Bridging the gap between small and large scales: a nonlinear post-Friedmann framework for relativistic structure formation

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Outline

- standard ΛCDM cosmology and a basic question
- non-linear Post-Friedmann ΛCDM: a new weak-field/post-Newtonian type approximation scheme for cosmology
- cosmological frame dragging from Newtonian N-body simulations
- other work and possible directions

Credits

- Irene Milillo, Daniele Bertacca, MB and Andrea Maselli (2015), *The missing link: a nonlinear post-Friedmann framework for small and large scales* [arXiv:1502.02985], Physical Review D, *in press*

- MB, Dan B. Thomas and David Wands (2014), *Computing General Relativistic effects from Newtonian N-body simulations: Frame dragging in the post-Friedmann approach,* Physical Review D, 89, 044010 [arXiv:1306.1562]

- Dan B. Thomas, MB and David Wands (2015), *The fully non-linear post-Friedmann frame-dragging vector potential: Magnitude and time evolution from N-body simulations* [arXiv:1501.00799]

- Dan B. Thomas, MB and David Wands (2014), *Relativistic weak lensing from a fully non-linear cosmological density field,* [arXiv:1403.4947]

- Dan B. Thomas, MB, Kazuya Koyama, Baojiu Li and Gong-bo Zhao (2015) *f(R) gravity on non-linear scales: The post-Friedmann expansion and the vector potential* [arXiv:1503.07204]

Standard ΛCDM Cosmology

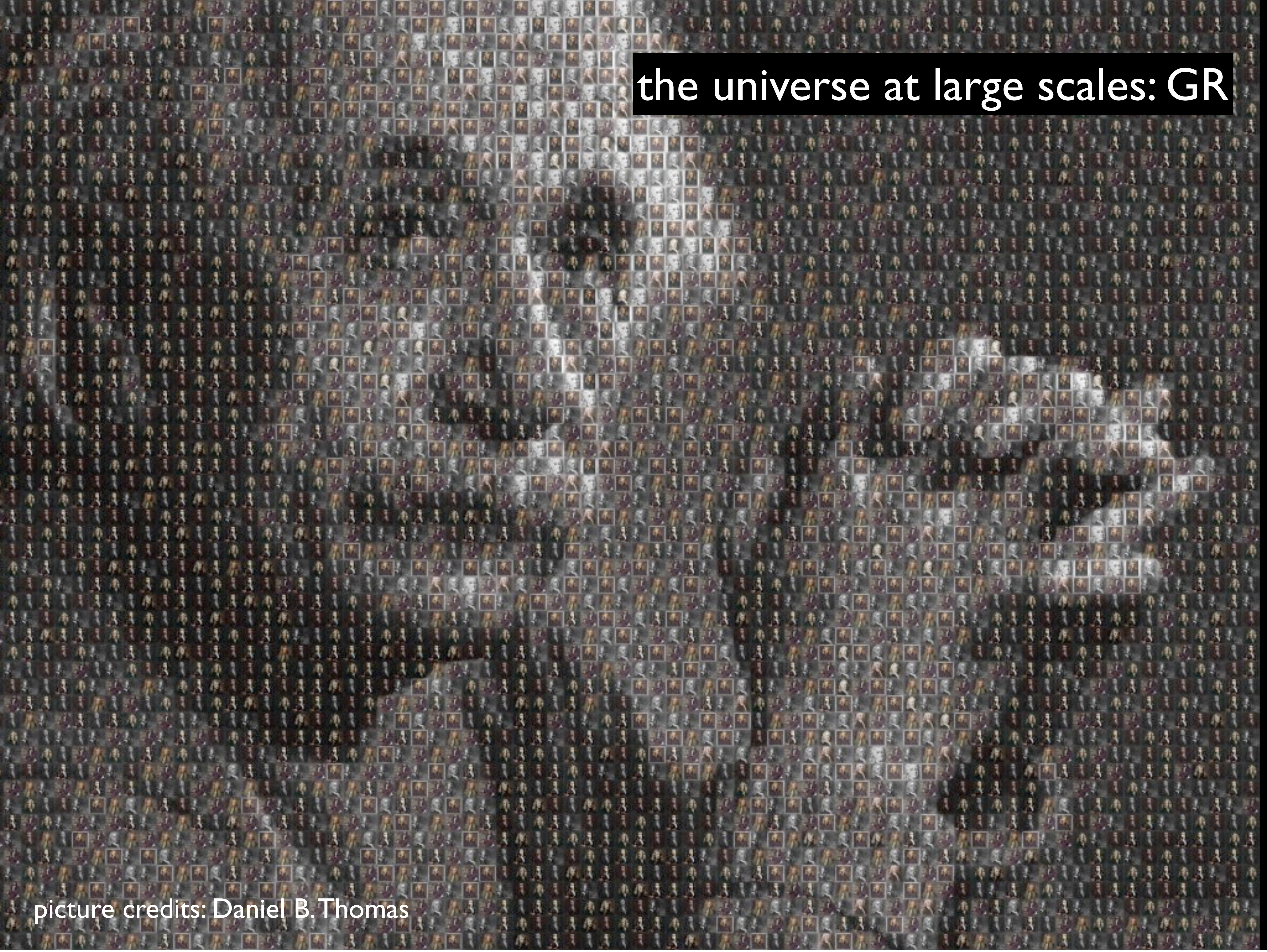
- Recipe for modeling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations (e.g. CMB), good for large scales I-order, II order, gradient expansion
  3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.

Questions on ΛCDM

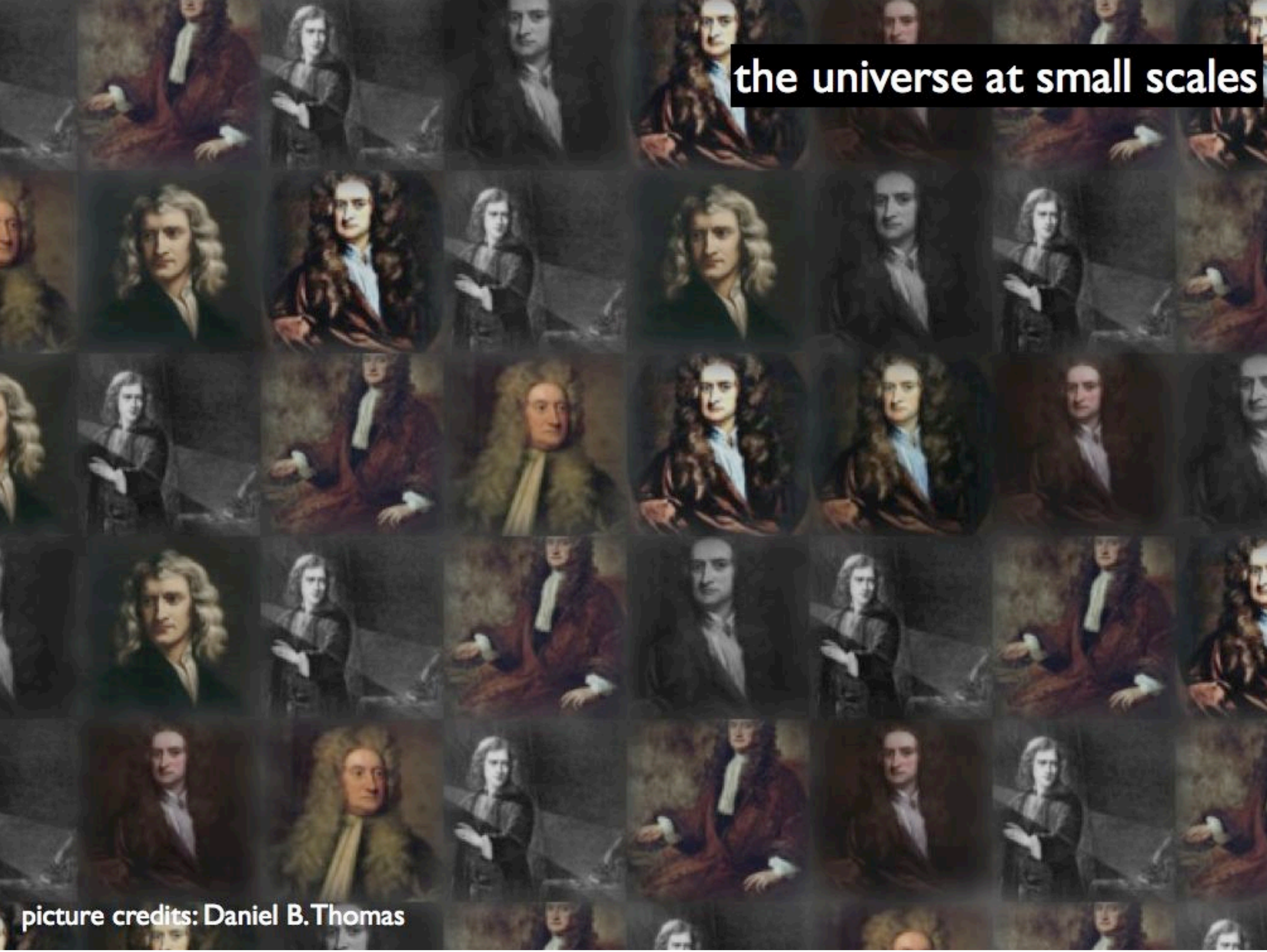
- Recipe for modelling based on 3 main ingredients:

