Separate Universe Simulations

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Outline

- Motivation
- Implementation
- Simulations
- Applications
  - Squeezed-limit *N*-point functions
  - Local halo bias
- Summary

Motivation

- In cosmology one basic question is how some quantity/observable responds to a change in the large-scale matter density, δ_ρ.
  - Most prominent example is the bias of tracers (e.g. galaxies) of the underlying matter density field:

$$N_g(\delta_\rho) = N_{g,0}\left(1 + b_1\delta_\rho + \frac{b_2}{2}\delta_\rho^2 + \ldots\right)$$

  - Other examples are the flux in Lyman-α, the matter power spectrum, halo profiles, number density of voids, …

Motivation

- Such a question is usually addressed by indirect approaches:
  - One could consider subvolumes of cosmological simulations. As the mean overdensity in the subvolumes varies, one could measure the desired quantity as a function of δ_ρ.
    Problems:
    - all other quantities also vary from subvolume to subvolume => a lot of noise!
    - results depend on the size of the subvolumes / smoothing scale
  - Bias parameters of tracers can be measured by clustering statistics like the power spectrum and higher N-point functions.
    Problems:
    - one needs large volumes to get statistically accurate results
    - only b_1 can be measured easily

Motivation

- The separate universe picture allows for an efficient simulation of the effect of a large-scale overdensity
  - The evolution inside a spherical uniform overdensity embedded in a (flat) FRW background is exactly equivalent to that inside a separate (curved) universe (Lemaitre 1933).
  - In order to simulate the separate universe with standard N-body codes, we need to find the mapping

$$\Omega_m, \Omega_\Lambda, H_0 \xrightarrow{\delta_\rho} \tilde{\Omega}_m, \tilde{\Omega}_\Lambda, \tilde{H}_0$$

So far this was only worked out for small δ_ρ (McDonald 2003, Sirko 2005, Li et al. 2014)

  - Advantages:
    - The random realization of the initial density fluctuations can be kept the same => sample variance cancels to a large extent
    - δ_ρ can be large and chosen by hand

Implementation

- The overdensity is absorbed in the modified background density: $\rho(t)\left(1+\delta_\rho(t)\right)=\tilde{\rho}(t)$

=> $$\frac{\Omega_m H_0^2}{a^3(t)}\left(1+\delta_\rho(t)\right)=\frac{\tilde{\Omega}_m \tilde{H}_0^2}{\tilde{a}^3(t)}$$

- Matching the scale factors at early times, for which $\delta_\rho \rightarrow 0$, we get: $\Omega_m H_0^2=\tilde{\Omega}_m \tilde{H}_0^2$
- Using the Friedmann equations and the continuity equations, we can derive

$$\tilde{K}=\frac{5}{3}\Omega_m H_0^2 \frac{\delta_\rho(t_i)}{a(t_i)}=\frac{5}{3}\Omega_m H_0^2 \frac{\delta_L(t_0)}{D(t_0)}$$

Implementation

- The mapping of the cosmological parameters defined with respect to $\tilde{a} = 1$ and $a = 1$, respectively:

$$\tilde{H}_0 = H_0(1+\delta_{H_0})$$
$$\tilde{\Omega}_m = \Omega_m(1+\delta_{H_0})^{-2}$$
$$\tilde{\Omega}_\Lambda = \Omega_\Lambda(1+\delta_{H_0})^{-2}$$

$$1+\delta_{H_0} = \sqrt{1-\frac{5}{3}\Omega_m\frac{\delta_L(t_0)}{D(t_0)}}$$

- As the physical densities $\Omega_{\mathrm{m}}H_0^2$ and $\Omega_{\mathrm{b}}H_0^2$ as well as the spectral index n_{s} and the initial amplitude A_{s} remain unchanged, the initial power spectrum is the same in both cosmologies.

Implementation

- Output scale factors
  - As $\tilde{a}(t) \neq a(t)$, the same physical time corresponds to different scale factors.
  - We define the difference as $\tilde{a}(t) = [1+\delta_a(t)]\, a(t)$
  - Due to mass conservation $1+\delta_\rho(t) = [1+\delta_a(t)]^{-3}$
- Comoving coordinates $x = [1+\delta_a(t)]\, \tilde{x}$
- Reference background density $\rho(t)(1+\delta_\rho(t)) = \tilde{\rho}(t)$

Simulations

- We simulate 16 realizations of a ~(500 Mpc/h)^3 box with
  - the fiducial cosmology:
    flat ΛCDM $\Omega_m = 0.27$, $H_0 = 70$ km/s Mpc^{-1}, $\sigma_8 = 0.8$, $n_s = 0.95$
  - 22 separate universe cosmologies with

    $\delta_L(t_0) = \pm$ (0.01, 0.02, 0.05, 0.07, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1)

    => $\tilde{\Omega}_m$ ranges from 0.17 to 0.66

    $\tilde{\Omega}_\Lambda$ ranges from 0.46 to 1.79

    $\tilde{H}_0$ ranges from 45 to 88 km/s Mpc^{-1}

- We output snapshots at redshifts in the fiducial cosmology: z = 0 and z = 2

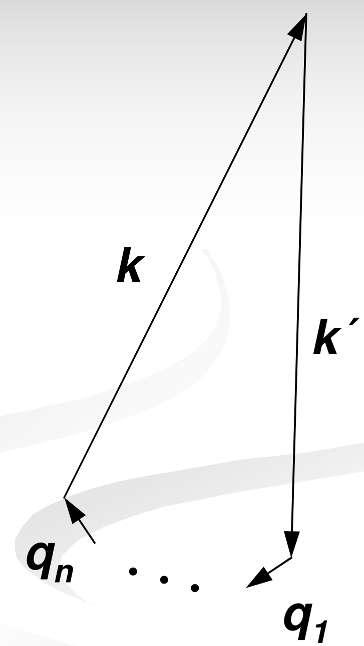
Application I
Squeezed-limit *N*-point functions

