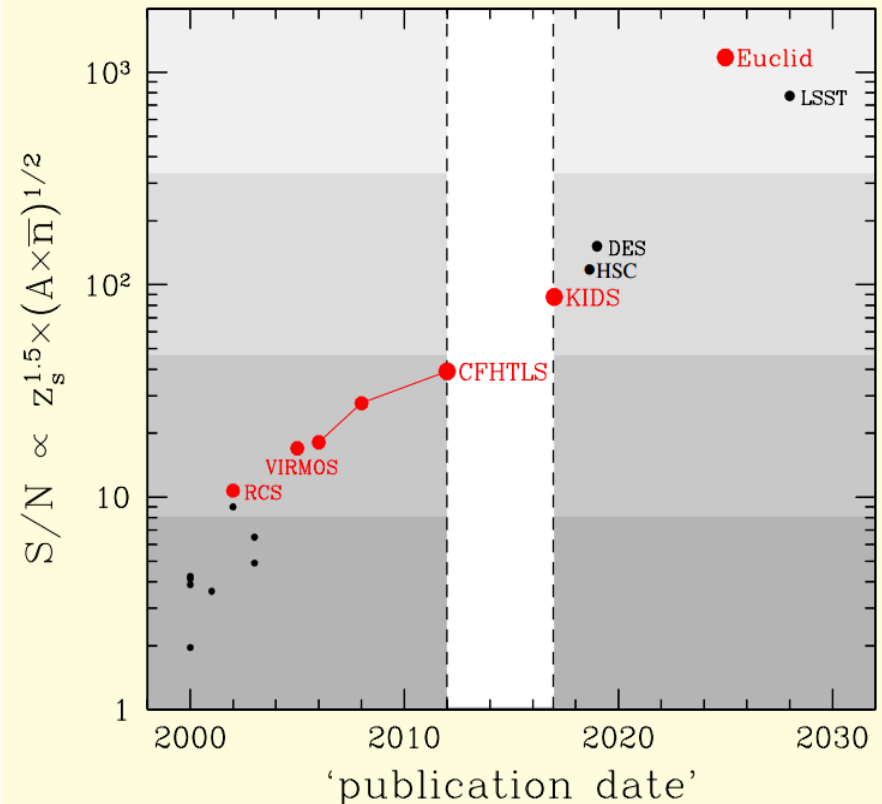


# N-body simulations can nowadays be used to directly constraint cosmological parameters

$$\Omega_m = 0.29 \pm 0.01$$
$$\sigma_8 = 0.81 \pm 0.01$$



From H. Hoekstra



# N-body simulations can and should be used to directly to constraint cosmological parameters

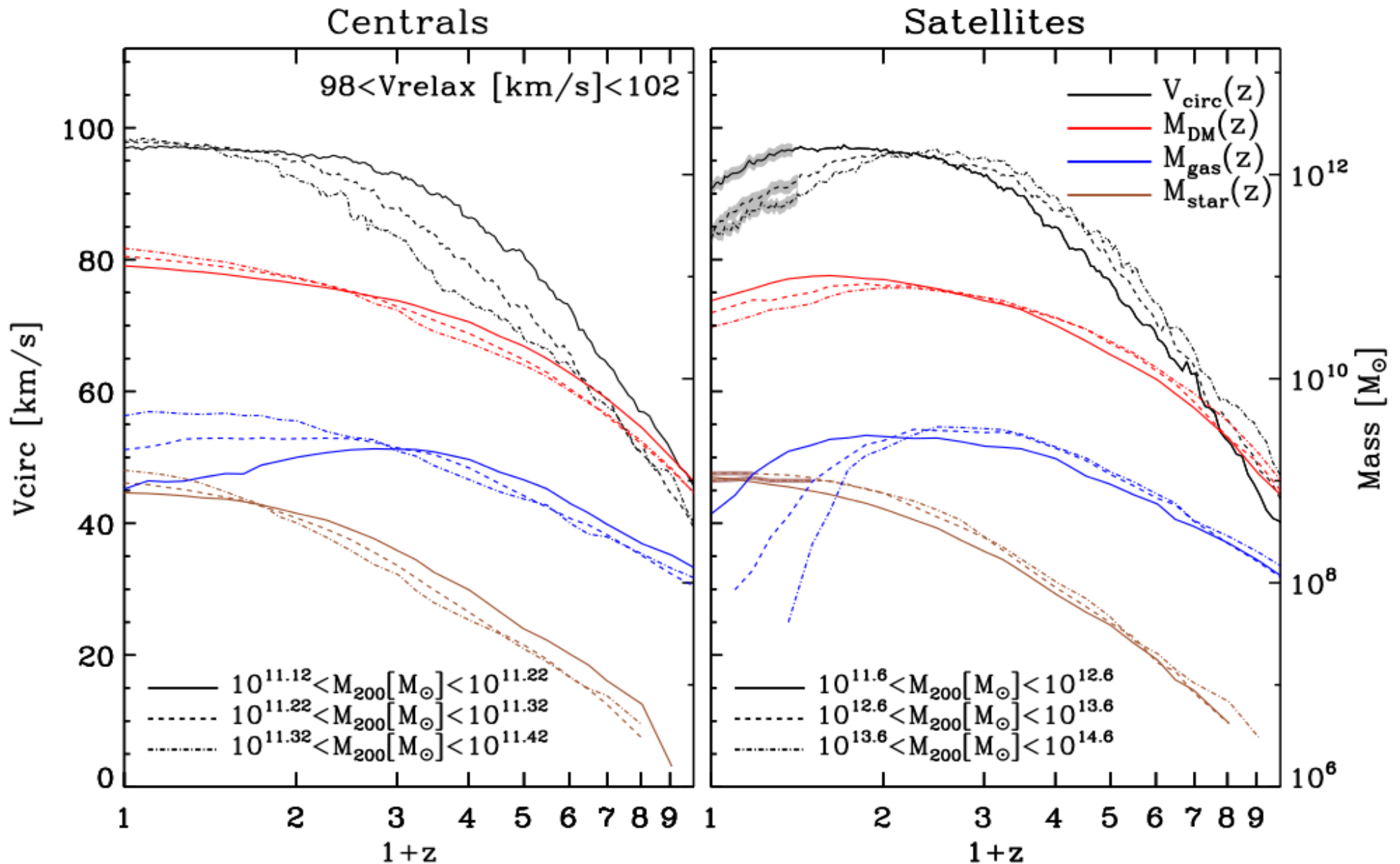
## The dark matter as a function of cosmology