1. Homogeneous isotropic background, FRW models
2. Relativistic Perturbations (e.g. CMB)
3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)

- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - We need to bridge the gap between 2 and 3


the universe at large scales: GR
picture credits: Daniel B. Thomas

the universe at small scales



picture credits: Daniel B. Thomas

Questions/Motivations

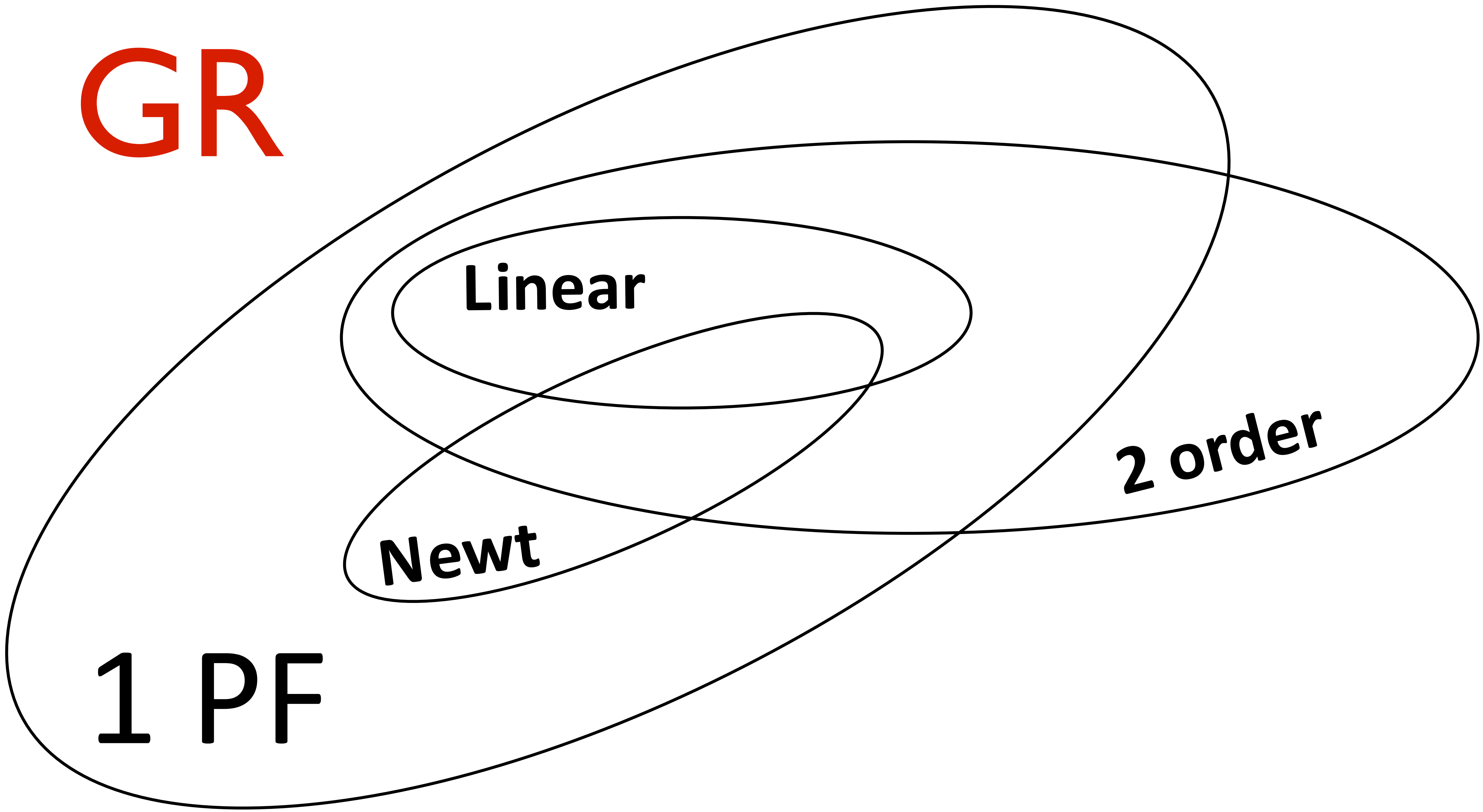
- Is the Newtonian approximation good enough to study non-linear structure formation?
  - surveys and simulations covering large fraction of H^{-1}
  - we are going to have more data: precision cosmology
  - we also need accurate cosmology: not only we want accurate observations, we also need accurate theoretical predictions (e.g.: Euclid target: N-body simulations wih 1% accuracy)
  - what if relativistic corrections are ~ few%?
    - We need to bridge the gap between small scale non-linear Newtonian approximation and large scale relativistic perturbation theory
    - We need a relativistic framework ("dictionary") to interprete N-body simulations [e.g. Chisari & Zaldarriaga (2011), Green & Wald (2012)]
    - We need to go beyond the standard perturbative approach, considering non-linear density inhomogeneities within a relativistic framework

non-linear post-Friedmann framework

- assume GR and a flat ΛCDM background
- perturbation theory is only valid for small δ
- current state:
  - non-linear relativistic approximate framework, incorporating fully non-linear Newtonian theory at small scales and standard relativistic perturbations at large scales ($\sim H^{-1}$ and beyond)
  - extract leading order relativistic corrections from standard Newtonian simulations
- future goals:
  - incorporate GR corrections in simulations
  - ▶ more accurate ΛCDM cosmology

post-Friedmann framework

- spaces of equations (not solutions!)