- The n[th]-order response functions of the power spectrum

$$R_n(k,t) = \frac{1}{P(k,t)} \frac{d^n P(k,t \mid \delta_L)}{d\delta_L^n}\bigg|_{\delta_L=0}$$

are equivalent to the angle-averaged squeezed-limit n+2-point functions

$$R_n(k) = \lim_{q_i \to 0} \frac{\int \frac{d^2\hat{q}_i}{4\pi} \langle \delta(k)\delta(k')\delta(q_1)\cdots\delta(q_n)\rangle_c'}{P(k)P_l(q_1)\cdots P_l(q_n)}$$



Application I
Squeezed-limit *N*-point functions

- The n[th]-order **full** response functions of the power spectrum

$$R_n(k,t) = \frac{1}{P(k,t)} \left.\frac{d^n P(k,t\,|\,\delta_L)}{d\delta_L^n}\right|_{\delta_L=0}$$

1) Growth-only effect:

$$G_n(k,t) = \frac{1}{P(k,t)} \left.\frac{d^n P(k,t\,|\,\delta_L)}{d\delta_L^n}\right|_{k}$$

2) Dilation effect:

$$P(k) = (1+\delta_a)^3\,\tilde{P}([1+\delta_a]k)$$

3) Reference density effect:

$$P(k) = (1+\delta_\rho)^2\,\tilde{P}(k)$$

- 2) and 3) together:

$$P(k) = (1+\delta_\rho)\,\tilde{P}([1+\delta_a]k)$$

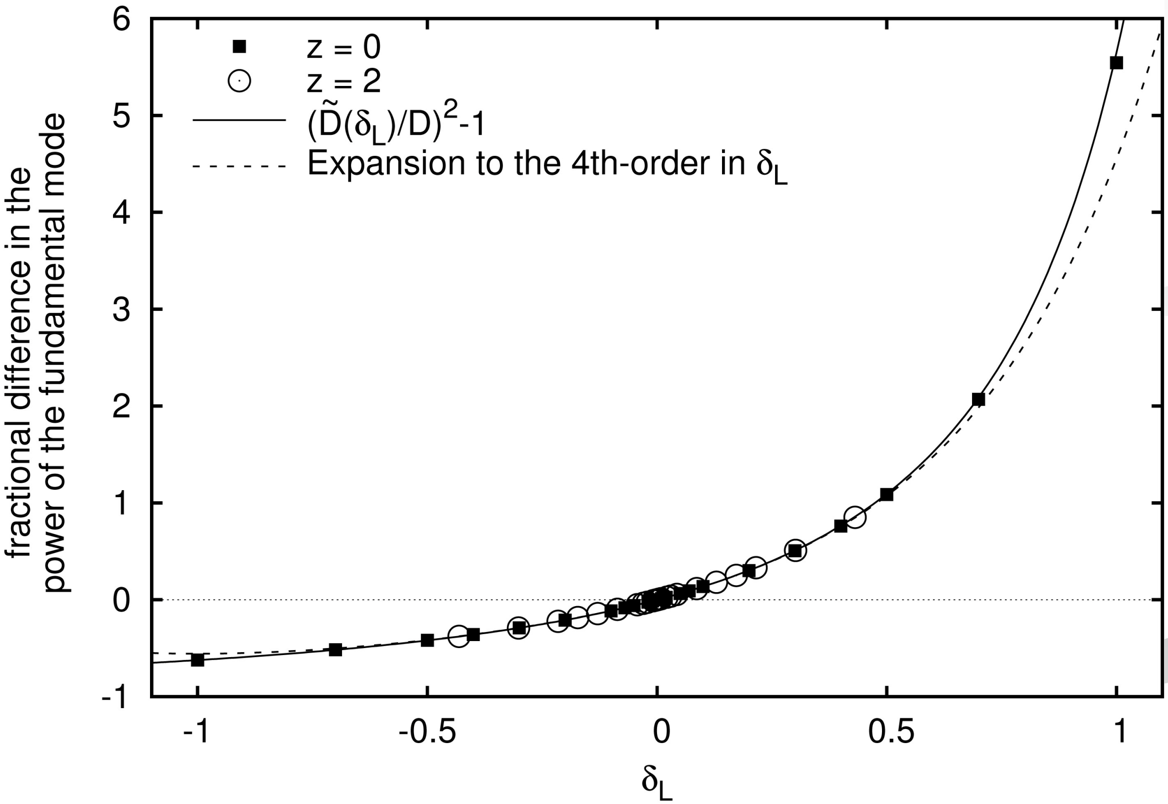
- Expansion in δ_L: $\delta_a = \sum_{n=1}^{\infty} e_n \delta_L^n$ and $\delta_\rho = \sum_{n=1}^{\infty} f_n \delta_L^n$

Application I
Squeezed-limit *N*-point functions

- How do we get the nth-order response?

