Relativistic Dynamics in N-body Simulations

based on Phys. Rev. **D88** 103527 (arXiv:1308.6524),
Class. Quant. Grav. **31** 234006 (arXiv:1408.3352),
and work in progress with D. Daverio, R. Durrer, and M. Kunz

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Newtonian N-body simulations have a long tradition in LSS modelling

- conceptually simple
- “good enough” given the limited precision of past LSS surveys

Models beyond ΛCDM may have relativistic sources of stress-energy perturbations

- Newtonian limit not always a good approximation

Increasing data quality imposes new challenge to take into account relativistic effects (e.g. in modelling RSD, WL. . .)

- advantage of conceptual simplicity is lost

Let’s go relativistic!

Strategy

- choose ansatz for the metric (perturbed FLRW)

$$ds^2 = a^2(\tau)\Big[-(1+2\Psi)\,d\tau^2 + (1-2\Phi)\,\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j - 2B_i dx^i d\tau\Big]$$

- metric components are evolved with Einstein's equations

$$G^\mu_\nu = 8\pi G T^\mu_\nu$$

- stress-energy tensor is determined by solving the EOM's of all sources of stress-energy

$$T^{\mu\nu}_{\rm m} = \sum_n m_{(n)} \frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_{(n)})}{\sqrt{-g}} \left(-g_{\alpha\beta}\frac{dx^\alpha_{(n)}}{d\tau}\frac{dx^\beta_{(n)}}{d\tau}\right)^{-\frac{1}{2}} \frac{dx^\mu_{(n)}}{d\tau}\frac{dx^\nu_{(n)}}{d\tau}$$

- instead of peculiar velocities, use relativistic momentum

$$\mathbf{v}-\mathbf{B}=\frac{(1+\Psi+\Phi)\mathbf{p}-\frac{1}{2}\mathbb{h}\cdot\mathbf{p}}{\sqrt{m^2+\mathbf{p}^2}}$$

gives "simple" equations valid for arbitrary $\mathbf{p}$:

- stress-energy tensor

$$T^0_0=-\frac{1+3\Phi}{a^3}\sum_n\delta^{(3)}(\mathbf{x}-\mathbf{x}_{(n)})\left[\sqrt{m^2_{(n)}+\mathbf{p}^2_{(n)}}+\mathbf{B}\cdot\mathbf{p}_{(n)}\right]$$

$$\Pi_{ij}\doteq\delta_{k(i}T^k_{j)}-\frac{1}{3}\delta_{ij}T^k_k=\left(\delta_{k(i}\delta_{j)l}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)$$
$$\times\frac{1+3\Phi}{a^3}\sum_n\delta^{(3)}(\mathbf{x}-\mathbf{x}_{(n)})\left[\frac{p^k_{(n)}p^l_{(n)}}{\sqrt{m^2_{(n)}+\mathbf{p}^2_{(n)}}}+\delta^{km}B_m p^l_{(n)}\right]$$

- geodesic equation

$$\mathbf{p}'=-\left(\mathcal{H}-\Phi'\right)\mathbf{p}-\sqrt{m^2+\mathbf{p}^2}\nabla\Psi-\nabla\left(\mathbf{B}\cdot\mathbf{p}\right)$$
$$-\frac{1}{2}\mathbb{h}'\cdot\mathbf{p}+\frac{\mathbf{p}(\mathbf{p}\cdot\nabla\Phi)-\mathbf{p}^2\nabla\Phi}{\sqrt{m^2+\mathbf{p}^2}}+\frac{1}{2}\frac{\nabla(\mathbf{p}\cdot\mathbb{h}\cdot\mathbf{p})-\mathbf{p}\cdot\nabla(\mathbb{h}\cdot\mathbf{p})}{\sqrt{m^2+\mathbf{p}^2}}$$

Einstein’s equations at leading order

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = -4\pi G a^2 \delta T^0_0$$

$$\left(\frac{\partial^2}{\partial x^i \partial x^j} - \frac{1}{3}\delta_{ij}\Delta\right)(\Phi - \Psi) = 8\pi G a^2 \Pi^{(S)}_{ij}$$

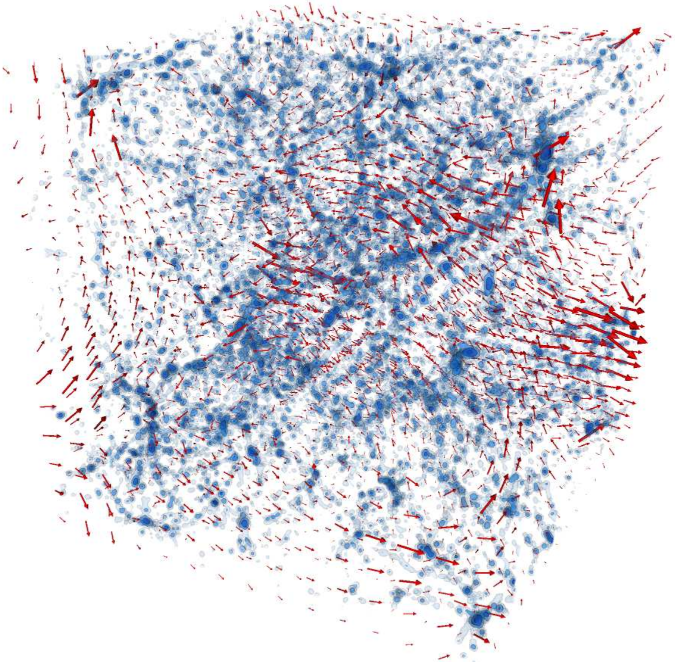
$$B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} = 8\pi G a^2 \Pi^{(V)}_{ij}$$

