Halo Power Spectrum and Bispectrum in the Effective Field Theory of Large Scale Structures

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with:

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Contents

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p} \cdot \nabla f - am\nabla\phi \cdot \nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Integral moments of the distribution function:

mass density field & mean streaming velocity field

$$\rho(\mathbf{x}) = ma^{-3}\int d^3p\, f(\boldsymbol{x},\boldsymbol{p}), \qquad v_i(\mathbf{x}) = \frac{\int d^3p\, \frac{p_i}{am} f(\boldsymbol{x},\boldsymbol{p})}{\int d^3p\, f(\boldsymbol{x},\boldsymbol{p})},$$

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

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and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Eulerian framework - fluid approximation:

$$\begin{aligned}\frac{\partial\delta}{\partial\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] &= 0\\ \frac{\partial v_i}{\partial\tau} + \mathcal{H}v_i + \mathbf{v}\cdot\nabla v_i &= -\nabla_i\phi - \frac{1}{\rho}\nabla_i(\rho\sigma_{ij}),\end{aligned}$$

where σ_{ij} is the velocity dispersion.

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p} \cdot \nabla f - am\nabla\phi \cdot \nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Eulerian framework - **pressureless perfect fluid** approximation:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \mathbf{v} \cdot \nabla v_i = -\nabla_i\phi.$$

Irrotational fluid: $\theta = \nabla \cdot \mathbf{v}$.

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p}\cdot\nabla f - am\nabla\phi\cdot\nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

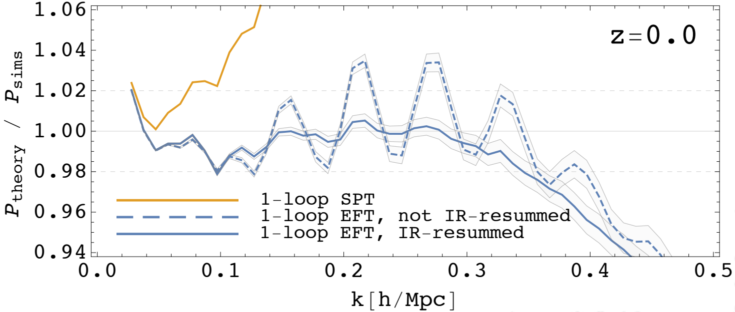
EFT approach introduces a tress tensor for the long-distance fluid:

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial\tau} + \mathcal{H}v_i + \mathbf{v}\cdot\nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_j(\tau_{ij}),$$

with given as $\tau_{ij} = p_0\delta_{ij} + c_s^2\delta\rho\delta_{ij} + O(\partial^2\delta,\ldots)$ [Carrasco et al. 2012]
-derived by smoothing the short scales in the fluid with
the smoothing filter $W(\Lambda)$.

EFTofLSS one-loop results for DM

$$P_{\text{EFT-1-loop}} = P_{11} + P_{\text{1-loop}} - 2(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{11}$$



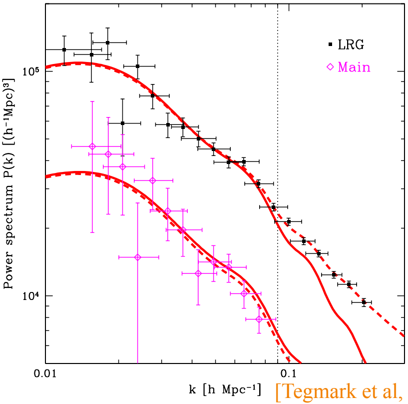
[first by Carrasco et al, 2012]

- Well defined and convergent expansion in k/k_{NL} (one parameter).
- IR resummation (Lagrangian approach) - BAO peak! [Senatore et al, 2014]

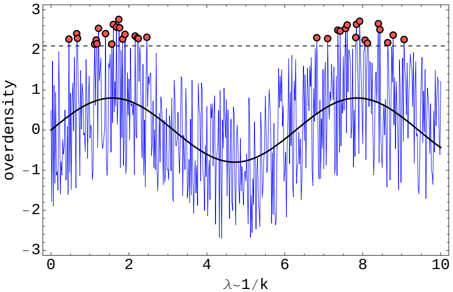
Galaxies and biasing of dark matter halos

Galaxies form at high density peaks of initial matter density:

rare peaks exhibit higher clustering!