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## The galaxy population

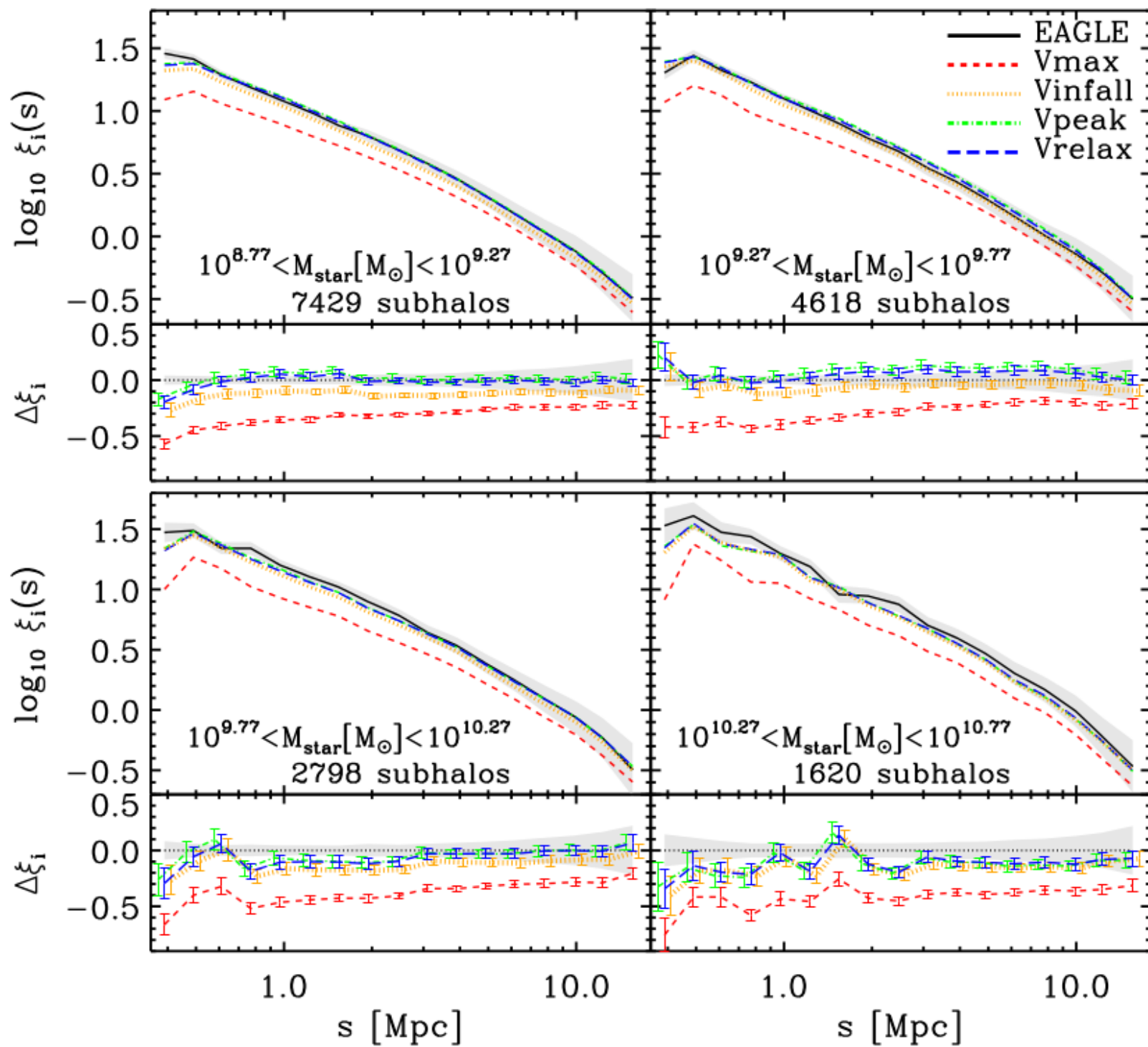
- Hydrodynamical simulations
- Semi-analytics models
- Halo Occupation distribution
- **Subhalo Abundance matching**

