Angular Power Spectrum Estimation in Radio Interferometric Observations

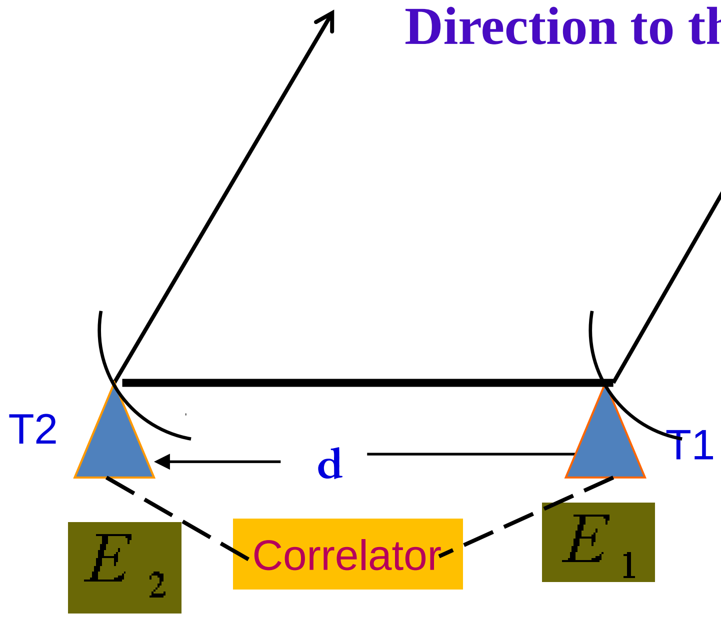
Samir Choudhuri

Indian Institute Technology Kharagpur, India

Collaborators:
Prof. Somnath Bharadwaj
SK. Saiyad Ali
Abhik Ghosh

2014MNRAS.445.4351C

Radio Interferometers: Visibilities



$$U = \frac{d}{\lambda}$$

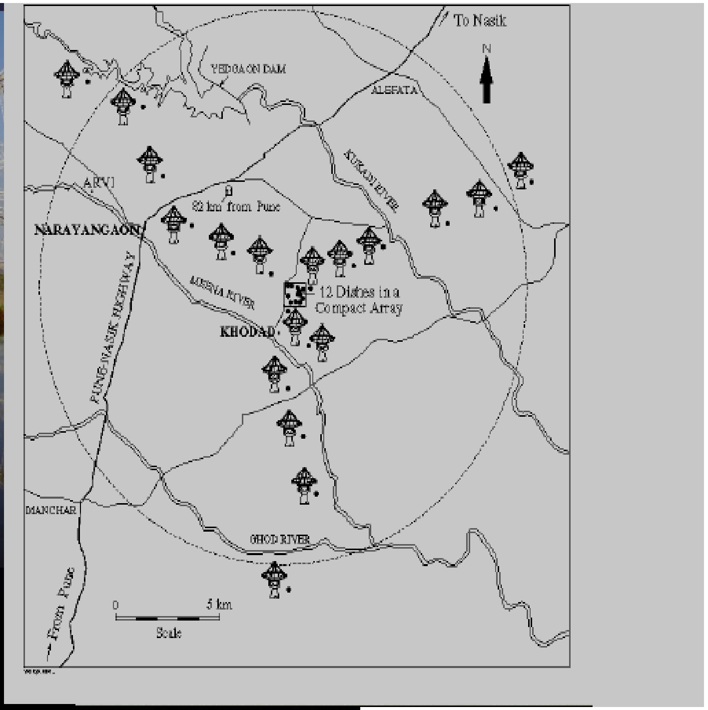
Field of view of the antenna – Small – Plane parallel approx.

$$V(U,\nu) = \iint A_{\upsilon}(\theta) I_{\upsilon}(\theta) e^{-2\pi i U.\theta} d^2\theta$$

$$V(\mathbf{U},\nu) = S(\mathbf{U},\nu) + F(\mathbf{U},\nu) + N(\mathbf{U},\nu)$$

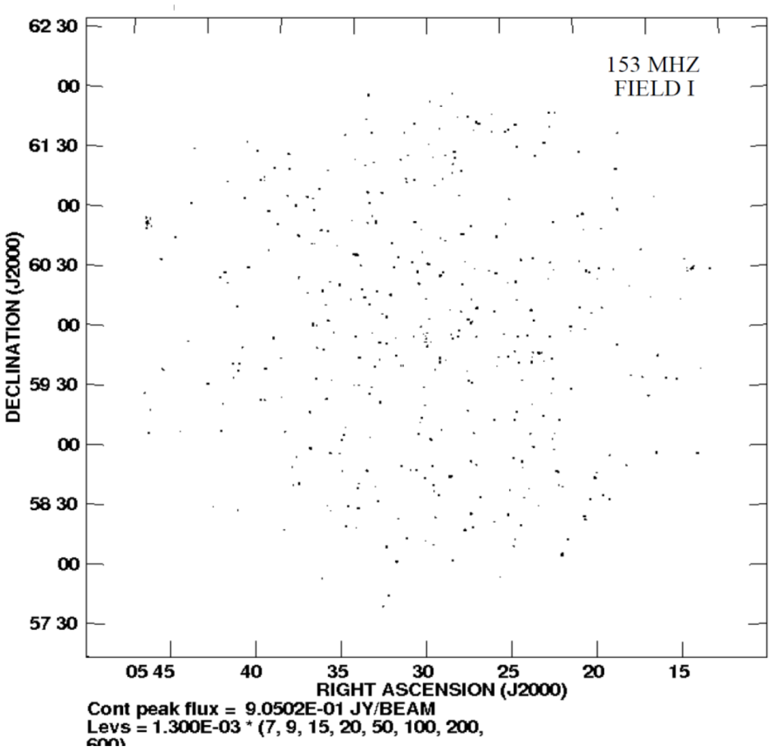
Giant Metrewave Radio Telescope, Pune,India

