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Towards Precision Cosmology:

The halo model and necessary modifications

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May 14, 2015

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Outline

- Introduction: 2-point statistics, Power spectrum.
- The halo model.
- Necessary modifications
    - Non-linear clustering of dark-matter,
    - Covariance matrix.

Introduction

- Cosmology is a statistical science as most of its observables are statistical in nature.
- The simplest statistic is the 2-point correlation function or the power spectrum of the matter density field.

$$\langle\tilde{\delta}(\mathbf{k})\tilde{\delta}(\mathbf{k}')\rangle = (2\pi)^3\delta^3_D(\mathbf{k}+\mathbf{k}')P(k)$$

- It underlies observables like galaxy clustering, weak lensing, Baryon Acoustic Oscillations (BAO), Lyman-alpha forest etc.
- Various estimators: perturbation theories, fitting formulae from simulations, **the halo model** etc.

The Halo Model

$$P(k) = P_{1h}(k) + P_{2h}(k),$$

$$P_{1h}(k) = \int d\nu f(\nu) \frac{M}{\bar{\rho}} |u(k|M)|^2,$$

$$P_{2h}(k) = \left[\int d\nu f(\nu) b(\nu) u(k|M) \right]^2 P_{\rm L}(k),$$

$$u(k|M) = \frac{4\pi}{M} \int_0^{R_{\rm vir}} {\rm d}r\, r^2\, \rho(r|M)\, \frac{\sin(kr)}{kr}$$

$$\int_0^{\infty} f(\nu) d\nu = 1 \qquad \int_0^{\infty} f(\nu) b(\nu) d\nu = 1$$

$$\nu(M, z) = \left(\frac{\delta_c}{\sigma(M, z)} \right)^2$$

- Ingredients:
  - Mass function: f,
  - Halo bias: b,
  - Average density profile of halos:
  - Linear Power Spectrum: $P_L(k)$

Ref: McClelland & Silk 1977; Seljak 2000; Ma & Fry 2000; Peacock & Smith 2000; Cooray & Sheth 2002

The modified Halo model

$$P(k,z) = P_{\rm zel}(k,z) + P_{\rm 1h}(k,z)$$

$$P_{\rm 1h}(k,z) = (A_0 - A_2 k^2 + A_4 k^4) F(k)$$

$$A_0 = 1529.87 \sigma_8^{3.9} \times (1 + [-0.22 n_{\rm eff} - 0.4]),$$

$$A_2 = 1299.75 \sigma_8^{3.0} \times (1 + [-1.58 n_{\rm eff} - 2.8]),$$

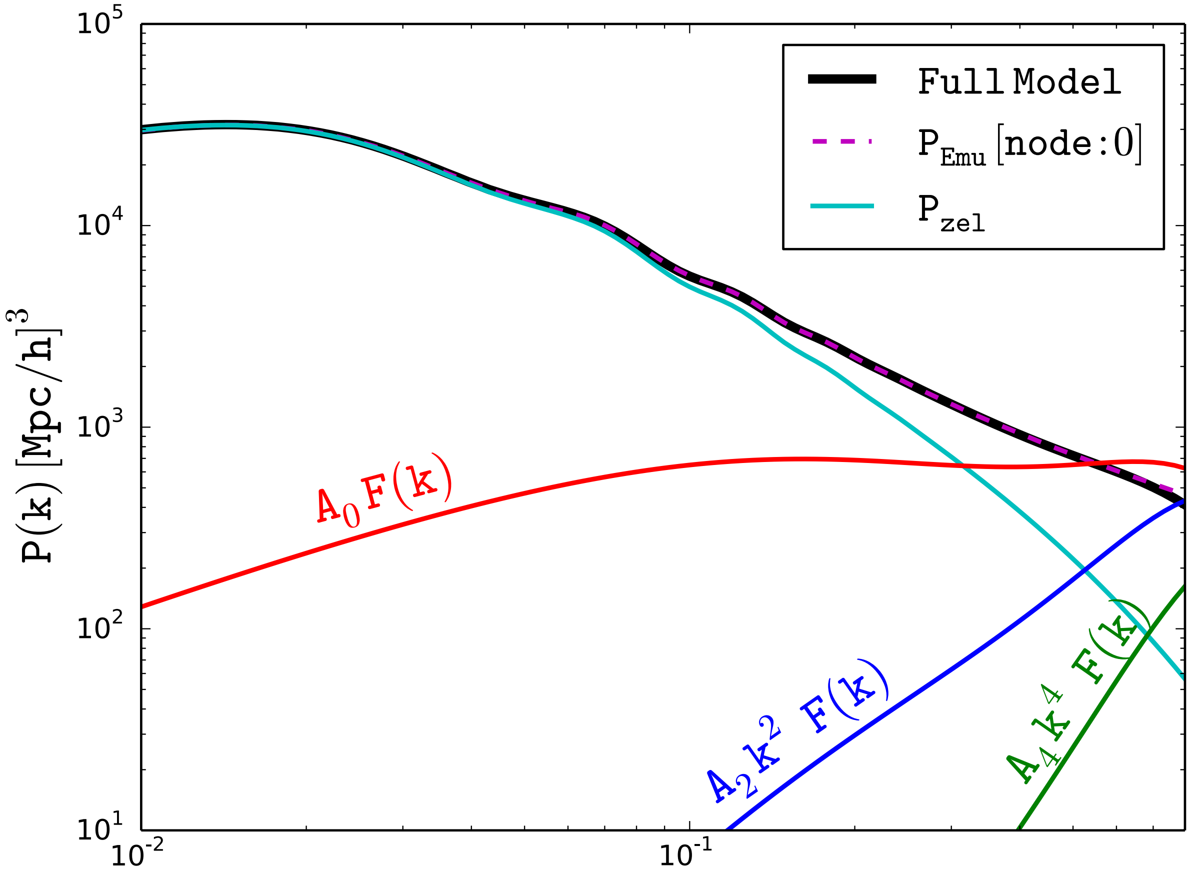
$$A_4 = 758.31 \sigma_8^{2.2} \times (1 + [-2.27 n_{\rm eff} - 4.2]),$$