Post-Newtonian cosmology

- post-Newtonian: expansion in 1/c powers (more later)
- various attempts and studies:
  - Tomita Prog. Theor. Phys. 79 (1988) and 85 (1991)
  - Matarrese & Terranova, MN 283 (1996)
  - Takada & Futamase, MN 306 (1999)
  - Carbone & Matarrese, PRD 71 (2005)
  - Hwang, Noh & Puetzfeld, JCAP 03 (2008)
- even in perturbation theory it is important to distinguish post-Newtonian effects, e.g. in non-Gaussianity and initial conditions. MB, J. C. Hidalgo, N. Meures, D. Wands, ApJ 785:2 (2014) [arXiv:1307:1478], cf. Bartolo et al. CQG 27 (2010) [arXiv: 1002.3759]

metric and matter I

starting point: the 1-PN cosmological metric
(cf. Chandrasekhar 1965)

$$g_{00} = -\left[1 - \frac{2U_N}{c^2} + \frac{1}{c^4}(2U_N^2 - 4U_P)\right] + O\left(\frac{1}{c^6}\right),$$

$$g_{0i} = -\frac{a}{c^3}B_i^N - \frac{a}{c^5}B_i^P + O\left(\frac{1}{c^7}\right),$$

$$g_{ij} = a^2\left[\left(1 + \frac{2V_N}{c^2} + \frac{1}{c^4}(2V_N^2 + 4V_P)\right)\delta_{ij} + \frac{1}{c^4}h_{ij}\right] + O\left(\frac{1}{c^6}\right),$$

we assume a Newtonian-Poisson gauge: B_i is solenoidal and h_{ij} is TT, at each order 2 scalar DoF in g_{00} and g_{ij}, 2 vector DoF in frame dragging potential B_i and 2 TT DoF in h_{ij} (not GW!)

metric and matter II

having in mind Newtonian cosmology it is natural to define the peculiar velocity v^i such that

$$u^i = \frac{dx^i}{cd\tau} = \frac{dx^i}{cdt}\frac{dt}{d\tau} = \frac{v^i}{ca}u^0$$

$$u^0 = 1 + \frac{1}{c^2}\left(U_N + \frac{1}{2}v^2\right) + \frac{1}{c^4}\left[\frac{1}{2}U_N^2 + 2U_P + v^2V_N + \frac{3}{2}v^2U_N + \frac{3}{8}v^4 - B_i^N v^i\right]$$

$$u_i = \frac{av_i}{c} + \frac{a}{c^3}\left[-B_i^N + v_iU_N + 2v_iV_N + \frac{1}{2}v_iv^2\right],$$

$$u_0 = -1 + \frac{1}{c^2}\left(U_N - \frac{1}{2}v^2\right) + \frac{1}{c^4}\left[2U_P - \frac{1}{2}U_N^2 - \frac{1}{2}v^2U_N - v^2V_N - \frac{3}{8}v^4\right].$$

$$T^\mu{}_\nu = c^2\rho u^\mu u_\nu,$$

$$T^0{}_0 = -c^2\rho - \rho v^2 - \frac{1}{c^2}\rho\left[2(U_N+V_N)v^2 - B_i^N v^i + v^4\right]$$

$$T^0{}_i = c\rho a v_i + \frac{1}{c}\rho a\left\{v_i[v^2 + 2(U_N+V_N)] - B_i^N\right\},$$

$$T^i{}_0 = -c\frac{1}{a}\rho v^i - \frac{1}{c}\frac{1}{a}\rho v^2 v^i,$$

$$T^i{}_j = \rho v^i v_j + \frac{1}{c^2}\rho\left\{v^i v_j[v^2 + 2(U_N+V_N)] - v^i B_j^N\right\},$$

$$T^\mu{}_\mu = T = -\rho c^2.$$

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$$T^\mu{}_\nu = c^2\rho u^\mu u_\nu,$$

$$\begin{aligned}
T^0{}_0 &= -c^2\rho - \rho v^2 - \frac{1}{c^2}\rho\left[2(U_N+V_N)v^2 - B_i^N v^i + v^4\right] \\
T^0{}_i &= c\rho a v_i + \frac{1}{c}\rho a\left\{v_i[v^2 + 2(U_N+V_N)] - B_i^N\right\}, \\
T^i{}_0 &= -c\frac{1}{a}\rho v^i - \frac{1}{c}\frac{1}{a}\rho v^2 v^i, \\
T^i{}_j &= \rho v^i v_j + \frac{1}{c^2}\rho\left\{v^i v_j[v^2 + 2(U_N+V_N)] - v^i B_j^N\right\}, \\
T^\mu{}_\mu &= T = -\rho c^2.
\end{aligned}$$

note:

ρ is a non-perturbative quantity

Newtonian ΛCDM, with a bonus

- insert leading order terms in E.M. conservation and Einstein equations
- subtract the background, getting usual Friedmann equations
- introduce usual density contrast by $\rho=\rho_b(1+\delta)$

from E.M. conservation:
Continuity & Euler equations

$$\dot{\delta} + \frac{v^i \delta_{,i}}{a} + \frac{v^i_{\ ,i}}{a}(\delta + 1) = 0\ ,$$
$$\dot{v}_i + \frac{v^j v_{i,j}}{a} + \frac{\dot{a}}{a} v_i = \frac{1}{a} U_{N,i}\ .$$

Poisson

$$G^0_{\ 0} + \Lambda = \frac{8\pi G}{c^4} T^0_{\ 0} \ \rightarrow \ \frac{1}{c^2}\frac{1}{a^2}\nabla^2 V_N = -\frac{4\pi G}{c^2}\bar{\rho}\delta$$

Newtonian ΛCDM, with a bonus

what do we get from the ij and 0i Einstein equations?

$$\text{trace of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4}T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{2}{a^2}\nabla^2(V_N - U_N) = 0\,,$$

zero "Slip"

$$\text{traceless part of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4}T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{1}{a^2}\left[(V_N - U_N)_{,i}{}^{,j} - \frac{1}{3}\nabla^2(V_N - U_N)\delta_i^j\right] = 0$$

bonus

$$G^0{}_i = \frac{8\pi G}{c^4}T^0{}_i \;\rightarrow\; \frac{1}{c^3}\left[-\frac{1}{2a^2}\nabla^2 B_i^N + 2\frac{\dot{a}}{a^2}U_{N,i} + \frac{2}{a}\dot{V}_{N,i}\right] = \frac{8\pi G}{c^3}\bar{\rho}(1+\delta)v_i$$

- Newtonian dynamics at leading order, with a bonus: the frame dragging potential B_i is not dynamical at this order, but cannot be set to zero: doing so would forces a constraint on Newtonian dynamics
- result entirely consistent with vector relativistic perturbation theory
- in a relativistic framework, gravitomagnetic effects cannot be set to zero even in the Newtonian regime, cf. Kofman & Pogosyan (1995), ApJ 442:

magnetic Weyl tensor at leading order

$$H_{ij} = \frac{1}{2c^3}\left[B^N_{\mu,\nu(i}\varepsilon_{j)}{}^{\mu\nu} + 2v_\mu(U_N + V_N)_{,\nu(i}\varepsilon_{j)}{}^{\mu\nu}\right]$$

Post-Friedmannian ΛCDM

next to leading order: the 1-PF variables

- resummed scalar potentials

$$\phi_P := -(U_N + \frac{2}{c^2} U_P),$$
$$\psi_P := -(V_N + \frac{2}{c^2} V_P),$$

- resummed gravitational potential
- resummed "Slip" potential

$$\phi_G := \frac{1}{2}(\phi_P + \psi_P),$$
$$\frac{1}{c^2} D_P := \frac{1}{2}(\phi_P - \psi_P);$$