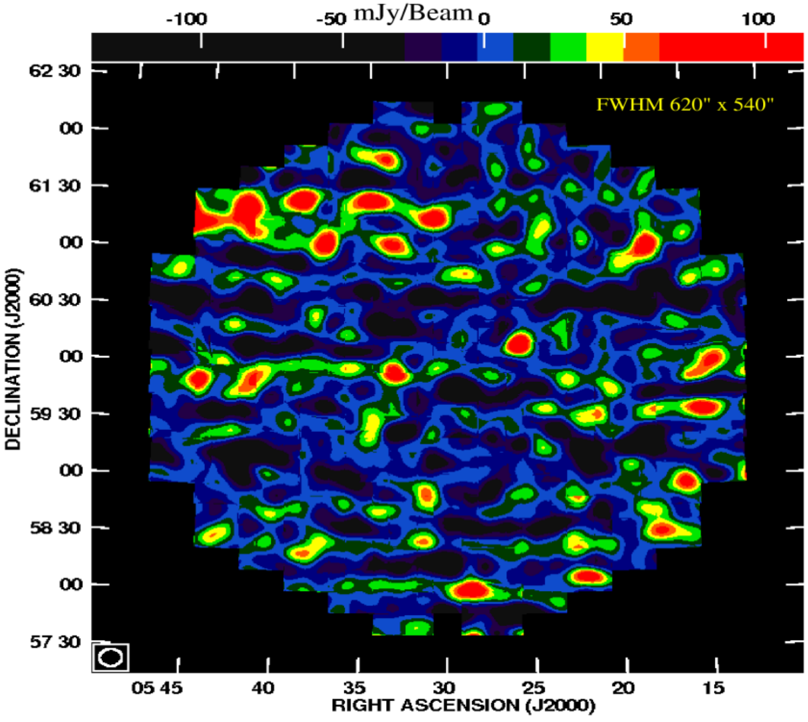
30 fixed antennas x 45m diameter



It is currently operating at several frequency band in the frequency range 150 -1420 MHz

Foregrounds





Point Sources

Diffuse

GMRT 150MHz Observation

Ghosh et al. 2012

Motivation

How to quantify these fluctuations ?

Angular Power Spectrum

Any *brightness temperature fluctuation* on the sky are usually described by an expansion in spherical harmonics.

$$\delta T(\nu, \hat{\mathbf{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(\nu) Y_{\ell m}(\hat{\mathbf{n}})$$

The *Angular Power Spectrum* defined as :

$$C_l = < a_{lm}(\nu) a_{lm}^{*}(\nu) >$$

How they are related?

$$\mathcal{V}(\mathbf{U},\nu) = \mathcal{S}(\mathbf{U},\nu) + \mathcal{N}(\mathbf{U},\nu)$$

↑

Entire Sky Signal

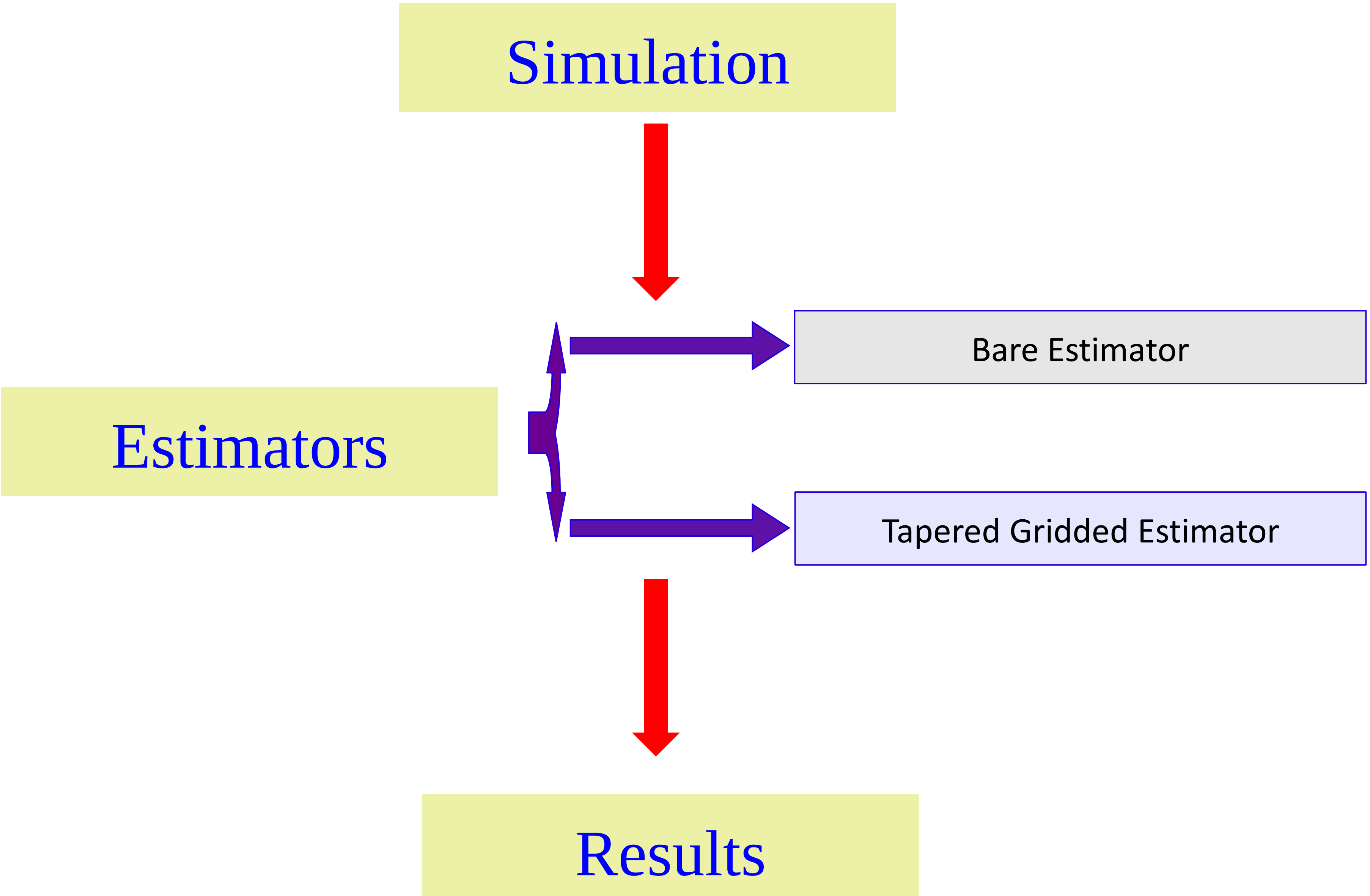
Two Visibility Correlation:

$$V_{2ij} \equiv \langle \mathcal{V}_i \mathcal{V}_j^* \rangle = V_0 \, e^{-|\Delta \mathbf{U}_{ij}|^2/\sigma_0^2} \, C_{\ell_i} + \delta_{ij} 2\sigma_n^2$$

$$S_2(\mathbf{U}, \mathbf{U} + \Delta \mathbf{U}) = \frac{\pi \theta_0^2}{2} \left(\frac{\partial B}{\partial T} \right)^2 \exp \left[- \left(\frac{\Delta U}{\sigma_0} \right)^2 \right] C_\ell$$

$$V_0 = \frac{\pi \theta_0^2}{2} \left(\frac{\partial B}{\partial T} \right)^2$$

Noise bias can be avoided by excluding self-correlation term.