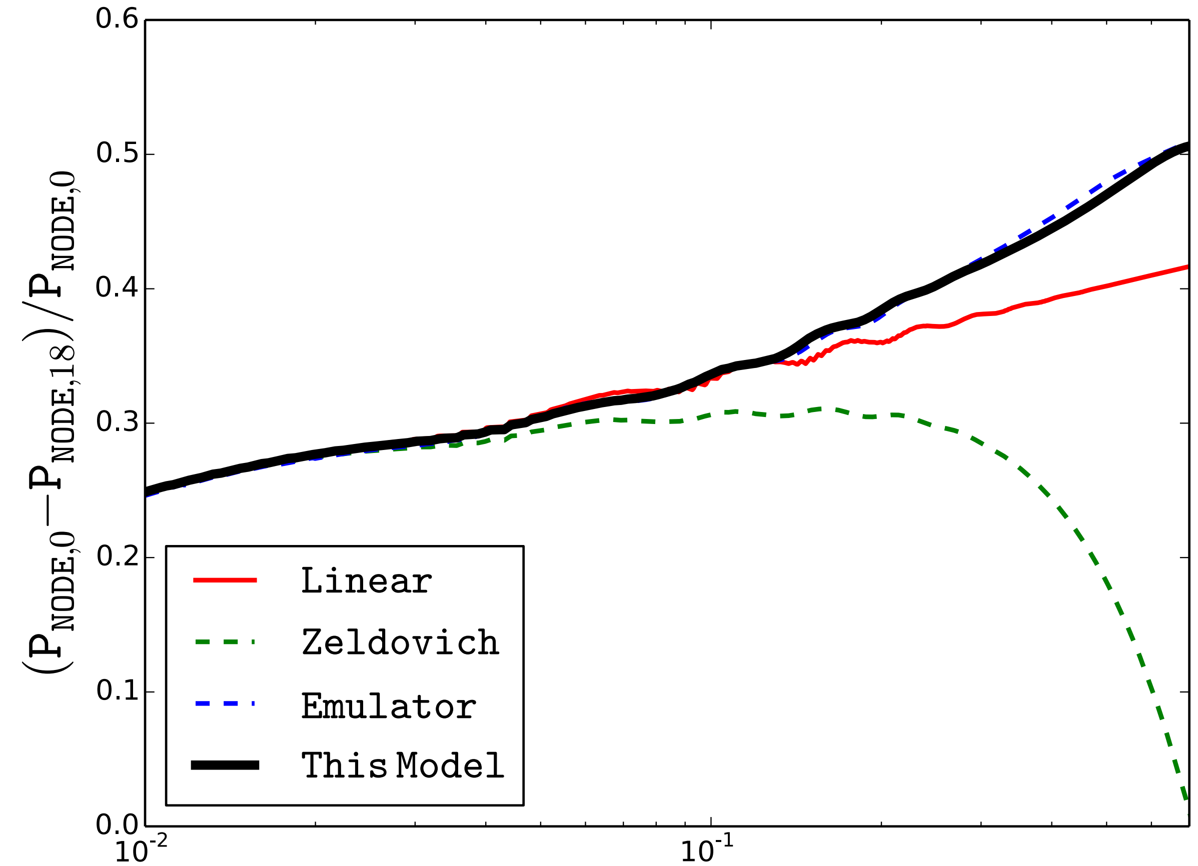
$$F(k) = \sum_{n=0}^{10} a_n k^n$$

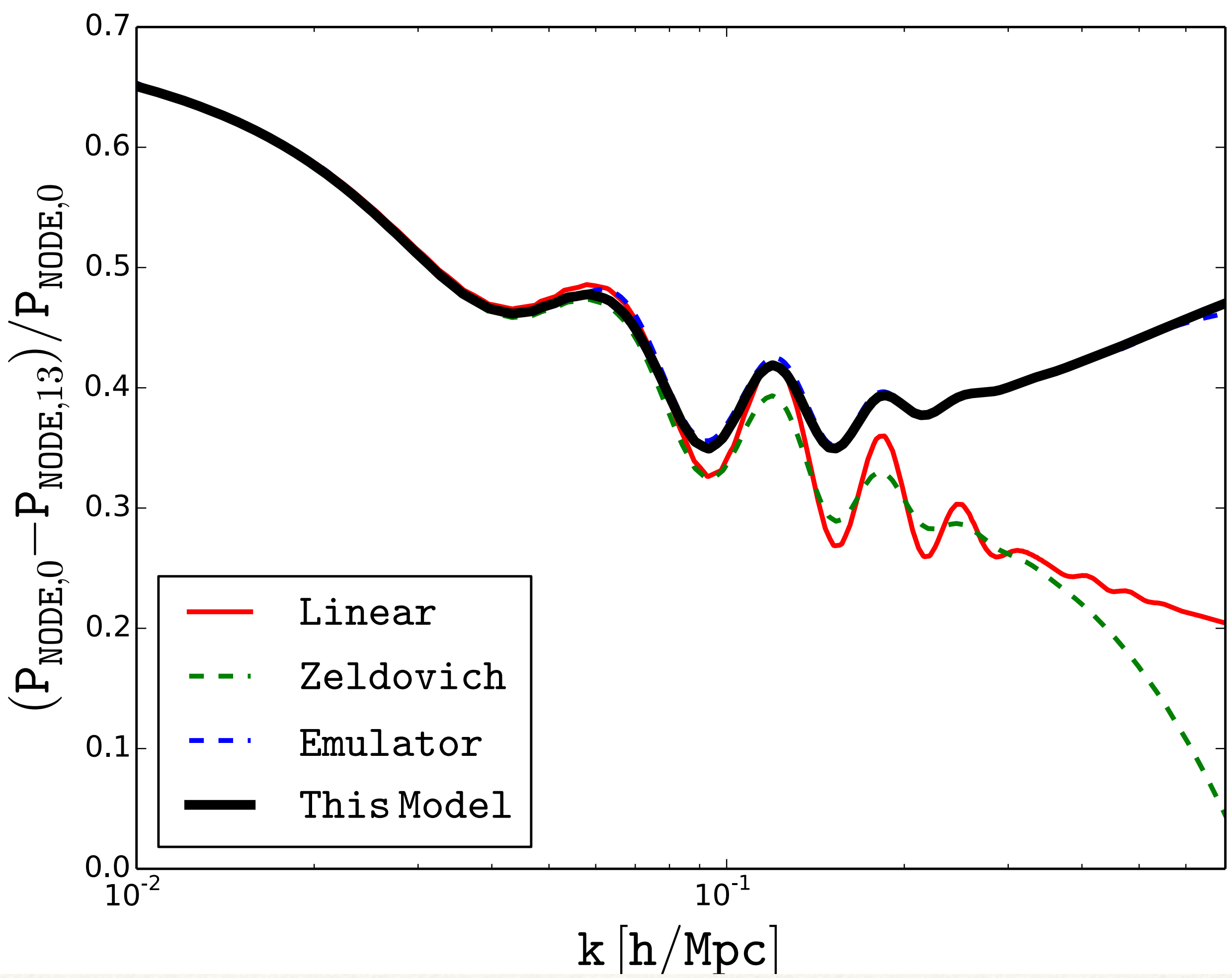
$$\sigma^2(M) = \frac{1}{2\pi^2} \int {\rm d}k\, k^2\, P_{\rm L}(k)\, |\bar{W}(kR)|^2$$