- resummed vector "frame dragging" potential

$$\omega_i = B_i^N + \frac{1}{c^2} B_i^P .$$

- Chandrasekhar velocity:

$$v_i^* = v_i - \frac{1}{c^2}\omega_i ,$$

Post-Friedmannian ΛCDM

The 1-PF equations: scalar sector

Continuity & Euler

$$\frac{d\delta}{dt} + \frac{v^{*i}{}_{,i}}{a}(\delta+1) - \frac{1}{c^2}\left[(\delta+1)\left(3\frac{d\phi_G}{dt} + \frac{v_k^*\phi_{G,k}}{a} + \frac{\dot a}{a}v^{*2}\right) - \frac{1}{a}\omega^j\delta_{,j}\right] = 0\,. \qquad (8.6)$$

$$\frac{dv_i^*}{dt} + \frac{\dot a}{a}v_i^* + \frac{1}{a}\phi_{G,i} + \frac{1}{c^2}\left[\frac{1}{a}\phi_{G,i}(4\phi_G + v^{*2}) - 3v_i^*\frac{d\phi_G}{dt} + \frac{1}{a}D_{P,i} - \frac{1}{a}v_i^* v_j^* \phi_G{}^{,j} - \frac{\dot a}{a}v^{*2}v_i^* + \frac{1}{a}\omega_{j,i}v^{*j} + \frac{1}{a}\omega^j v^{*i}_{,j}\right] = 0\,,$$

generalized Poisson: a non-linear evolution eq. for ϕ_G

$$\frac{1}{c^2}\frac{2}{3}\nabla^2\phi_G + \frac{1}{c^4}\left[a^2\left(\ddot\phi_G + 2\frac{\dot a}{a}\dot\phi_G + 2\frac{\ddot a}{a}\phi_G - \left(\frac{\dot a}{a}\right)^2\phi_G\right) + \frac{2}{3}\nabla^2\phi_G^2 - \frac{3}{2}\phi_{G,i}\phi_G{}^{,i}\right] = \frac{1}{c^2}\frac{8\pi G}{3}a^2\bar\rho\delta$$
$$+\frac{1}{c^4}4\pi G a^2\bar\rho(1+\delta)v^{*2}\,.$$

non-dynamical "Slip"

$$\frac{1}{c^4}\frac{2}{3}\nabla^2\nabla^2 D_P = -\frac{1}{c^4}\left[\left(\phi_{G,i}\phi_G{}^{,j}\right)^{,i}_{,j} - \frac{1}{3}\nabla^2\left(\phi_{G,i}\phi_G{}^{,i}\right)\right] - \frac{1}{c^4}4\pi G a^2\bar\rho\left[(1+\delta)\left(v_i^* v^{*j} - \frac{1}{3}v^{*2}\delta_i^j\right)\right]^{,i}_{,j}.$$

so far so good...

- at leading order, we have obtained Newtonian cosmology equations
- the corresponding metric is a consistent approximate solution of EFE in the Newtonian regime, valid for scales $\ll H^{-1}$
- how about large linear scales?

linearized equations

linearized equations for the resummed variables:
standard scalar and vector perturbation equations
in the Poisson gauge

$$\nabla^2\psi_P - \frac{3}{c^2}a^2\left[\frac{\dot a}{a}\dot\psi_P + \left(\frac{\dot a}{a}\right)^2\phi_P\right] = 4\pi G\bar\rho a^2\delta\ ,$$

$$-\nabla^2(\psi_P - \phi_P) + \frac{3}{c^2}a^2\left[\frac{\dot a}{a}(\dot\phi_P + 3\dot\psi_P) + 2\frac{\ddot a}{a}\phi_P + \left(\frac{\dot a}{a}\right)^2\phi_P + \ddot\psi_P\right] = 0$$

$$\nabla^2\left(\frac{\dot a}{a}\phi_P + \dot\psi_P\right) = -4\pi G a\bar\rho\theta\ ,$$

$$\frac{1}{c^2}\nabla^2\nabla^2(\phi_P - \psi_P) = 0\ ,$$

$$\dot\delta + \frac{\theta}{a} - \frac{3}{c^2}\dot\psi_P = 0\ ,$$

$$\dot\theta + \frac{\dot a}{a}\theta + \frac{1}{a}\nabla^2\phi_P = 0\ .$$

cf. Ma & Bertschinger, ApJ (1994)

nonlinear post-Friedmann framework: applications

frame-dragging potential from N-body simulations

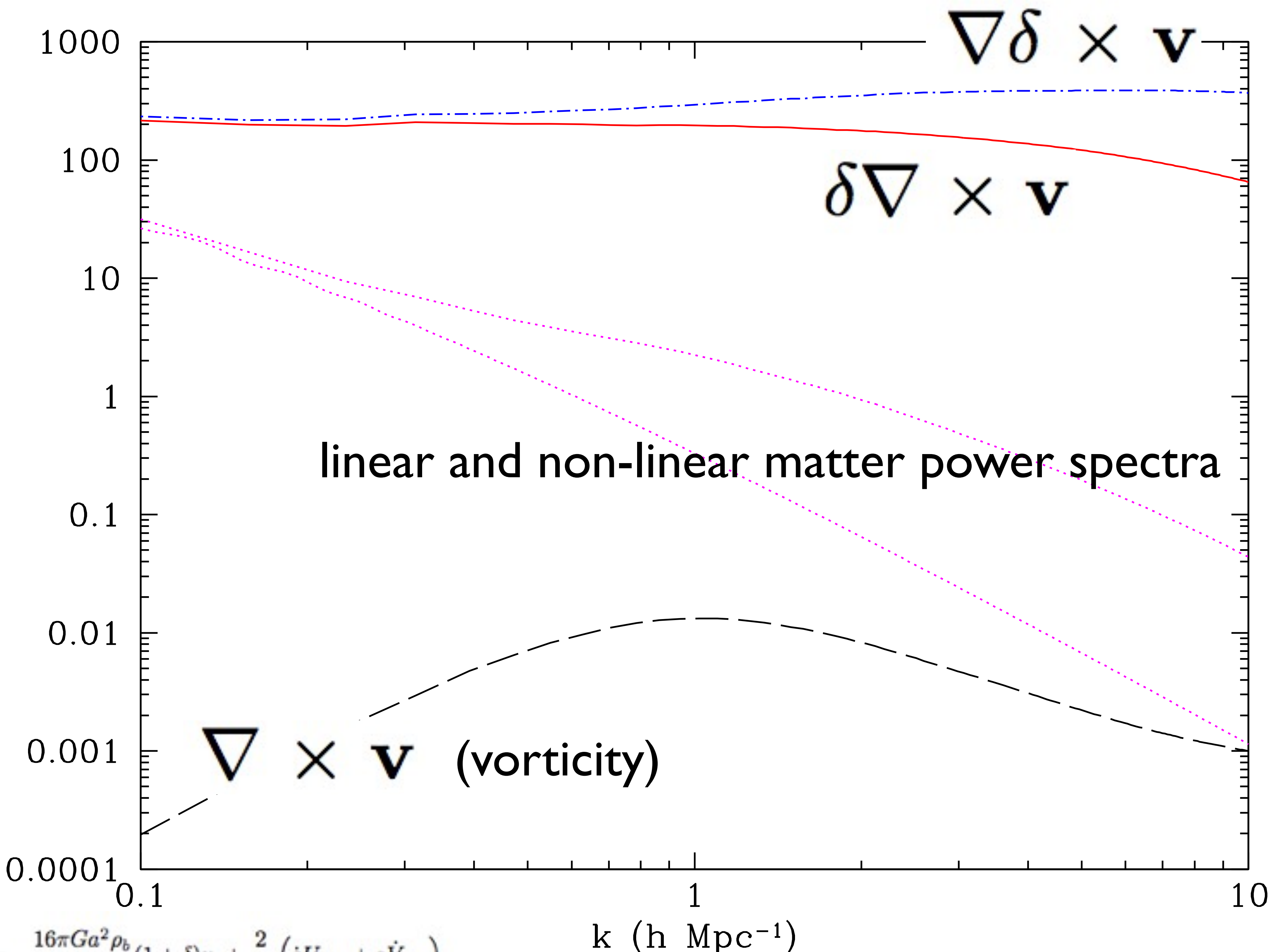
- Simulations at leading order in the post-Friedmann expansion
- dynamics is Newtonian, but a frame-dragging vector potential is sourced by the vector part of the Newtonian energy current