Power spectrum measured using the comoving scales and the reference density of the respective cosmology

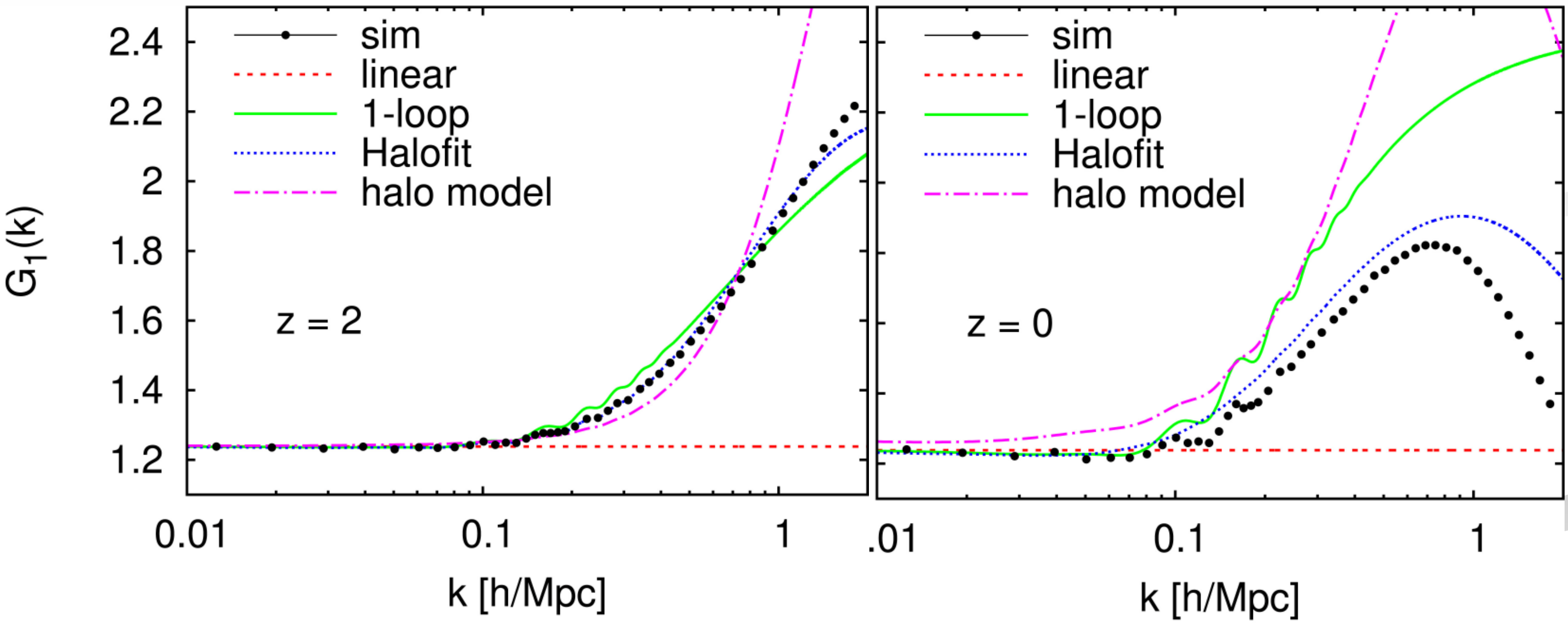
$$G_n(k,t) = \frac{1}{P(k,t)} \frac{d^n P(k,t \mid \delta_L)}{d\delta_L^n}\bigg|_k$$



Application I
Squeezed-limit *N*-point functions

- The n^th-order **growth-only** response functions of the power spectrum

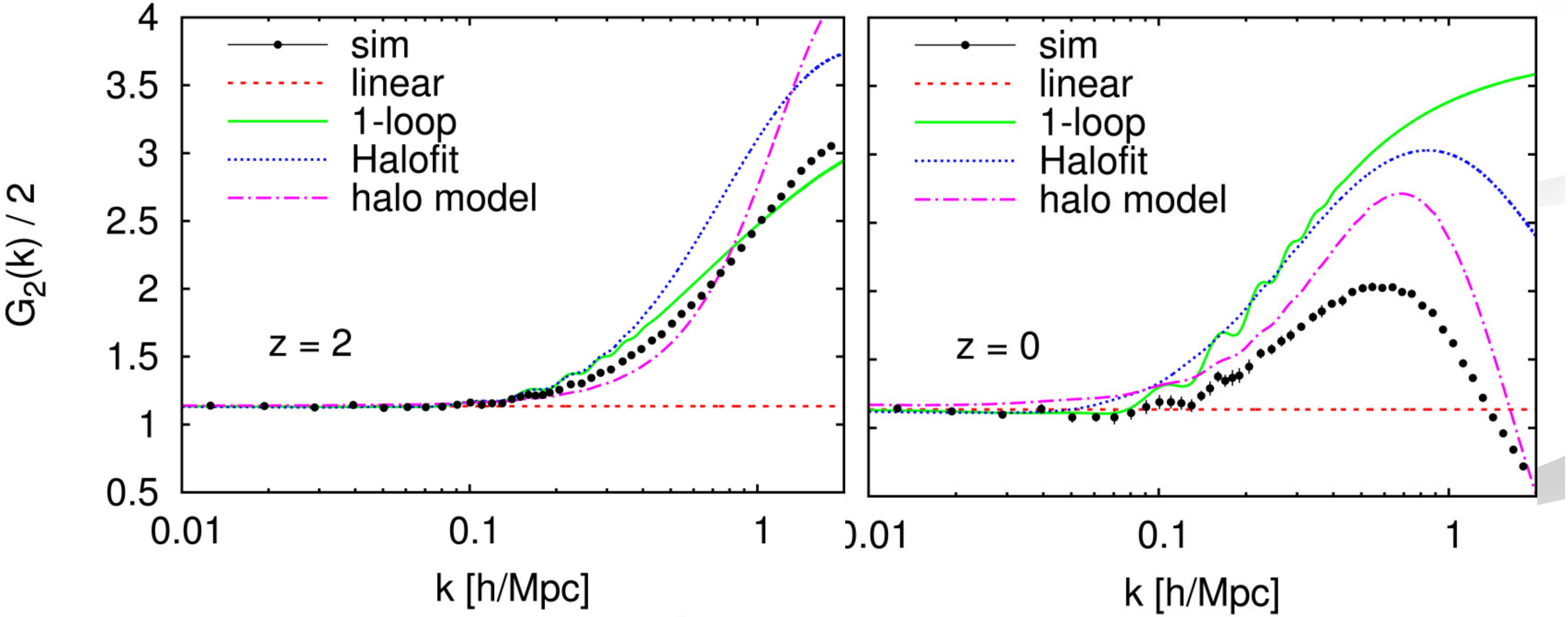
$$G_n(k,t) = \frac{1}{P(k,t)} \frac{d^n P(k,t\,|\,\delta_L)}{d\delta_L^n}\bigg|_k$$