Simulation
Estimators
Bare Estimator
Tapered Gridded Estimator
Results

Simulation

Simulation

We generate the $\Delta\tilde{T}(\mathbf{U})$,

$$\Delta\tilde{T}(\mathbf{U}) = \sqrt{\frac{\Omega\, C_\ell^M}{2}}[x(\mathbf{U}) + iy(\mathbf{U})],$$

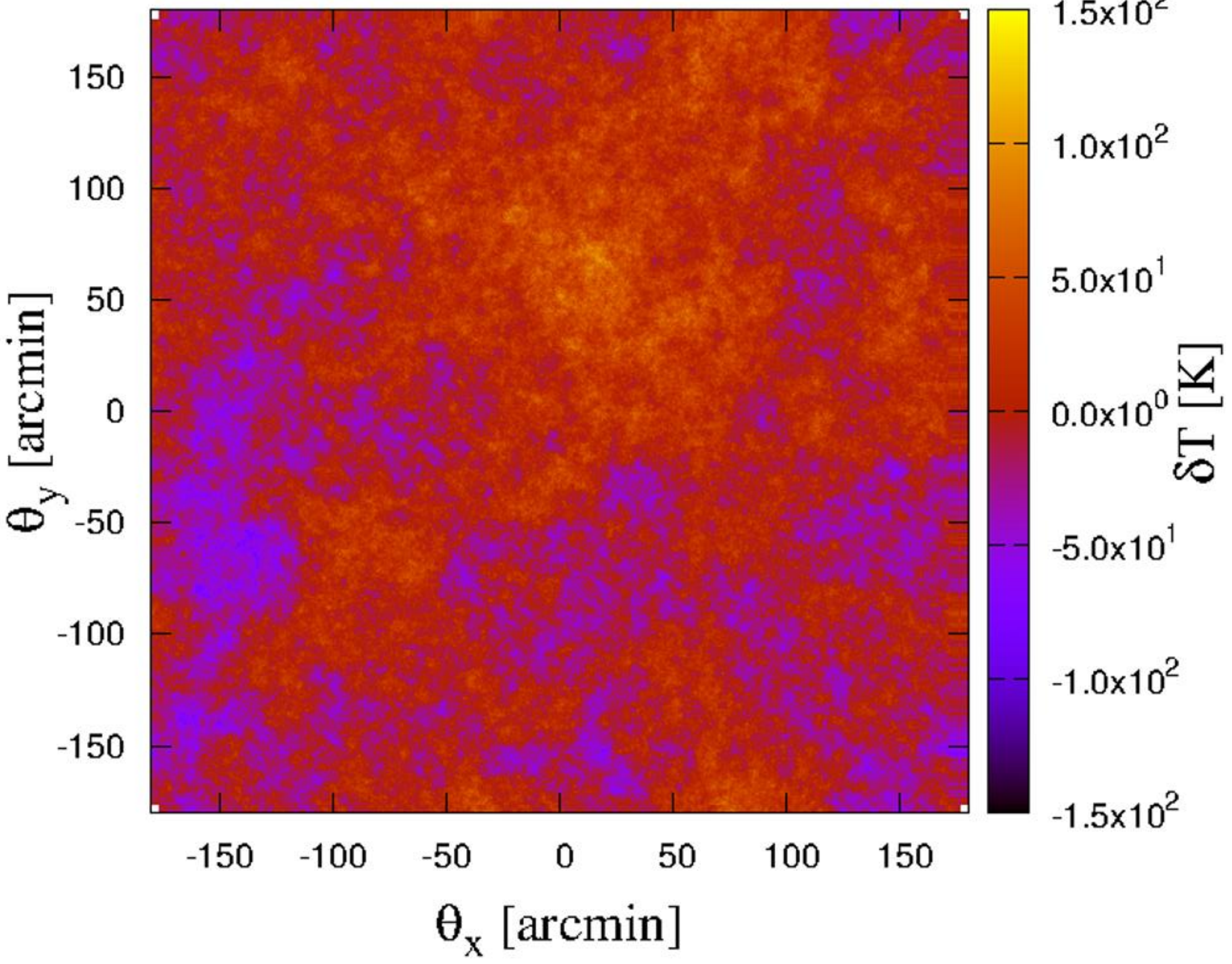
$$C_\ell^M = A_{150} \times \left(\frac{1000}{\ell}\right)^\beta \quad A_{150} = 513\,\mathrm{mK}^2 \quad \beta = 2.34$$

