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Momentum conservation factor in DPS

S.P. Baranov, A.V. Lipatov, M.A. Malyshev, A.M. Snigirev, N.P. Zotov

D.V.Skobeltsyn Institute of Nuclear Physics

M.V.Lomonosov Moscow State University

Moscow, Russia, 119991

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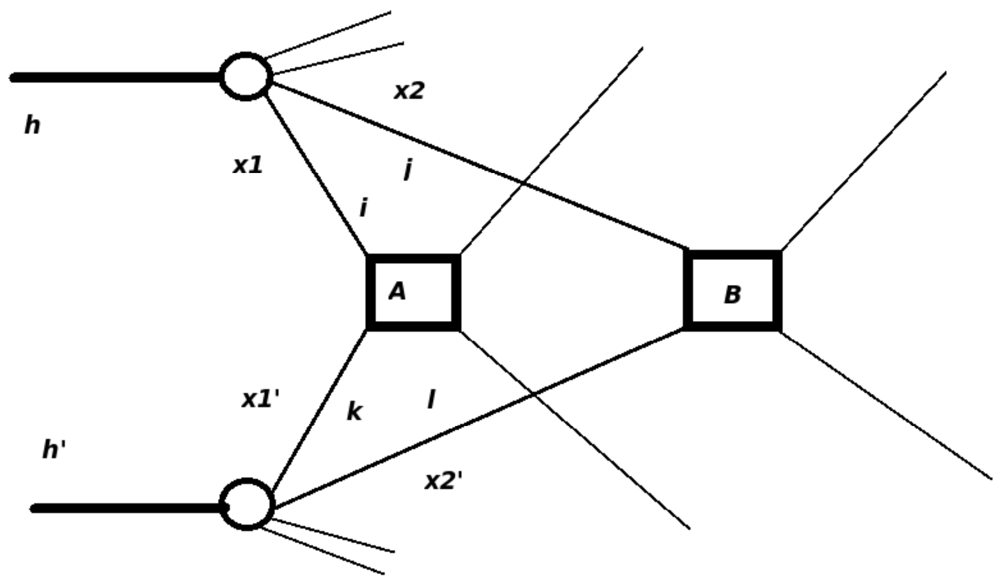
For hard double parton scattering with subprocesses A and B



the cross section of DPS is usually expressed in the simple form

$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2}\frac{\sigma_{\rm SPS}^{A}\sigma_{\rm SPS}^{B}}{\sigma_{\rm eff}},$$

which is obtained from more general starting formula:

(Paver, Treleani,..., Blok,...., Diehl,...).

$$\sigma_{DPS}^{AB} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) \hat{\sigma}_{ik}^{A}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}_{jl}^{B}(x_2, x_2^{'}, Q_2^2)$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2 b_1 d^2 b_2 d^2 b,$$

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two parton undergoing hard processes A and B at the scales Q_1 and Q_2.

$\hat{\sigma}_{ik}^{A}$ and $\hat{\sigma}_{jl}^{B}$ are the parton-level subprocess cross sections.

The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain all the information when probing the hadron in two different points simultaneously, via the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2),$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1,$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (not calculated in pQCD).

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2),$$

the cross section of double parton scattering can be expressed in the simple form

$$\sigma_{\mathrm{DPS}}^{\mathrm{AB}} = \frac{m}{2} \frac{\sigma_{\mathrm{SPS}}^{A} \sigma_{\mathrm{SPS}}^{B}}{\sigma_{\mathrm{eff}}},$$

$$\pi R_{\mathrm{eff}}^2 = \sigma_{\mathrm{eff}} = [\int d^2b (T(\mathrm{b}))^2]^{-1}$$

is the effective interaction transverse area (effective cross section). R_{eff} is an estimate of the size of the hadron.

The momentum (*instead of the mixed (momentum and coordinate)*) representation is more convenient sometimes:

$$\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{q}; Q_1^2, Q_2^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}) \hat{\sigma}^B_{jl}(x_2, x_2^{'})$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; -\mathrm{q}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} \frac{d^2 q}{(2\pi)^2}.$$

Here the transverse vector q is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such dependence arises because the diffe - rence of parton transverse momenta within the parton pair is not conserved.