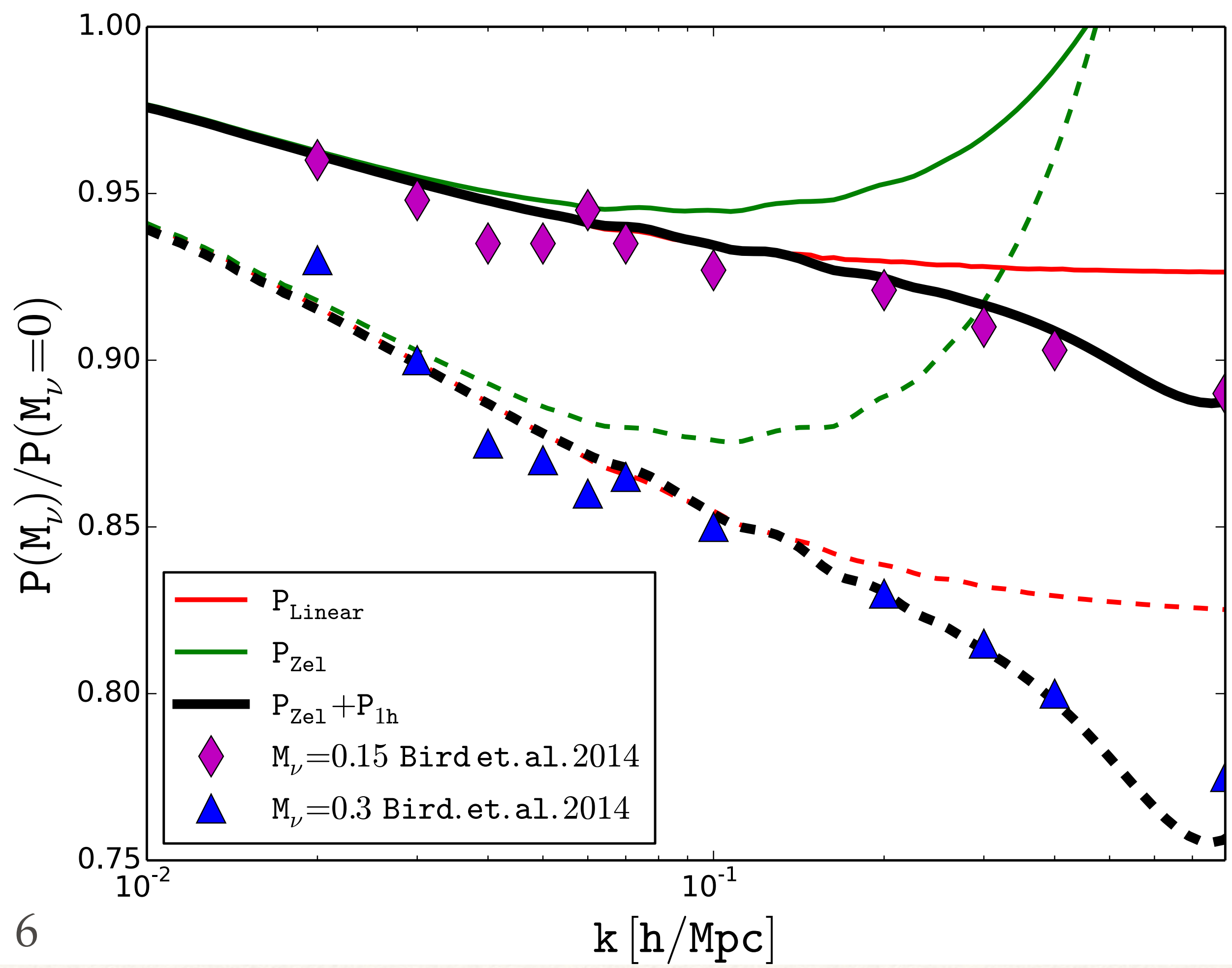
$n_{\rm eff}$ is the slope of $P_L(k)$ at BAO scale (~0.2 h/Mpc)

- Ingredient:
  - Linear Power Spectrum: $P_L(k)$
- Estimating coefficients:
  - Fit to simulations (Emulator),
  - the halo model.


Full Model
P_Emu [node:0]
P_zel
A_0 F(k)
A_2 k^2 F(k)
A_4 k^4 F(k)
P(k) [Mpc/h]^3
10^5
10^4
10^3
10^2
10^1
10^-2
10^-1


Linear
Zeldovich
Emulator
This Model
(P_NODE,0 − P_NODE,18)/P_NODE,0
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^-2
10^-1