Ghosh et al. 2012

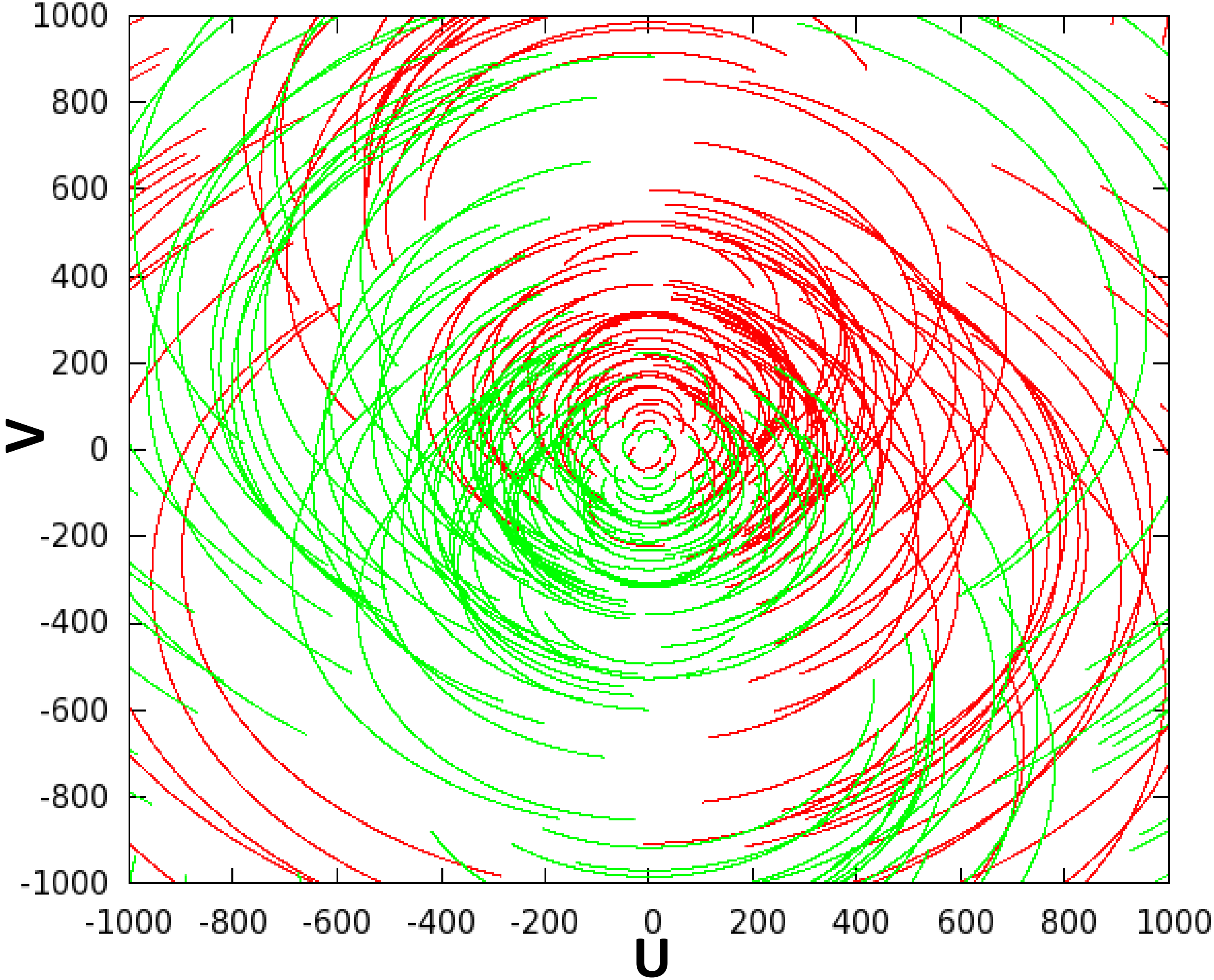
Then we use FFT to generate $\delta T(\vec{\theta})$ in the image plane.

Simulations has been done considering GMRT 150 MHz Observations.

Diffuse Emission (sky plane)

