The effective cross section of double parton scattering in a Light-Front quark model

Matteo Rinaldi[1]

In collaboration with:

Sergio Scopetta[1], **Marco Traini**[2], **Vicente Vento**[3]

[1]Dep. of Physics and Geology, Perugia University and INFN, Perugia, Italy

[2]Dep. of Physics Trento University and INFN-TIFPA, Italy

[3]Dep. of Theor. Physics, Valencia University and IFIC, Valencia, Spain

Outlook

- Double parton scattering (DPS) and double parton distribution functions (dPDFs)

- The 3D proton structure in single & double parton scatterings

- Double parton correlations (DPCs) in double parton distribution functions

- dPDFs in constituent quark models

  M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013)
  M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014)

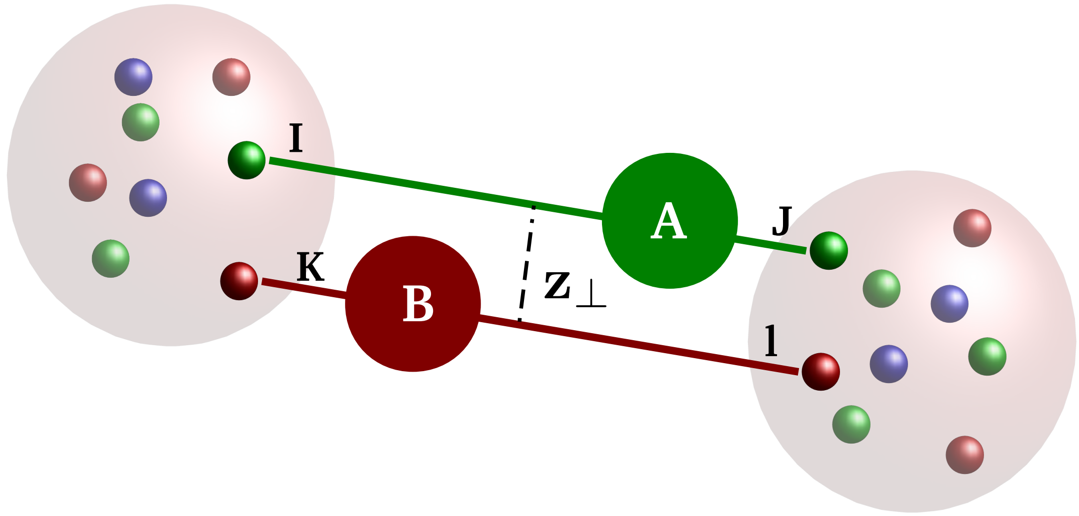
- Calculation of the "effective X-section"

  M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016), arXiv:1506.05742 [hep-ph]

- Conclusions

DPS and dPDFs from multi parton interactions

Multi parton interaction (MPI) can contribute to the, *pp* and *pA*, cross section @ the LHC:



The cross section for a DPS event can be written in the following way:
(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))

dPDF

Momentum scale

$$d\sigma = \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(x_1,x_3,\mu_A)\hat{\sigma}_{kl}(x_2,x_4,\mu_B)\int d\tilde{z}_\perp F_{ik}(x_1,x_2,z_\perp,\mu_A,\mu_B)F_{jl}(x_3,x_4,z_\perp,\mu_A,\mu_B)$$

Momentum fraction carried by the parton inside the hadron

Transverse distance between the two partons

DPS processes are important for fundamental studies, *e.g.* the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON

How 3-Dimensional structure of a hadron can be investigated?

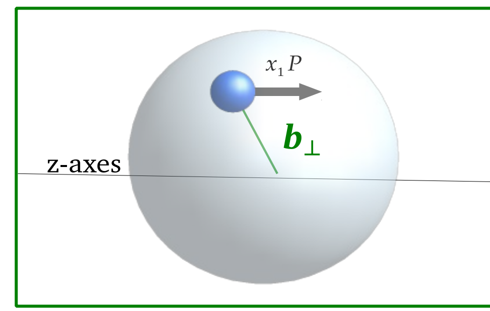
The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS, double parton sattering …), measuring different kind of Parton Distributions, providing different kind of information:

DVCS — ***Generalized Parton Distributions in impact parameter space***

$$\mathcal{H}(x_1, \boldsymbol{b}_\perp) \qquad \mathcal{E}(x_1, \boldsymbol{b}_\perp)$$

longitudinal momentum fraction carried by the parton

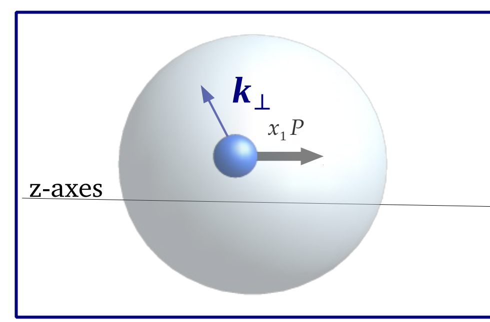
transverse distance between the parton and center of proton



SIDIS.. — ***Transverse Momentum Dependent parton distribution functions***

$$f_1(x_1, \boldsymbol{k}_\perp) \;\; g_{1L}(x_1, \boldsymbol{k}_\perp)\, h_1(x_1, \boldsymbol{k}_\perp) \;\; f_{1T}^{\perp}(x_1, \boldsymbol{k}_\perp)..$$

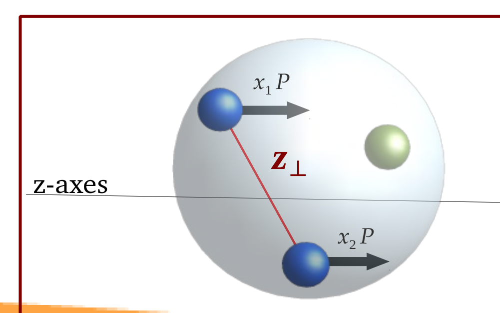
transverse component of the parton momentum



1 BODY

DPS — ***Double Parton Distribution Functions***

$$F_{UU}(x_1, x_2, \boldsymbol{z}_\perp) \;\; F_{LL}(x_1, x_2, \boldsymbol{z}_\perp)$$



2 BODY

Parton correlations and dPDFs

@ LHC kinematics it is often used a factorized form of the dPDFs: $(\mathbf{x_1}, \mathbf{x_2}) - \mathbf{z}_\perp$ factorization:

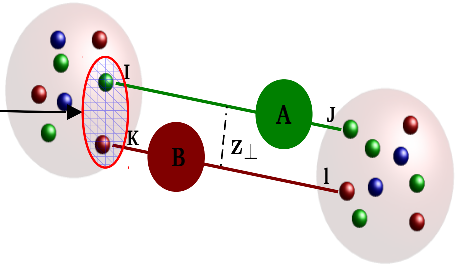
$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

* Here and in the following: $\mu = \mu_A = \mu_B$

and $\mathbf{x_1}, \mathbf{x_2}$ factorization:

$$F_{ij}(x_1, x_2, \mu) = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF}}\, q_j(x_2, \mu)}\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

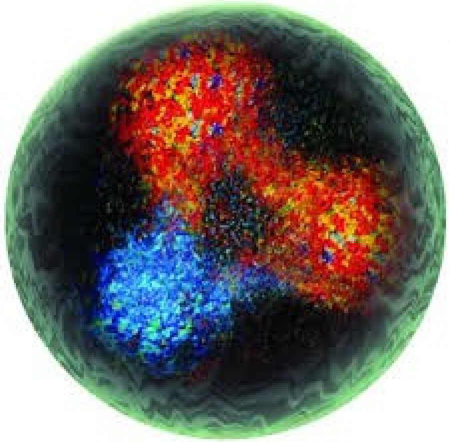
NO CORRELATION ANSATZ



In this scenario, parton correlations inside the proton are neglected!

- In principle, they are present!
- Several authors addressing this issue:
  Many published papers: Calucci and Treleani (1999), Korotkikh and Snigirev (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012-2014), Schweitzer, Strikman and Weiss (2013), Gaunt and Szczurek (2015).......
- dPDFs are non-perturbative quantities ⟺ DPCs not calculated directly from QCD

DPCs in constituent quark models (CQM)



- Main features:
  - potential model
  - effective particles
  - particles are strongly bound and correlated

- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region at low energy scale, while LHC data are available at small x
- At very low x , due to the large population of partons, the role of correlations may be less relevant BUT theoretical microscopic estimates are necessary

pQCD evolution of the calculated dPDFs is necessary to move towards the experimental kinematics!