[Tegmark et al, 2006]



- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \ldots$
- Understanding bias is crucial for understanding the galaxy clustering

Earlier approaches to halo biasing

Local biasing model: halo field is a function of just DM density field

$$\delta_{\rm h} = c_\delta \delta + c_{\delta^2}\left(\delta^2 - \left\langle \delta^2 \right\rangle\right) + c_{\delta^3}\delta^3 + \ldots$$

[Fry & Gaztanaga, 1993]

Non-local (in space) relation of the halo density field to the dark matter

[McDonald & Roy, 2008]

$$\begin{aligned}\delta_{\rm h}(\mathbf{x}) = {} & c_\delta \delta(\mathbf{x}) + c_{\delta^2}\delta^2(\mathbf{x}) + c_{\delta^3}\delta^3(\mathbf{x}) \\ & + c_{s^2}s^2(\mathbf{x}) + c_{\delta s^2}\delta(\mathbf{x})s^2(\mathbf{x}) + c_\psi \psi(\mathbf{x}) + c_{st}s(\mathbf{x})t(\mathbf{x}) + c_{s^3}s^3(\mathbf{x}) \\ & + c_\epsilon \epsilon + \ldots ,\end{aligned}$$

with effective ('Wilson') coefficients c_l and variables:

$$s_{ij}(\mathbf{x}) = \partial_i\partial_j\phi(\mathbf{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta(\mathbf{x}), \qquad t_{ij}(\mathbf{x}) = \partial_i v_j - \frac{1}{3}\delta^{\rm K}_{ij}\theta(\mathbf{x}) - s_{ij}(\mathbf{x}),$$

$$\psi(\mathbf{x}) = [\theta(\mathbf{x}) - \delta(\mathbf{x})] - \frac{2}{7}s(\mathbf{x})^2 + \frac{4}{21}\delta(\mathbf{x})^2,$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

Effective field theory of biasing

Non-local (space and time) relation of the halo density field to the dark matter

[Senatore 2014]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\; H(t')\; \Big[\bar{c}_\delta(t,t')\; : \delta(\boldsymbol{x}_{\rm fl},t') : \\ + \bar{c}_{\delta^2}(t,t')\; : \delta(\boldsymbol{x}_{\rm fl},t')^2 : + \bar{c}_{s^2}(t,t')\; : s^2(\boldsymbol{x}_{\rm fl},t') : \\ + \bar{c}_{\delta^3}(t,t')\; : \delta(\boldsymbol{x}_{\rm fl},t')^3 : + \bar{c}_{\delta s^2}(t,t')\; : \delta(\boldsymbol{x}_{\rm fl},t') s^2(\boldsymbol{x}_{\rm fl},t') : + \dots \\ + \bar{c}_\epsilon(t,t')\, \epsilon(\boldsymbol{x}_{\rm fl},t') + \bar{c}_{\epsilon\delta}(t,t')\; : \epsilon(\boldsymbol{x}_{\rm fl},t')\delta(\boldsymbol{x}_{\rm fl},t') : + \dots \\ + \bar{c}_{\partial^2\delta}(t,t')\; \frac{\partial^2_{x_{\rm fl}}}{k_M^2} \delta(\boldsymbol{x}_{\rm fl},t') + \dots \Big]$$

Novice consideration of non-local in time formation, which depends on fields evaluated on past history on past path:

$$\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\; \boldsymbol{v}(\tau'', \boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

Effective field theory of biasing

New physical scale $k_M \sim 2\pi \left(\frac{4Pi}{3}\frac{\rho_0}{M}\right)^{1/3}$, which can be different then k_{NL}. We look at the correlations at $k \ll k_M$.

Each order in perturbation theory we get new bias coefficients:

$$\delta_{\mathrm{h}}(k,t) = c_{\delta,1}\left[\delta^{(1)}(k,t) + \text{flow terms}\right] + c_{\delta,2}\left[\delta^{(2)}(k,t) + \text{flow terms}\right] + \dots$$

Emergence of degeneracy: choice of most convenient basis

Turns out that at one loop 2-pt and tree level 3-pt function LIT and non-LIT are degenerate- this is no longer the case at higher loops or when 4-pt function is considered.

Effective field theory of biasing

Independent operators in the`Basis of Descendants':

(1)st order: $\left\{\mathbb{C}^{(1)}_{\delta,1}\right\}$

(2)nd order: $\left\{\mathbb{C}^{(2)}_{\delta,1},\ \mathbb{C}^{(2)}_{\delta,2},\ \mathbb{C}^{(2)}_{\delta^2,1}\right\}$

(3)rd order: $\left\{\mathbb{C}^{(3)}_{\delta,1},\ \mathbb{C}^{(3)}_{\delta,2},\ \mathbb{C}^{(3)}_{\delta,3},\ \mathbb{C}^{(3)}_{\delta^2,1},\ \mathbb{C}^{(3)}_{\delta^2,2},\ \mathbb{C}^{(3)}_{\delta^3,1}, \mathbb{C}^{(3)}_{\delta,3_{c_s}}\ \mathbb{C}^{(3)}_{s^2,2}\right\}$