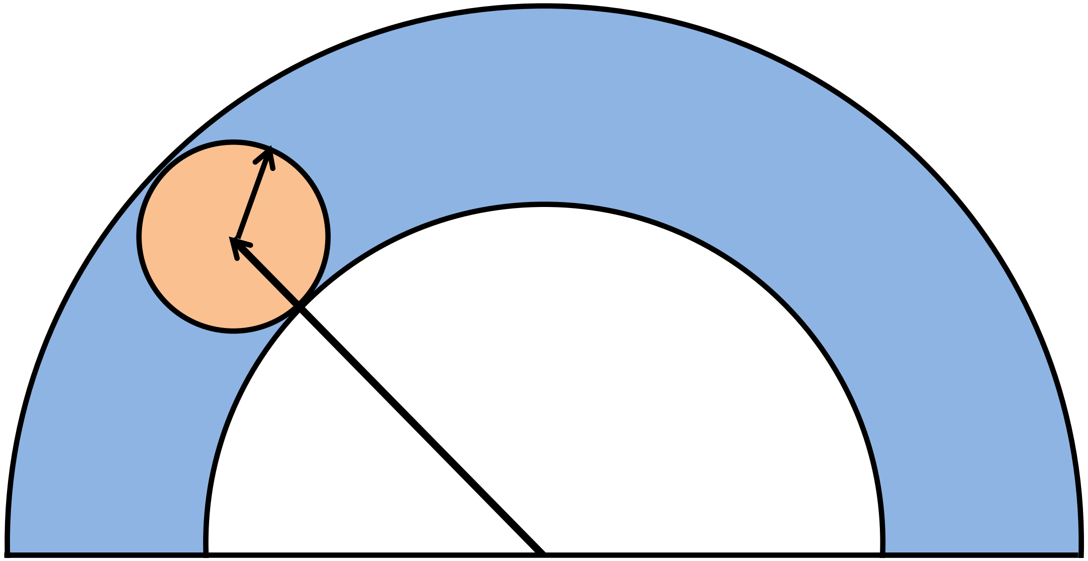


GMRT Baseline Distribution



Estimators

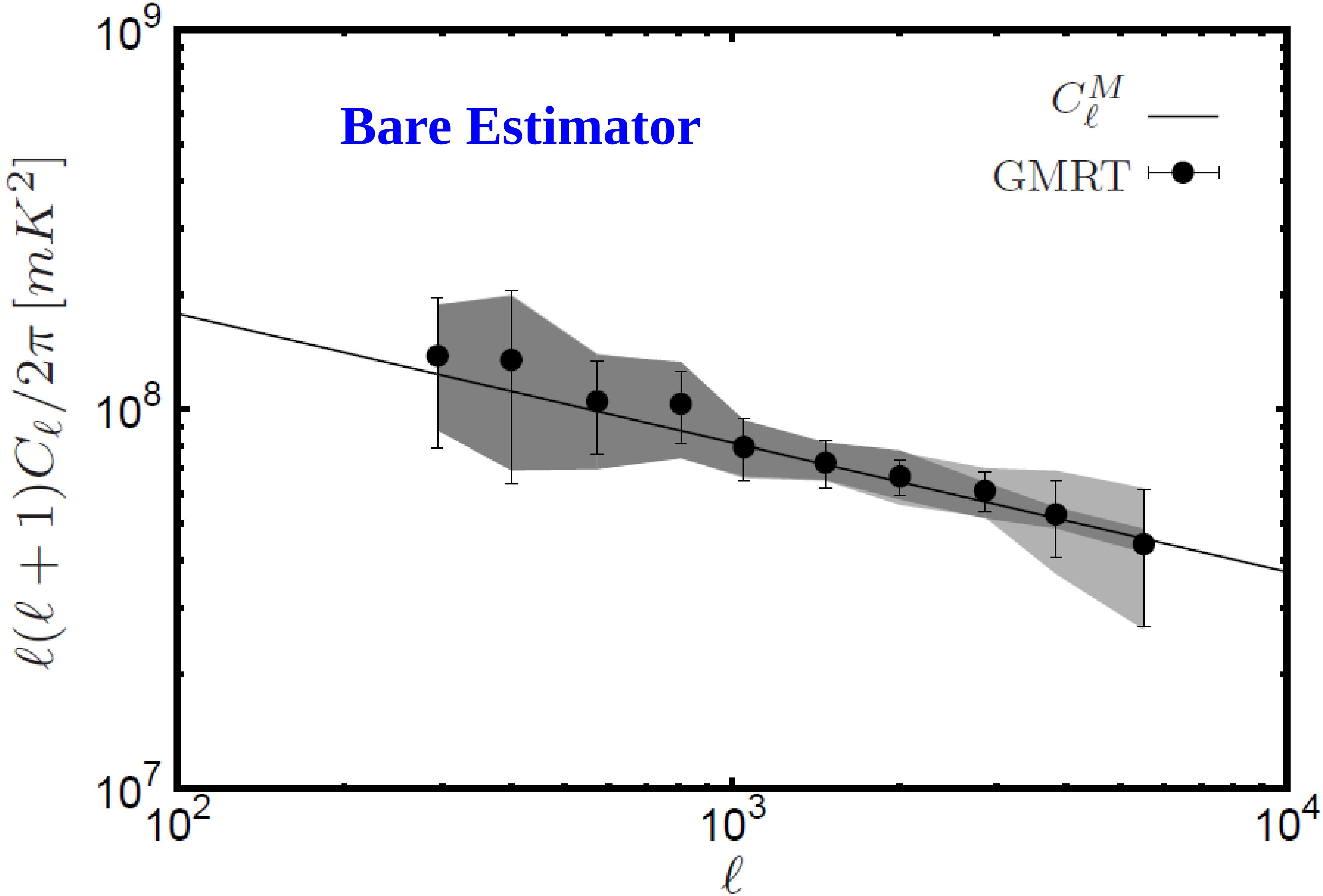
Bare Estimator



The Bare Estimator $\hat{E}_B(a)$ is defined as

$$\hat{E}_B(a) = \frac{\sum_{i,j} w_{ij} \mathcal{V}_i \mathcal{V}_j^*}{\sum_{i,j} w_{ij} V_0 e^{-|\Delta \mathbf{U}_{ij}|^2/\sigma_0^2}} \qquad w_{ij} = (1 - \delta_{ij}) K_{ij}$$

Correlation length, $\sigma_0 = \frac{0.76}{\theta_{FWHM}}$ For GMRT, $\sigma_0 = 16.6$



Bare Estimator
C_ℓ^M
GMRT
10^9
10^8
10^7
10^2
10^3
10^4
$\ell(\ell+1)C_\ell/2\pi\,[mK^2]$
ℓ

Disadvantage:

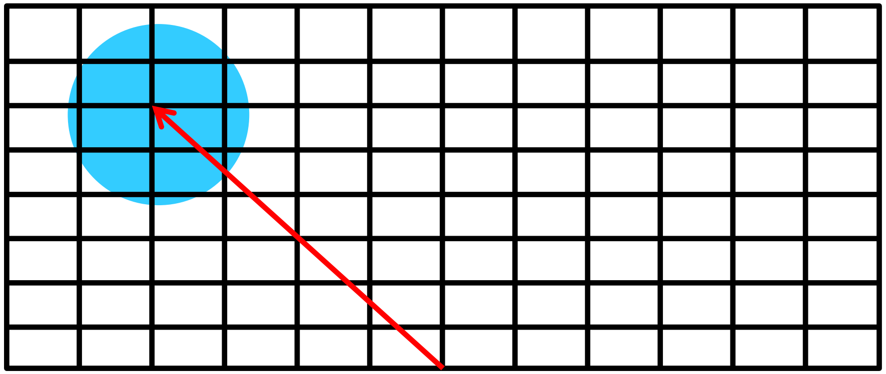
The Bare Estimator deals directly with the visibilities and the computational time for the pairwise correlation scales proportional to N^2, where N is the total number of visibilities in the data.