# Testing SHAM in hydrodynamical simulations



# Redshift space clustering

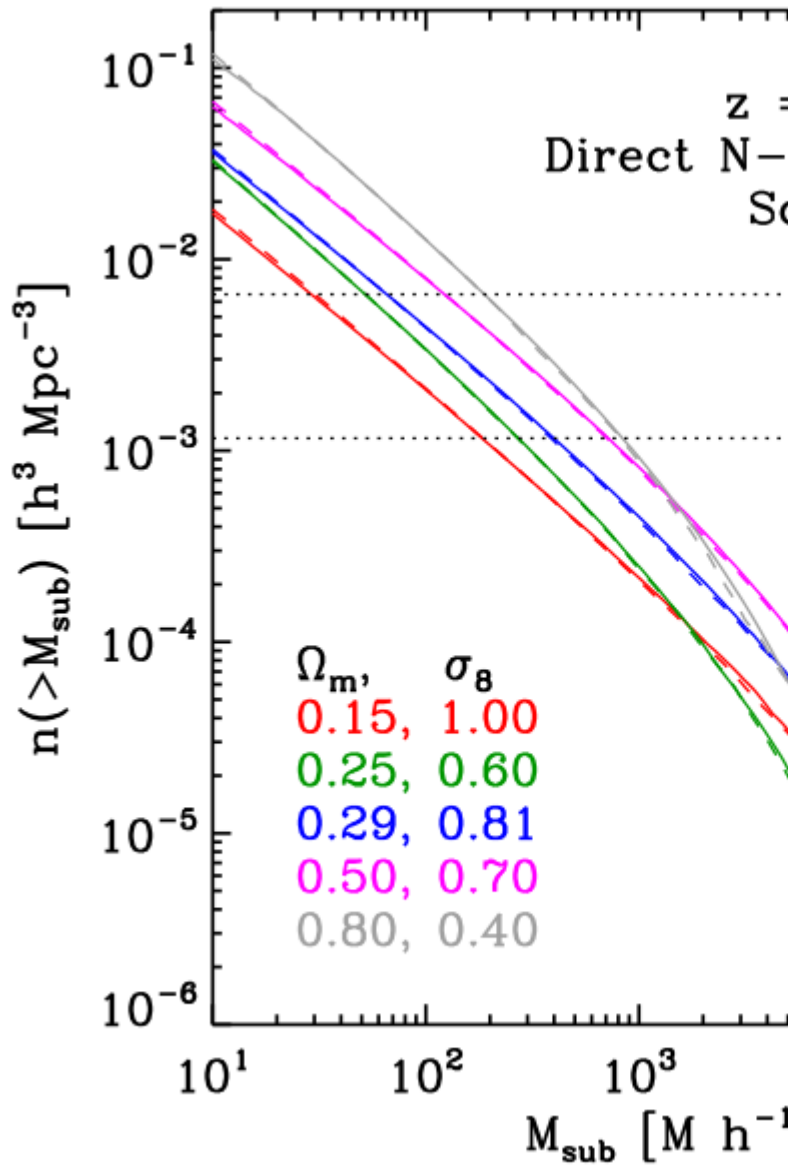
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# Can we put these two ingredients together?

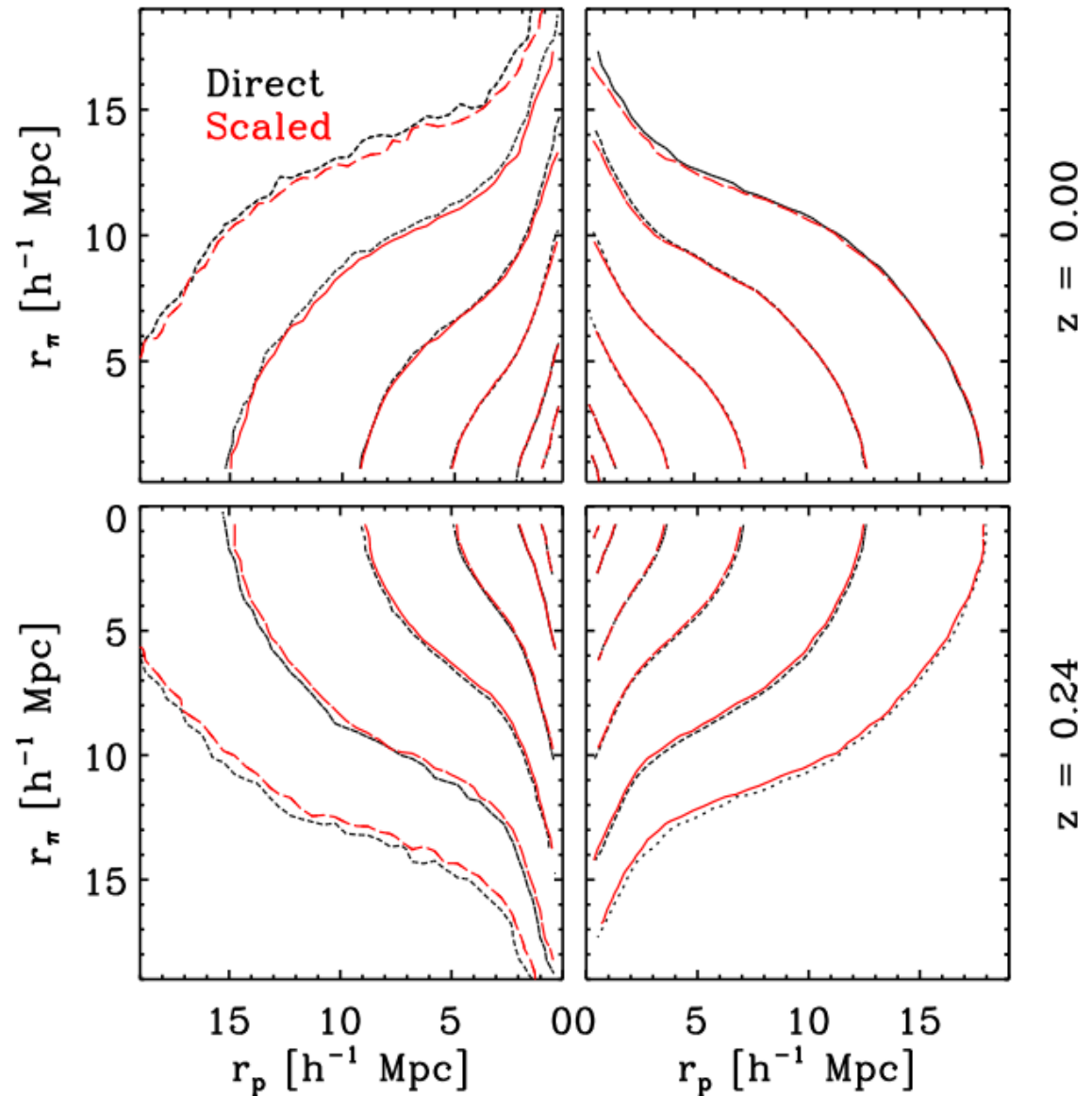
**The dark matter as a function of cosmology**