Application I
Squeezed-limit *N*-point functions

- The n[th]-order **growth-only** response functions of the power spectrum

$$G_n(k,t) = \frac{1}{P(k,t)} \frac{d^n P(k,t\,|\,\delta_L)}{d\delta_L^n}\bigg|_k$$



Application I
Squeezed-limit *N*-point functions

- The nth-order **growth-only** response functions of the power spectrum

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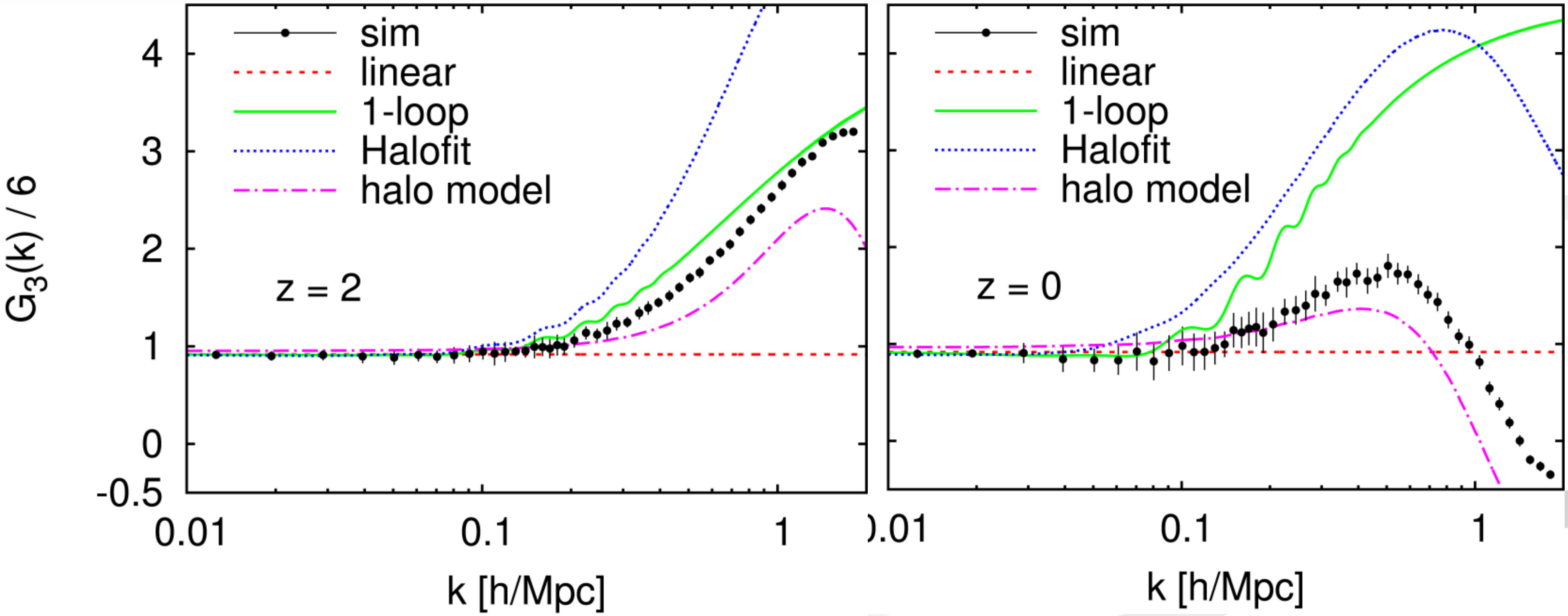


Application I
Squeezed-limit *N*-point functions

- The n-th-order **full** response functions of the power spectrum

$$R_n(k,t) = \frac{1}{P(k,t)} \left. \frac{d^n P(k,t \mid \delta_L)}{d\delta_L^n} \right|_{\delta_L=0}$$

1) Growth-only effect:

$$G_n(k,t) = \frac{1}{P(k,t)} \left. \frac{d^n P(k,t \mid \delta_L)}{d\delta_L^n} \right|_k$$

2) Dilation effect:

$$P(k) = (1+\delta_a)^3 \tilde{P}([1+\delta_a]k)$$

3) Reference density effect:

$$P(k) = (1+\delta_\rho)^2 \tilde{P}(k)$$

- 2) and 3) together:

$$P(k) = (1+\delta_\rho) \tilde{P}([1+\delta_a]k)$$

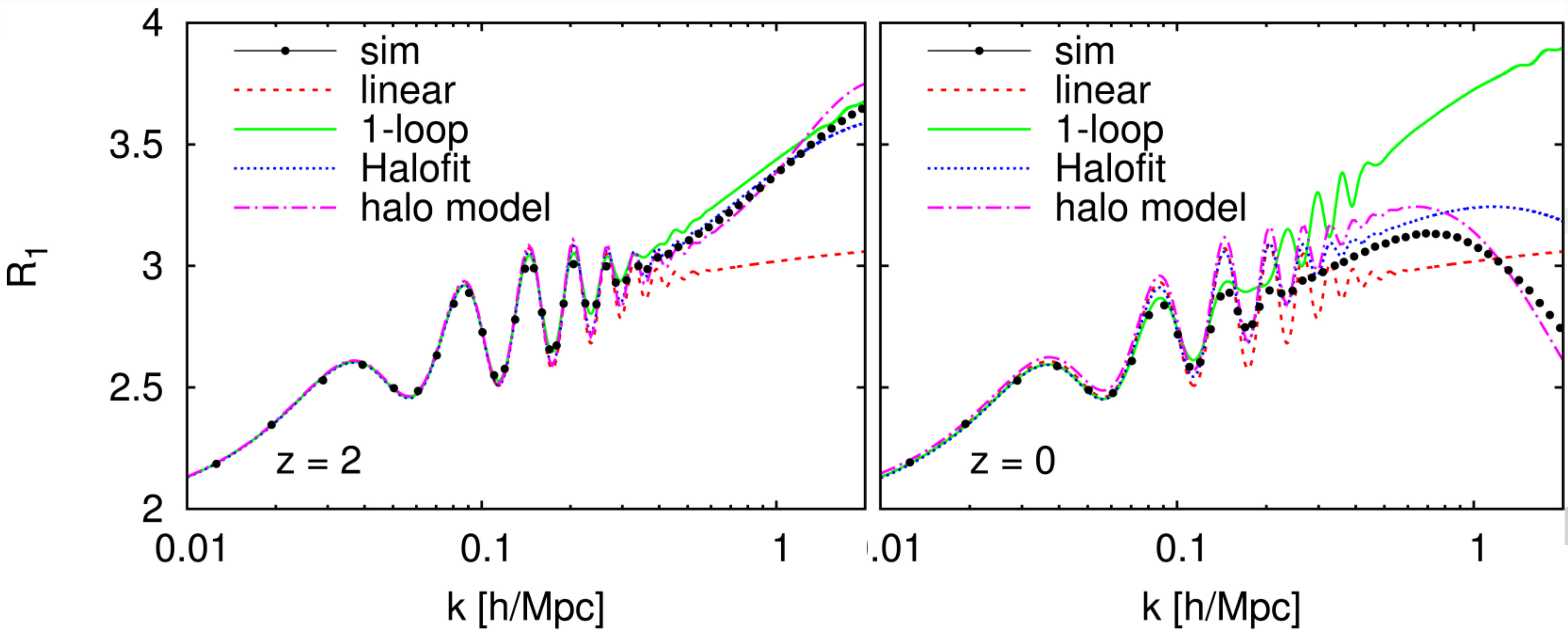
- Expansion in δ_L:

$$\delta_a = \sum_{n=1}^{\infty} e_n \delta_L^n \quad \text{and} \quad \delta_\rho = \sum_{n=1}^{\infty} f_n \delta_L^n$$

Application I
Squeezed-limit *N*-point functions

- The 1st-order **full** response functions of the power spectrum

$$R_1(k) = f_1 + e_1 \frac{kP'(k)}{P(k)} + G_1(k)$$



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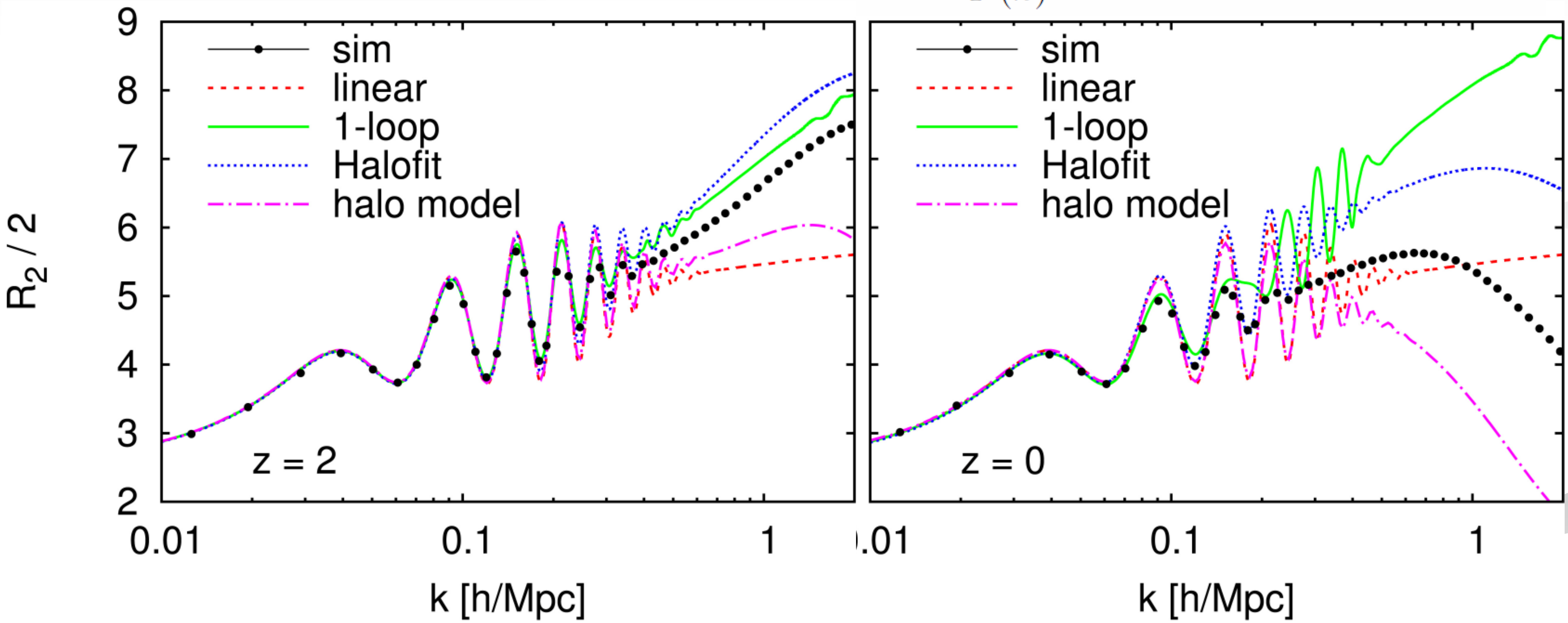
Application I
Squeezed-limit *N*-point functions