$$\nabla \times \nabla^2 \mathbf{B}^N = -(16\pi G\bar{\rho}a^2)\nabla \times [(1+\delta)\mathbf{v}].$$

frame-dragging potential from N-body simulations

- first calculation of an intrinsically relativistic quantity in fully non-linear cosmology
- three runs of N-body simulations with 1024^3 particles and 160 h^{-1} Mpc (Gadget-2)
- publicly available Delauney Tessellation Field Estimator (DTFE) used to extract the velocity field. cf. Pueblas & Scoccimarro (2009)
- MB, D. B. Thomas and D. Wands, Physical Review (2014), 89, 044010 [arXiv:1306.1562] - Dan B. Thomas, MB and David Wands (2015) [arXiv:1501.00799]

power spectra: sources

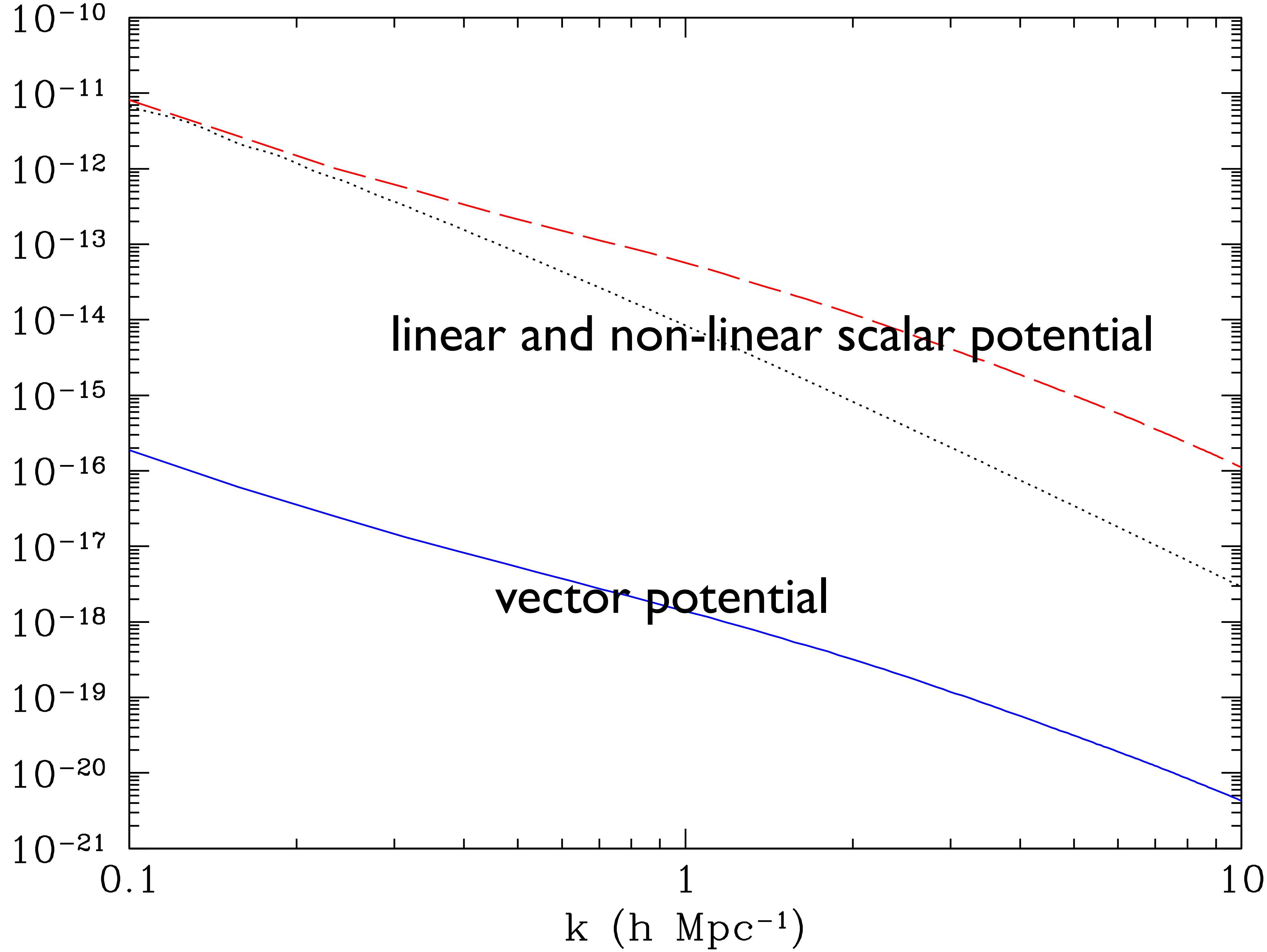


$$\frac{1}{c^3}\nabla^2 P_i^N = -\frac{16\pi G a^2 \rho_b}{c^3}(1+\delta)v_i + \frac{2}{c^3}\left(\dot{a}U_{N,i} + a\dot{V}_{N,i}\right)$$

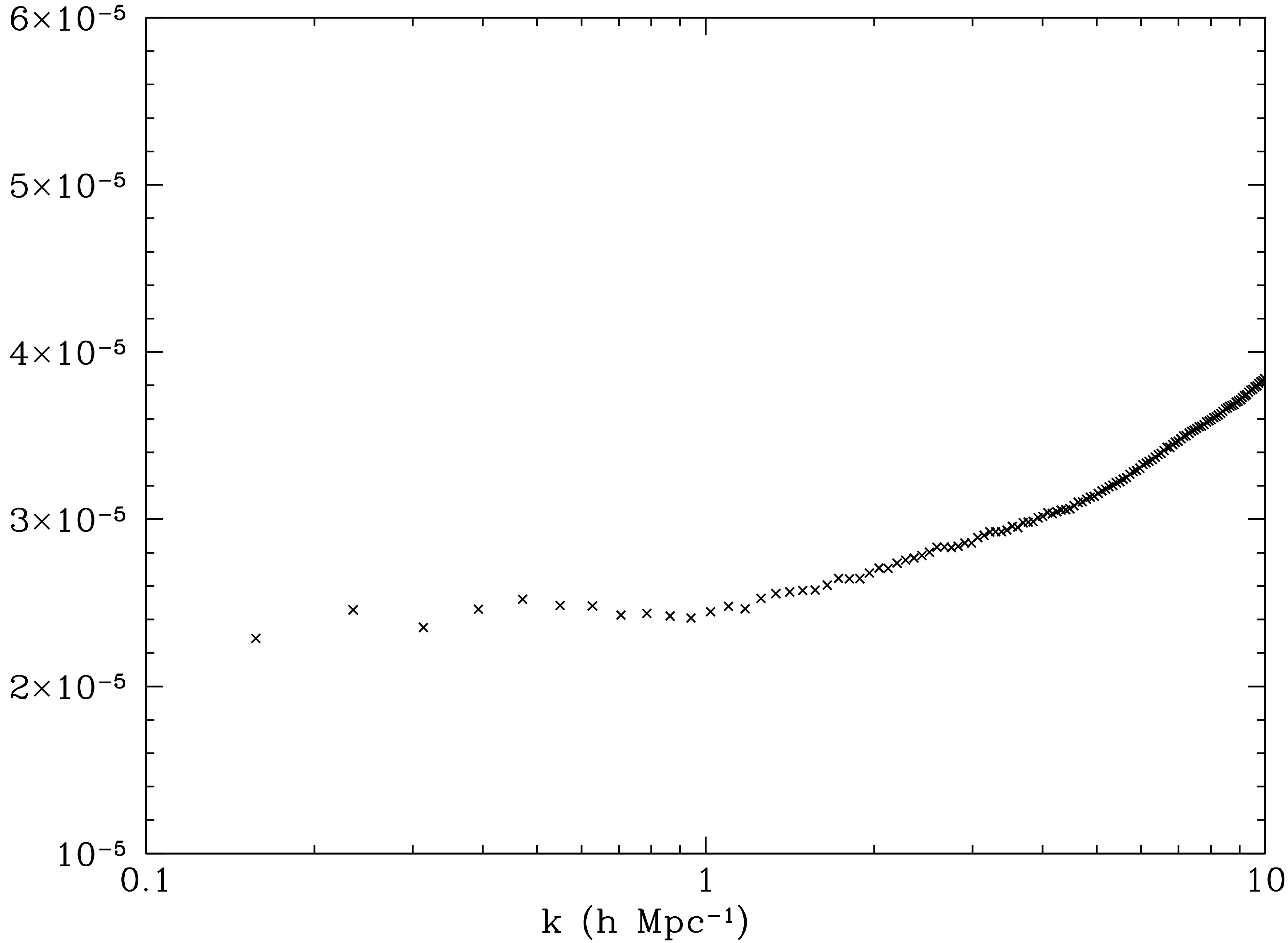
$$\nabla \times \nabla^2 \vec{P}^N = -\left(16\pi G \rho_b a^2\right)\nabla \times [(1+\delta)\vec{v}]$$