**The galaxy population**



$n = 1.16 \times 10^{-3}$

$n = 6.5 \times 10^{-3}$

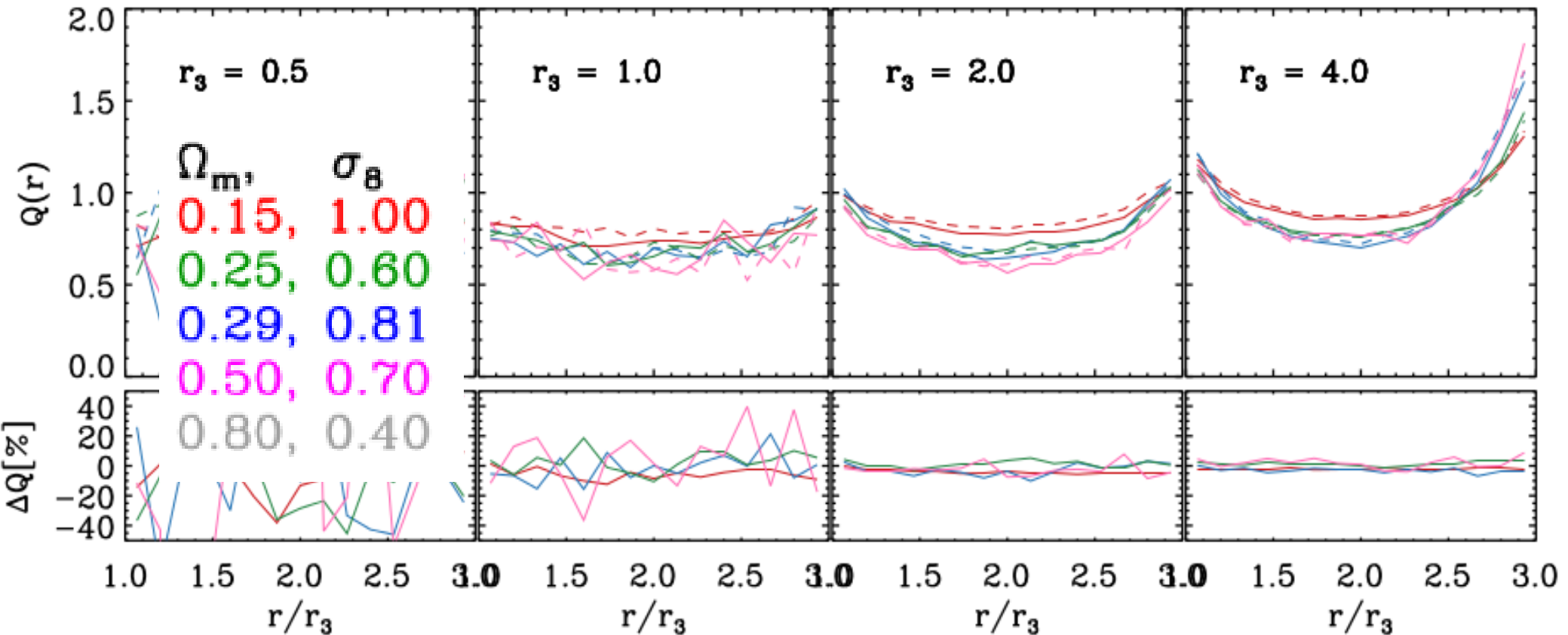


# Can we push this further?

## 3pt correlation functions in *redshift space*

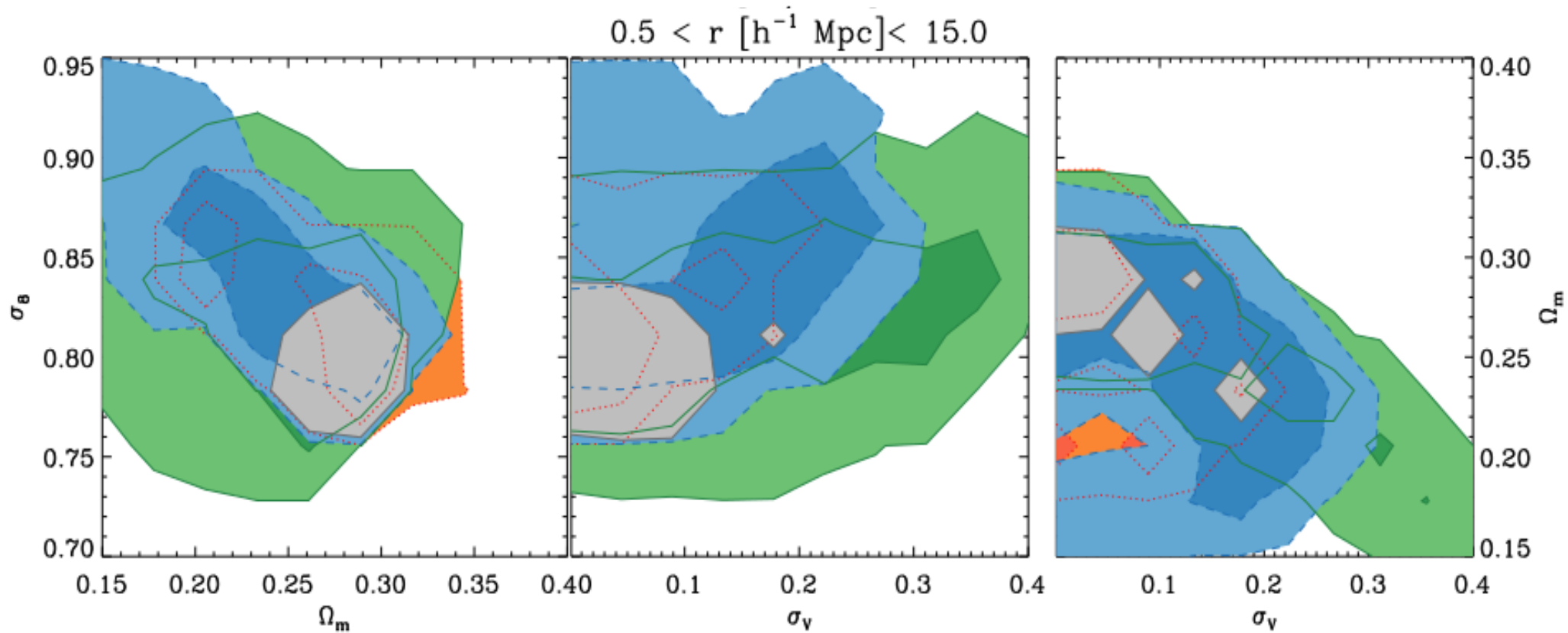
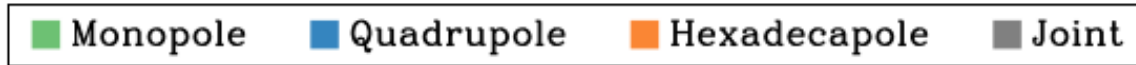
*Different triangular configurations can be predicted to the same accuracy*

$$Q(s, u, \theta) \equiv \frac{\zeta(s, u, \theta)}{\xi(r_1)\xi(r_2) + \xi(r_2)\xi(r_3) + \xi(r_3)\xi(r_1)}$$



