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# Inclusive three jet production at the LHC as a new BFKL probe

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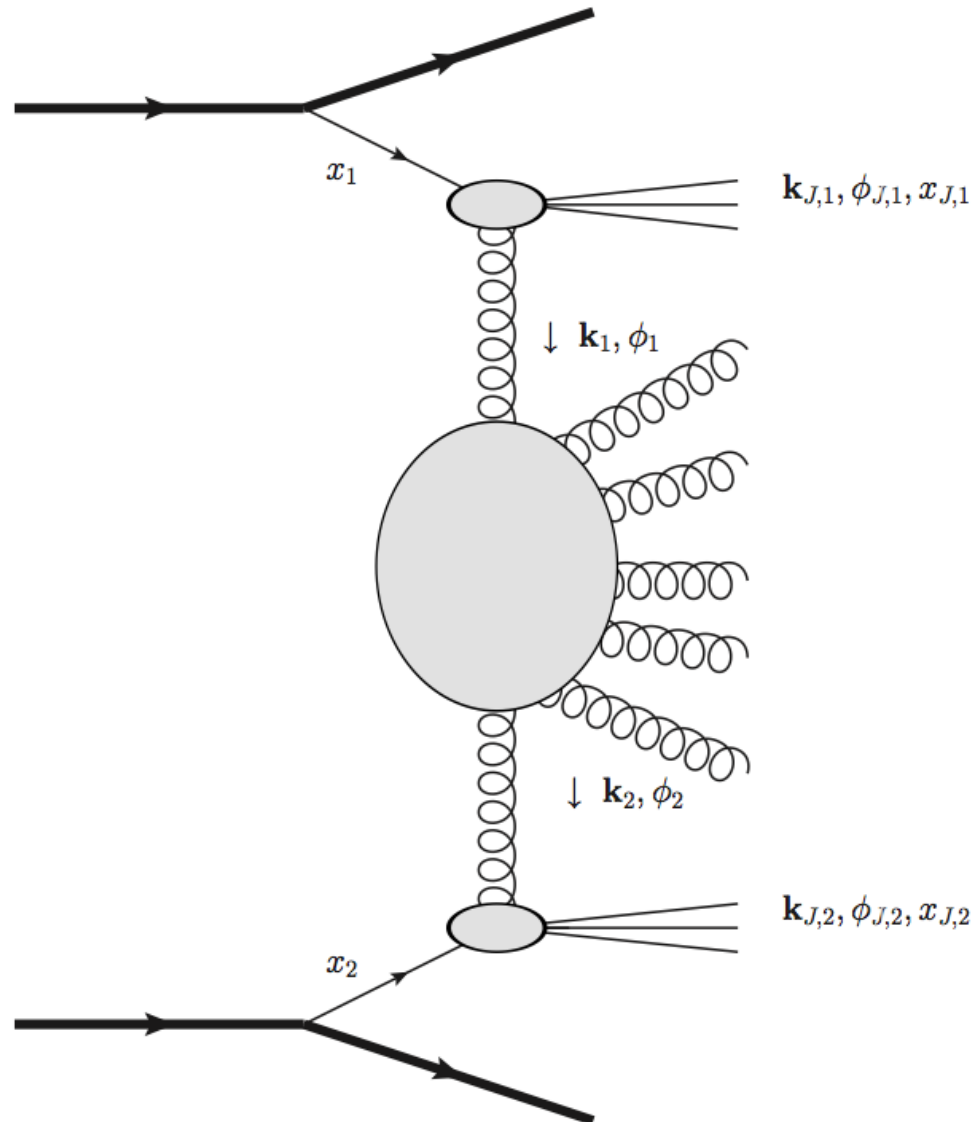
In collaboration with F. Caporale, B. Murdaca and A. Sabio Vera  
[arXiv:1508:07711](https://arxiv.org/abs/1508.07711)

7th International Workshop on MPI at the LHC,  
23 - 27 November 2015, Trieste, Italy