Stochastic: $\left\{\mathbb{C}_{\epsilon},\ \mathbb{C}^{(1)}_{\delta\epsilon,1}\right\}$

We compare $P^{1-\text{loop}}_{\text{hh}}$, $P^{1-\text{loop}}_{\text{hm}}$, $B^{\text{tree}}_{\text{hhh}}$, $B^{\text{tree}}_{\text{hhm}}$, $B^{\text{tree}}_{\text{hmm}}$ statistics

Renormalization! (takes care of short distance physics has at long distances of interest)

In practice, $\tilde{c}_{\delta,1}$ is a bare parameter, the sum of a finite part and a counterterm:

$$\tilde{c}_{\delta,1} = \tilde{c}_{\delta,1,\,\text{finite}} + \tilde{c}_{\delta,1,\,\text{counter}},$$

After renormalization we end up with using 7 finite bias parameters b_i (coefficients in EFT).

Observables: $P_{\mathbf{hm}}$, $P_{\mathbf{hh}}$, $B_{\mathbf{hmm}}$, $B_{\mathbf{hhm}}$, $B_{\mathbf{hhh}}$

Example: Halo-Matter Power Spectrum (one loop)

$$
\begin{aligned}
P_{hm}(k) =& b_{\delta,1}(t) \Bigg(P_{11}(k) + 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q})\, \widehat{c}^{(2)}_{\delta,1,s}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q})\, P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&\qquad + 3P_{11}(k)\int \frac{d^3q}{(2\pi)^3} \left(F_s^{(3)}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) + \widehat{c}^{(3)}_{\delta,1,s}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) \right) P_{11}(q) \Bigg) \\
&+ b_{\delta,2}(t)\, 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) \left(F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) - \widehat{c}^{(2)}_{\delta,1,s}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) \right) \\
&\qquad \times P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&+ b_{\delta,3}(t) 3P_{11}(k) \int \frac{d^3q}{(2\pi)^3} \left(\widehat{c}^{(3)}_{\delta,3,s}(\boldsymbol{k},-\boldsymbol{q},\boldsymbol{q}) \right) P_{11}(q) \\
&+ b_{\delta^2}(t) 2\int \frac{d^3q}{(2\pi)^3} F_s^{(2)}(\boldsymbol{k}-\boldsymbol{q},\boldsymbol{q}) P_{11}(q) P_{11}(|\boldsymbol{k}-\boldsymbol{q}|) \\
&+ \left(b_{c_s}(t) - 2(2\pi) c^2_{s(1)}(t) b_{\delta,1}(t) \right) \frac{k^2}{k^2_{\rm NL}} P_{11}(k)
\end{aligned}
$$

Error estimates and bias fits

Error bars of the theory are given by the higher loop estimates:

e.g. $\Delta P_{hm} \sim (2\pi)\, b_1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}(k)$.

This determines the theory reach $k_{\rm max}$.

$k_{\rm max}$ $[h/\mathrm{Mpc}]$	bin0	bin1
mm	$0.22 - 0.31$	$0.22 - 0.31$
hm	$0.24 - 0.35$	$0.22 - 0.35$
hh	$0.19 - 0.32$	$0.17 - 0.30$
mmm	$0.14 - 0.22$	$0.14 - 0.22$
hmm	$0.13 - 0.22$	$0.13 - 0.22$
hhm	$0.13 - 0.22$	$0.13 - 0.22$
hhh	$0.13 - 0.21$	$0.13 - 0.21$

Fits to N-body simulations:

bin_1: k_{min}=0.04h/Mpc, k_{max}=0.11h/Mpc						
hm	hh	hmm	hhm	hhh	χ^2	p
+	+	−	−	−	0.0372	1.000
+	+	+	−	−	0.662	0.9937
+	+	−	+	−	0.615	0.9982
+	+	−	−	+	0.730	0.9724
+	+	+	+	−	0.849	0.8911
+	+	+	−	+	0.846	0.8963
+	+	−	+	+	1.17	0.09115
+	+	+	+	+	1.13	0.1105

Most of the constraint comes form the 3-pt function.

Fits to 3-pt and 4-pt function would enable full predictivity for 2-pt function.