# Application: Main SDSS sample

Angulo, Marin & White, in prep



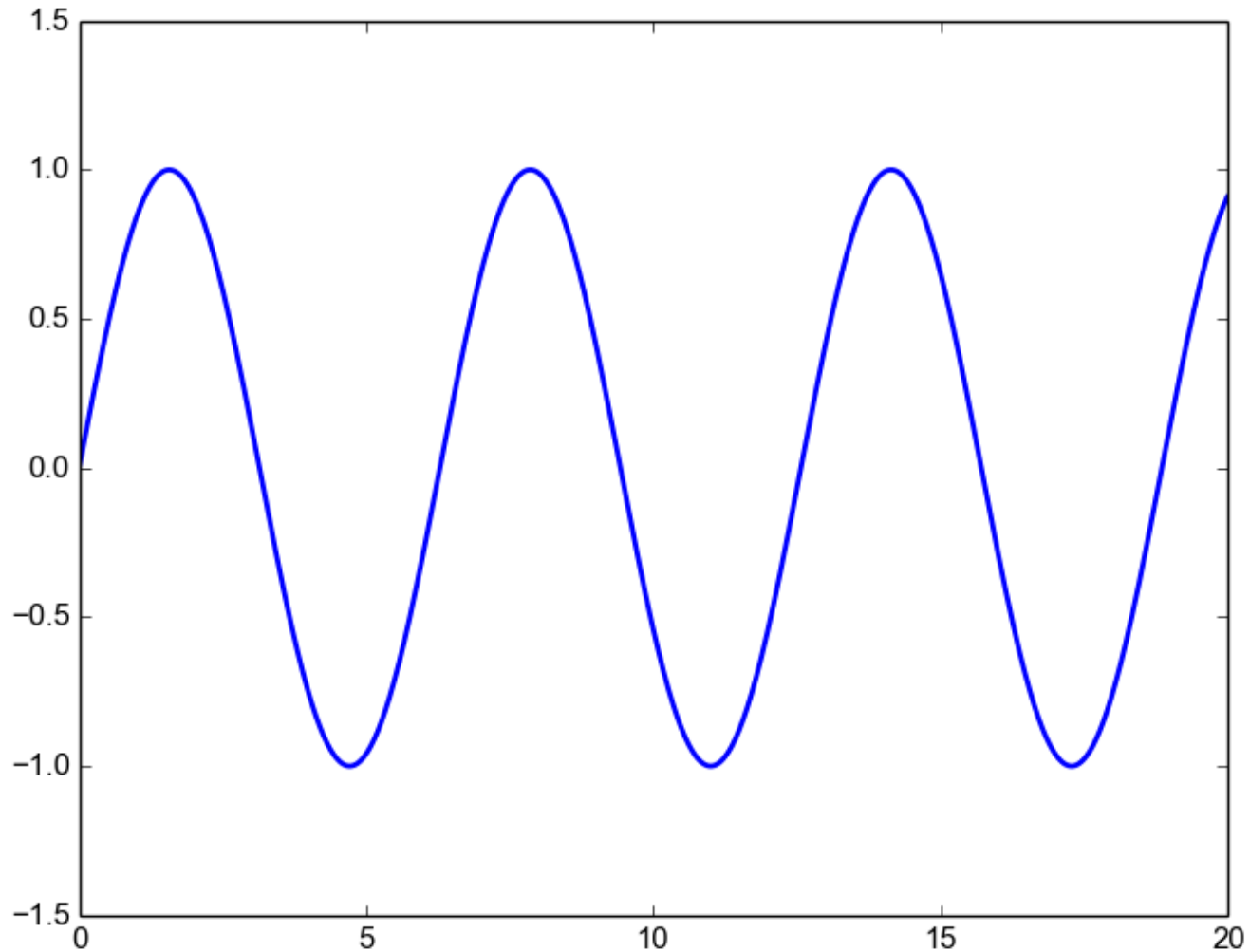
**After BAO and RSD, future surveys will extract information from the largest cosmological scales**