- The 2nd-order **full** response functions of the power spectrum

$$\frac{R_2(k)}{2} = f_2 + e_2 \frac{kP'(k)}{P(k)} + e_1^2 \frac{k^2 P''(k)}{2P(k)} + \frac{G_2(k)}{2} + f_1 e_1 \frac{kP'(k)}{P(k)} + f_1 G_1(k) + e_1 \frac{kP'(k)}{P(k)} G_1(k) + e_1 k G_1'(k)$$



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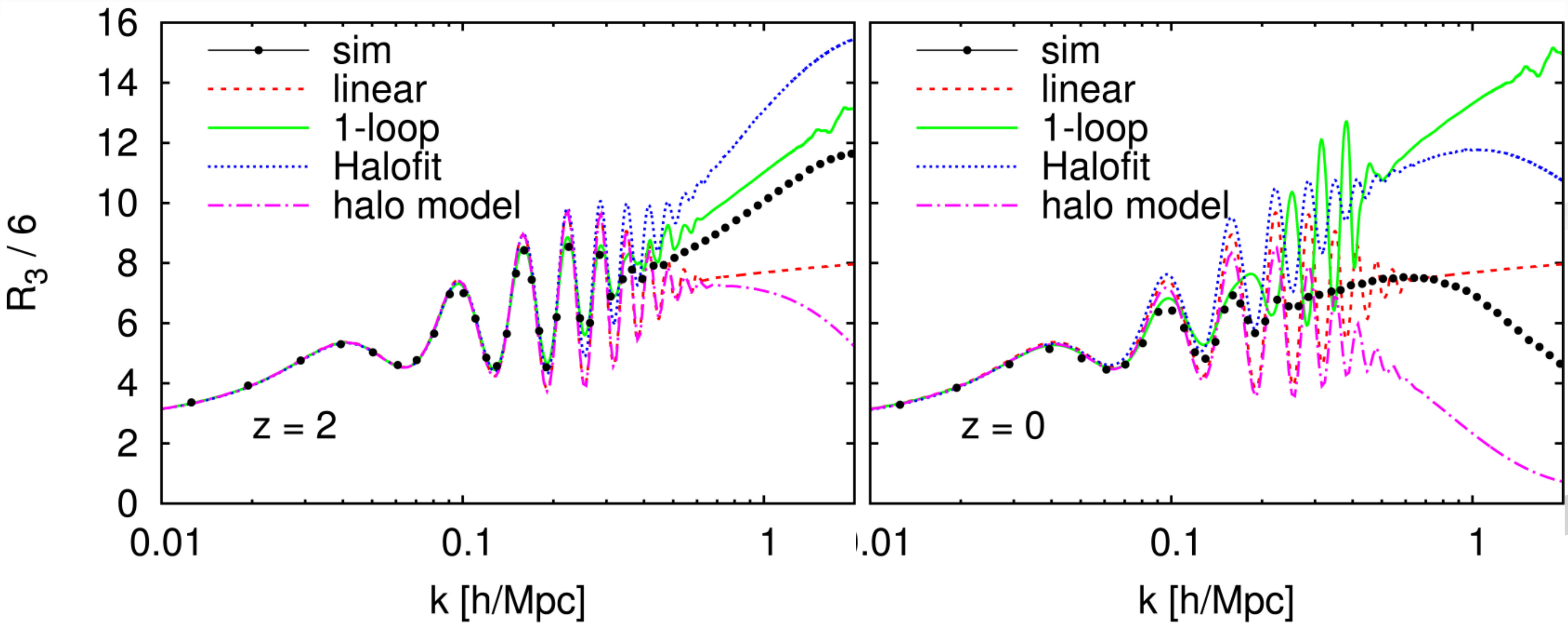
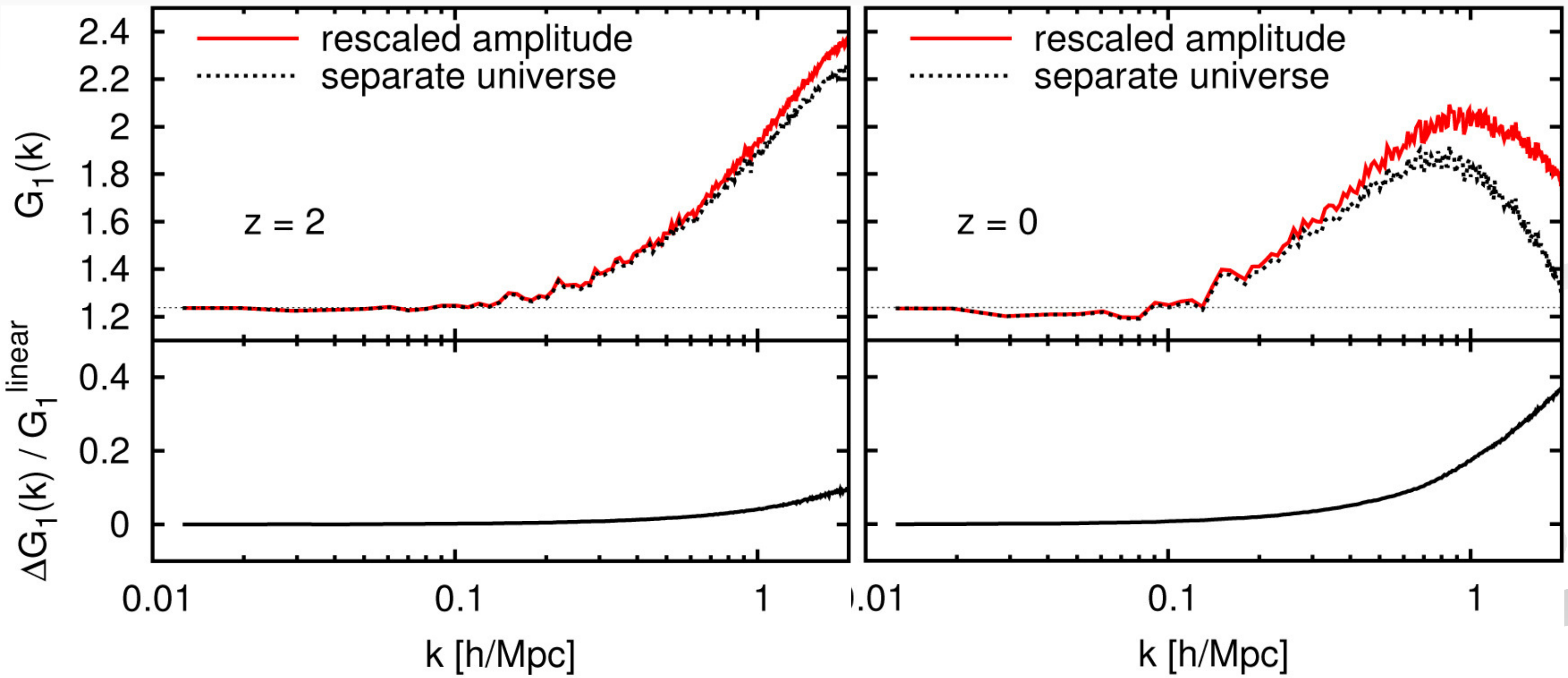
Application I
Squeezed-limit *N*-point functions