EFT of biased tracers: bias fits

Error bars of the theory are given by the higher loop estimates:

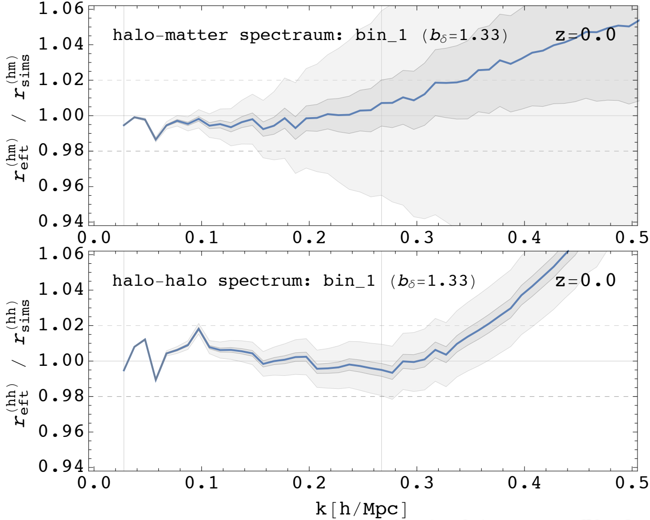
e.g. $\Delta P_{hm} \sim (2\pi)\, b_1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}(k)$.

This determines the theory reach $k_{\rm max}$.

	bin0	bin1
$b_{\delta,1}$	1.00 ± 0.01	1.32 ± 0.01
$b_{\delta,2}$	0.23 ± 0.01	0.52 ± 0.01
$b_{\delta,3}$	0.48 ± 0.12	0.66 ± 0.13
b_{δ^2}	0.28 ± 0.01	0.30 ± 0.01
b_{c_s}	0.72 ± 0.16	0.27 ± 0.17
$b_{\delta\epsilon}$	0.31 ± 0.08	0.76 ± 0.17
Const_ϵ	5697 ± 108	10821 ± 169

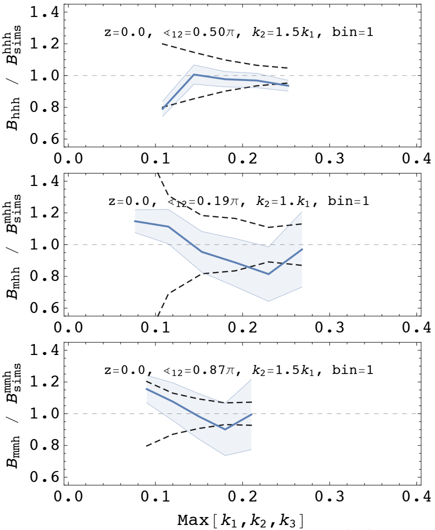
Halo Power Spectrum results (bin 1)

Comparison to N-body simulations: Power Spectrum fitted up to $k < 0.26\mathrm{Mpc}/h$ and Bispectrum up to $k < 0.11\mathrm{Mpc}/h$



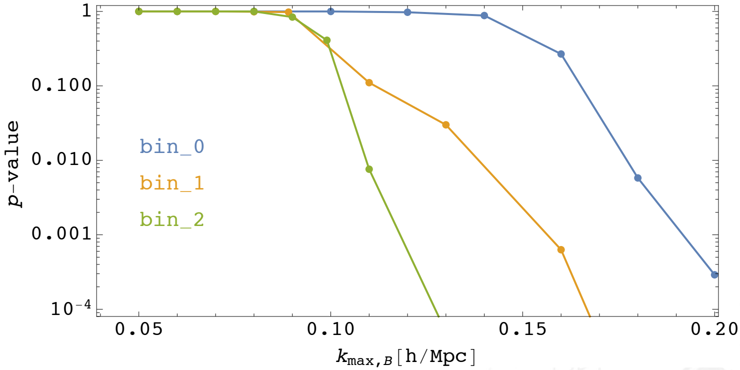
Halo Bispectrum results (bin 1)

Comparison to N-body simulations: Power Spectrum fitted up to $k < 0.26\text{Mpc}/h$ and Bispectrum up to $k < 0.11\text{Mpc}/h$



Bispectrum p-values

Characteristic sharp drop in the p-value after the maximal Bispectrum scale $k_{\max,B}$



Within these scales our EFT results fit the data well, and then fail after crossing this scales.