- Non-Gaussianities
- General Relativity effects
- Neutrino Masses

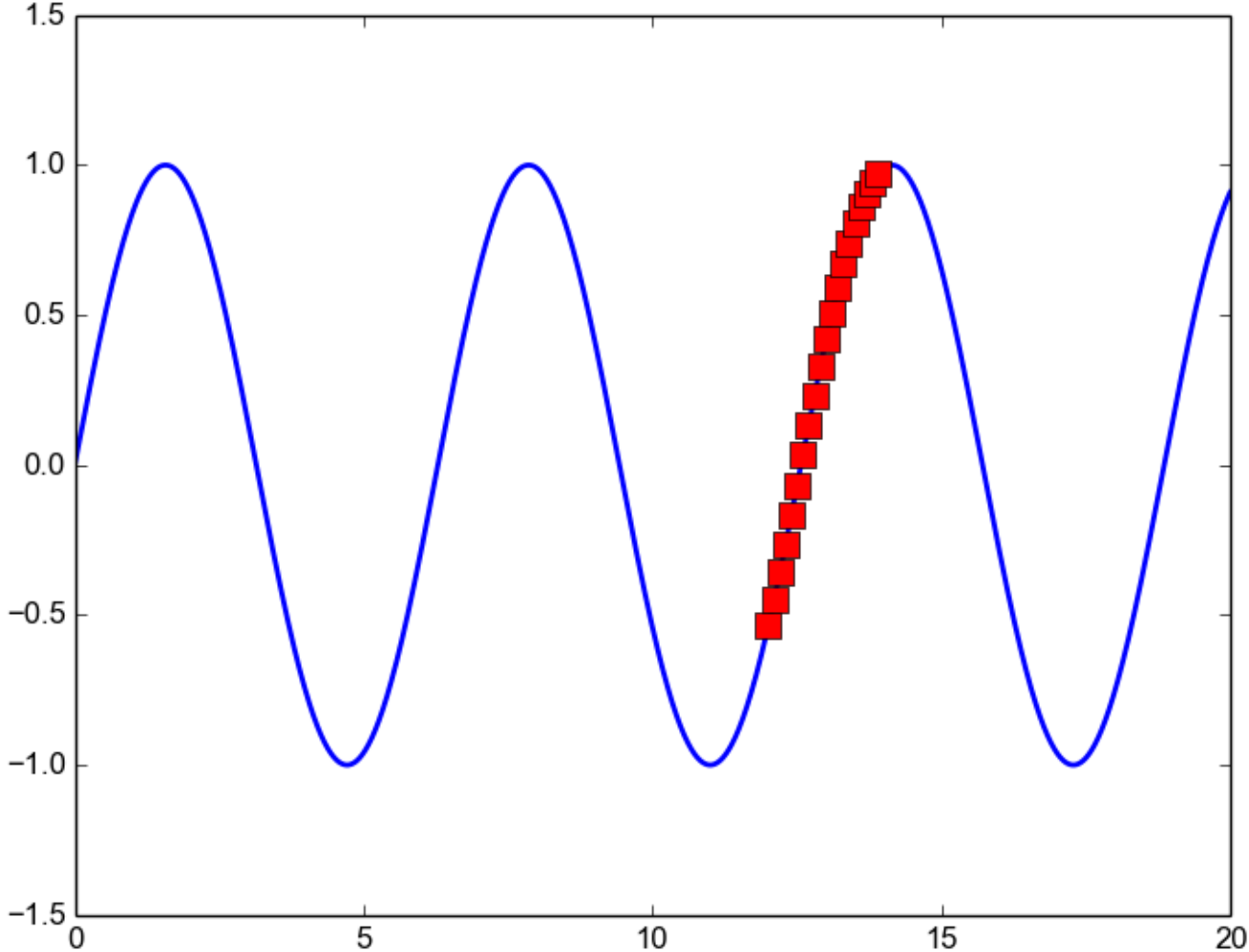
**A forward modelling would also make simpler to model complex observational setups**



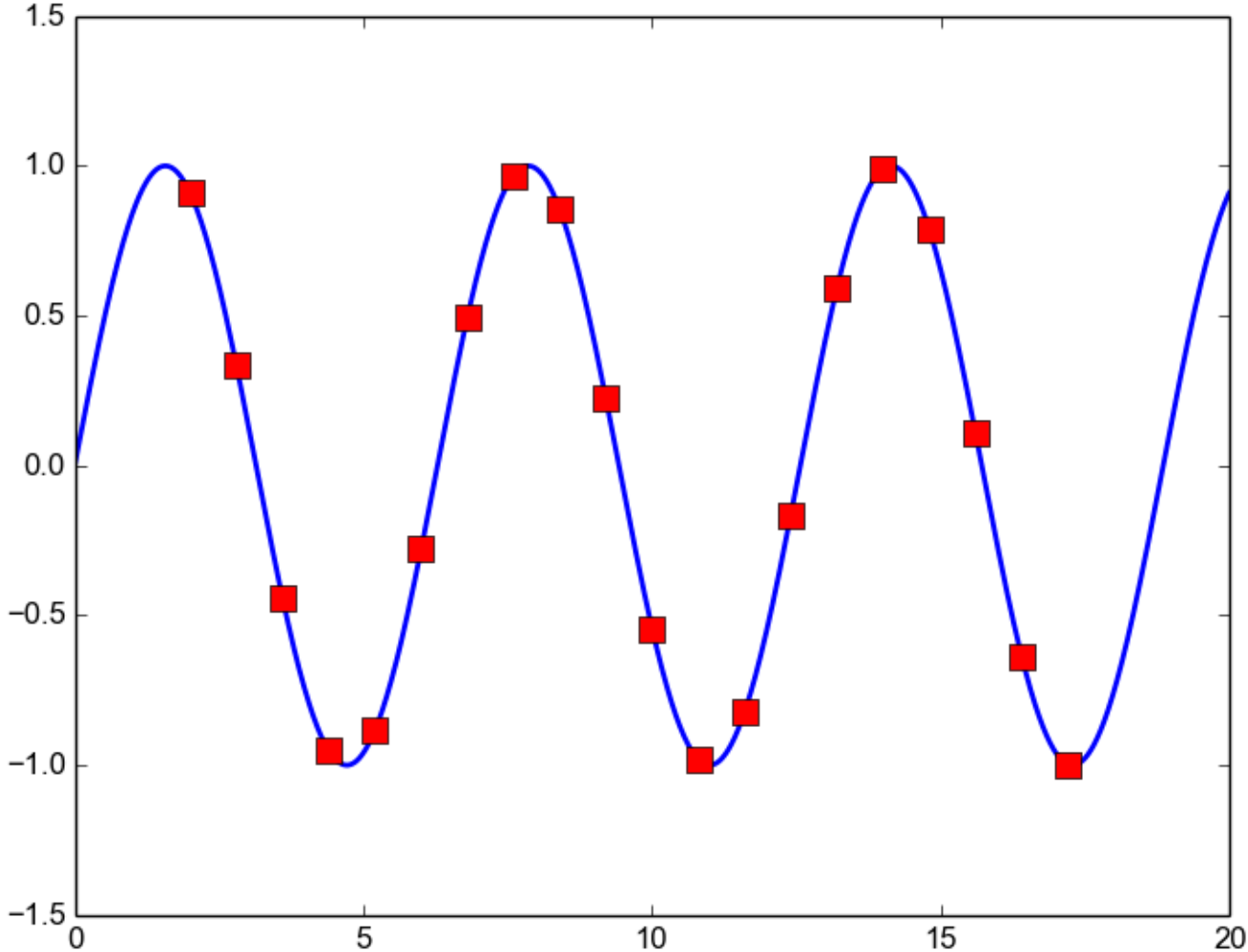
# How do we optimally measure those scales?