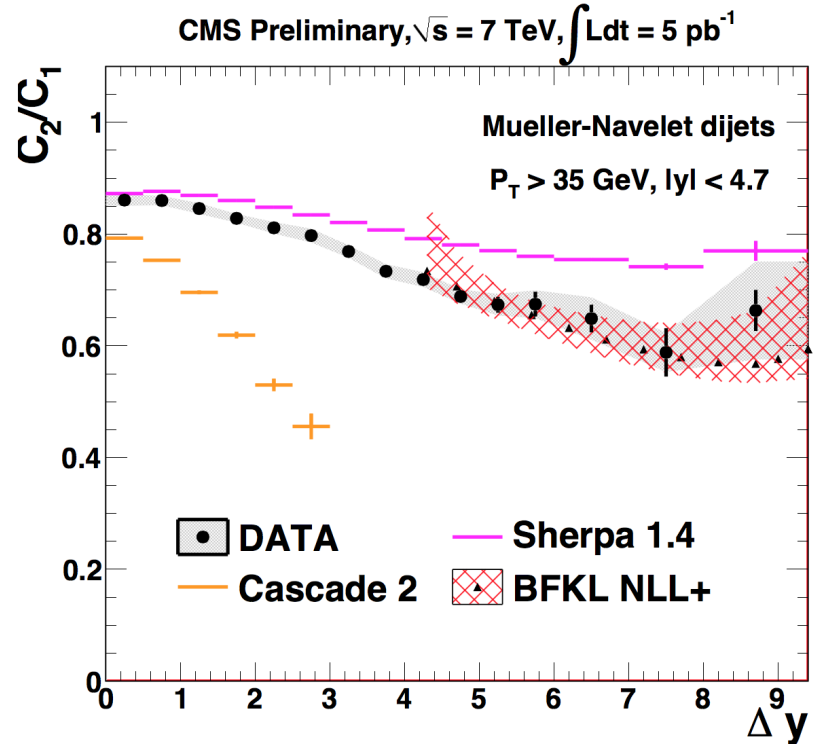
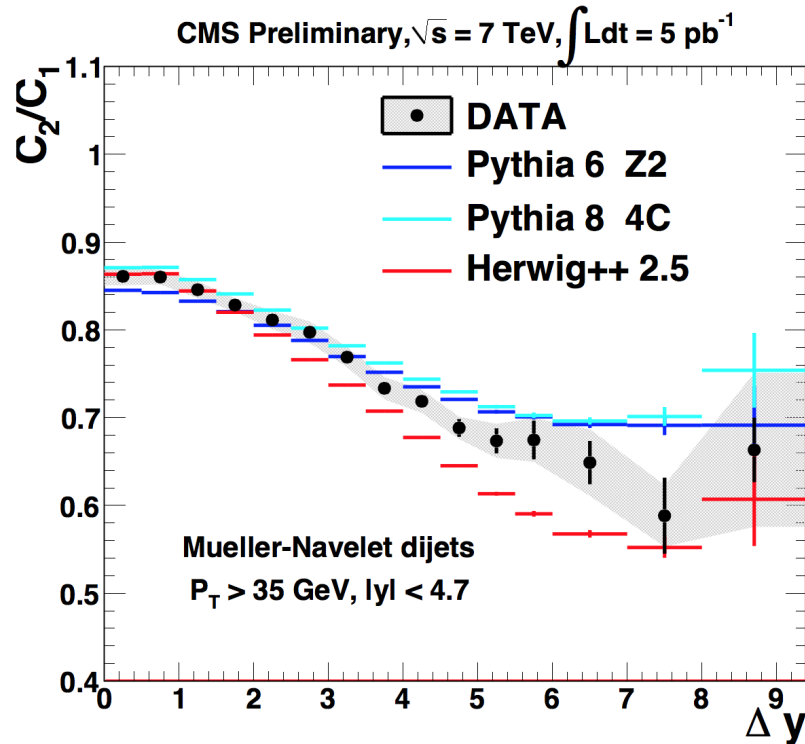
# BFKL phenomenology