$$\nabla \times [(1+\delta)\vec{v}] = (\nabla\delta) \times \vec{v} + (1+\delta)\nabla \times \vec{v}$$

scalar and vector potentials



ratio of the potentials



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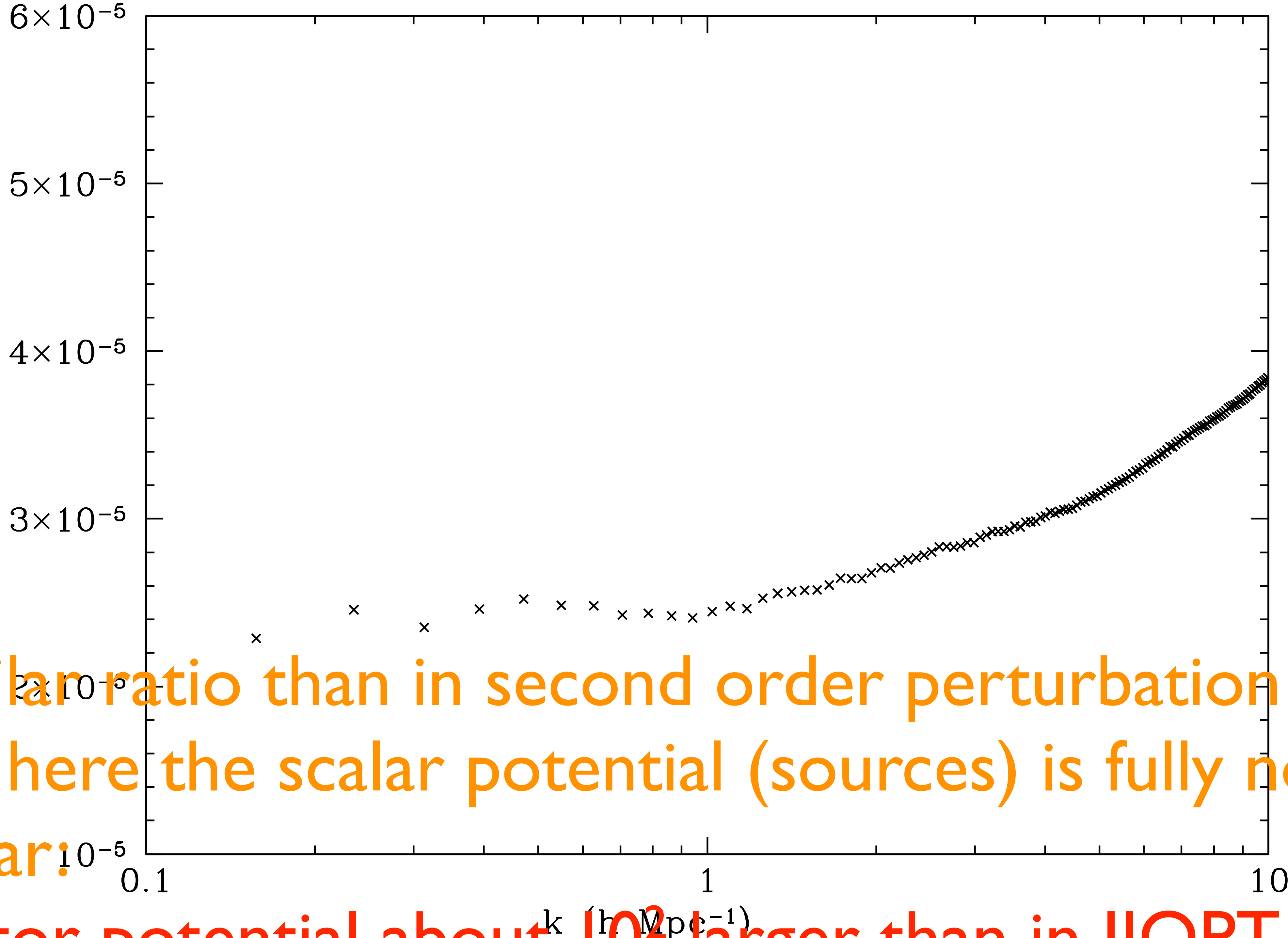


similar ratio than in second order perturbation theory but here the scalar potential (sources) is fully non-linear:

vector potential about 10^2 larger than in IIOPT

cf. Lu, Ananda, Clarkson & Maartens (2009)

post-F: other work

- weak lensing: D. B. Thomas, M. Bruni and D. Wands, [arXiv: 1403.4947]
  - lensing computed up to c^{-4} valid on fully non-linear scales; effects on convergence/weak lensing E-modes negligible, currently probably not detectable; B-modes estimate says it is very small.
  - need thinking about other possible detectable effects
- extended paper with more details on the simulations and the vector potential; Thomas, Bruni & Wands [arXiv:1501.00799]
- post-F f(R) expansion and vector potential, Thomas et [arXiv: 1503.07204] cf. Clifton and Dunsby [arXiv:1501.04004]

post-F vector potential in f(R)

