Double parton scattering in the ultraviolet

addressing the double counting problem

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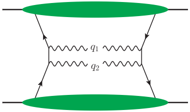
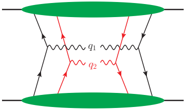
Reminder: single vs. double hard scattering (SPS vs. DPS)

- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$





single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q^2

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q^2$

double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q^2$

- for transv. momenta $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

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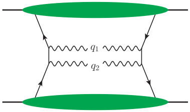

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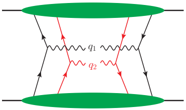


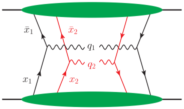
double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q^2$

- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- depending on process: enhancement or suppression
  from parton type (quarks vs. gluons), coupling constants, etc.

Double parton scattering

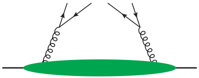
$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



$C =$ combinatorial factor
$\hat{\sigma}_i =$ parton-level cross sections
$F(x_1, x_2, \boldsymbol{y}) =$ double parton distribution (DPD)
$\boldsymbol{y} =$ transv. distance between partons

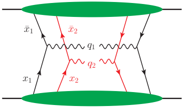
- at higher orders in α_s get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F
- analogous formulation for measured $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$
  $\rightsquigarrow$ transverse-momentum dependent DPDs
- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2}\ \text{splitting fct} \otimes \text{usual PDF}$$



Double parton scattering: ultraviolet problem

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1 \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



$$\begin{aligned} C &= \text{combinatorial factor} \\ \hat{\sigma}_i &= \text{parton-level cross sections} \\ F(x_1, x_2, \boldsymbol{y}) &= \text{double parton distribution (DPD)} \\ \boldsymbol{y} &= \text{transv. distance between partons} \end{aligned}$$

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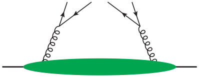
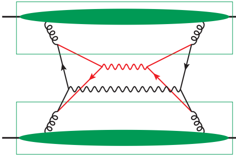
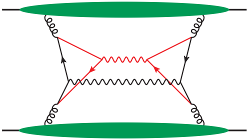


gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$
in fact, formula not valid for $|\boldsymbol{y}| \sim 1/Q$

...and more problems





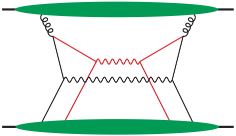
- double counting problem between double scattering with splitting and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: "1 vs 2"

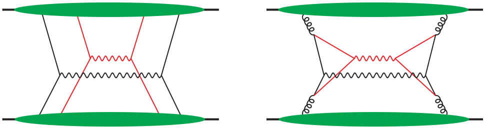
$$\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \times F_{\text{non-split}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



A consistent solution

MD, J. Gaunt work in progress



- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\, \Phi(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and non-splitting contributions

  analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    (much simpler computation than σ_{SPS} at given order)

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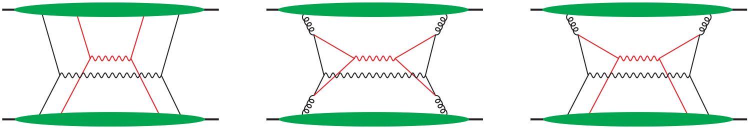

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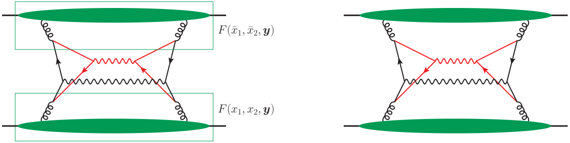
  analogous regulator for transverse-momentum dependent DPDs

- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1vs1 + 1vs2)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    (much simpler computation than σ_{SPS} at given order)
  - can also include twist 2 $\times$ twist 4 contribution
    and double counting subtraction for "1 vs 2" term

Subtraction formalism at work



$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \lesssim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \ \sigma \approx \sigma_{\text{SPS}}$ dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub}
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \ \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- same argument for 1 vs 2 term and $\sigma_{\text{tw}2\times\text{tw}4}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory
  J. Collins, Foundations of Perturbative QCD, Chapt. 10

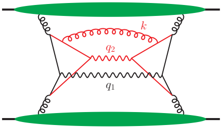
Added benefit: DGLAP logarithms

- define DPDs as matrix elements of renormalised twist-two operators:

  $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\mathcal{O}_1(\mathbf{0}; \mu_1)\, \mathcal{O}_2(\boldsymbol{y}; \mu_2)|p\rangle \qquad f(x; \mu) \sim \langle p|\mathcal{O}(\mathbf{0}; \mu)|p\rangle$

  $\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

$$\frac{d}{d\log\mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F \qquad \text{for } i = 1, 2$$



- for $Q_1 \ll Q_2$ higher orders in box graph give logarithms $\alpha_s^n \log^n(Q_2/Q_1)$ of DGLAP type from region $Q_1 \ll |\boldsymbol{k}_1| \ll \cdots \ll |\boldsymbol{k}_n| \ll Q_2$
  - resummed by DPD evolution if take $\nu \sim \mu_1 \sim Q_1$, $\mu_2 \sim Q_2$ and appropriate initial conditions, e.g. $F = F_{\text{split}} + F_{\text{non-split}}$

$$F_{\text{split}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2\Lambda^2} \quad \text{with} \quad 1/y^{*2} = 1/y^2 + 1/y_{\text{max}}^2$$

$$F_{\text{non-split}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0)\, \Lambda^2 e^{-y^2\Lambda^2}/\pi$$