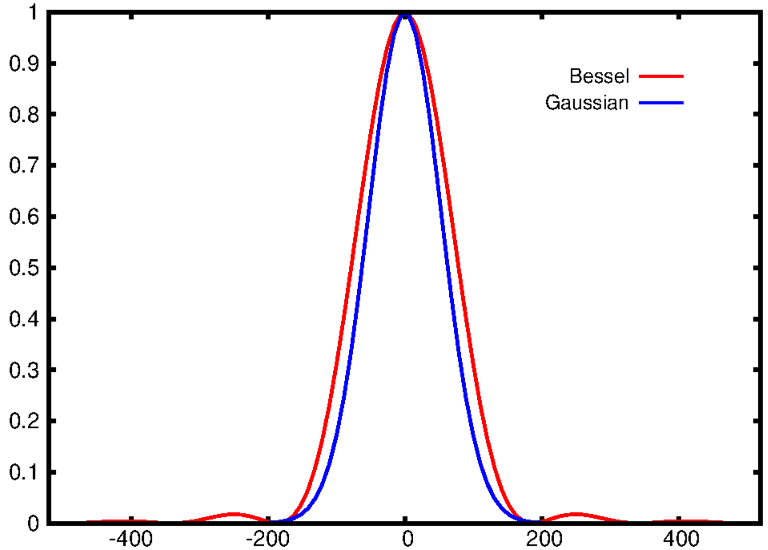
Tapered Gridded Estimator

We define tapered Gridded Estimator as,

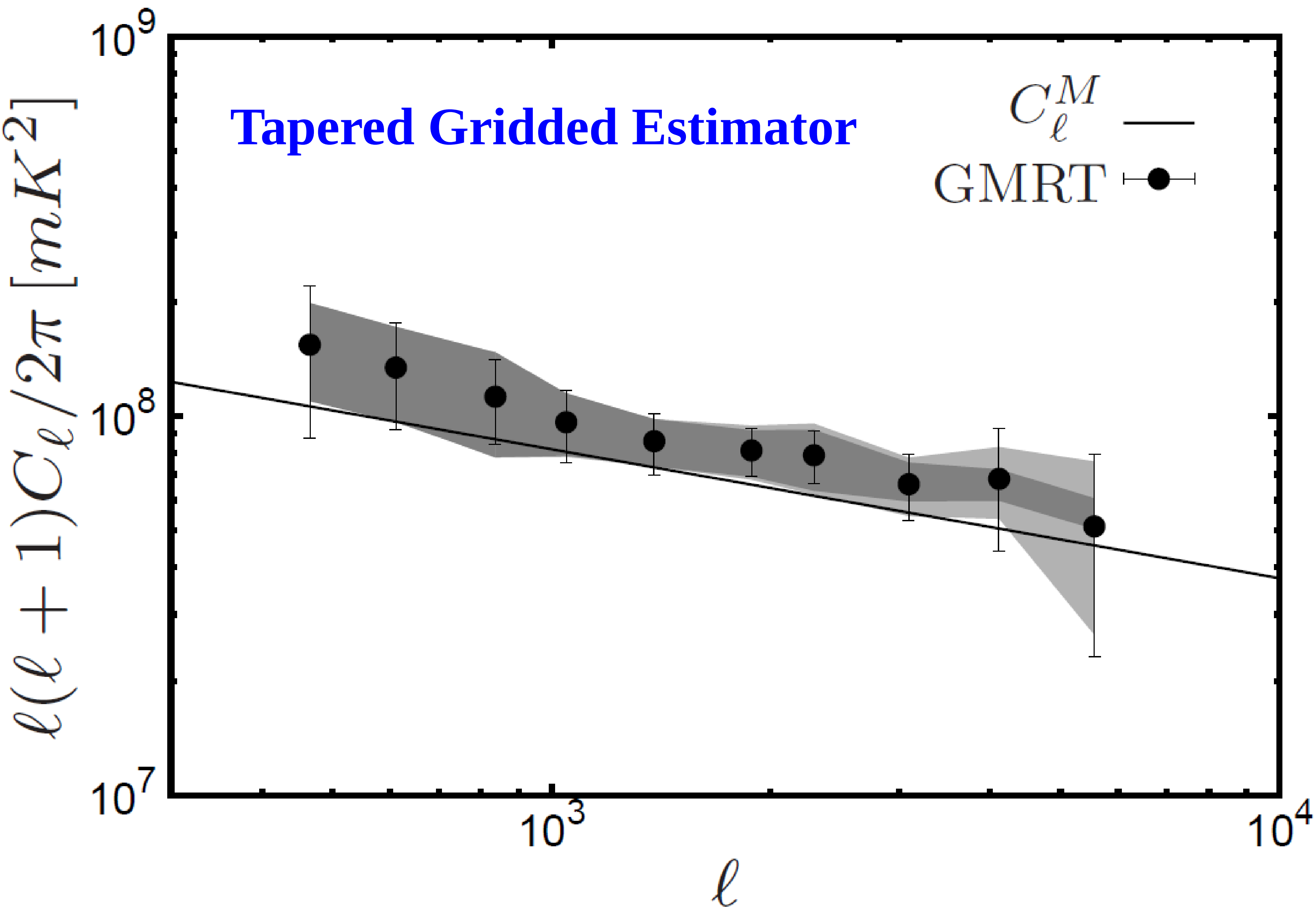
$$\hat{E}_g = \frac{(\mathcal{V}_{cg}\mathcal{V}_{cg}^{*} - \sum_i \mid \tilde{w}(\mathbf{U}_g - \mathbf{U}_i) \mid^2 \mid \mathcal{V}_i \mid^2)}{(\mid K_{1g} \mid^2 V_1 - K_{2gg}V_0)}$$

$$\mathcal{V}_c(\mathbf{U}) = \tilde{w}(\mathbf{U}) \otimes \mathcal{V}(\mathbf{U})$$