- CQM calculations are able to reproduce the gross-feature of experimental PDFs in the valence region
- Results can be quite general. In DIS Physics, CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

The Light-Front approach

Relativity can be implemented, for a CQM, by using a Light-Front (LF) approach yielding, among other good features, the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

- Full Poincaré covariance
- fixed number of on-mass-shell particles

Among the 3 possibles forms of RHD we have chosen the LF one since there are several advantages. The most relevant are the following:

- ✓ 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\mathbf{P}^+$, $\mathbf{P}_\perp$, iii) Rotation around z.
- ✓ The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- ✓ in a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- ✓ The IMF (Infinite Momentum Frame) description of DIS is easily included.

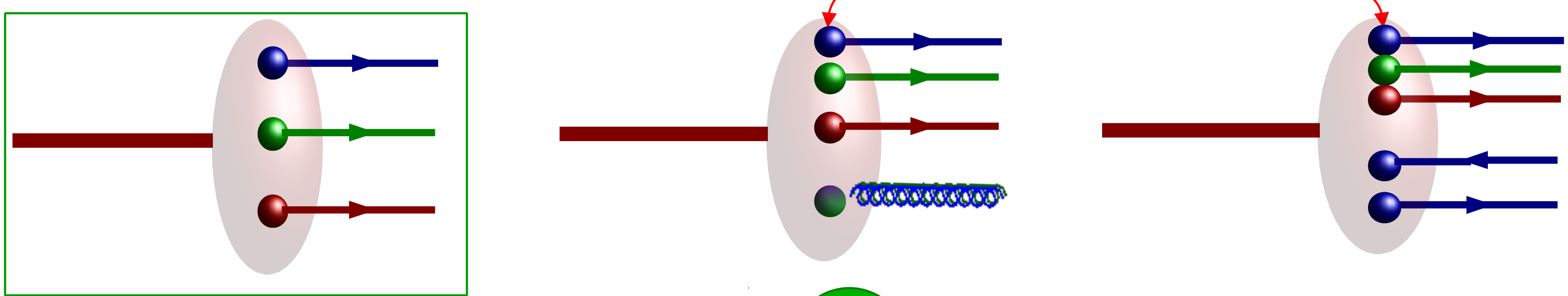
The LF approach is extensively used for hadronic studies (e.m. form factors, PDFs, GPDs, TMDs........)

A Light-Front wave function representation

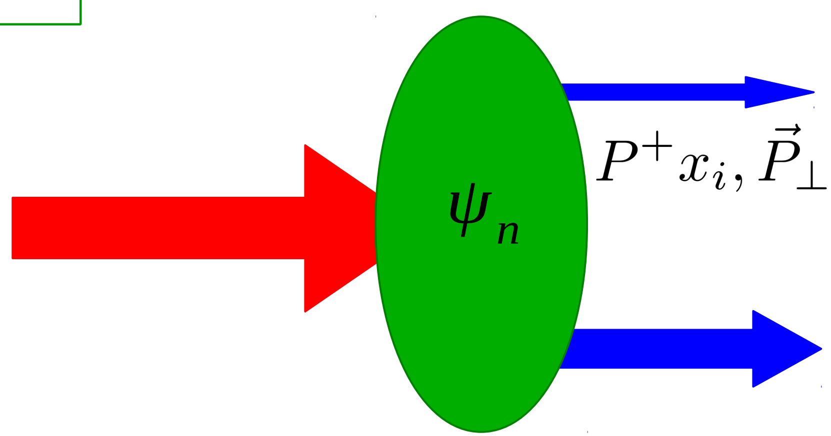
The proton wave function can be represented in the following way:

see *e.g.*: **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+\ \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqq\ g}|qqq\ g + \psi_{qqq\ q\bar{q}}|qqq\ q\bar{q}\rangle$$



p: $P^+\ \vec{P}_\perp$



$P^+ x_i, \vec{P}_\perp x_i, \vec{k}_{\perp i}$

$$\begin{cases} x_i = \dfrac{k_i^+}{P^+} \\ \sum_i^n x_i = 1 \\ \sum_i^n \vec{k}_{\perp i} = 0 \end{cases}$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i)$ ⟷ Invariant under LF boosts!

A Light Front wave function representation

It is possible to connect the front-form description of states and the canonical, instant-form one:

See *e.g.*: **B. D. Keister, W. N. Polyzou Adv. Nucl .Phys. 20, 225 (1991)**

$$|\vec{k}_\perp, \lambda, \tau\rangle_{[l]} \propto \sqrt{2k_0} \sum_{\lambda'} D^{1/2}_{\lambda\lambda'}(R_{il}(\vec{k})) |\vec{k}_\perp, \lambda', \tau\rangle_{[i]}$$

→ Melosh rotations

A relation between hadron wave functions $\psi^{[l]}_\lambda, \ \psi^{[i]}_\lambda$ can be obtained, *e.g.* for n=3:

$$\psi^{[l]}_\lambda(\beta_1, \beta_2, \beta_3) \propto \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right] \sum_{\mu_1\mu_2\mu_3} D^{1/2*}_{\mu_1\lambda_1}(R_{il}(\vec{k}_1)) D^{1/2*}_{\mu_2\lambda_2}(R_{il}(\vec{k}_2)) D^{1/2*}_{\mu_3\lambda_3}(R_{il}(\vec{k}_3)) \psi^{[i]}_\lambda(\alpha_1, \alpha_2, \alpha_3)$$

$$\begin{cases} \beta_i = x_i, \vec{k}_i, \lambda_i, \tau_i \\ \alpha_i = \vec{k}_i, \mu_i, \tau_i \\ \omega_i = k_{0i} \\ M_0 = \sum_i \omega_i \end{cases}$$

dPDFs in a Light-Front approach

Extending the procedure developed in S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003) for GPDs, we obtained the following expression of the dPDF in momentum space, often called ${}_2$GPDs from the Light-Front description of quantum states in the intrinsic system:

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3$$

$$F_{ij}(x_1, x_2, k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, k_\perp)\Phi(\{\vec{k}_i\}, -k_\perp)$$

$$\times \quad \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

Conjugate to $z_\perp$

$$M_0 = \sum_i \sqrt{\vec{k}_i^2 + m^2}$$

GOOD SUPPORT

$$\mathrm{x}_1 + x_2 > 1 \Rightarrow F_{ij}(x_1, x_2, k_\perp) = 0$$

$$\Phi(\{\vec{k}_i\}, \pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

Now we need a model to properly describe the hadron wave function in order to estimate the LF ${}_2$GPDs

$$\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3) = D^{\dagger 1/2}(R_{il}(\vec{k}_1)) D^{\dagger 1/2}(R_{il}(\vec{k}_2)) D^{\dagger 1/2}(R_{il}(\vec{k}_3)) \psi^{[i]}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

Melosh rotation

Instant form proton w.f.
We need a CQM!

A Hyper-central CQM

We have chosen the following CQM for its capability to basically describe the hadron spectrum, despite of its simplicity (**P. Faccioli, M. Traini, V. Vento, Nucl. Phys. A 656, 400-420 (1999)**)

$$\psi^{[i]} = \frac{1}{\pi\sqrt{\pi}} \Psi(k_\xi) \times SU(6)_{spin-isospin}$$

$$k_\xi = \sqrt{2(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)}$$

Where the function $\Psi(k_\xi)$ is solution of the Mass equation:

$$(M_0 + V)\Psi(k_\xi) \equiv \left(\sum_i^3 \sqrt{\vec{k}_i^2 + m^2} - \frac{\tau}{\xi} + \kappa_l \xi \right) \Psi(k_\xi) = M\Psi(k_\xi)$$

$$\tau = 3.30, \qquad \kappa_l = 1.80 \text{ fm}^{-2}$$

$$\Psi(k_\xi) = \sum_{\nu=0}^{16} c_\nu \frac{(-1)^\nu}{\alpha^3} \left[\frac{2\nu!}{(\nu+2)!} \right]^{1/2} e^{-k_\xi^2/(2\alpha^2)} \sum_{m=0}^{\nu} \frac{(-1)^m}{m!} \frac{(\nu+2)!}{(\nu-m)!(m+2)!} \left(\frac{k_\xi^2}{\alpha^2} \right)^m$$

$$\alpha = 7.9 \text{ fm}^{-1}$$

c_ν: expansion coefficients

This model has been used in:

P. Faccioli, M. Traini, V. Vento, NPA 656, 400-420 (1999)
S. Boffi, B. Pasquini and M. Traini, NPB 649, 243 (2003)
S. Boffi, B. Pasquini and M. Traini, NPB 680, 147-163 (2004)...
M. Traini, PRD89, 034021 (2014)

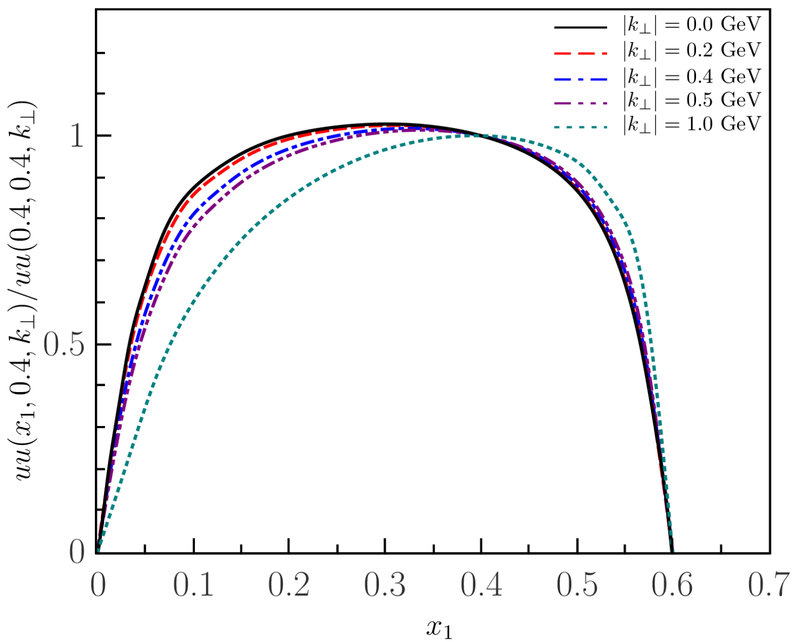
Numerical Results

(M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014))

Here, ratios, sensitive to correlations, are shown in order to test the factorization ansatz!

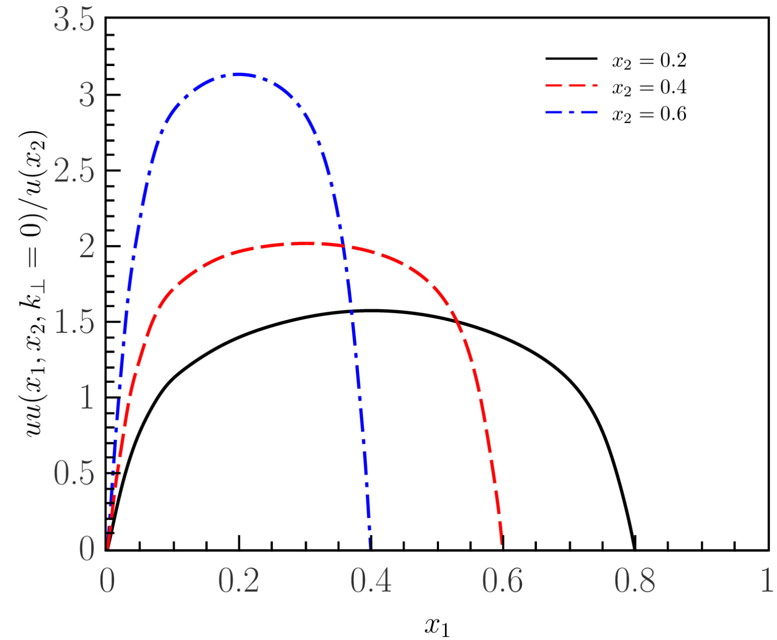
$$\frac{uu(x_1, 0.4, k_\perp)}{uu(0.4, 0.4, k_\perp)}$$

Factorization: $(x_1, x_2) - k_\perp$



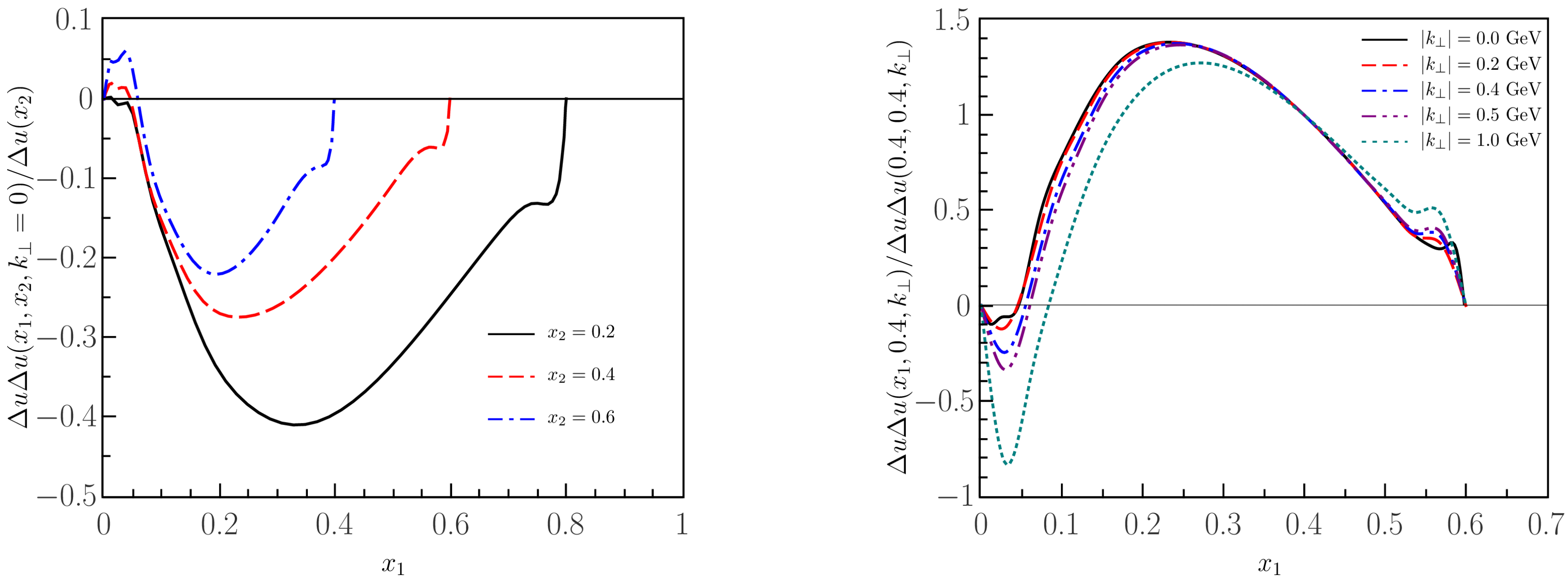
$$\frac{uu(x_1, x_2, k_\perp = 0)}{u(x_2)}$$

Factorization: $x_1 - x_2$



- Thanks to the good support of the calculated $_2$GPDs, the symmetry, due to the particle indistinguishability, is found! $\mathbf{uu(x_1, x_2, k_\perp = 0) = uu(x_2, x_1, k_\perp = 0)}$
- The $(x_1, x_2) - k_\perp$ and $x_1 - x_2$ factorizations are violated!

Results for spin correlations



$$u_{\uparrow(\downarrow)}u_{\uparrow(\downarrow)}(x_1,x_2,k_\perp)3(\sqrt{3})^3\int\prod_{i=1}^{3}d\vec{k}_i\delta\left(\sum_{i=1}^{3}\vec{k}_i\right)\delta\left(x_1-\frac{k_1^+}{M_0}\right)\delta\left(x_2-\frac{k_2^+}{M_0}\right)\Phi^*(\{\vec{k}_i\},k_\perp)\frac{1\pm\sigma_3(1)}{2}\frac{1\pm\sigma_3(2)}{2}\Phi(\{\vec{k}_i\},-k_\perp)$$

Here we have calculated: $\Delta u\Delta u(x_1,x_2,k_\perp)=\sum_{i=\uparrow,\downarrow}u_iu_i-\sum_{i\neq j=\uparrow,\downarrow}u_iu_j;$

$$|\Delta u\Delta u|\leq uu$$

Positivity bound

(defined in **M. Diehl et Al, JHEP 1203, 089 (2012), M. Diehl and T. Kasemets, JHEP 1305, 150 (2013))**

This particular distribution, different from zero also in an unpolarized proton, contains more information on spin correlations, which could be important at small x and large *t* (LHC) !