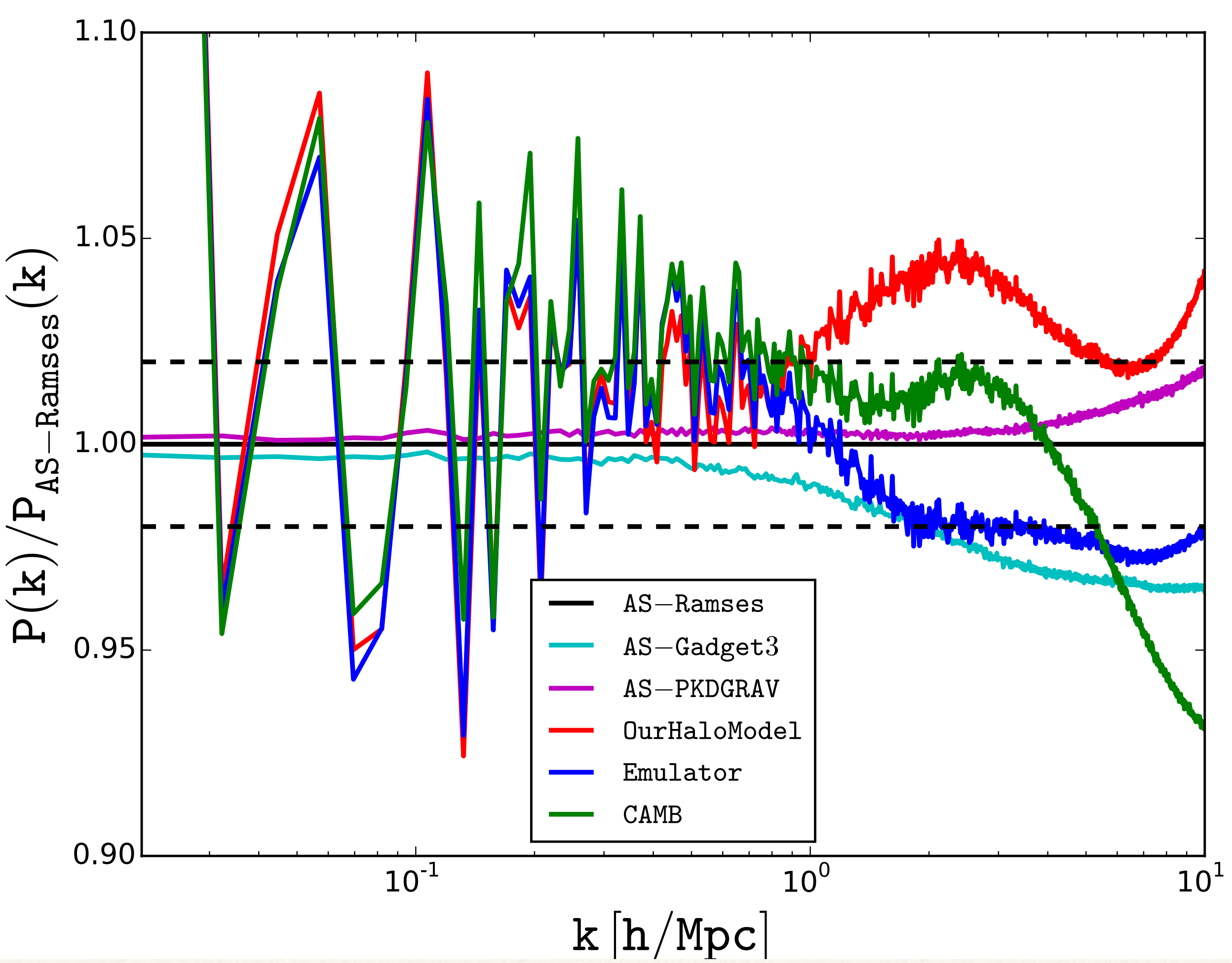

Linear
Zeldovich
Emulator
This Model
(P_NODE,0 − P_NODE,13)/P_NODE,0
k [h/Mpc]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^-2
10^-1

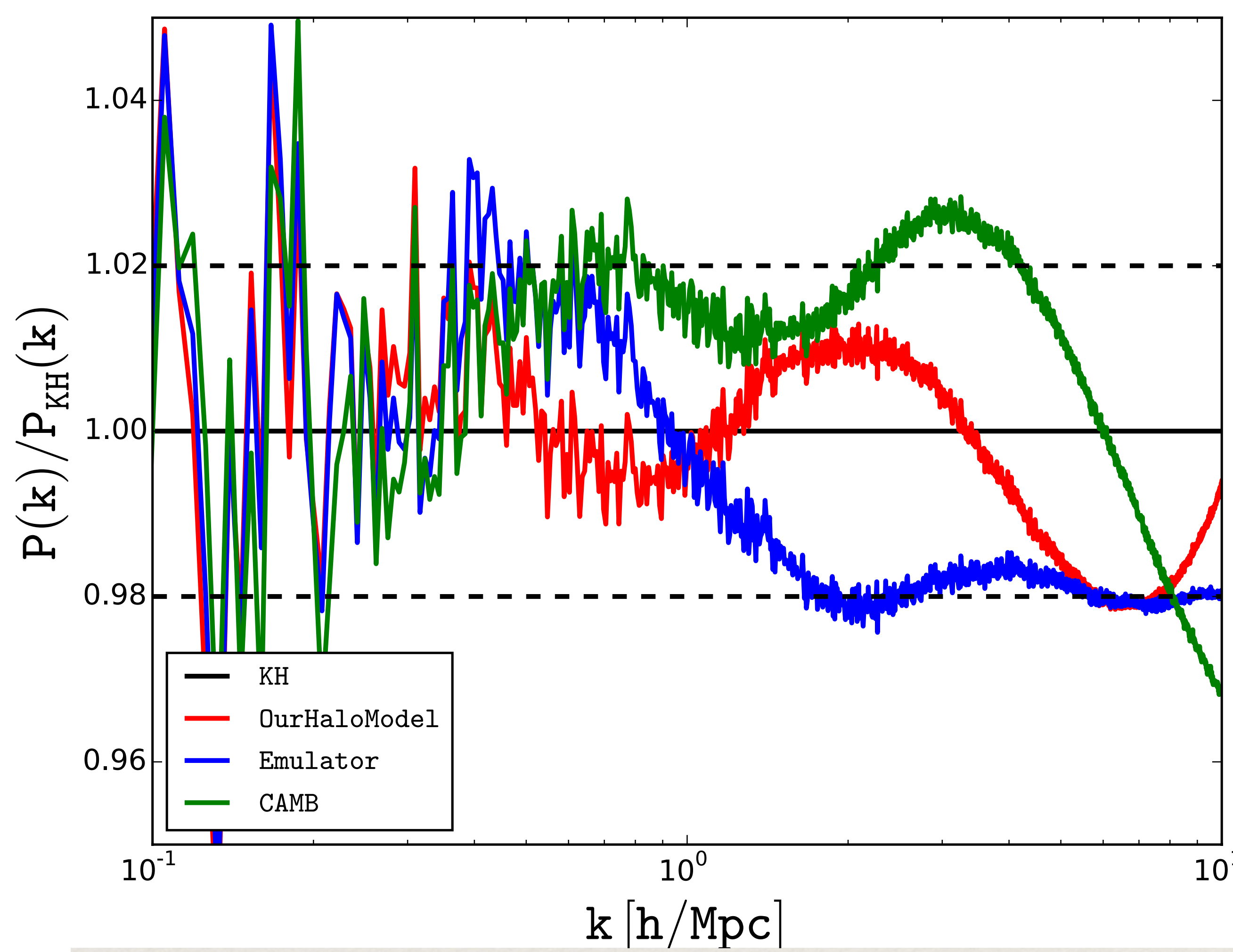

P_Linear
P_Zel
P_Zel + P_1h
M_ν=0.15 Bird et. al. 2014
M_ν=0.3 Bird. et. al. 2014
P(M_ν)/P(M_ν=0)
k [h/Mpc]
1.00
0.95
0.90
0.85
0.80
0.75
10^-2
10^-1

Improved model (in prep)…

$$P(k) = P_{Zel}(k) + P_{1h}(k)$$

$$P_{1h}(k) = A_0 \frac{1 + R_{2u}k^2}{1 + R_{2d}k^2 + R_{4d}k^4}$$





KH: Heitmann et al. 2014
AS: Schneider et al. 2015

Publicly available code (in prep.)