Adding baryonic effects

- baryon effects (on DM) in EFT framework recently studied [Lewandowski et al. 2014]
- baryons at large distances described as additional fluid component (short distance physics is encoded in an effective stress tensor)

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\ H(t') \left[\bar{c}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\boldsymbol{x}_{\text{fl}},t')}{H(t')^2} + \bar{c}_{\delta_b}(t,t')\, w_b\, \delta_b(\boldsymbol{x}_{\text{fl}\,b}) \right.$$
$$+ \bar{c}_{\partial_i v_c^i}(t,t')\ w_c\ \frac{\partial_i v_c^i(\boldsymbol{x}_{\text{fl}\,c},t')}{H(t')} + \bar{c}_{\partial_i v_b^i}(t,t')\ w_b\ \frac{\partial_i v_b^i(\boldsymbol{x}_{\text{fl}\,b},t')}{H(t')}$$
$$+ \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\ \frac{\partial_i\partial_j\phi(\boldsymbol{x}_{\text{fl}},t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\boldsymbol{x}_{\text{fl}},t')}{H(t')^2} + \dots$$
$$+ \bar{c}_{\epsilon_c}(t,t')\ w_c\ \epsilon_c(\boldsymbol{x}_{\text{fl}\,c},t') + \bar{c}_{\epsilon_b}(t,t')\ w_b\ \epsilon_b(\boldsymbol{x}_{\text{fl}\,b},t')$$
$$\left. + \bar{c}_{\epsilon_c\partial^2\phi}(t,t')\ w_c\ \epsilon_c(\boldsymbol{x}_{\text{fl}\,c},t')\frac{\partial^2\phi(\boldsymbol{x}_{\text{fl}},t')}{H(t')^2} + \bar{c}_{\epsilon_b\partial^2\phi}(t,t')\ w_b\ \epsilon_b(\boldsymbol{x}_{\text{fl}\,b},t')\frac{\partial^2\phi(\boldsymbol{x}_{\text{fl}},t')}{H(t')^2} \dots \right]$$

where $\boldsymbol{x}_{\text{fl}}$ is defined by Poisson equation and:

$$\boldsymbol{x}_{\text{fl}\,b}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_b(\tau'',\boldsymbol{x}_{\text{fl}}(\boldsymbol{x},\tau,\tau''))\,, \quad \boldsymbol{x}_{\text{fl}\,c}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_c(\tau'',\boldsymbol{x}_{\text{fl}}(\boldsymbol{x},\tau,\tau''))$$

Adding Non-Gaussianities

We assume that non-G. correlations are present only in the initial conditions and effect can be described by the squeezed limit, $k_L \ll k_S$ of correlation functions.

After horizon re-rentry, but still early enough to neglect all gravitational non-linearities, the primordial density fluctuation are given by

$$\delta^{(1)}(\boldsymbol{k}_S, t_{\rm in}) \simeq \delta_g(\boldsymbol{k}_S) + f_{\rm NL}\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in})\delta_g(\boldsymbol{k}_S - \boldsymbol{k}_L, t_{\rm in}) \; ,$$

where $\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in}) = \frac{3}{2}\frac{H_0^2\,\Omega_m}{D(t_{\rm in})}\frac{1}{k_S^2\,T(k)}\left(\frac{k_L}{k_S}\right)^{\alpha}\delta_g(\boldsymbol{k}_L, t_{\rm in})$ and where $T(k)$ is the transfer function.

In the presence of primordial non-Gaussianities, additional components:

$$\begin{aligned}\delta_h(\boldsymbol{x}, t) \simeq\, & f_{\rm nl}\,\tilde{\phi}(\boldsymbol{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in}) \int^t dt'\, H(t') \left[\bar{c}^{\,\tilde{\phi}}(t, t') + \bar{c}^{\,\tilde{\phi}}_{\partial^2\phi}(t, t')\,\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl}, t')}{H(t')^2} + \dots\right] \\ & + f_{\rm nl}^2\,\tilde{\phi}(\boldsymbol{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in})^2 \int^t dt'\, H(t') \left[\bar{c}^{\,\tilde{\phi}^2}(t, t') + \bar{c}^{\,\tilde{\phi}^2}_{\partial^2\phi}(t, t')\,\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl}, t')}{H(t')^2} + \dots\right] + \dots\end{aligned}$$

Summary

- EFT gives a consistent expansion in $(k/k_{\rm NL})^2$, and for halos also in $(k/k_{\rm M})^2$, nonlocal effect in time and space included
- EFT approach is well suited for galaxy clustering (one-loop power spectra $k \sim 0.3h/\rm{Mpc}$, tree level bispectra $k \sim 0.1 - 0.15h/\rm{Mpc}$)
- Consistent description of five different observables ($P_{\rm hm}$, $P_{\rm hh}$, $B_{\rm hmm}$, $B_{\rm hhm}$, $B_{\rm hhh}$) with seven bias parameters.

Outlook:

- Higher loops calculations in order to extend the $k_{\rm max}$, and higher statistics (e.g. 4-pt function - great potential)
- Calculation of observables taking into account baryons, non-Gaussianities and RSD …
- Generalization of the formalism in order include GR effects (become important as surveys grow).