- The 3rd-order **full** response functions of the power spectrum



Application I
Squeezed-limit *N*-point functions

- Comparison to **rescaled-initial-amplitude** simulations



Application II
Local halo bias

- Count the number of halos per mass bin in each simulation

$$N_h(M \mid \delta_\rho)$$

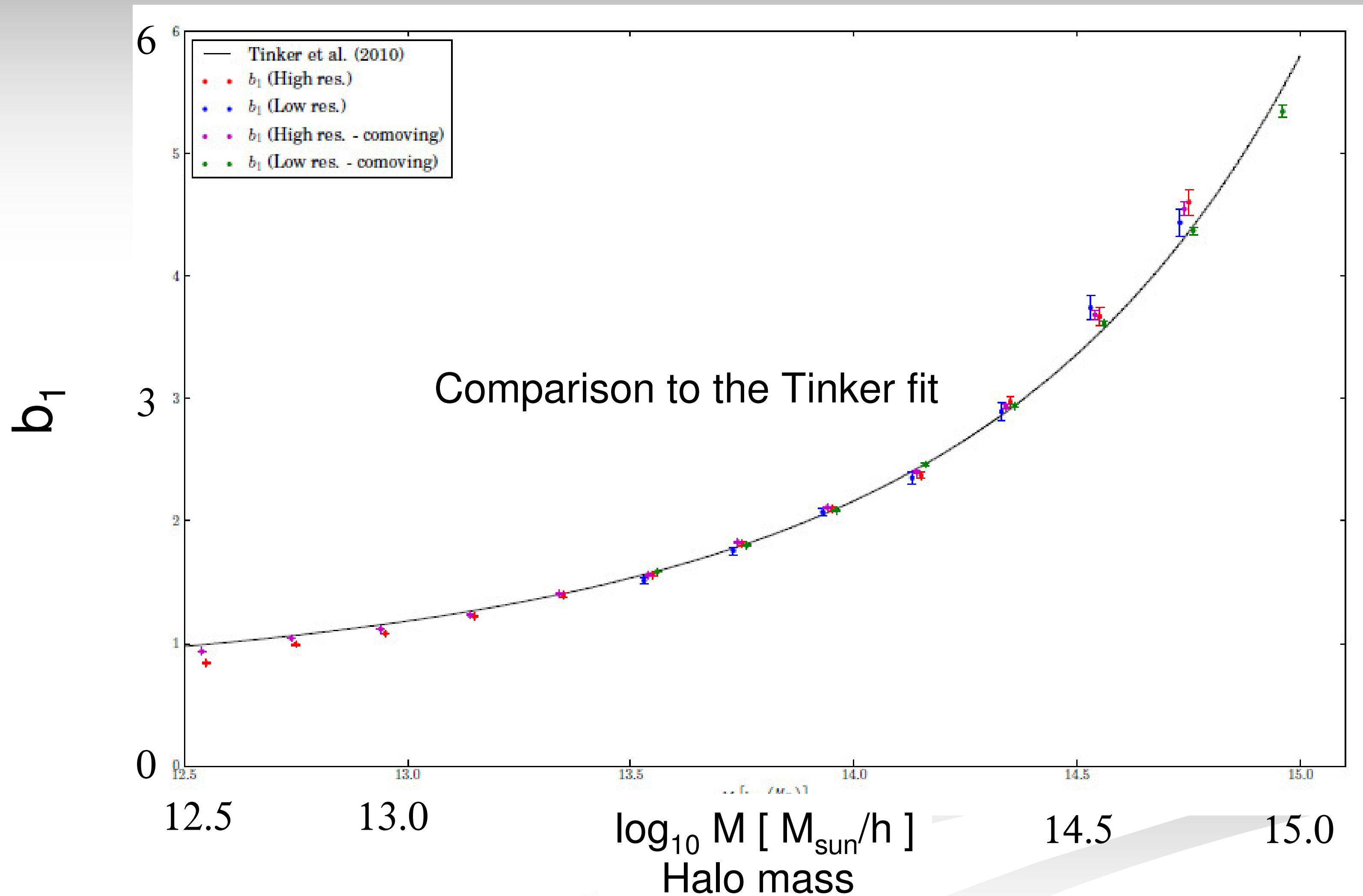
- Compute the fractional difference to the fiducial cosmology