- LHC has produced and will further produce an abundance of data
- This is the best time to investigate the applicability of the BFKL resummation program within the context of a hadron collider
- In the last years: the big hit from the theory/experimental side was the study of Mueller-Navelet jets
- We need new observables: apart from the usual “growth with energy” signal, we should consider azimuthal angle dependencies

# Mueller-Navelet jets

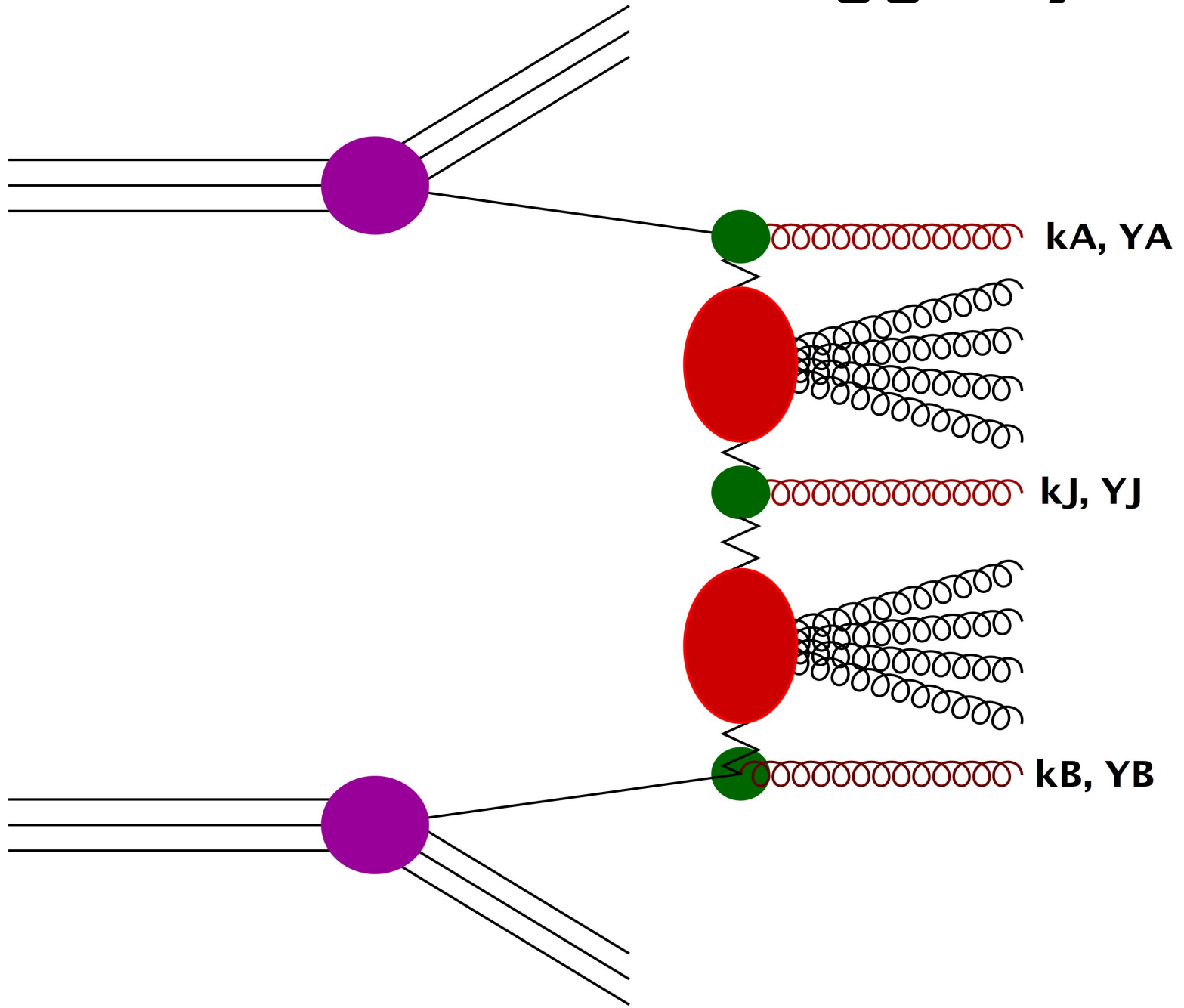


# Mueller-Navelet dijets azimuthal decorrelations

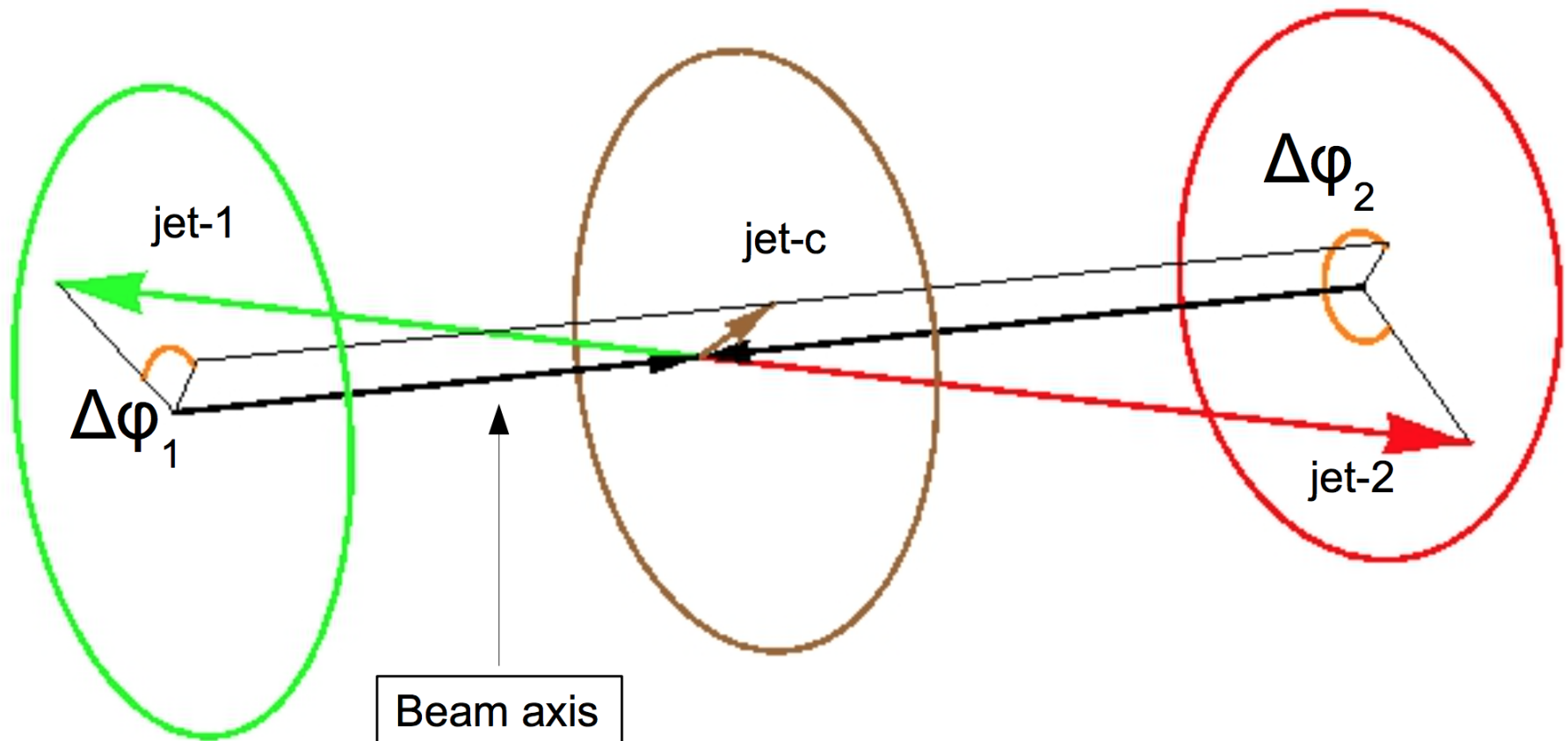


$$C_n = \langle \cos(n(\pi - \Delta\phi)) \rangle$$

# An event with three tagged jets

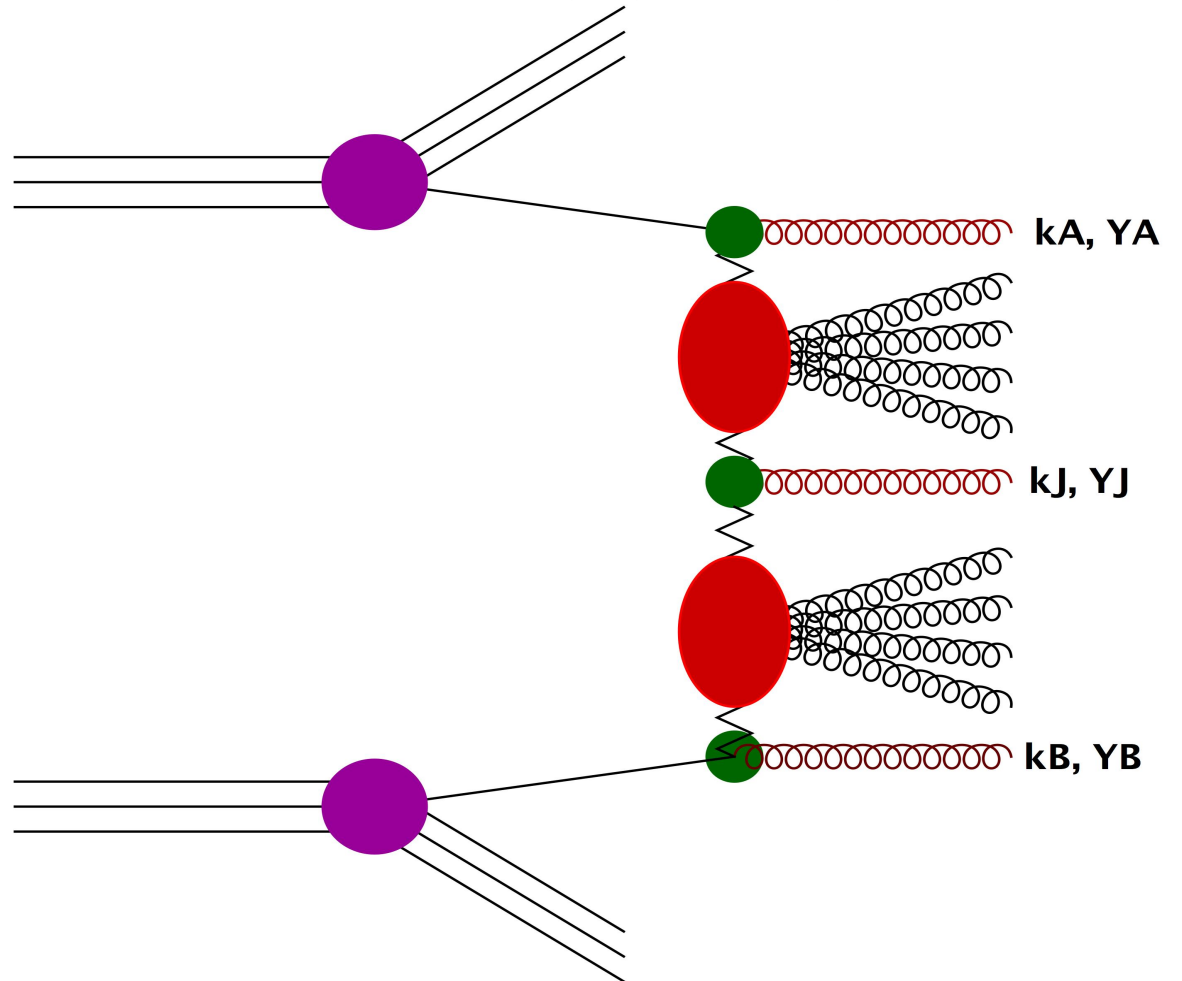


