Minkowski functionals as a tracer of cosmic inhomogeneity

Alexander Wiegand

Harvard-Smithsonian Center for Astrophysics

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A. Wiegand et al. MNRAS 443:241, 2014

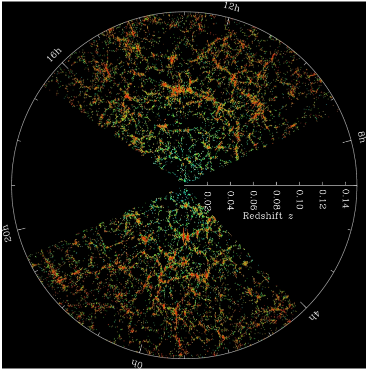
Outline

1. Analysing structure with galaxy surveys
2. Minkowski functionals and germ-grain model
3. SDSS DR7 LRG analysis
4. Conclusion

Galaxy surveys

- Measurements of the galaxy distribution have been pushed to higher and higher redshifts

survey	galaxies	z	year
2dFGRS	200k	0.2	2003
SDSS I	700k	0.3	2005
SDSS II	900k	0.5	2008
WiggleZ	240k	1.0	2012
SDSS III	1.5M	0.8	2014
MS DESI	35M	1.7	2020
Euclid	50M/2B	2	2025



Slice of SDSS data from http://www.sdss.org/

- Excellent data to improve our understanding of the origin and evolution of cosmic structure

Connected correlation functions

- Standard analysis uses two pt. correlations

$$\xi_2 = \langle \delta(\mathbf{x}_1)\,\delta(\mathbf{x}_2) \rangle$$

- For Gaussian: $\xi_n = 0 \forall n \geq 3$
- but higher order correlations become important in late universe

$$\begin{aligned}
\xi_3 &= \langle \delta(\mathbf{x}_1)\,\delta(\mathbf{x}_2)\,\delta(\mathbf{x}_3) \rangle \\
\xi_4 &= \langle \delta(\mathbf{x}_1)\,\delta(\mathbf{x}_2)\,\delta(\mathbf{x}_3)\,\delta(\mathbf{x}_4) \rangle - \xi_2(\mathbf{x}_{12})\,\xi_2(\mathbf{x}_{34}) \\
&\quad - \xi_2(\mathbf{x}_{13})\,\xi_2(\mathbf{x}_{24}) - \xi_2(\mathbf{x}_{14})\,\xi_2(\mathbf{x}_{23})
\end{aligned}$$

Germ-grain model

- In 3D there are 4 independent Minkowski functionals:

geometric quantity		μ	V_μ	W_μ	v'_μ
volume	V	0	V	V	$V/V_{\mathcal{D}}$
surface area	A	1	$A/6$	$A/3$	$A/6N$
integral mean curv.	H	2	$H/3\pi$	$H/3$	$H/3\pi N$
Euler characteristic	χ	3	χ	$\frac{4\pi}{3}\chi$	χ/N

- Problem: Trivial for a set of points
  $\Rightarrow$ Find a prescription to make bodies
- Decorate every galaxy with a ball of radius R
- Study the functionals of these bodies as a function of R

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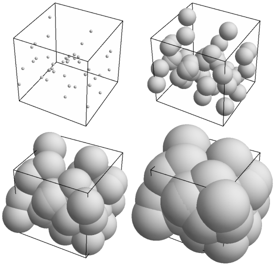


Dependence on correlation functions

- The densities of the functionals

$$\begin{aligned} v_0 &= 1 - e^{-\varrho_0 \overline{V}_0} \\ v_1 &= \varrho_0 \overline{V}_1 e^{-\varrho_0 \overline{V}_0} , \\ v_2 &= \left(\varrho_0 \overline{V}_2 - \frac{3\pi}{8} \varrho_0^2 \overline{V}_1^2 \right) e^{-\varrho_0 \overline{V}_0} , \\ v_3 &= \left(\varrho_0 \overline{V}_3 - \frac{9}{2} \varrho_0^2 \overline{V}_1 \overline{V}_2 + \frac{9\pi}{16} \varrho_0^3 \overline{V}_1^3 \right) e^{-\varrho_0 \overline{V}_0} \end{aligned}$$

- are related to the distribution's correlation functions ξ_{n+1}

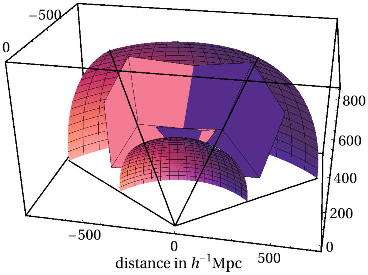
$$\overline{V}_\mu = V_\mu(B) + \sum_{n=1}^{\infty} \frac{(-\varrho_0)^n}{(n+1)!} \int_{\mathcal{D}} \mathrm{d}^3 x_1 .. \mathrm{d}^3 x_n \xi_{n+1}(0, \mathbf{x}_1, .. \mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} .. \cap B_{\mathbf{x}_n})$$