Covariance Matrix - a limit to the measurement

$$\mathrm{Cov}(P(k_i), P(k_j)) = \langle P(k_i)P(k_j) - \langle P(k_i)\rangle\langle P(k_j)\rangle,$$

$$\mathrm{Cov}(P(k_i), P(k_j)) = P(k_i)P(k_j)\left(\frac{2}{N_i}\delta_{ij} + \left(\frac{\sigma_{A_0}}{A_0}\right)^2\right)$$

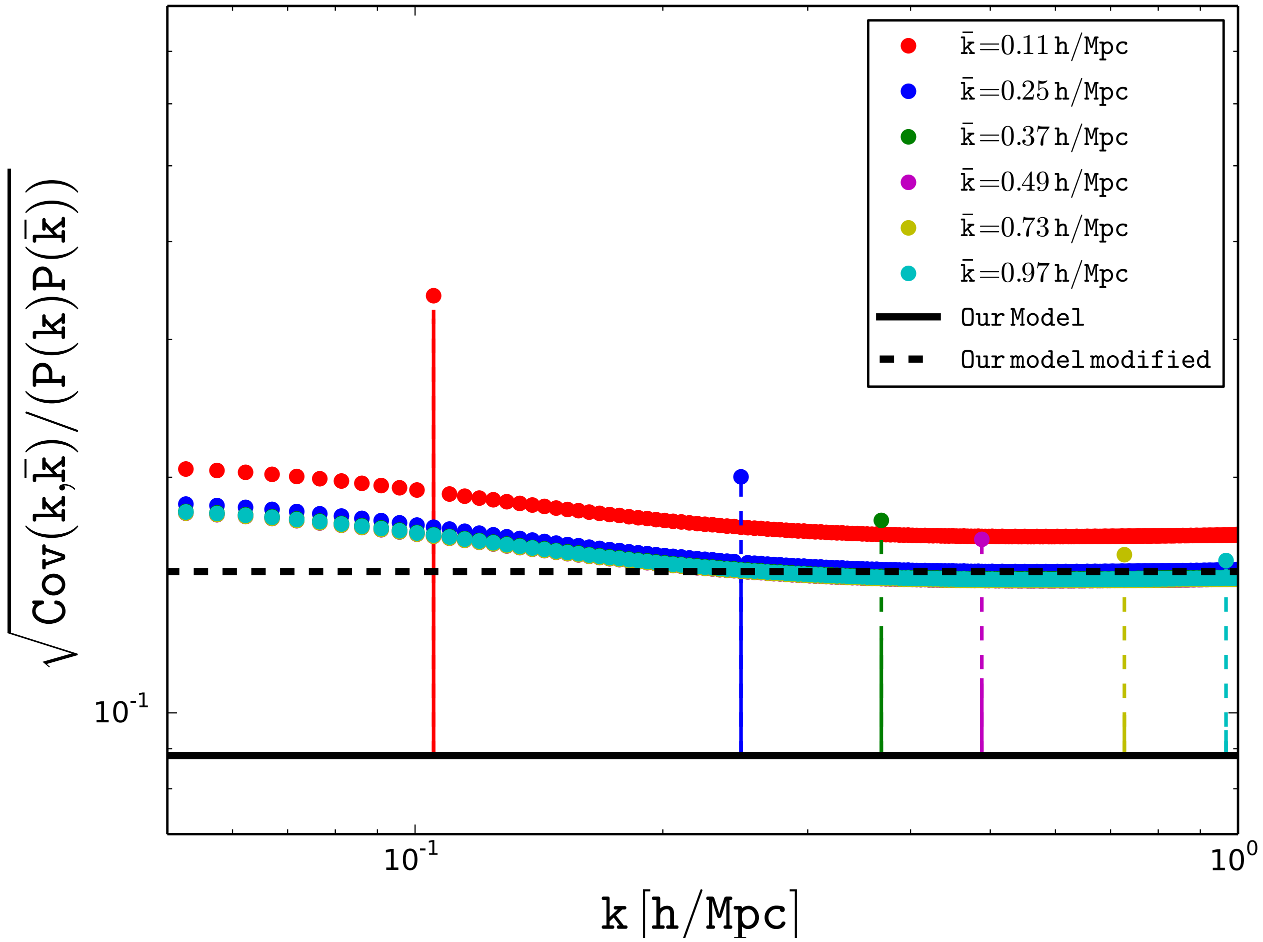
$$\left(\frac{\sigma_{A_0}}{A_0}\right)^2 = \frac{\int f(\nu)d\nu M^3}{[\int f(\nu)d\nu M]^2\bar{\rho}V},$$