Also in this case, both factorizations, $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ are strongly violated!

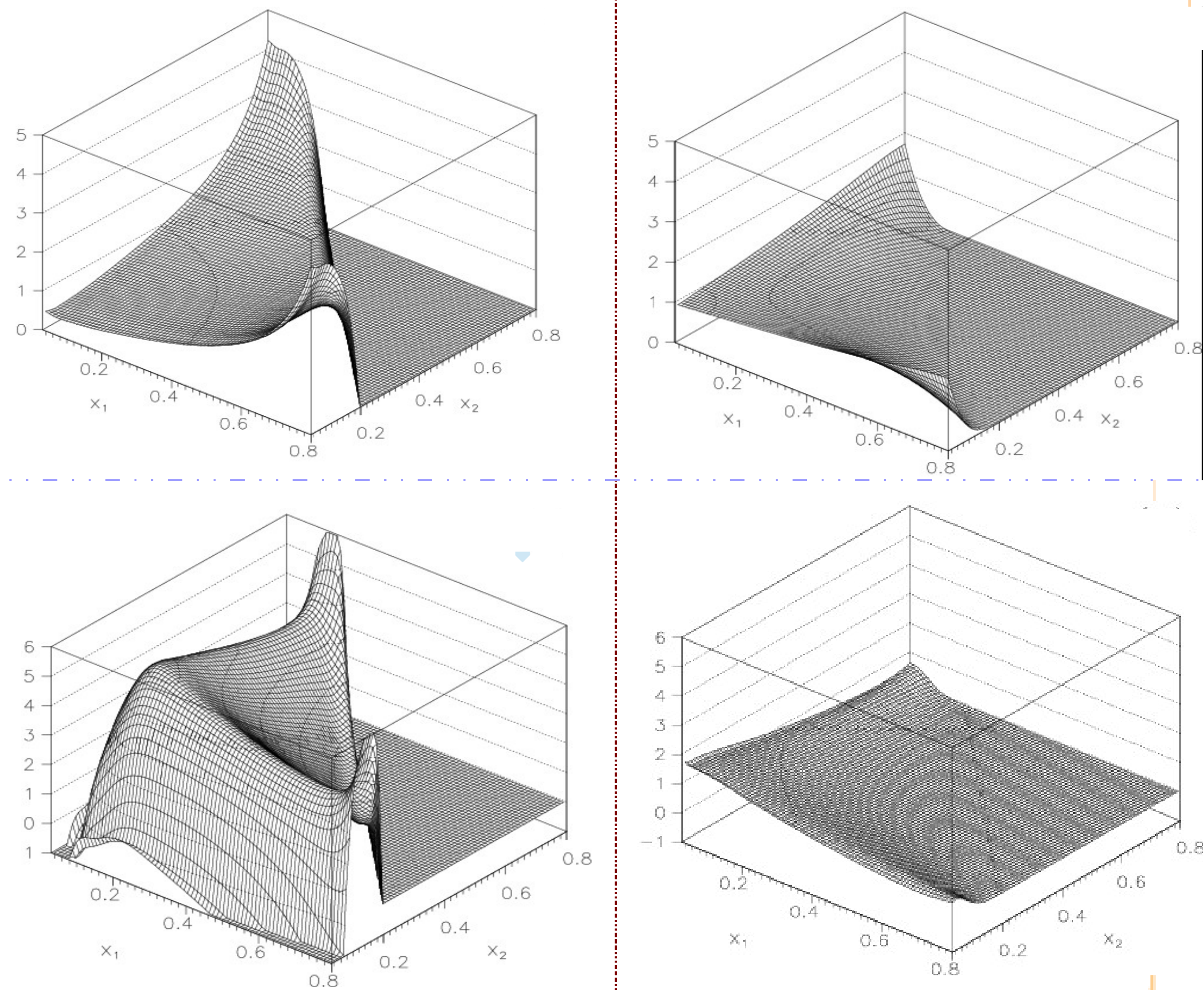
A pQCD evolution of the LF $_2$GPDs: the non-singlet sector

$Q^2 = \mu_0^2 \simeq 0.1 \text{ GeV}^2$ $\qquad$ $Q^2 = 10 \text{ GeV}^2$

$$\mathbf{r_u(x_1, x_2; Q^2) = C_u \frac{uu(x_1, x_2; Q^2)}{u(x_1; Q^2) u(x_2; Q^2)}}$$

$$\mathbf{r_{\Delta u}(x_1, x_2; Q^2) = C_{\Delta u} \frac{\Delta u \Delta u(x_1, x_2; Q^2)}{\Delta u(x_1; Q^2) \Delta u(x_2; Q^2)}}$$

$$C_i = \frac{[\int dx F_i]^2}{\int dx_1 dx_2 F_{ii}(x_1, x_2, k_\perp = 0)}$$



All these ratios would be 1 if there were no correlations!

The Effective X-section

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section": σ_{eff}

This object can be defined through the "pocket formula":

$$\sigma_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

Sensitive to correlations → σ_{eff}

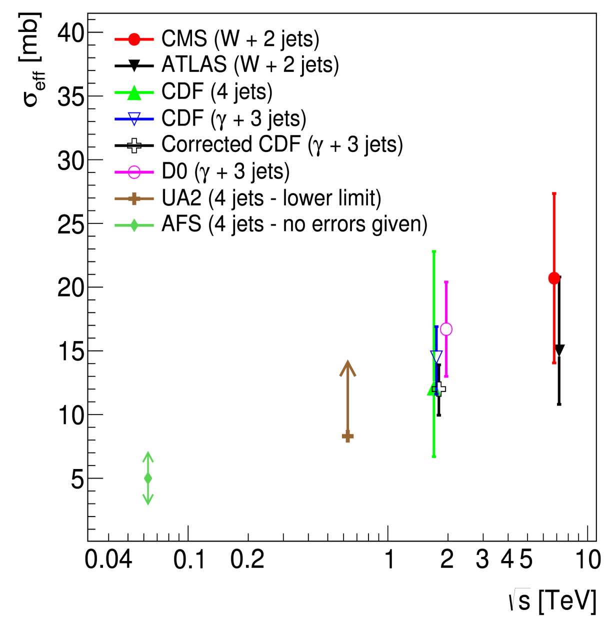
Combinatorial factor ← $\frac{m}{2}$

Differential cross section for the process: pp' $\rightarrow A(B) + X$ ← $\sigma_B^{pp'}$

Differential cross section for a DPS event: pp' $\rightarrow A + B + X$ ← σ_{double}^{pp}

….EXPERIMENTAL STATUS:

- Difficult extraction, approved analysis of for the production of same sign *WW* @LHC (RUN 2)
- the model dependent extraction of σ_{eff} from data is consistent with a "constant" , nevertheless there are large errorbars:
- different ranges in x_i accessed in different experiments!



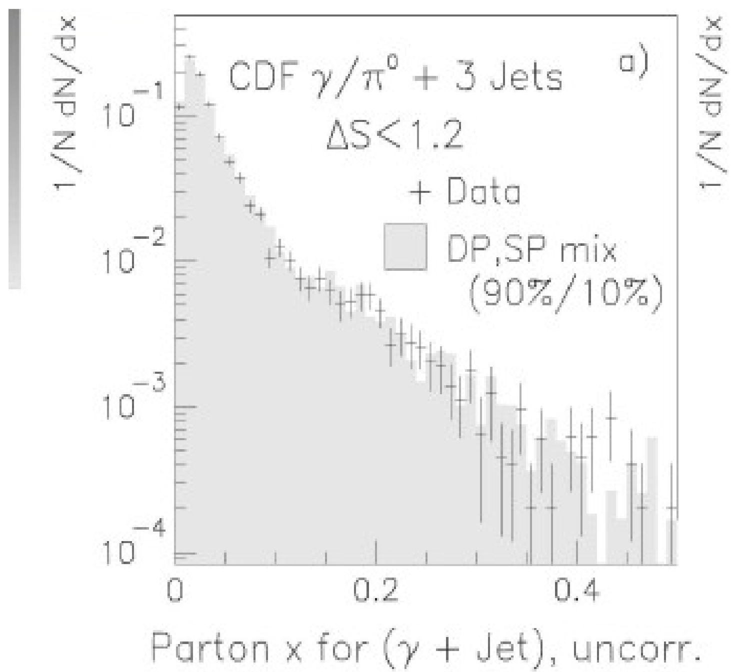
High x for hard jets (heavy particles detected, large partonic *s*):

AFS ⟶ $y \sim 0;\ x_1 \sim x_2;\ 0.2 < x_{1,2} < 0.4$

CDF ⟶ $0.02 < x_{1,2,3,4} < 0.4$

} **valence region included!**

σ_{eff} : x DEPENDENCE (?)