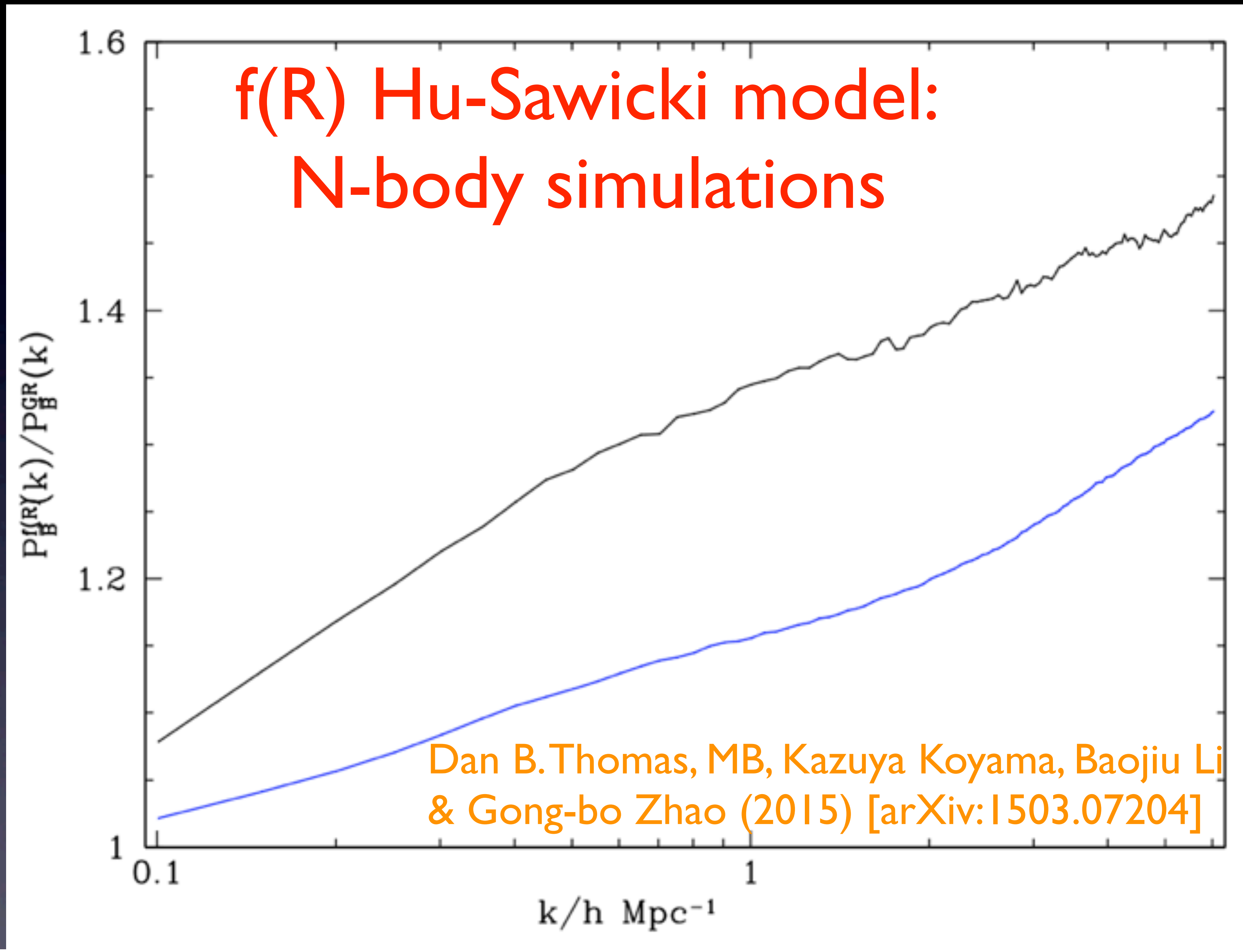
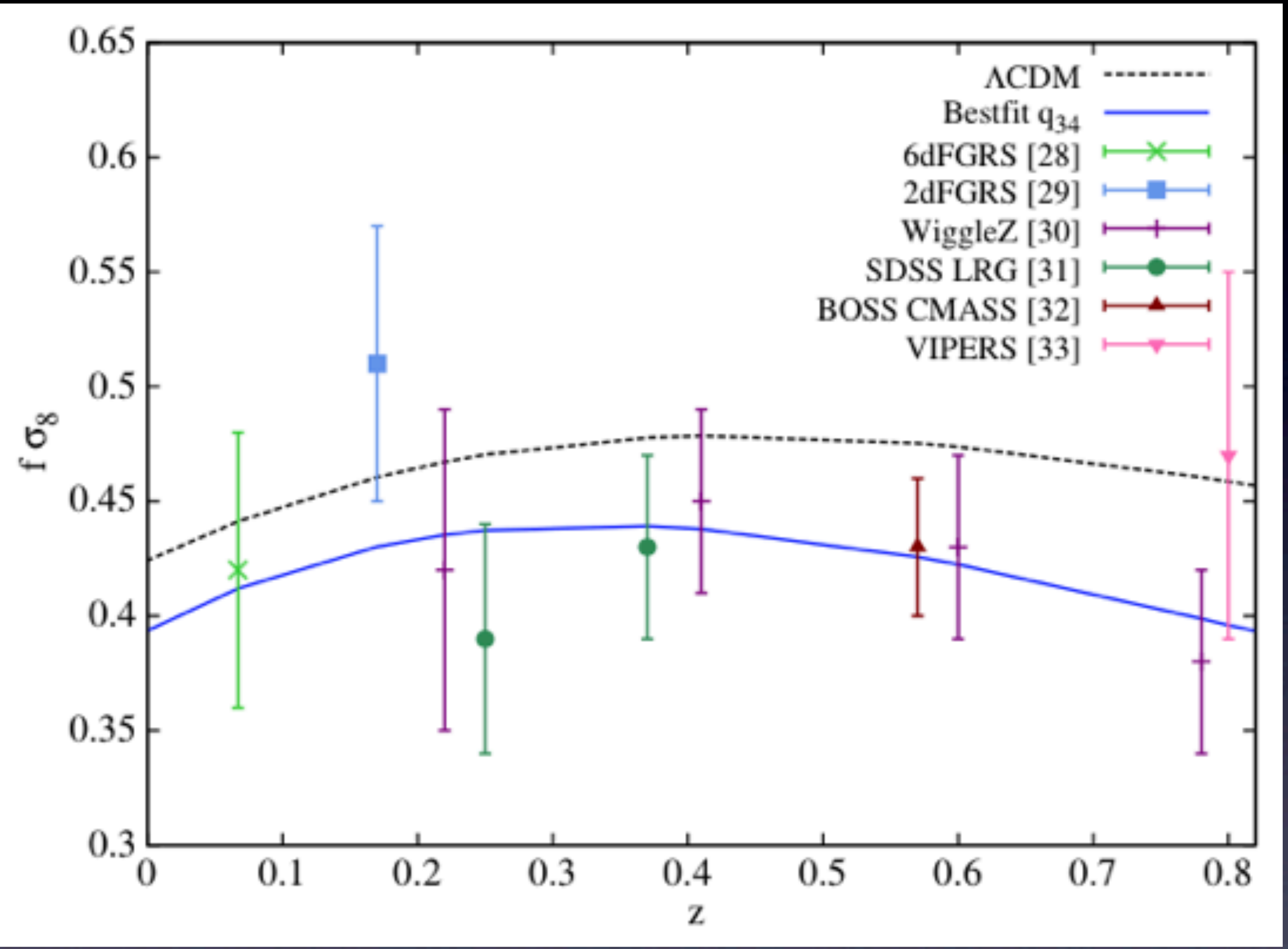


FIG. 3: The ratio of the vector potential power spectrum in $f(R)$ gravity to that in GR, for $|f_{R_0}| = 10^{-5}$. The blue curve shows the ratio at redshift one, and the black curve shows the ratio at redshift zero.

iVCDM



- iVCDM (Salvatelli, Said, MB & Wands, PRL 113, 181301, 2014): in view of simulations, compute leading order post-F for iVCDM from Einstein field equations, Maselli et al, in progress

Summary

- Non-linear GR effects worth investigating in view of future surveys
- PF: at leading Newtonian order in the dynamics, consistency of Einstein equations requires a non-zero gravito-magnetic vector potential
- PF framework provides a straightforward relativistic interpretation of Newtonian simulations: quantities are those of Newton-Poisson gauge at leading order
- linearised equations coincide with I-order relativistic perturbation theory in Poisson gauge (probably OK up to II-order, except sub-dominant terms)
- 2 scalar potentials, become 1 in the Newtonian regime and in the linear regime, valid at horizon scales: slip non-zero in relativistic mildly non-linear (intermediate scales?) regime

Outlook

- applications of Post Friedmann formalism in many directions:
  - extend link to SPT up to II order, derive post-F in comoving-synchronous gauge and link with fluid flow approach
  - need to apply approx. methods to solve eqs. (e.g. 2LP theory), then consider modifying N-body codes
  - derive Newtonian approx. for iVCDM and compute frame dragging
  - quantify Slip in ΛCDM, iVCDM and other models