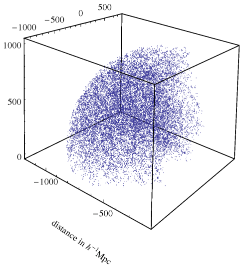
- with the functionals of a ball given by

$$V_0 = \frac{4\pi}{3} R^3 \; ; \; V_1 = \frac{2}{3} \pi R^2 \; ; \; V_2 = \frac{4}{3} R \; ; \; V_3 = 1$$

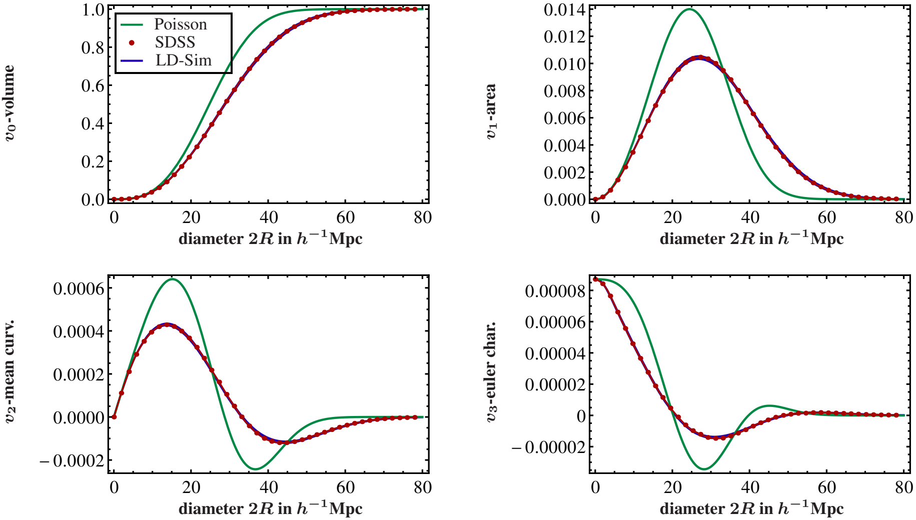
The samples considered

- Consider luminous red galaxy sample of Kazin et al. (2010)
- Choose controllable boundary ra $\in [132°, 235°]$, dec $\in [-1°, 60°]$
- We used two samples:
  - »dim sample«
    $L < -21.2$
    redshift $z \in [0.16, 0.35]$
    number of galaxies $41,375$
  - »bright sample«
    $L < -21.8$
    redshift $z \in [0.16, 0.44]$
    number of galaxies $22,386$
- Largest cubes in sample has sidelength of $452h^{-1}$Mpc