CDF, F. Abe et al.

PRD 56, 3811 (1997)

Shaded area: Montecarlo *without* correlations in x

Data well described (?) for x_1, x_2, x_1', x_2' in $[0.02, 0.4]$ (also in the valence region...)

May be not enough accuracy for high x? No x dependence?

Actually, our understanding is that, in the valence region, x dependence has to be seen. Let the model guide us...

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, arXiv:1506.05742 [hep-ph], PLB 752, 40 (2016)

$$\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\,\sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

This quantity can be written in terms of PDFs and dPDFs!

In terms of PARTON DISTRIBUTIONS, $\sigma_{A(B)}^{pp'}$ and σ_{double}^{pp} can be written as follows:

$$\sigma_{A(B)}^{pp'}(x_1, x_1', \mu_1) = \sum_{i,k} F_i^p(x_1,\mu_1) F_k^{p'}(x_{1'},\mu_1)\hat{\sigma}_{ik}^{A(B)}(x_1, x_{1'}, \mu_1)$$

$i,k = \{q, \bar{q}, g\}$

Standard PDF

Proportional to colour coefficient and universal function: $C_{ij}\bar{\sigma}(x,x')$

$$\sigma_{double}^{pp}(x_1, x_1', x_2, x_2', \mu) = \frac{m}{2}\sum_{i,j,k,l}\hat{\sigma}_{ik}^{A}(x_1,x_1',\mu)\hat{\sigma}_{jl}^{B}(x_2,x_2',\mu) \times \int \frac{d\vec{k}_\perp}{(2\pi)^2} F_{ij}(x_1,x_2,k_\perp,\mu) F_{kl}(x_1',x_2',-k_\perp,\mu)$$

$_2$GPDs

Finally, combining the previous equations in the "pocket formula", one obtains:

Here the scale is omitted

$$\sigma_{\mathbf{eff}}(\mathbf{x_1},\mathbf{x_1'},\mathbf{x_2},\mathbf{x_2'}) = \frac{\sum_{\mathbf{i,k,j,l}}\mathbf{F_i(x_1)F_k(x_1')F_j(x_2)F_l(x_2')C_{ik}C_{jl}}}{\sum_{\mathbf{i,j,k,l}}\mathbf{C_{ik}C_{jl}}\int \mathbf{F_{ij}(x_1,x_2;k_\perp)F_{kl}(x_1',x_2';-k_\perp)}\frac{\mathbf{dk_\perp}}{(\mathbf{2\pi})^\mathbf{2}}}$$

Non trivial x-dependence

Factorization and effective X-section I

M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089

The dPDF is formally defined through the Light-cone correlator:

$$F_{ij}(x_1,x_2,\vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^{2} dl_i^- e^{ix_i l_i^- p^+}\right] \langle p|O(z,l_1)|X\rangle\langle X|O(0,l_2)|p\rangle\Big|_{l_1^+=l_2^+=z^+=0}^{\vec{l}_{1\perp}=\vec{l}_{2\perp}=0}$$

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+}|p'\rangle\langle p'|$$

$$F_{ij}(x_1,x_2,\vec{z}_\perp) \sim \int d\vec{b}(x_1,0,\vec{b}+\vec{z}_\perp)\tilde{f}(x_2,0,\vec{b})$$

GPDs in impact parameter space

$_2$GPDs $$F_{ij}(x_1,x_2,\vec{k}_\perp) \sim f(x_1,0,\vec{k}_\perp)f(x_2,0,\vec{k}_\perp)$$ GPDs

In GPDs, the variables $\vec{b}$ and x are correlated → Correlations between $\vec{z}_\perp$ and $x_1,\ x_2$ could be present in dPDFs !

Factorization and effective X-section II

In particular, at very low *x*, where gluon contributions dominate, one can also assume:

$$f_g(x,0,\vec{k}_\perp) \sim g(x) f(\vec{k}_\perp)$$

$$F_{gg}(x_1,x_2,\vec{k}_\perp) \sim g(x_1) g(x_2) f^2(\vec{k}_\perp)$$

Using this approximation, one finds the following expression for σ_{eff} :

$$\sigma_{eff} \propto \frac{1}{\int d\vec{k}_\perp f^4(k_\perp)}$$

***x* independent**

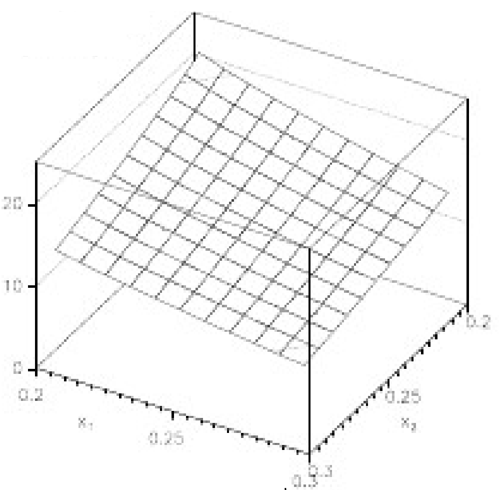
B. Blok, Y. Dokshitzer, L. Frankfurt and M. Strikman, Eur. Phys. J C 72, 1963 (2012)

Numerical results

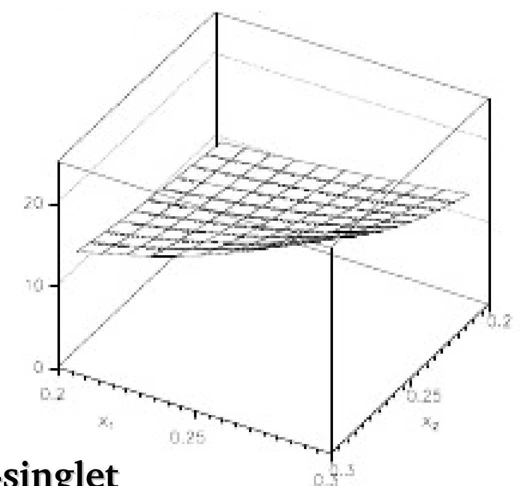
M. R., S. Scopetta, M. Traini and V.Vento, arXiv:1506.05742 [hep-ph], PLB 752, 40 (2016)

Our predictions of σ_{eff} in the valence region at different energy scales:

$\sigma_{eff}(x_1, x_2, \mu_0^2)$

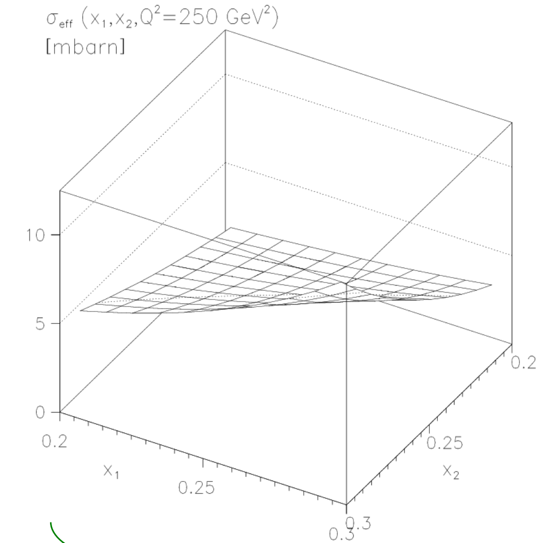


$\sigma_{eff}(x_1, x_2, Q^2 = 250 \text{ GeV}^2)$

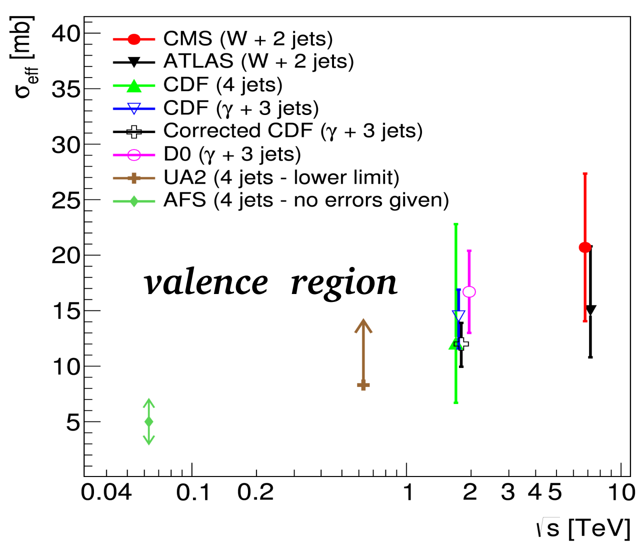


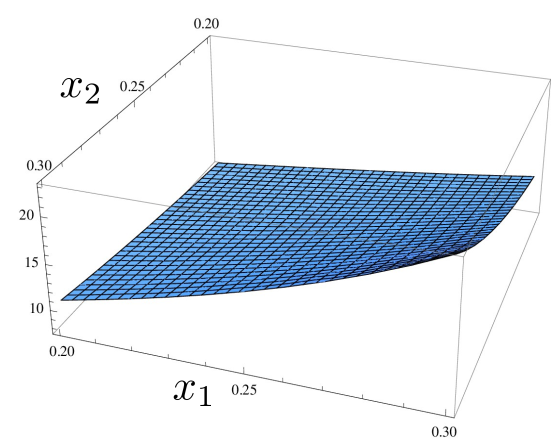
Non-singlet

$\overline{\sigma_{eff}} \sim 11$ mb



Non-singlet & gluons





PRELIMINARY

Non-singlet & sea

The old data lie in the obtained range of σ_{eff}

Conclusions

A CQM calculation of the dPDFs with a fully covariant approach:

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014)

- symmetry in the exchange of two partons in the dPDFs correctly restored
- violations of both the $(x_1, x_2) - k_\perp$ and x_1, x_2 factorizations for the polarized and unpolarized $_2$GPDs
- at very small x, the role of correlations is less important after evolution to experimental scales
- Spin correlations are still important also after pQCD evolution

Calculation of the effective X-section

M. R., S. Scopetta, M. Traini and V.Vento, arXiv:1506.05742 [hep-ph], PLB 752, 40 (2015)

- Calculation of the effective X-section at the hadronic and at high energy scales
- **x-dependent quantity obtained!** Qualitatively in agreement with data
- The x-dependence of the “effective X-section” could give information on the

3d structure of the proton!

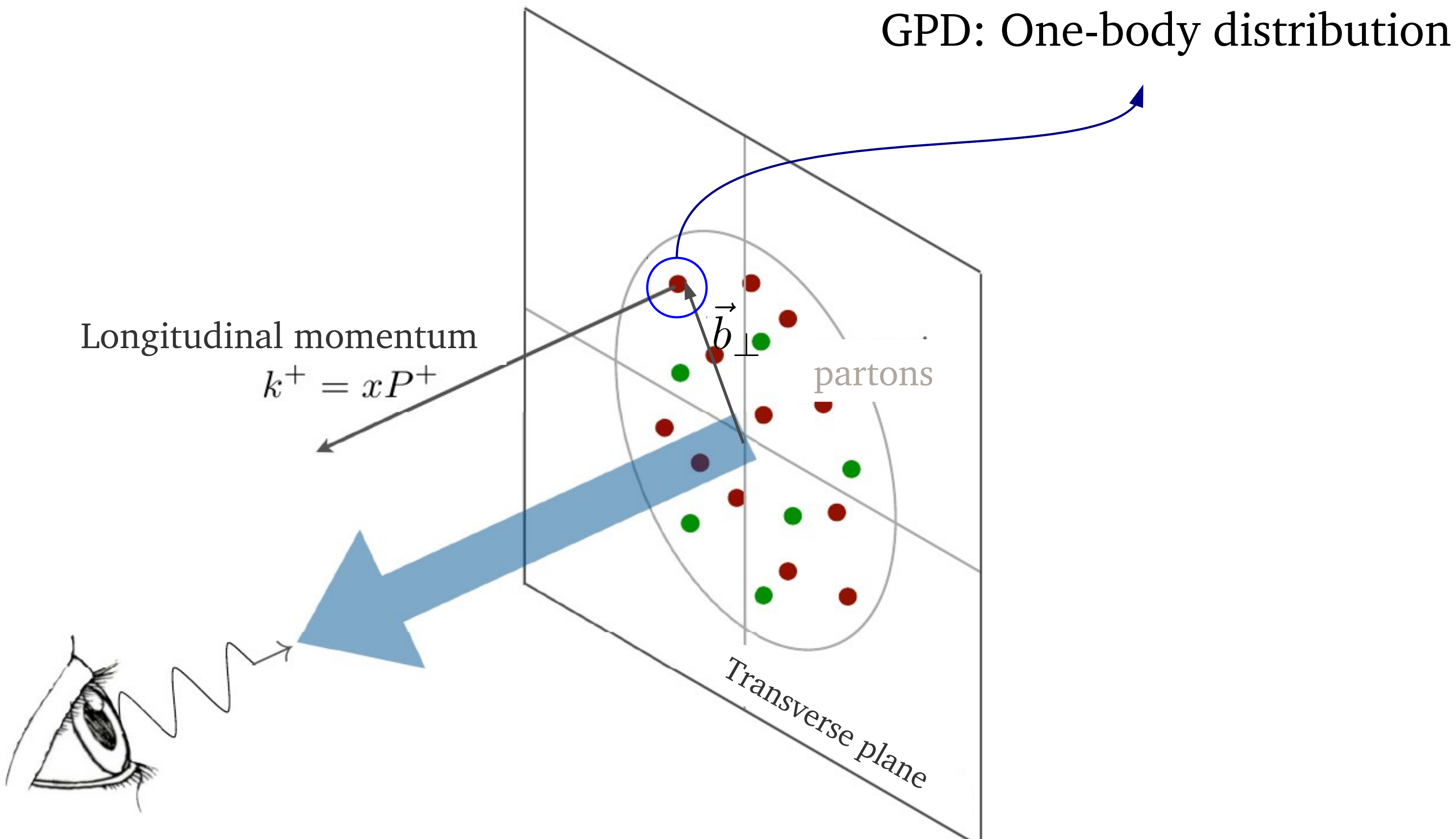
What are we working on

- pQCD evolution of the calculated $_2$GPDs taking into account the sea contribution; PRELIMINARY
- analysis of the inhomogeneous contribution in the pQCD evolution;
- Non perturbative Gluons and sea quarks (higher Fock states) to be included into the scheme.