$$\delta_{N_h}(M \mid \delta_\rho) = \frac{N_h(M \mid \delta_\rho) - N_{h,0}(M)}{N_{h,0}(M)}$$

- Fit a polynomial in δ_ρ to the measurements

$$\delta_{N_h} = b_1 \delta_\rho + \frac{b_2}{2}\delta_\rho^2 + \frac{b_3}{3!}\delta_\rho^3 + \cdots$$

Application II
Local halo bias



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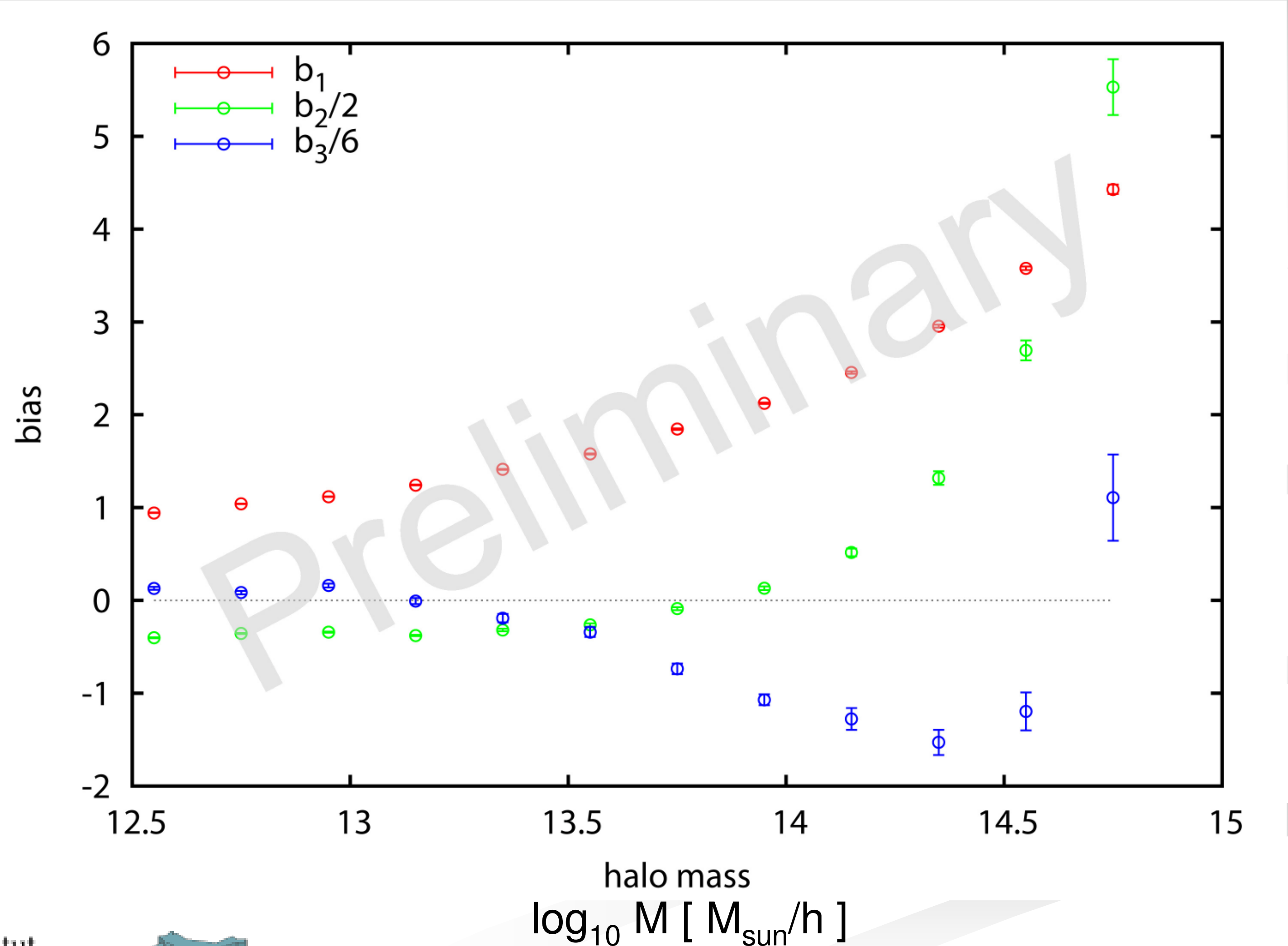
Application II
Local halo bias



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Summary

- Separate universe picture is intuitive
- It singles out the effect of a uniform overdensity
- It is straightforward to implement in N-body simulations (and basically in any cosmological structure formation technique)
- Allows for efficient and precise computation of
  - the angle-averaged squeezed-limit N-point functions
  - the local halo bias parameters
  - …
- Easy estimation of cosmic variance effects
- Does not include effects of the tidal field or density gradients