# An event with three tagged jets



# An event with three tagged jets

Assuming that  $Y_A > y_J > Y_B$  and also that  $k_A$  and  $k_B$  are fixed we can write for the differential cross section:



$$\frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2\vec{p}_A \int d^2\vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ \times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B)$$

Starting point

What to do next?



# What to do next?

1. Integrate over the angle difference of  $k_A$  and  $k_B$  and also over the angle of the central jet

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$$\begin{aligned} & \int_0^{2\pi} d\Delta\phi \cos(M\Delta\phi) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} \\ &= \bar{\alpha}_s \sum_{L=0}^M \int_0^\infty dp^2 \int_0^{2\pi} d\theta \frac{(-1)^M \binom{M}{L} (k_J^2)^{\frac{L-1}{2}} (p^2)^{\frac{M-L}{2}} \cos(L\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)}^M} \\ & \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_M(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, p_B^2, y_J - Y_B) \end{aligned}$$

$$\Delta\phi \equiv \theta_A - \theta_B - \pi$$

1. Integrate over the angle difference of  $k_A$  and  $k_B$  and also over the angle of the central jet

$$\begin{aligned}
 & \int_0^{2\pi} d\Delta\phi \cos(M\Delta\phi) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} \\
 &= \bar{\alpha}_s \sum_{L=0}^M \int_0^\infty dp^2 \int_0^{2\pi} d\theta \frac{(-1)^M \binom{M}{L} (k_J^2)^{\frac{L-1}{2}} (p^2)^{\frac{M-L}{2}} \cos(L\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)^M}} \\
 &\times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_M(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, p_B^2, y_J - Y_B)
 \end{aligned}$$

where:

$$\begin{aligned}
 \phi_n(p_A^2, p_B^2, Y) &= 2 \int_0^\infty d\nu \cos\left(\nu \ln \frac{p_A^2}{p_B^2}\right) \frac{e^{\bar{\alpha}_s \chi_{|n|}(\nu) Y}}{\pi \sqrt{p_A^2 p_B^2}}, \\
 \chi_n(\nu) &= 2\psi(1) - \psi\left(\frac{1+n}{2} + i\nu\right) - \psi\left(\frac{1+n}{2} - i\nu\right)
 \end{aligned}$$