Direct link to LHC Physics

The 3D proton structure

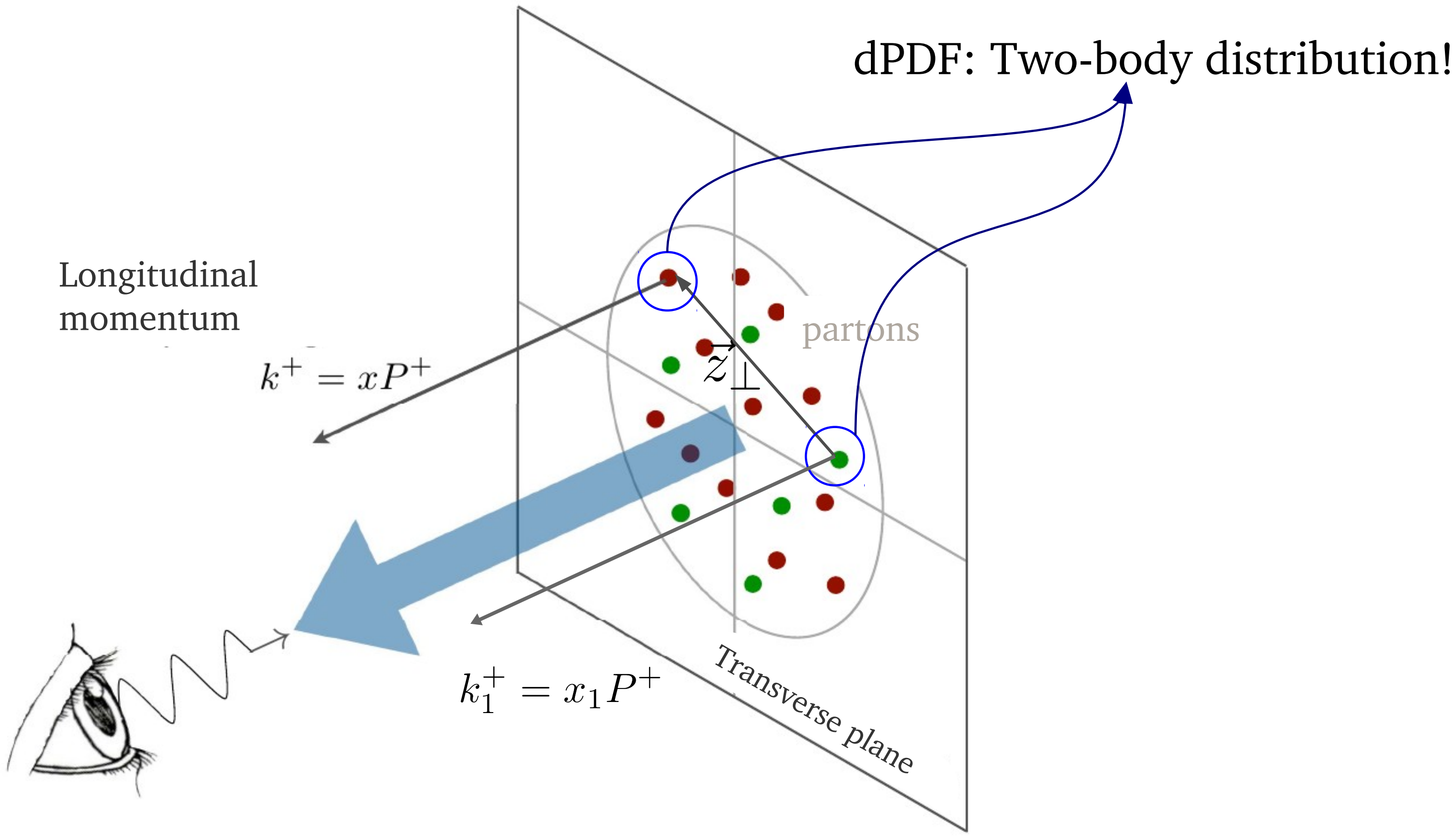
From Generalized Parton Distribtions (GPDs) in coordinate space, one obtains the **"PROBABILITY OF FINDING A PARTON WITH LONGITUDINAL MOMENTUM FRACTION $\mathbf{x}$ AND TRANSVERSE DISTANCE, FROM THE CENTER OF THE PROTON, $\mathbf{b}_\perp$ "**, being the conjugate variable to $\mathbf{\Delta}_\perp$, the momentum transferred.



Alessandro Bacchetta's slide

The 3D proton structure

dPDFs contain, w.r.t. the GPDs, new details on the 3D partonic structure, being two-body densities, sensitive to correlations, in principle.



New way to access information on the non-perturbative structure of the PROTON!

NR model calculations of quark-quark dPDFs

- A Non Relativistic (NR) reduction allows one to calculate it, in momentum space, in terms of intrinsic wave functions (WFs):

A.V. Manhoar and W.J. Waalewijn, PRD 85 114009 (2012))

$$F_{12}(x_1, x_2, \vec{k}_\perp) = 3\int d\vec{k}_1 d\vec{k}_2 \delta\left(x_1 - \frac{k_1^-}{P^-}\right)\delta\left(x_2 - \frac{k_2^-}{P^-}\right)$$

$$a^\pm = a_0 \pm a_3$$

$$\times \psi^*\left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2}\right)\hat{P}_{q_1}(1)\hat{P}_{q_2}(2)\psi\left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2}\right)$$

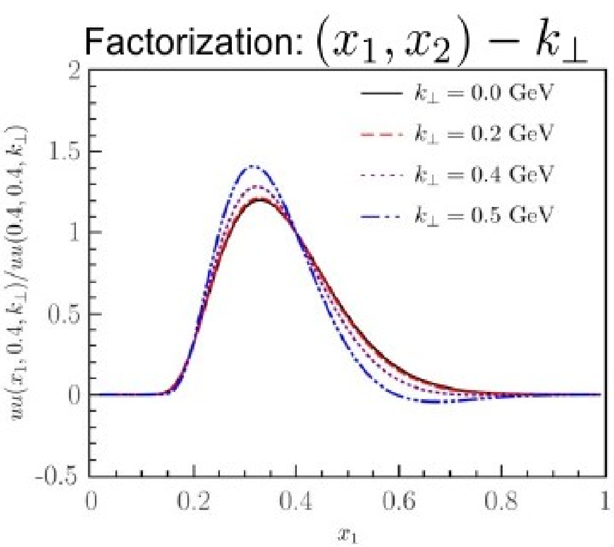
We need to choose the WF corresponding to a suitable CQM to perform the calculation

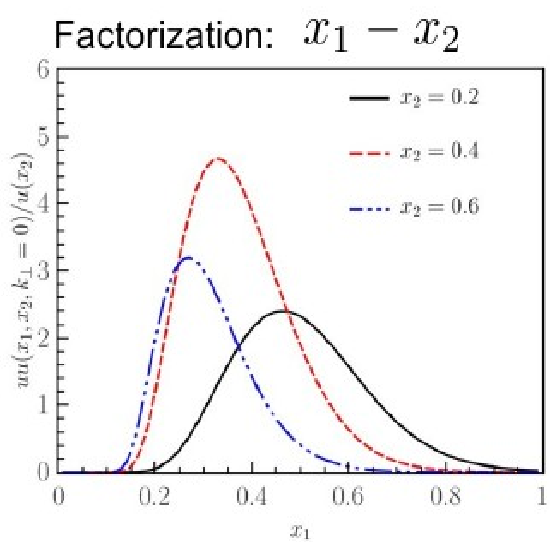
Flavor projector: $\hat{P}_{u(d)} = \frac{1 \pm \tau_3(i)}{2}$

Conjugated variable to $z_\perp$

For two quarks of flavours q_1= u, q_2=u:

(M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013))





The factorization ansatz are violated in the IK model!

Are improvements necessary? Yes!

To pQCD evolve

Overcome the so called "bad support problem ":
the dPDFs should be zero when: $x_2 + x_1 > 1$
This condition is not realized in the described CQM!

Implement the Relativity

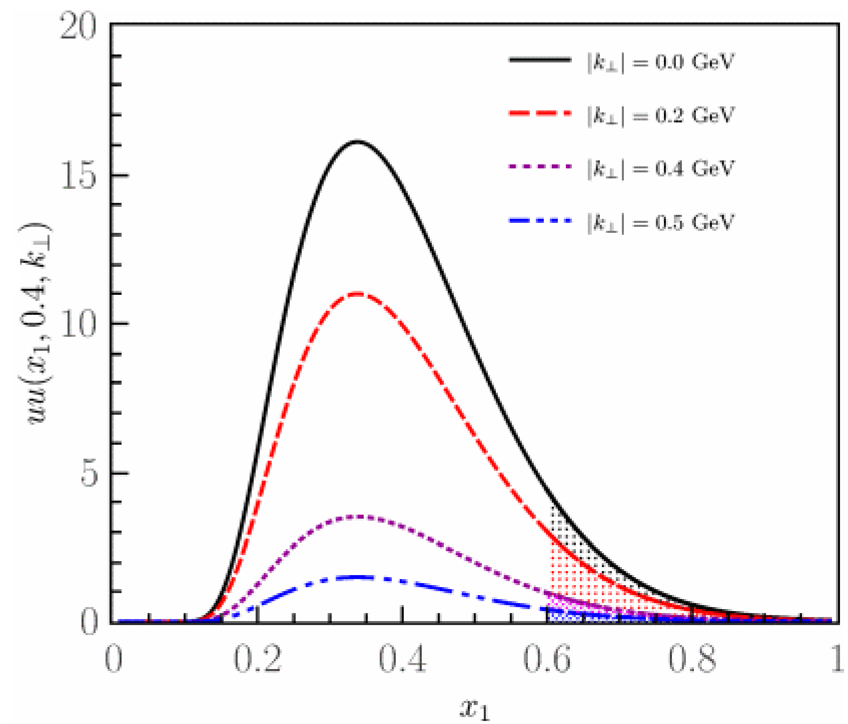
In this approach:

$$\delta\left(x_i - \frac{k_i^+}{P^+}\right) \Rightarrow \delta\left(x_i - \frac{k_i^+}{M}\right)$$

Proton Mass!