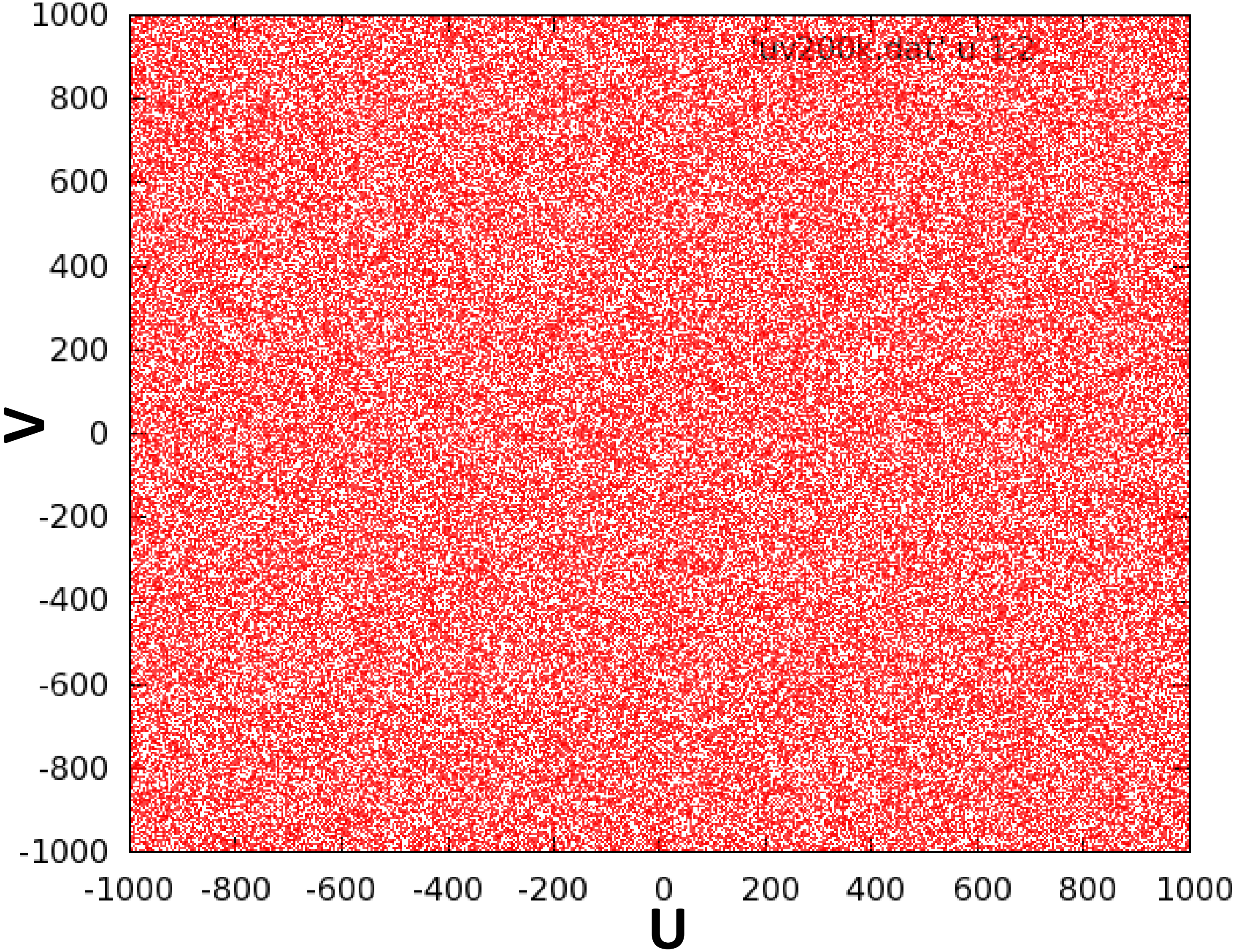
θ in arc-min

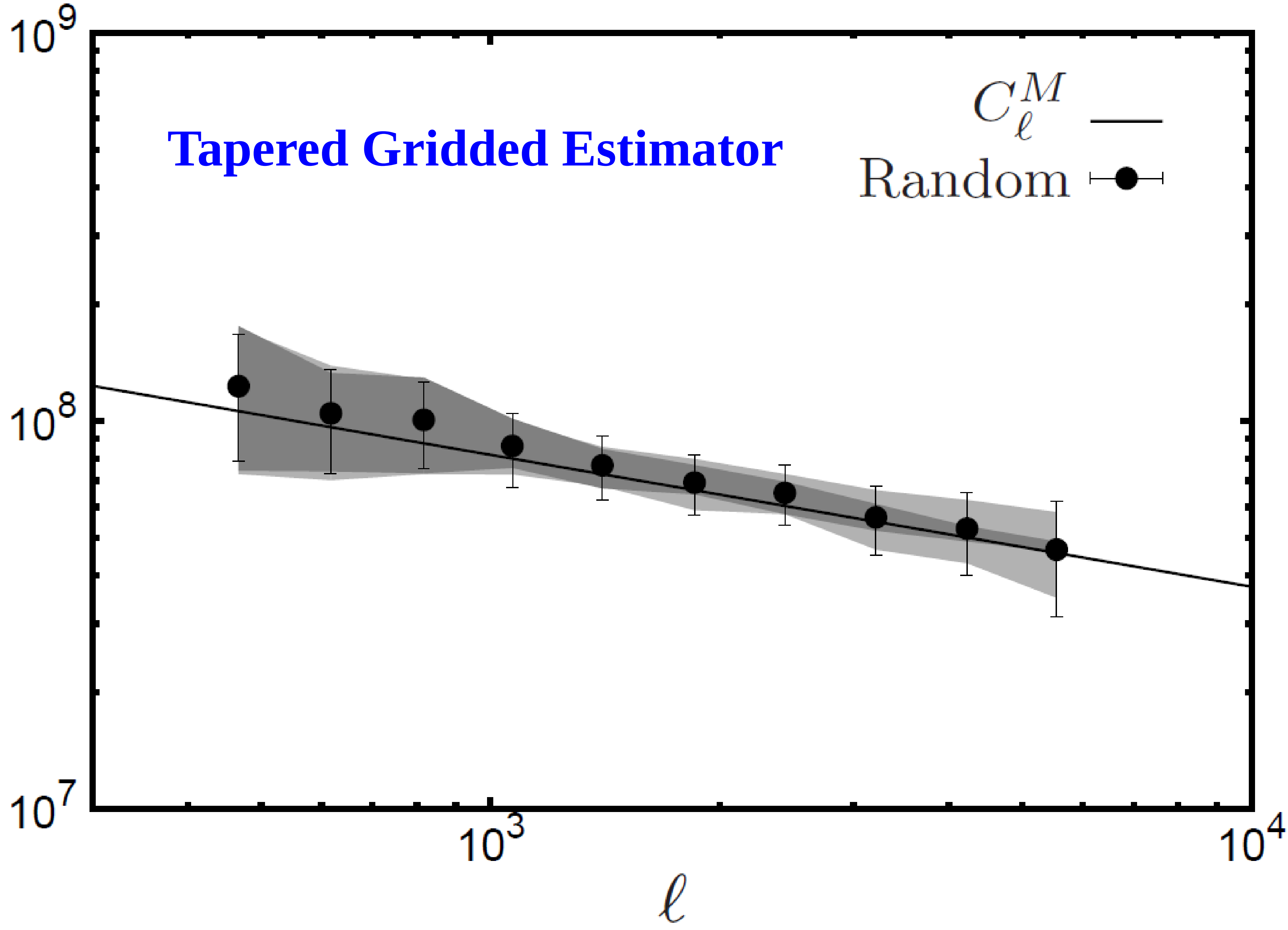


Tapered Gridded Estimator
C_ℓ^M
GMRT
10^9
10^8
10^7
10^3
10^4
$\ell(\ell+1)C_\ell/2\pi\ [mK^2]$
ℓ

Why Overestimate?