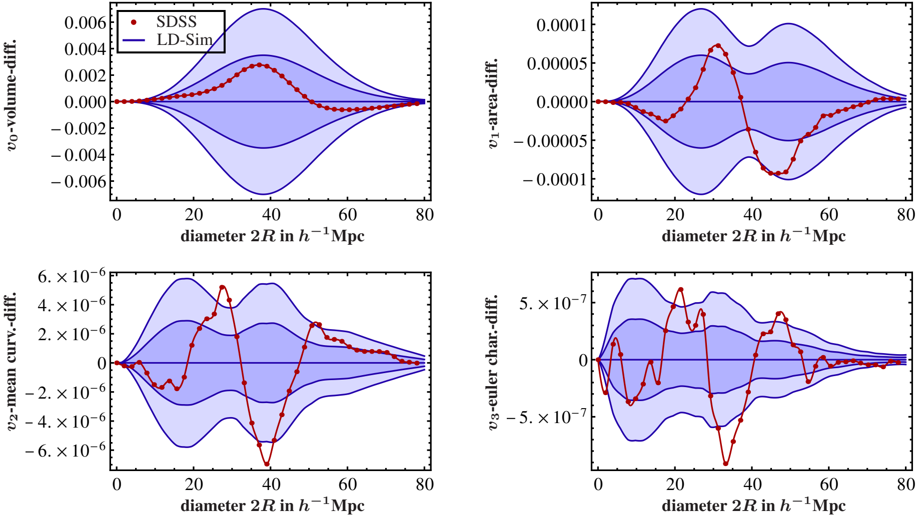




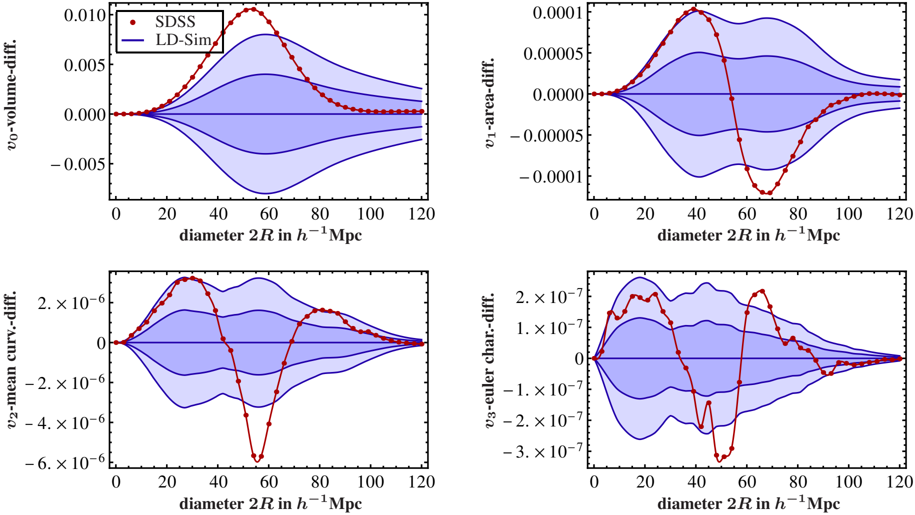
Minkowski functional densities - dim sample



Differences for the dim sample



Differences for the bright sample



Higher order coefficients

- In general: Consider the modified Minkowski functionals

$$\overline{V}_\mu = V_\mu(B) + \sum_{n=1}^{\infty} \frac{(-\varrho_0)^n}{(n+1)!} \int_{\mathcal{D}} \mathrm{d}^3x_1..\mathrm{d}^3x_n \xi_{n+1}(0, \mathbf{x}_1, ..\mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} .. \cap B_{\mathbf{x}_n})$$

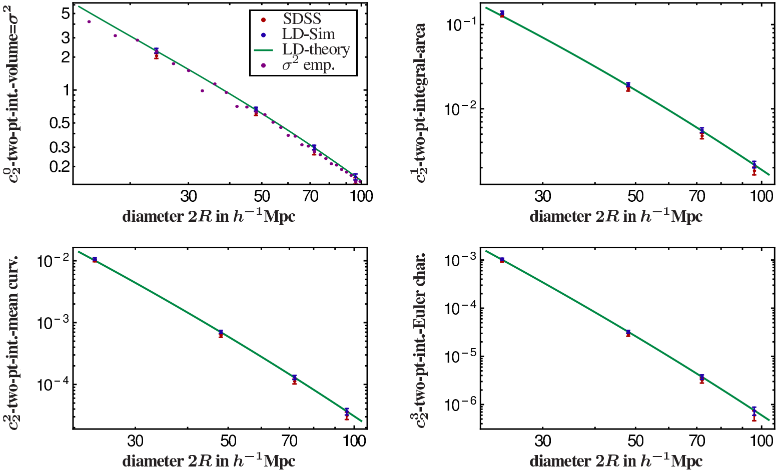
  as a power series in the density ϱ_0

$$\overline{V}_\mu = \sum_{n=0}^{\infty} \frac{b_{n+1}^\mu}{(n+1)!} (-\varrho_0)^n$$

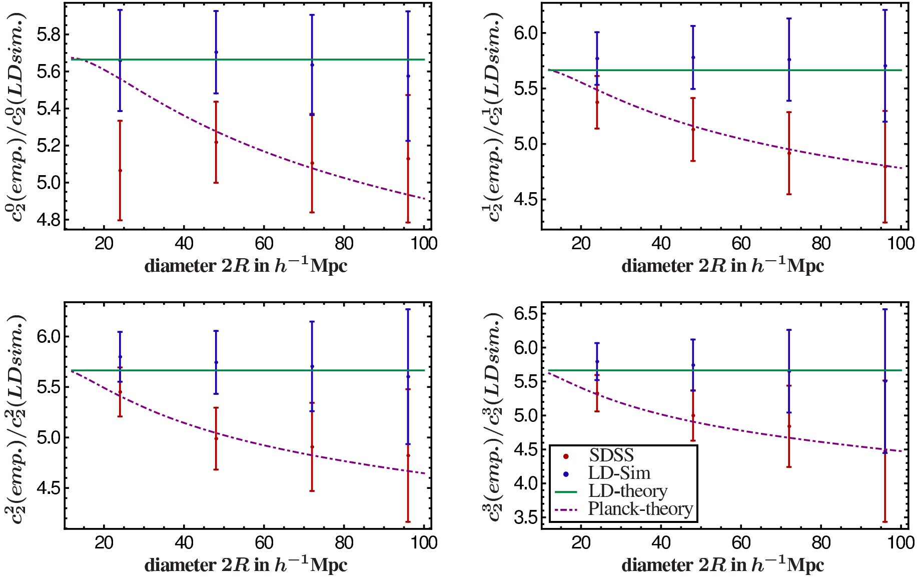
  with coefficients $b_1^\mu = V_\mu(B)$ and