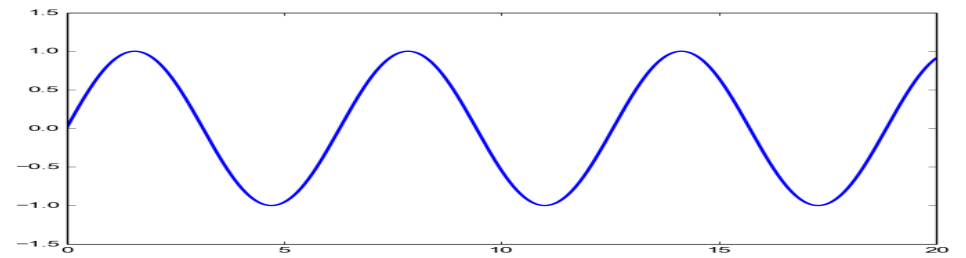
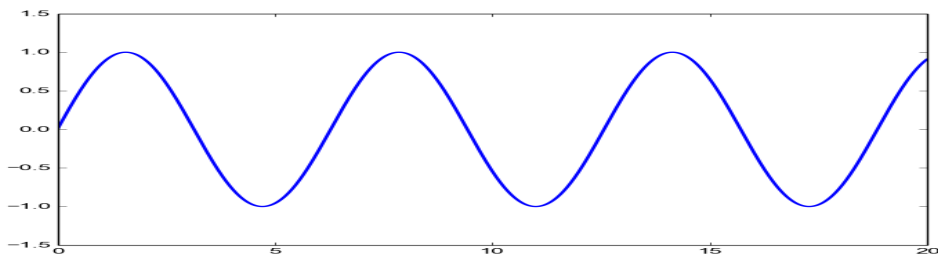
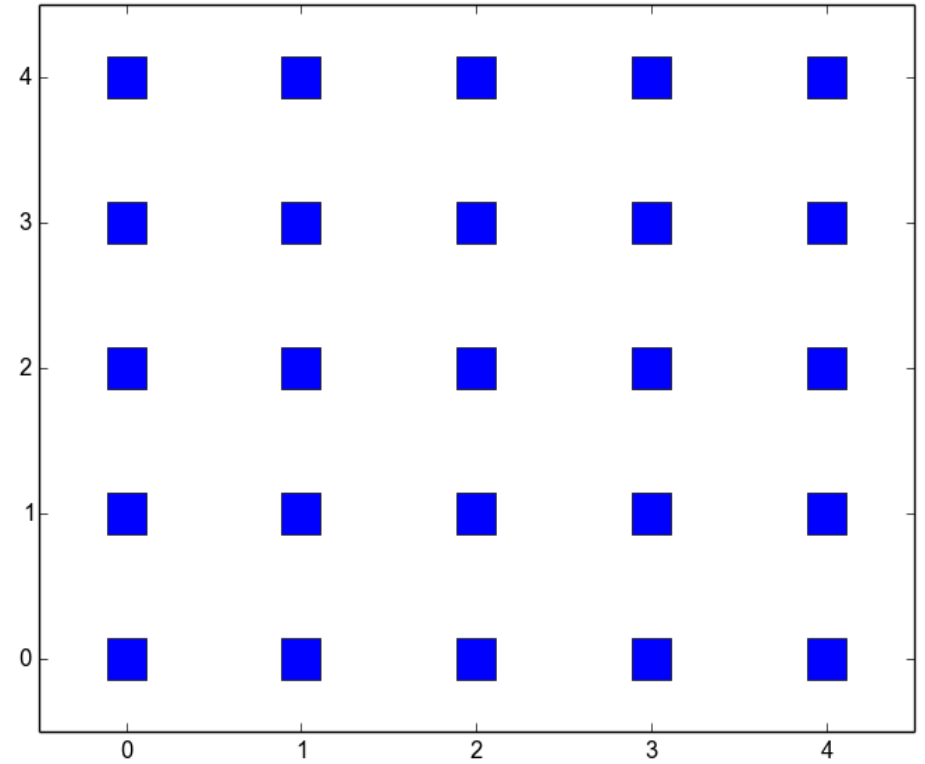
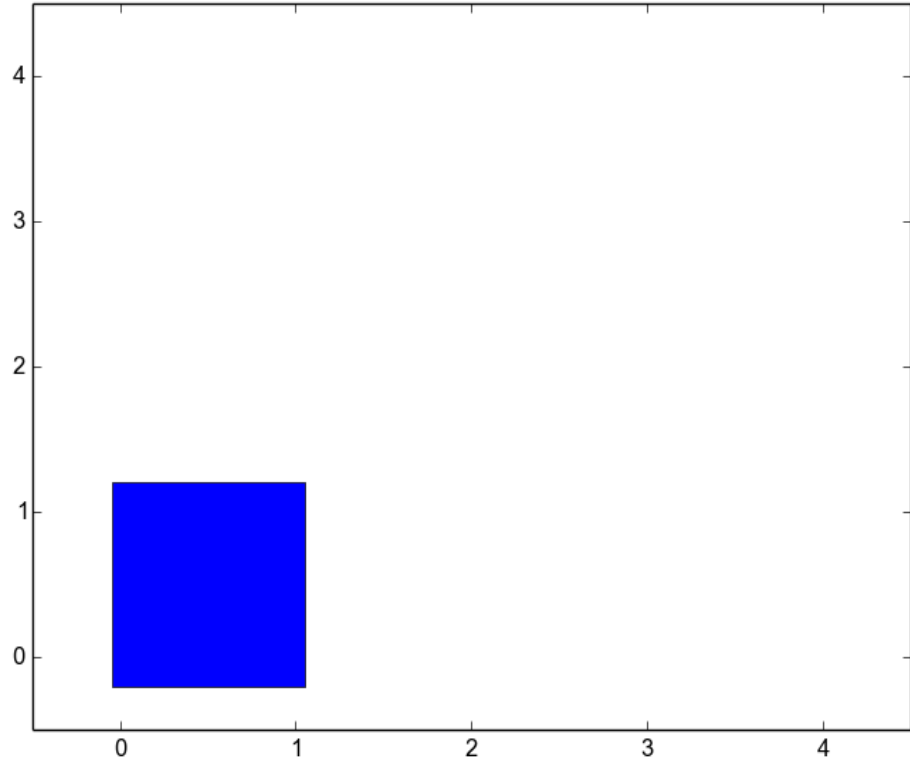
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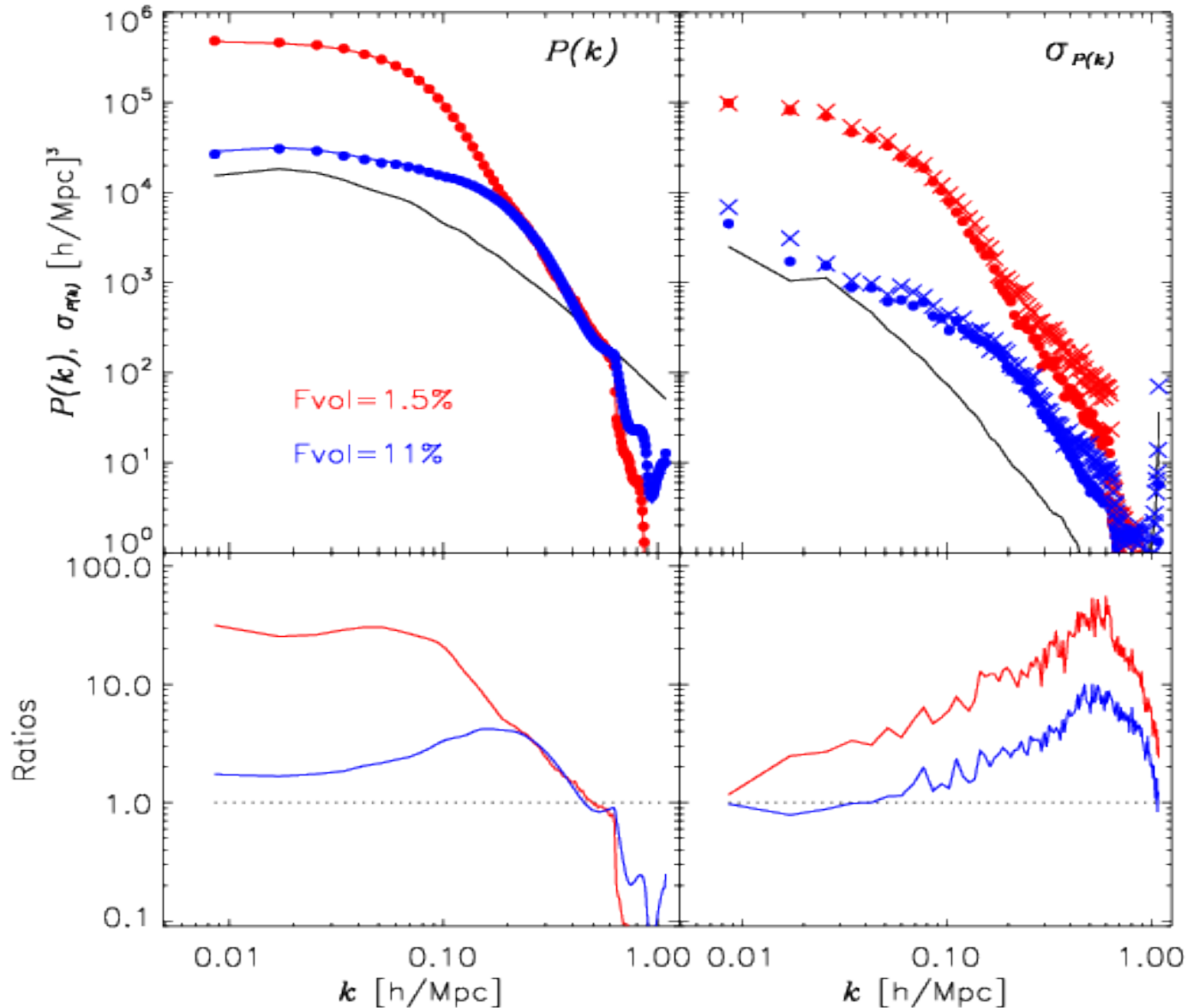
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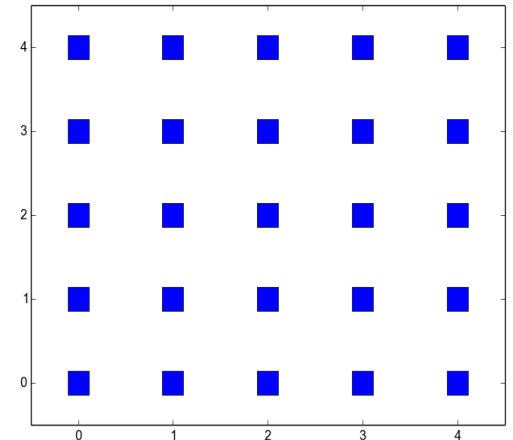
# Continuous v/s sparse sampling



**$k < 0.1$  h/Mpc scales can be measured in 10% of the time**  
 **$k < 0.01$  h/Mpc scales can be measured in 1% of the time**



$dx = 5 \text{ Mpc/h}$



$L = 1200 \text{ Mpc/h}$

# Summary

- Modern N-body simulations are essential to address current and future challenges in cosmology. The exaflop limit and 10 trillion particle runs are expected by 2020
- In a formative era, simulations were essential to probe that the Universe we observed can be explained by simple initial conditions and the laws of physics
- In a consolidation era, simulations have provided us for very accurate predictions for the properties of structure
- In the next era, N-body results could be used directly in cosmological analyses