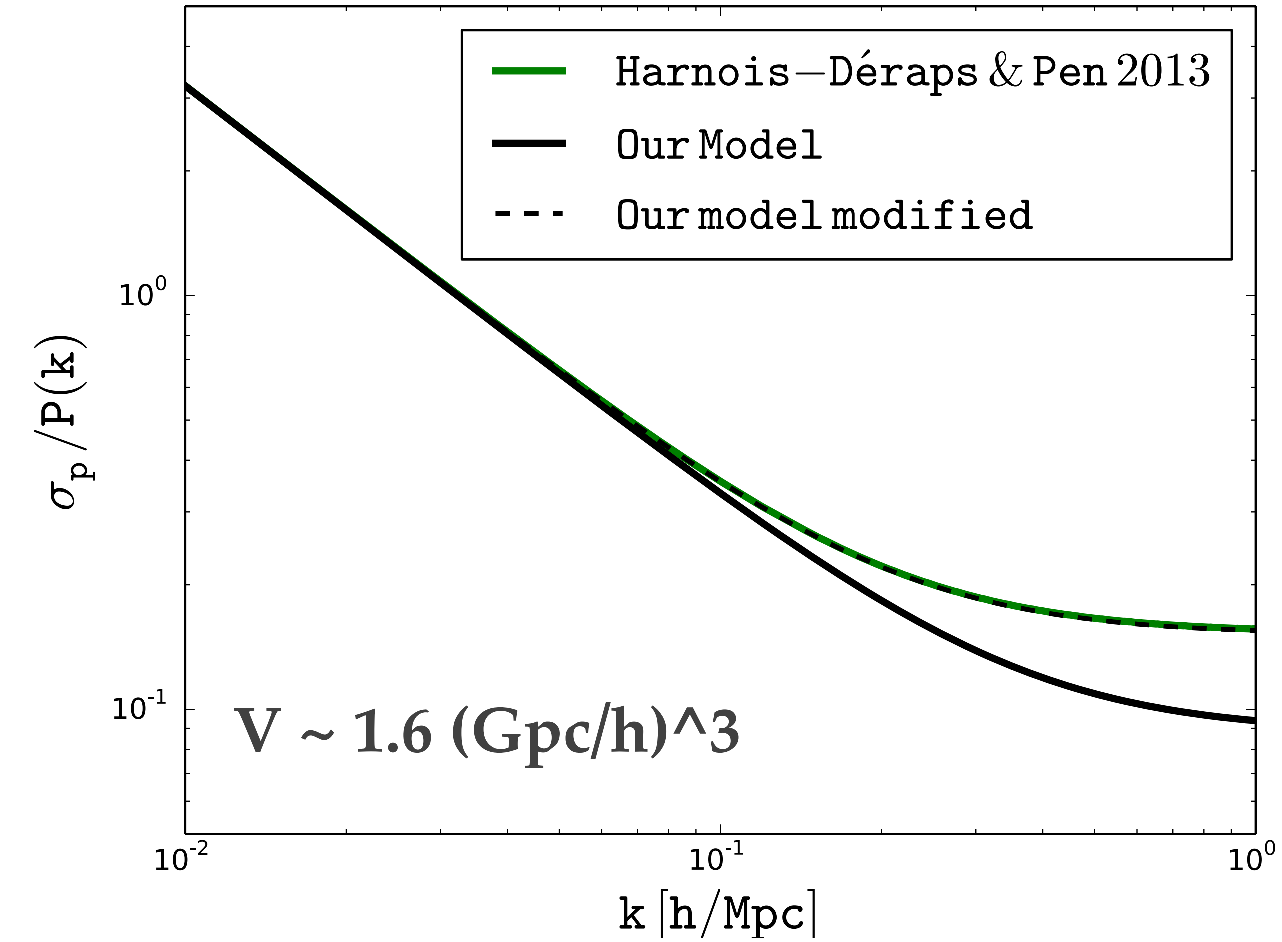
~0.01^2

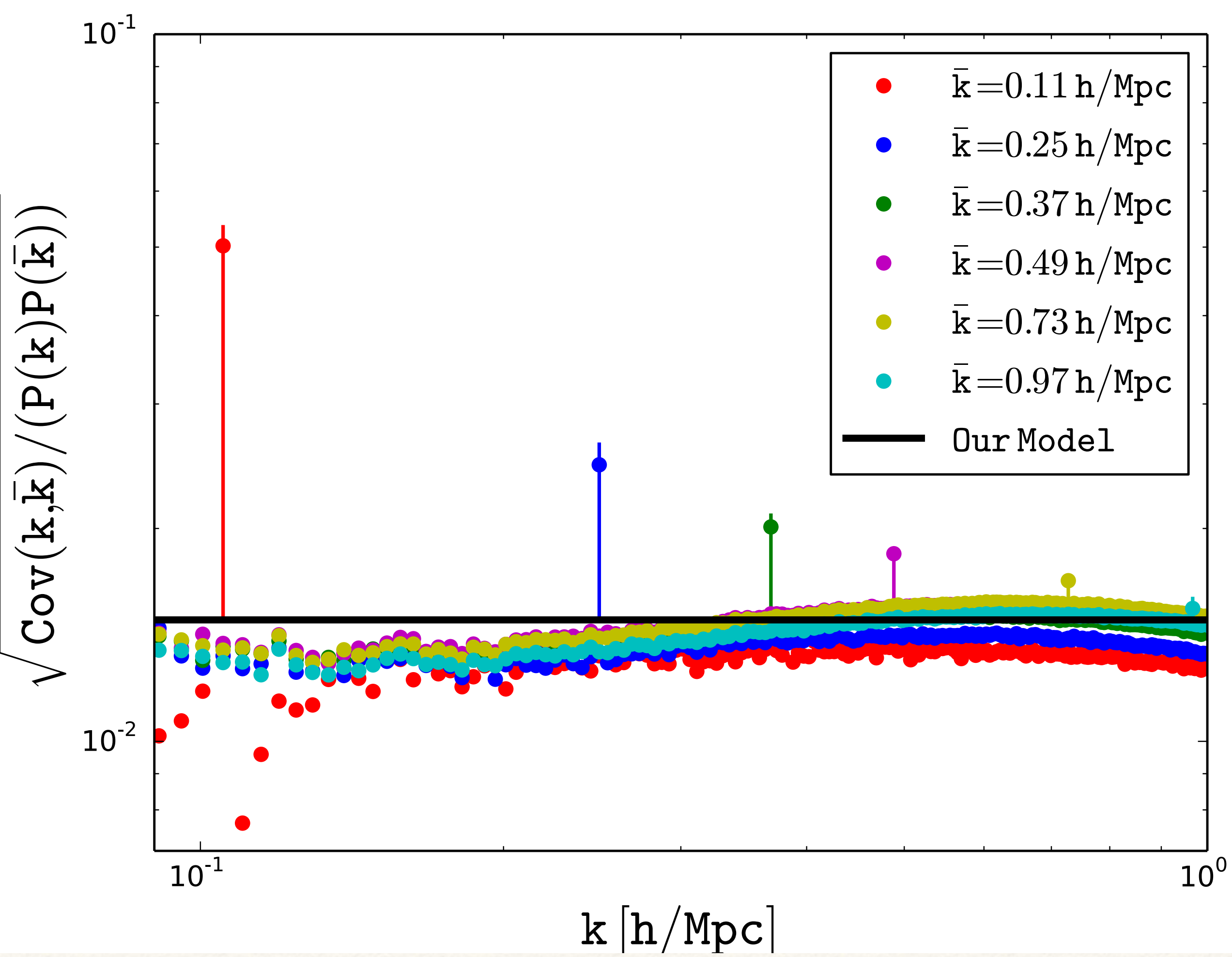
$$\frac{\sigma_{A_0}}{A_0} = \frac{\delta_{A_0}}{[(V/1h^{-1}\mathrm{Gpc})^3]^{1/2}}, \quad \delta_{A_0} = 0.0079(h^{-1}\mathrm{Gpc})^{3/2},$$

(Fitting with Li et al. 2014)

$$\mathrm{Cov}(P(k_i), P(k_j)) = P(k_i)P(k_j)V^{-1}\left(\frac{4\pi^2}{k_i^2\Delta k}\delta_{ij} + \delta_{A_0}^2\right)$$


$\bar{k}=0.11$ h/Mpc
$\bar{k}=0.25$ h/Mpc
$\bar{k}=0.37$ h/Mpc
$\bar{k}=0.49$ h/Mpc
$\bar{k}=0.73$ h/Mpc
$\bar{k}=0.97$ h/Mpc
Our Model
Our model modified
$\sqrt{\mathrm{Cov}(k,\bar{k})/(P(k)P(\bar{k}))}$
k [h/Mpc]


Harnois−Déraps & Pen 2013
Our Model
Our model modified
$\sigma_p/P(k)$
V ~ 1.6 (Gpc/h)^3
k [h/Mpc]