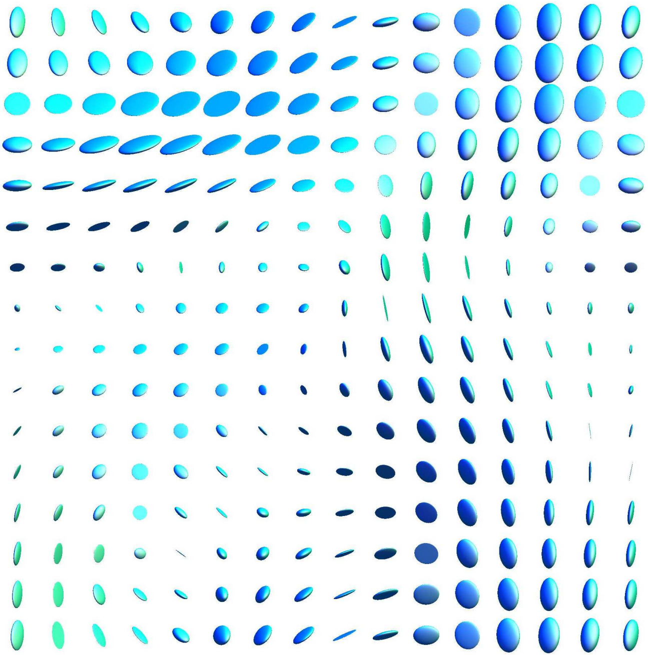
$$\frac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \frac{1}{2}\Delta h_{ij} = 8\pi G a^2 \Pi^{(T)}_{ij}$$

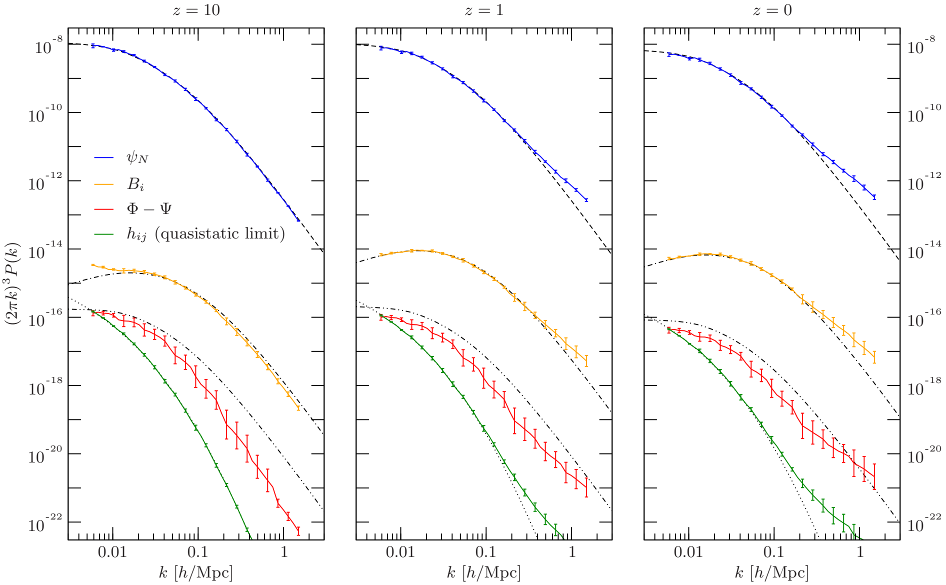
Including “shortwave corrections” . . .

$$\left(1+4\Phi\right)\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi + \frac{3}{2}\left(\nabla\Phi\right)^2 = -4\pi G a^2 \delta T^0_0$$

$$\begin{gathered}
\left(\frac{\partial^2}{\partial x^i \partial x^j} - \frac{1}{3}\delta_{ij}\Delta\right)\left[\left(\Phi-\Psi\right)\left(1+\Phi-\Psi\right)+\Phi^2\right]+ \\
B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} + \\
\frac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \frac{1}{2}\Delta h_{ij} + \\
2\Psi\Phi_{,ij} - \frac{2}{3}\delta_{ij}\Psi\Delta\Phi - \left(\Phi-\Psi\right)_{,i}\left(\Phi-\Psi\right)_{,j} + \\
\frac{1}{3}\delta_{ij}\delta^{kl}\left(\Phi-\Psi\right)_{,k}\left(\Phi-\Psi\right)_{,l} = 8\pi G a^2 \Pi_{ij}
\end{gathered}$$







z = 10
z = 1
z = 0
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
$(2\pi k)^3 P(k)$
ψ_N
B_i
$\Phi - \Psi$
h_{ij} (quasistatic limit)
0.01
0.1
1
k [h/Mpc]

Summary

- N-body simulations within a GR framework are feasible
- unified relativistic treatment is a clear, logical and transparent way to address the most general observables with minimal restrictions on the cosmological model
- technology should be useful for simulations with relativistic sources (dynamical DE, neutrinos, . . .)