“take home message”

- it is important to consider relativistic effects in structure formations, even at small/intermediate scales
- at large scales: matter power spectrum

MB, Crittenden, Koyama, Maartens, Pitrou & Wands, *Disentangling non-Gaussianity, bias and GR effects in the galaxy distribution*, arXiv:1106.3999, PRD 85 (2012)

see Bonvin & Durrer PRD 84 (2011) and Challinor & Lewis PRD 84 (2011)

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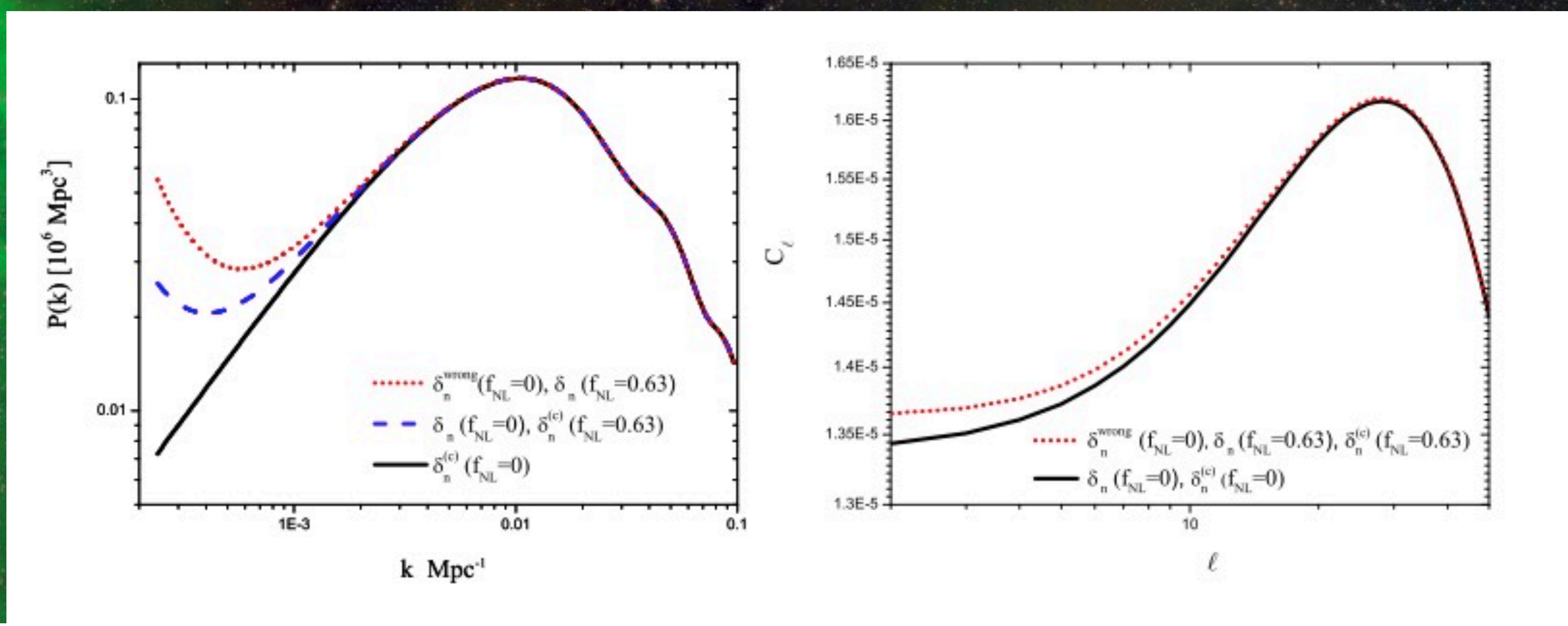
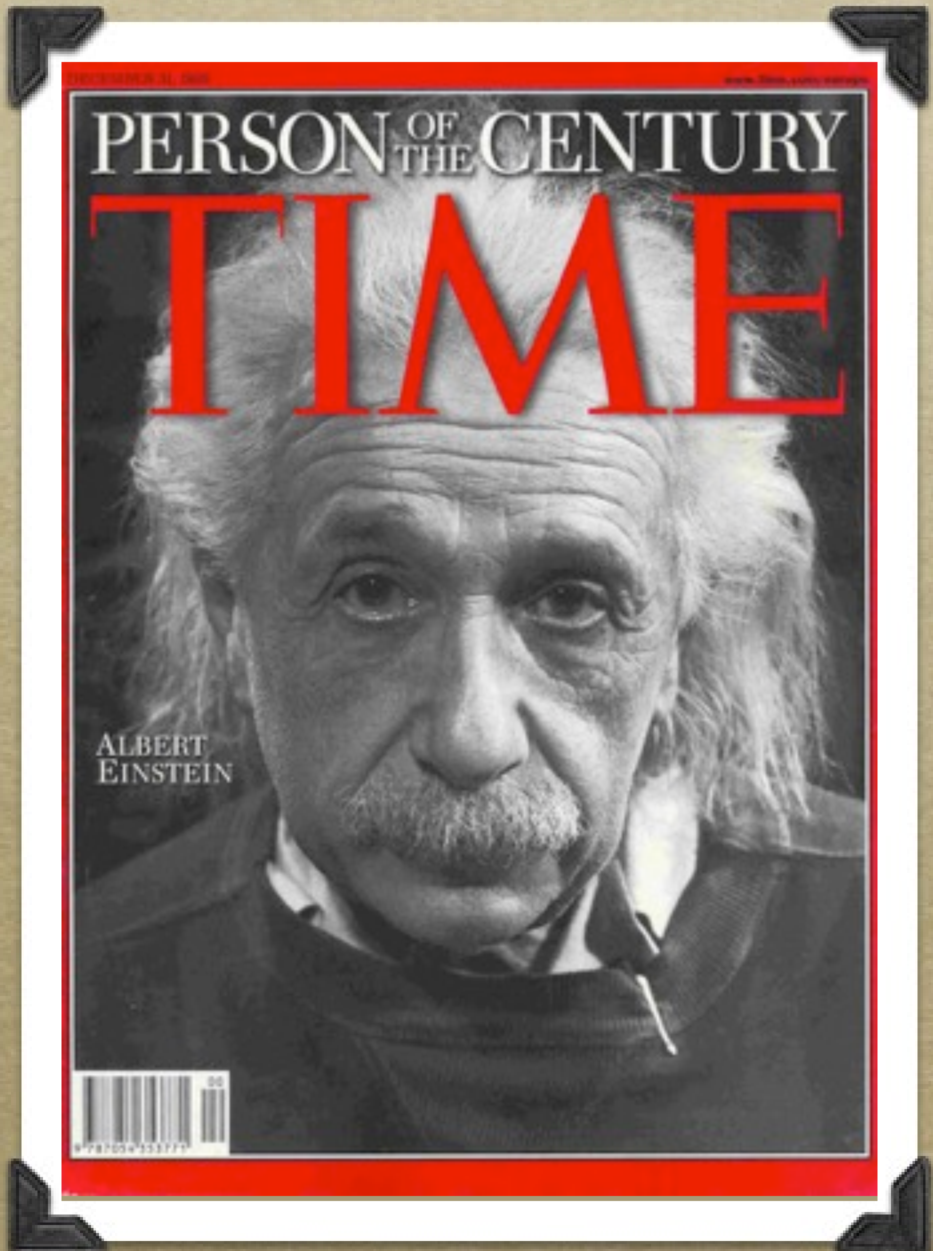


A Century of GR and Cosmology

- A Century of GR
- GR: first context for development of a physical theory of cosmology
- Newtonian Cosmology came later
- GR effects in LSS important



TIME cover, January 2000