$\bar{k}=0.11$ h/Mpc
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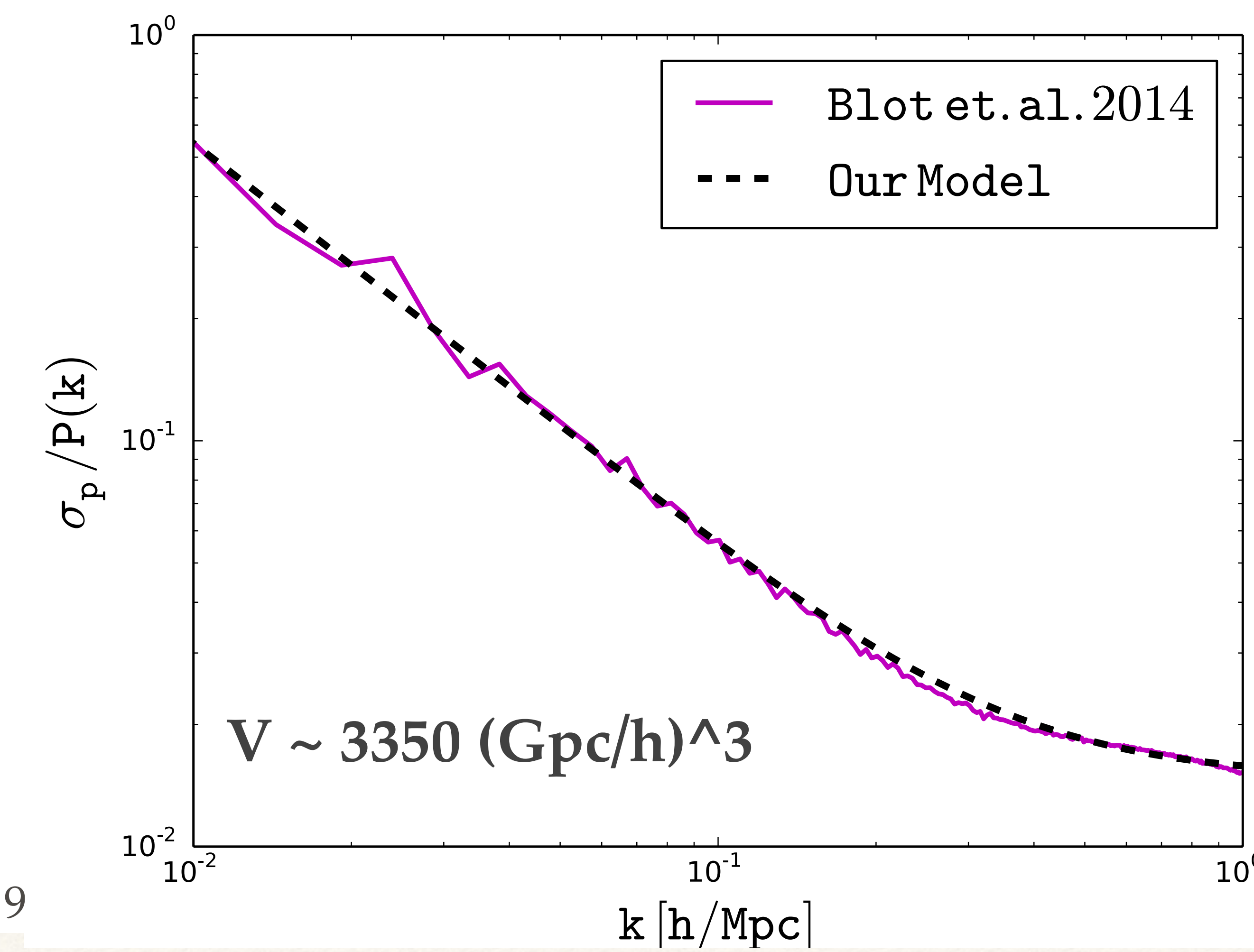
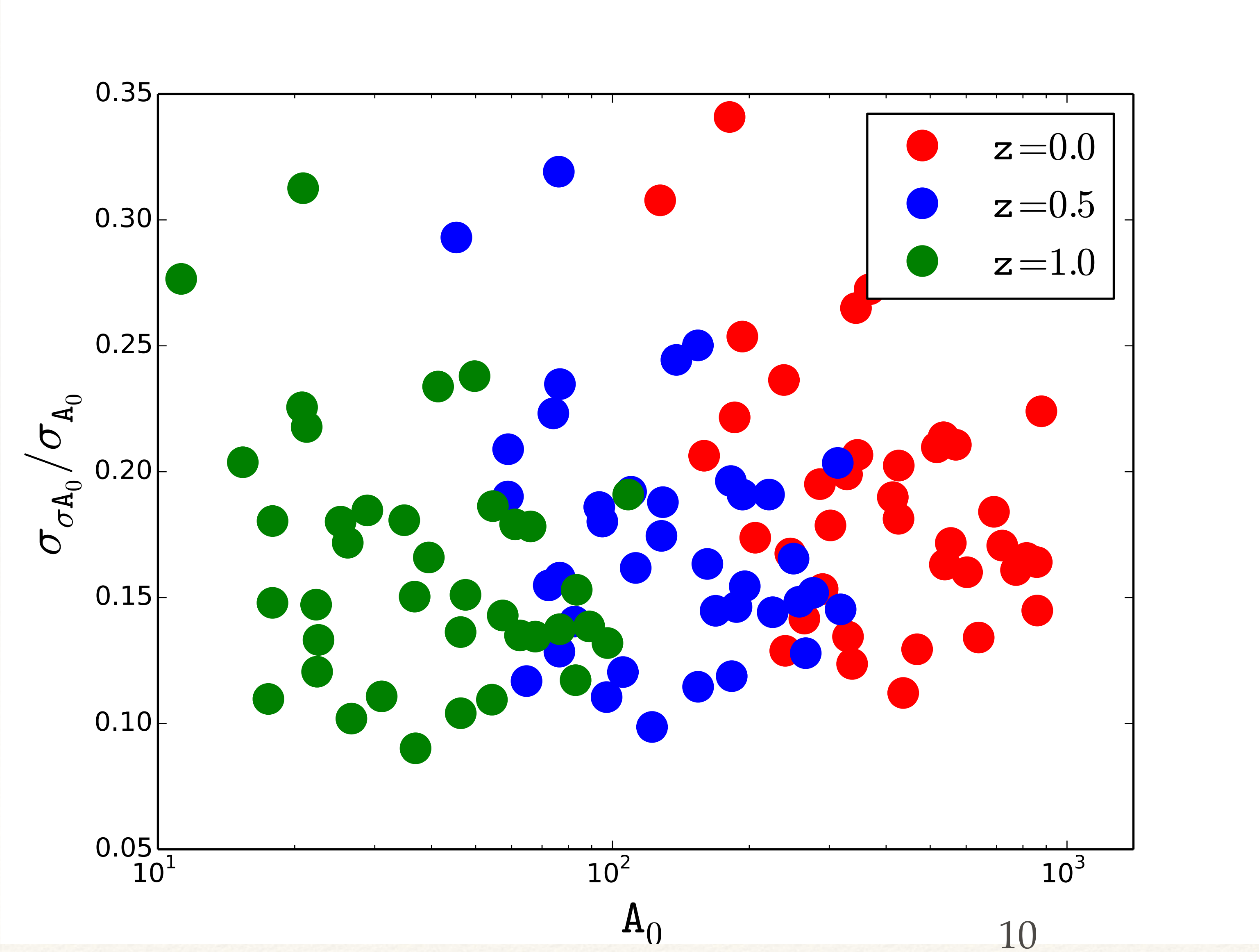
$\bar{k}=0.49$ h/Mpc
$\bar{k}=0.73$ h/Mpc
$\bar{k}=0.97$ h/Mpc
Our Model
$\sqrt{\mathrm{Cov}(k,\bar{k})/(P(k)P(\bar{k}))}$
k [h/Mpc]