so that one finds:

$$\sum_i x_i > 1$$



we are not describing properly the off-shell "*i*-" parton inside the hadron

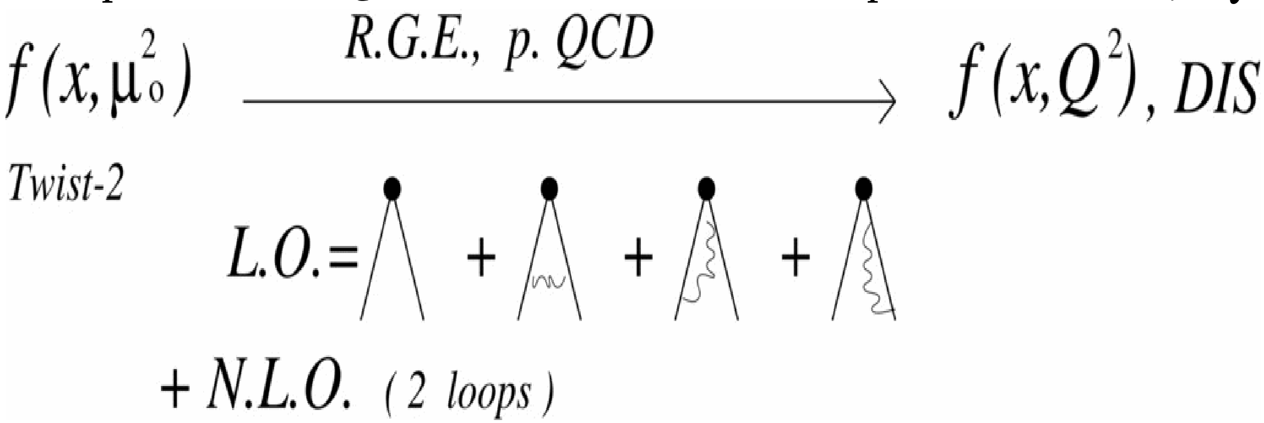
we need a new approach: the Light-Front

Model calculations of PDFs in the valence region

- In order to consistently compare data of twist-2 PDFs with the predictions of a CQM, one has to follow a 2-steps procedure:
  (firstly suggested by R.L. Jaffe and G.G. Ross, PLB 398 (1980) 313)

  1. evaluate in the model the twist-2 part of the corresponding observable, which has to be related to a low momentum scale, μ_0^2

  2. perform a perturbative QCD evolution to the DIS experimental scale, Q^2



Caveat: in the simplest CQM picture, ALL the gluons and sea quarks are perturbatively generated

Our first choice: the Isgur and Karl (IK) model

IK is based on a One Gluon Exchange (OGE) correction to the harmonic oscillator (H.O.), generating a hyperfine interaction which breaks SU(6). Nucleon state (up to the 2nd energy shell):

$$|N\rangle = a\boxed{|^2S_{1/2}\rangle_S} + b|^2S'_{1/2}\rangle_S + c|^2S_{1/2}\rangle_M + d|^4D_{1/2}\rangle_M$$

$$|^{2S+1}X\rangle_t, \quad t = A, M, S = \text{ symmetry type}$$

$$\begin{cases} a = 0.931,\ b = 0.274,\ c = 0.233,\ d = 0.067 & \text{From spectroscopy} \\ a = 1,\ b = c = d = 0 & \text{H.O. is recovered} \end{cases}$$

IK is a suitable framework for a first CQM calculation of DPCs:

- IK is the prototype of any other CQM, low energy properties of the nucleon (spectrum and electromagnetic form factors at small momentum transfer) are basically reproduced
- Gross features of the standard PDFs are reproduced

The model results correspond to a low momentum scale (hadronic scale, μ_0). There are only valence quarks: the scale has to be very low ($\mu_0 \simeq 0.300$ GeV according to NLO pQCD). Data are taken at a high momentum scale t. QCD evolution needed!

pQCD evolution of dPDFs calculations

The evolution equations for the dPDFs are based on a generalization of the DGLAP equations used, *e.g.*, for the single PDFs (**Kirschner 1979, Shelest, Snigirev, Zinovev 1982**).

Introducing the Mellin moments:

$$\langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} = \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n_1-2}\, x_2^{n_2-2}\, x_1 x_2 F_{q_1,q_2}(x_1, x_2, Q^2) \;,$$

defining the moments of the quark-quark NS splitting functions at LO as follows:

$$P_{NS}^{(0)}(n_1) = \int dx\; x^{n_1} P_{NS}^{(0)}(x) \;,$$

using the modified DGLAP evolution equations, without the inhomogeneous term, since we are evaluating the valence dPDFs, one gets

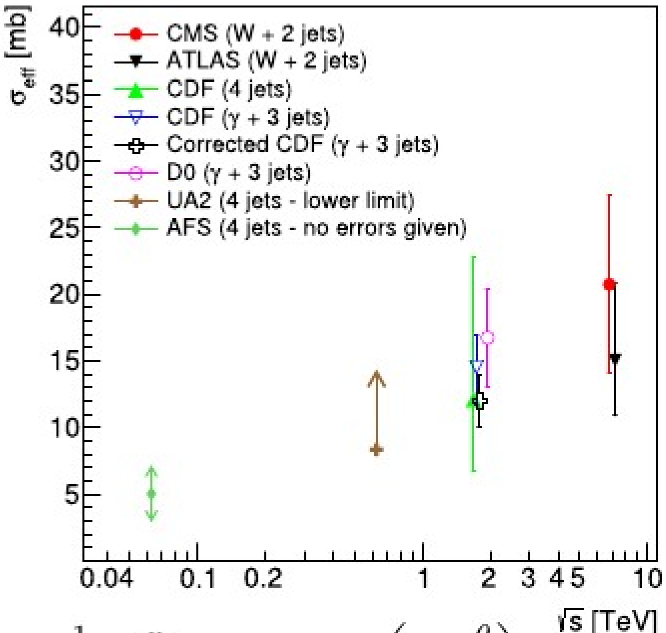
$$\langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} = \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)}\right)^{\frac{-P_{NS}^{(0)}(n_1) - P_{NS}^{(0)}(n_2)}{\beta_0}} \langle x_1 x_2 F_{q_1,q_2}(\mu_0^2)\rangle_{n_1,n_2}$$

The dPDF at any high energy scale is obtain by inverting the Mellin transformation:

$$\begin{aligned} x_1 x_2 F_{q_1,q_2}(x_1, x_2, Q^2) &= \frac{1}{2\pi i}\oint_{\mathcal{C}} dn_1 \,\frac{1}{2\pi i}\oint_{\mathcal{C}} dn_2 \\ &\times \; x_1^{(1-n_1)}\, x_2^{(1-n_2)}\, \langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} \end{aligned}$$

σ_{eff}: experimental situation

- Difficult extraction;
  σ_{pp}^{double} not measured...
  see talks later today and tomorrow
- Older data at lower $\sqrt{S}$
- “constant” (large errorbars)
- Different ranges in x_i accessed
  in different experiments.



Kinematics:

$$x_{1,2} = \sqrt{\tau} e^{\pm y} \quad \tau = x_1 x_2 = \frac{s}{S} \quad y = \frac{1}{2} \ln \frac{x_1}{x_2} \simeq \eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

High x for hard jets (heavy particles detected, large partonic s)
For example: AFS, $y \simeq 0$, $x_1 = x_2$ in $[0.2, 0.3]$
CDF: x_1, x_2, x_1', x_2' in $[0.02, 0.4]$

Valence region included...

Sergio Scopetta's slide

Averaged X-section

In order to obtain a simple number of σ_{eff}, which can be compared with the experimental x-independent value, one can reduce our calculation by using the fully uncorrelated ansatz:

$$F_{uu}(x_1, x_2, k_\perp) = u(x_1)u(x_2)f_{uu}(k_\perp)$$

Where the "effective form-factor" is introduced:

$$\begin{aligned} f_{uu}(k_\perp) &= \frac{1}{4}\int dx_1 dx_2\; F_{uu}(x_1, x_2, k_\perp) \\ &= \int \underbrace{d\vec{k}_1 d\vec{k}_2 d\vec{k}_3}_{\text{Quark's momenta}}\; \Psi^\dagger(\vec{k}_1+\vec{k}_\perp, \vec{k}_2, \vec{k}_3)\,\boxed{\Psi(\vec{k}_1, \vec{k}_2+\vec{k}_\perp, \vec{k}_3)}\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3) \end{aligned}$$

Quark's momenta

Proton w.f.

Using these approximations the σ_{eff} expression is:

$$\sigma_{eff} = \frac{81}{64\int f_{uu}^2(\vec{k}_\perp)\frac{d\vec{k}_\perp}{(2\pi)^2}} \sim 10.9 \text{ mb}$$