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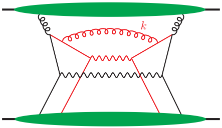
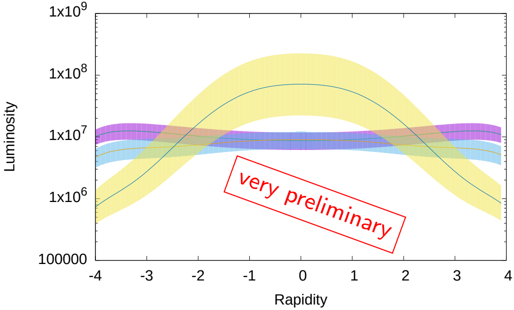


- lowest order 1 vs 2 term $\propto \log(Q/\Lambda)$
  additional logs $\alpha_s^n \log^{n+1}(Q/\Lambda)$ from $\Lambda \ll |\boldsymbol{k}_1| \ll \cdots \ll |\boldsymbol{k}_n| \ll Q$
  - again resummed by DPD evolution if take $\nu \sim \mu_1 \sim \mu_2 \sim Q$ with same initial conditions for F
  - with $\nu \sim Q$ have no $\log(Q/\Lambda)$ in $\sigma_{\text{tw2}\times\text{tw4}} - \sigma_{\text{sub (1vs2)}}$
    provides justification to omit this term while keeping 1 vs 2 in σ_{DPS}
- after Fourier trf. our σ_{DPS} is very similar to M Ryskin, A Snigirev 2011, 2012

DPS parton luminosities
for illustration, model parameters not tuned

- plot $\int d^2\boldsymbol{y}\, \Phi(\nu y) F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$ vs. rapidity of q_1 with q_2 central and $Q_1 = Q_2 = M_W$ at $\sqrt{s} = 14\,\mathrm{TeV}$
- bands for 2 vs 2 (violet), 1 vs 2 (blue) and 1 vs 1 (yellow) with scales $\nu = \mu_1 = \mu_2 = 0.5\,M_W \ldots 2\,M_W$

$u\bar{u}$



- 1 vs 1 term has strong cutoff dependence $\propto \nu^2$ if is important must add $-\,\sigma_{\text{sub (1vs1)}} + \sigma_{\text{SPS}}$

DPS parton luminosities for illustration, model parameters not tuned

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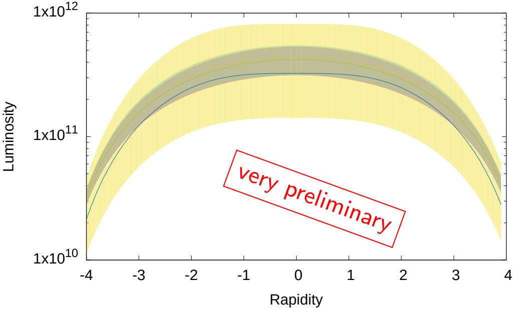
gg



- 1 vs 1 important, but not as much as for $u\bar{u}$

DPS parton luminosities for illustration, model parameters not tuned

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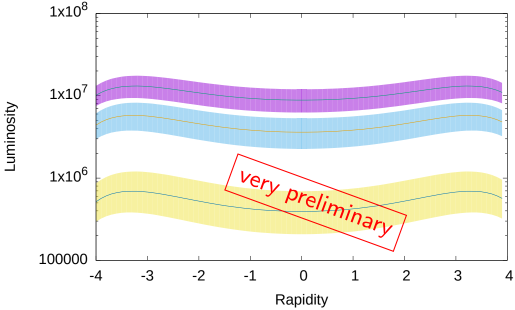
$u\bar{d}$



- $u\bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$, e.g. by $u \to ug \to ud\bar{d}$

A comment on sum rules

- $F(x_1, x_2, \boldsymbol{y})$ follows homogeneous DGLAP equation
  no splitting term $\rightsquigarrow$ does not conserve sum rules for $\int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})$
  J Gaunt, J Stirling 2009
- is irrelevant if cannot satisfy sum rules at some scale μ
  - if def. $F(x_1, x_2, \boldsymbol{y})$ by min. subtraction of UV divergences
    $\rightsquigarrow \int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y}) = \infty$
    due to splitting at short distances
    i.e. same physics that would provide inhomogenous term in evolution
- to use sum rules as constraint for DPD modelling
  must subtract infinite splitting contribution such that result
  - fulfills sum rule
  - enters in factorisation formula for cross section

This is not the case in any known scheme
$\Rightarrow$ at present sum rules have no theory justification

Summary

- double parton scattering important
  in specific kinematics/for specific processes
- recent progress:
  towards a systematic formulation of factorisation in QCD
- solution for UV problem of DPS $\leftrightarrow$ double counting with SPS
  - simple UV regulator for DPS using distance y between partons
  - simple subtraction term to avoid double counting order by order in perturbation theory

  naturally includes "1 vs 2" contributions
  and correctly resums DGLAP logarithms
- distinction between "splitting" and "non-splitting" in DPD
  necessary in ansatz for DPD (inevitable model dependence)
  but not in formulation of factorisation