Blot et.al. 2014
Our Model
$\sigma_p/P(k)$
V ~ 3350 (Gpc/h)^3
k [h/Mpc]

Variance of Covariance Matrix

$$\left(\frac{\sigma(\sigma_{A_0})}{\sigma_{A_0}}\right)^2 = \frac{\int f(\nu)d\nu M^7}{[(\int f(\nu)d\nu M^3]^2\bar{\rho}V},$$



Need a volume of ~ 500-5000 (Gpc/h)^3 to converge to 1%

Covariance matrix - improved model …

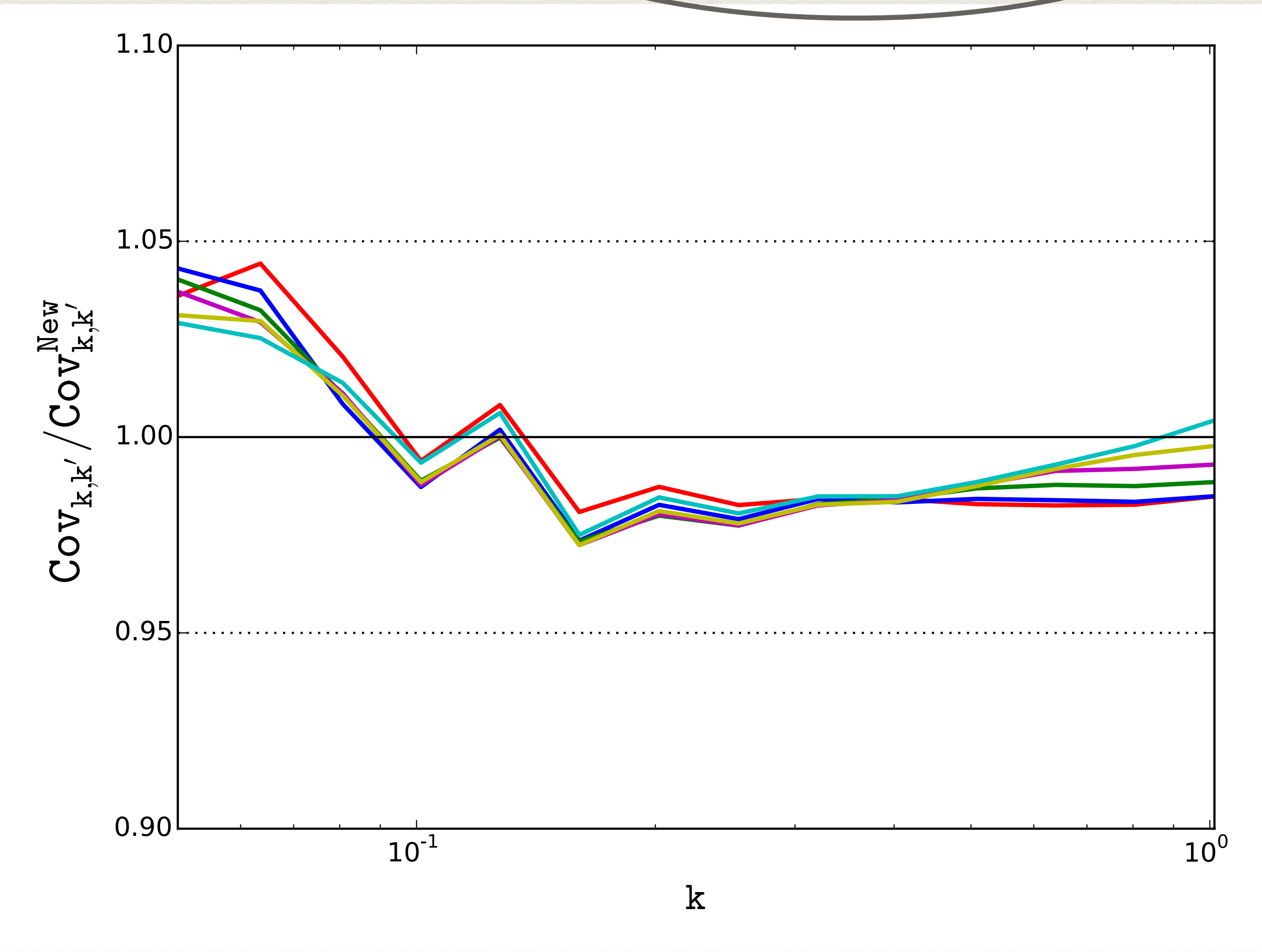
$$C_{ij} = C_{ij}^{\rm G} + C_{ij}^{\rm T0} + \sigma_b^2 \frac{\partial P(k_i)}{\partial \delta_b} \frac{\partial P(k_j)}{\partial \delta_b}$$

Li et al. 2014

Gaussian

Non-Gaussian

Super-sample variance



Conclusions

- The halo model can be modified in different ways to achieve accuracy and precision in modelling the clustering of matter in the Universe.
- In the first modification, we achieve percent level accuracy in P(k) up to k~0.7 h/Mpc.
- In an improved model (in prep.), we achieve ~3% accuracy in P(k) up to k~10 h/Mpc.
- Simple form of Covariance matrix, remarkable agreement with simulations.
- The non-Gaussian and super-sample contribution to the covariance matrix can be modelled together with a single vector.

Thank you for your attention…