Random UV Distribution





Tapered Gridded Estimator
C_ℓ^M
Random
10^9
10^8
10^7
10^3
10^4
ℓ

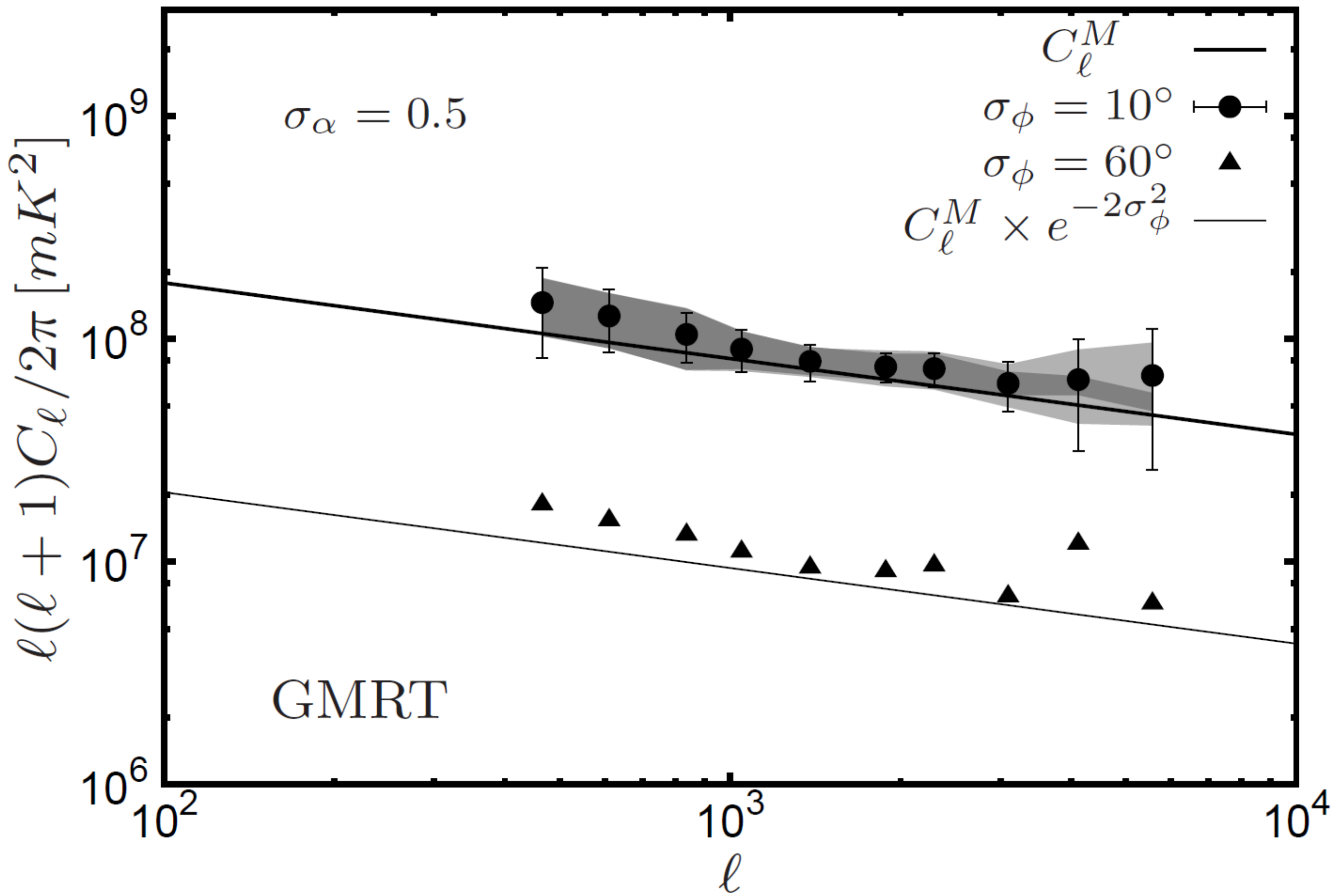
Instrumental Effect

(a) Gain Error
(b) W-term Effect

Gain Error

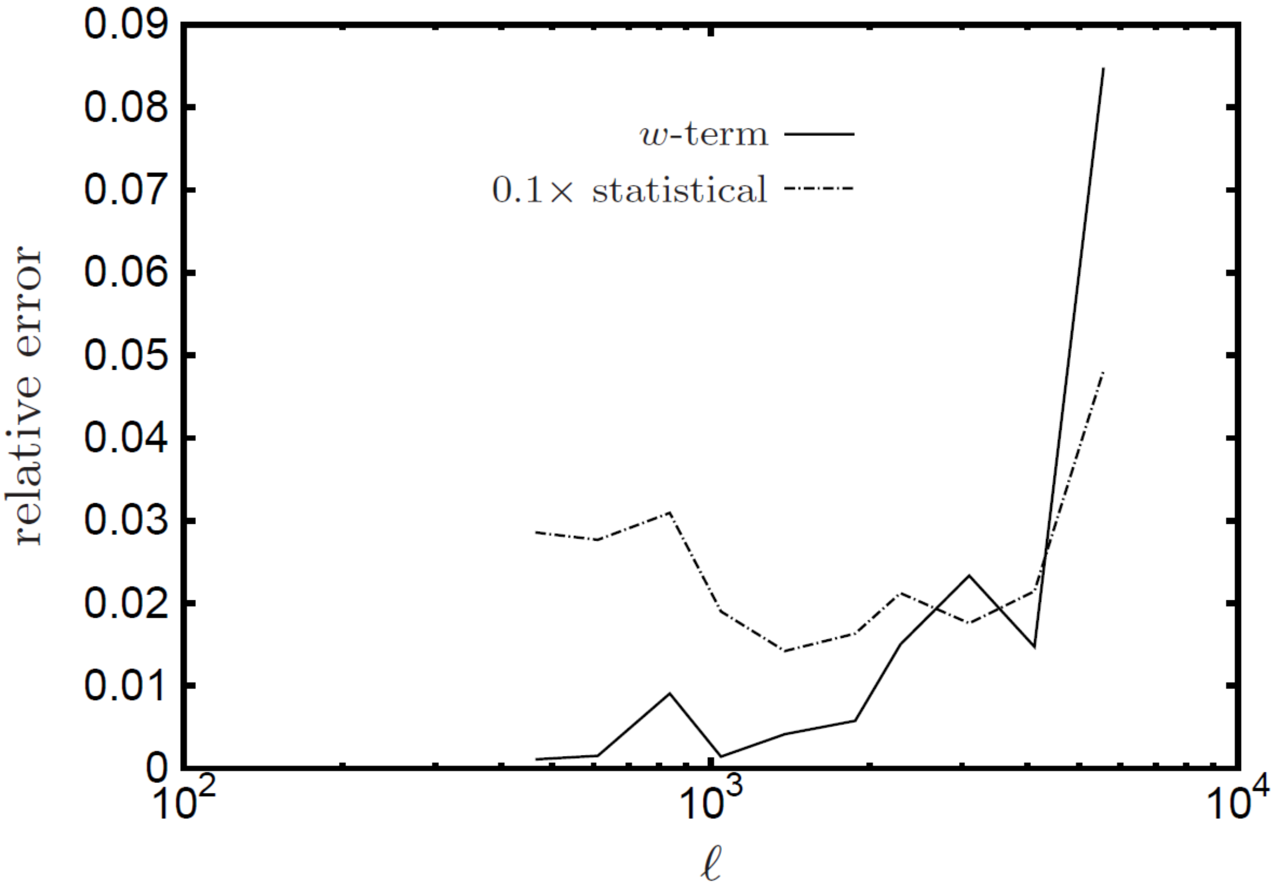
$$\mathcal{V}(\mathbf{U}_{ab}) = g_a g_b^* [\mathcal{S}(\mathbf{U}_{ab}) + \mathcal{N}(\mathbf{U}_{ab})]$$

$$g_a = (1 + \alpha_a) e^{i\phi_a}$$



The W-term Effect

$$\mathcal{S}(u,v,w) = \int dl dm \frac{\delta I(l,m)\mathcal{A}(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}$$



SUMMARY

1. We have introduced two estimators for quantifying the angular power spectrum of the sky brightness temperature. We find that the Bare Estimator is able to recover the input model to a good level of precision. For the GMRT estimated angular power spectrum from the Tapered Gridded Estimator is largely within the 1σ errors from the input model.

2. We studied the effect of gain error and find that expectation value of the estimators only depends on the phase error.

3. We find that the w-term does not cause a very big change in the estimated C_ℓ at the scales of our interest.

THANK YOU