$$b_{n+1}^\mu = \int_{\mathcal{D}} \xi_{n+1}(0, \mathbf{x}_1, \ldots \mathbf{x}_n) V_\mu(B \cap B_{\mathbf{x}_1} \cap \cdots \cap B_{\mathbf{x}_n}) \mathrm{d}^3x_1 \ldots \mathrm{d}^3x_n$$

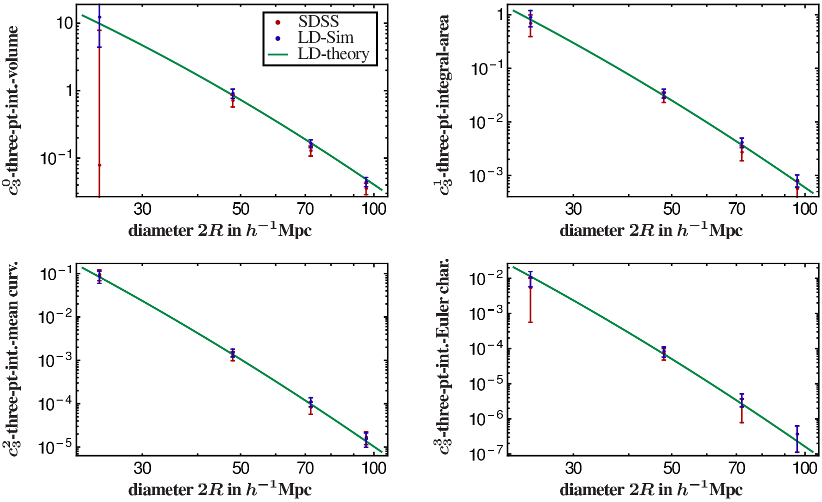
Integrals of the two-point function



Comparison with theoretical expectation

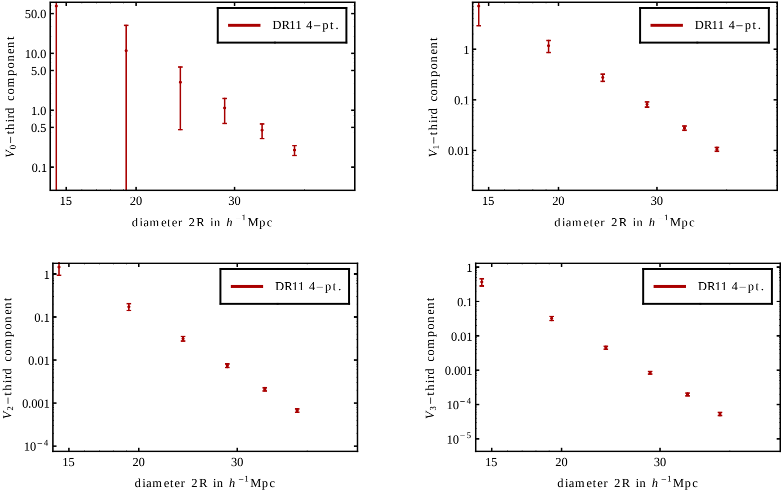


Integrals of the three-point function



$$b_3^\mu = \int_{\mathcal{D}} \mathrm{d}^3x_1 \mathrm{d}^3x_2 \xi_3\left(0, \mathbf{x}_1, \mathbf{x}_2\right) V_\mu\left(B \cap B_{\mathbf{x}_1}(R) \cap B_{\mathbf{x}_2}(R)\right)$$

Integrals of the four-point function



$$b_4^{\mu} = \int_{\mathcal{D}} \mathrm{d}^3 x_1 \mathrm{d}^3 x_2 \mathrm{d}^3 x_3 \xi_4 \left(0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) V_{\mu} \left(B \cap B_{\mathbf{x}_1}(R) \cap B_{\mathbf{x}_2}(R) \cap B_{\mathbf{x}_3}(R)\right)$$

Conclusion

- Excellent observational data on galaxy structure is (becoming) available
- Higher order correlations can be probed by germ-grain Minkowski functionals
- Minkowski functionals of the SDSS LRGs differ from simulations
- Difference partly related to different cosmologies $\Rightarrow$ Constrain cosmic parameters?