The main problems are

* to make the correct calculation of the two-parton functions $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ **WITHOUT simplifying factorization assumptions** (*which are not sufficiently justified and should be revised: (Blok, Dokshitzer, Frankfurt, Strikman; Diehl, Schafer; Gaunt, Stirling; Ryskin, Snigirev;...)*)

* to find (observe) longitudinal momentum parton correlations and deviation from the factorization form of DPS cross section.

In particular, the factorization ansatz

$$\boldsymbol{D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2)}$$

is justified for small longitudinal fractions only, where the evident restriction on the total parton momenta $x_1 + x_2 < 1$ can be neglected.

Setting the boundary condition in the form of the theta-function

$$\Theta(1 - x_1 - x_2)$$

would result in a step-like discontinuty at the edge of the space.

In more accurate approach:
(Snigirev, Gaunt, Stirling, Chang,....,Rinaldi,..., Golec-Biernat,...).

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2)(1 - x_1 - x_2)^n$$

the kinematical constrains are smoothly put in with the correction factor

$$(1 - x_1 - x_2)^n,$$

where $n > 0$ is a parameter to be fixed phenomenologically.

One often chooses $n = 2$.
This choice of the space factor can be partly justified in the framework of perturbative QCD *(Snigirev)*
+ gives dPDFs which satisfy the momentum sum rules reasonably well. *(Gaunt, Stirling)*.

Some pQCD issues:

In the collinear approximation the two-parton distribution functions, $\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ with the two hard scales set equal, satisfy the generalized DGLAP evolution equations
(Kirshner; Shelest, Snigirev, Zinovjev).

$$\frac{dD_i^{j_1 j_2}(x_1, x_2, t)}{dt} = \sum_{j_1{}'} \int_{x_1}^{1-x_2} \frac{dx_1{}'}{x_1{}'} D_i^{j_1{}' j_2}(x_1{}', x_2, t) P_{j_1{}' \to j_1}(\frac{x_1}{x_1{}'})$$

$$+ \sum_{j_2{}'} \int_{x_2}^{1-x_1} \frac{dx_2{}'}{x_2{}'} D_i^{j_1 j_2{}'}(x_1, x_2{}', t) P_{j_2{}' \to j_2}(\frac{x_2}{x_2{}'})$$

$$+ \sum_{j'} D_i^{j'}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}(\frac{x_1}{x_1 + x_2}),$$

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln(\frac{Q^2}{\Lambda_{QCD}^2})}{\ln(\frac{\mu^2}{\Lambda_{QCD}^2})}\right], \quad b = \frac{33 - 2n_f}{12\pi}$$

The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales $\mu^2(t=0)$ may be written in the form:

$$D_h^{j_1 j_2}(x_1,x_2,t) \;=\; D_{h1}^{j_1 j_2}(x_1,x_2,t) \;+ D_{h(QCD)}^{j_1 j_2}(x_1,x_2,t),$$

where

$$D_{h1}^{j_1 j_2}(x_1,x_2,t) =$$

$$\sum_{j_1{}' j_2{}'} \int\limits_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int\limits_{x_2}^{1-z_1} \frac{dz_2}{z_2}\; D_h^{j_1{}' j_2{}'}(z_1,z_2,0) D_{j_1{}'}^{j_1}(\frac{x_1}{z_1},t) D_{j_2{}'}^{j_2}(\frac{x_2}{z_2},t)\ ,$$

$$D_{h(QCD)}^{j_1 j_2}(x_1,x_2,t) =$$

$$\sum_{j' j_1{}' j_2{}'} \int\limits_0^t dt' \int\limits_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int\limits_{x_2}^{1-z_1} \frac{dz_2}{z_2}\; D_h^{j'}(z_1+z_2,t')$$

$$\frac{1}{z_1+z_2} P_{j' \to j_1{}' j_2{}'}\Big(\frac{z_1}{z_1+z_2}\Big) D_{j_1{}'}^{j_1}(\frac{x_1}{z_1},t-t') D_{j_2{}'}^{j_2}(\frac{x_2}{z_2},t-t').$$

The first term is the solution of homogeneous evolution equation (independent evolution of two branches), where the input dPDF is generally NOT known at the low scale $\mu(t=0)$. For this non-perturbative dPDF at low z_1, z_2 one may assume the factorization $D_h^{j_1' j_2'}(z_1, z_2, 0) \simeq D_h^{j_1'}(z_1, 0) D_h^{j_2'}(z_2, 0)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$).

This *leads to*

$$\boldsymbol{D_{h1}^{ij}(x_1, x_2, t) \simeq D_h^{i}(x_1, t) D_h^{j}(x_2, t)}$$

the factorization hypothesis usually used in current estimations.

Thus, the factorization is not in contradiction to pQCD-evolution (homogeneous) at small x_1, x_2.

Moreover, the additional second term has also the factorization property (!!) in the case of one slow ($x_1 \sim 0$) and one fast ($x_2 =$ finite) parton.

$$\frac{\text{second (nonhom.)}}{\text{first (hom., fact.)}} \sim \frac{1}{\langle \text{(q+g) initial multiplicity} \rangle}$$

Finite x_1, x_2:

at the parton level the behaviour of dPDFs near the kinematical boundary is under control:

moment representation —–> exact solution (nonhomogeneous) —–>

large moments —–> inverse Mellin transformation

Results only:

1) The limit $x_1 + x_2 \to 1$

$$\boldsymbol{D_i^{j_1 j_2}(x_1, x_2, t) \sim A_i^{j_1 j_2}(1 - x_1 - x_2)^{2C_F t + \delta_{i,j_1 j_2}},}$$

where $A_i^{j_1 j_2}$ have at most a logarithmic dependence on $(1 - x_1 - x_2)$. The exponents $\delta_{i,j_1 j_2}$ are some integer numbers given in Table 5 of Ref. *(Konishi, Ukawa, Veneziano, Nucl. Phys. B157 (1979), 45 –jet calculus)*

2) The analog of a double-Regge limit

$$\boldsymbol{1 - x_1 \ll 1, \quad 1 - x_1 - x_2 \ll 1 - x_1}$$

$$\boldsymbol{D_i^{j_1 j_2}(x_1, x_2, t) \sim H(t)(1 - x_1)^{k_{i,j_1 j_2}}(1 - x_1 - x_2)^{2C_F t + h_{i,j_1 j_2}},}$$

up to logarithmic terms. The exponents $k_{i,j_1 j_2}$ and $h_{i,j_1 j_2}$ can be computed and are given in Table 6 of Ref. *(KUV)*

Hadron level:

1) If nonperturbative input for dPDFs has a power of zero at $x_1 + x_2 \to 1$
is weaker than perturbative evolution (second)term, mpi15
then all results (at finite x_1, x_2) above are valid !

2) Moreover, in asymptotic: at large enough $t(Q)$
the second term is dominant and again results above are valid !

Thus, at x close to 1 dPDFs include the factors

$$\boldsymbol{(1 - x_1 - x_2),\ (1 - x_1),\ (1 - x_2)}$$

with the exponents depending on parton types.

These exponents are known at the parton level and can be calculated in principle at the hadron level fixing the asymptotic form of initial conditions near this kinematical boundary.

The problem of specifying the initial conditions for the evolution equations, which would obey exactly momentum sum rules and have the correct asymptotic behavior near the kinematical boundaries is not trivial and is under extensively study
(Snigirev, Gaunt, Stirling, Chang,....,Rinaldi,..., Golec-Biernat,...).

Phenomenology:

How to probe

$$(1 - x_1 - x_2)$$

assotiated W(Z)D production at the LHCb condition ?:

heavy system + large rapidity —> large enough x

Hint:

there is some discrepancy in assotiated ZD production at the LHCb
(LHCb Collab. JHEP 14 01 (2014) 091).

	measured	SPS	SPS	DPS
$Z + D^0$	**2.5**	0.85	0.64	**3.28**
$Z + D^+$	**0.44**	0.37	0.28	**1.29**

measured (pb) $<$ DPS (pb)

The correction due to the limited partonic phase space allows us to avoid the discrepancy above.

Numericaly the corrections from the limited partonic phase spase amount to a factor 2 in the total rates at the LHCb conditions with ZD or WD associated production taken as examples.

After applying the correction factor

$$(1 - x_1 - x_2)^n, \qquad [n = 2(3)]$$

the agreement with data becomes rather satisfactory
(Specification in calculations, results and plots in Zotov's talk + in preparation).