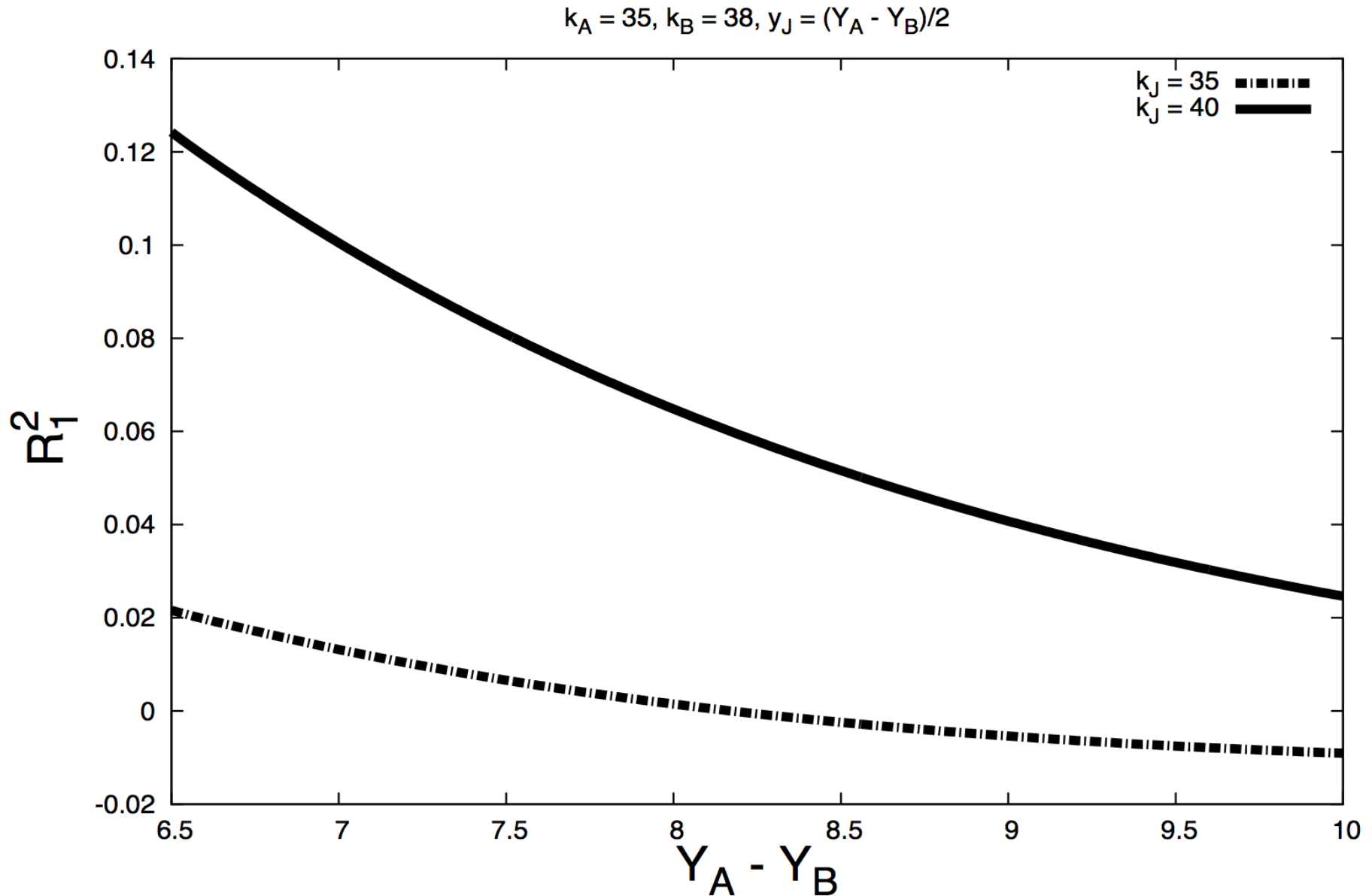
1. Integrate over the angle difference of  $k_A$  and  $k_B$  and also over the angle of the central jet

$$\begin{aligned}
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 &= \bar{\alpha}_s \sum_{L=0}^M \int_0^\infty dp^2 \int_0^{2\pi} d\theta \frac{(-1)^M \binom{M}{L} (k_J^2)^{\frac{L-1}{2}} (p^2)^{\frac{M-L}{2}} \cos(L\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta)^M}} \\
 & \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_M(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta, p_B^2, y_J - Y_B)
 \end{aligned}$$

$$\langle \cos(M(\theta_A - \theta_B - \pi)) \rangle = \frac{\int_0^{2\pi} d\Delta\phi \cos(M\Delta\phi) \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}{\int_0^{2\pi} d\Delta\phi \int_0^{2\pi} d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}$$

$$\mathcal{R}_N^M = \frac{\langle \cos(M(\theta_A - \theta_B - \pi)) \rangle}{\langle \cos(N(\theta_A - \theta_B - \pi)) \rangle}$$

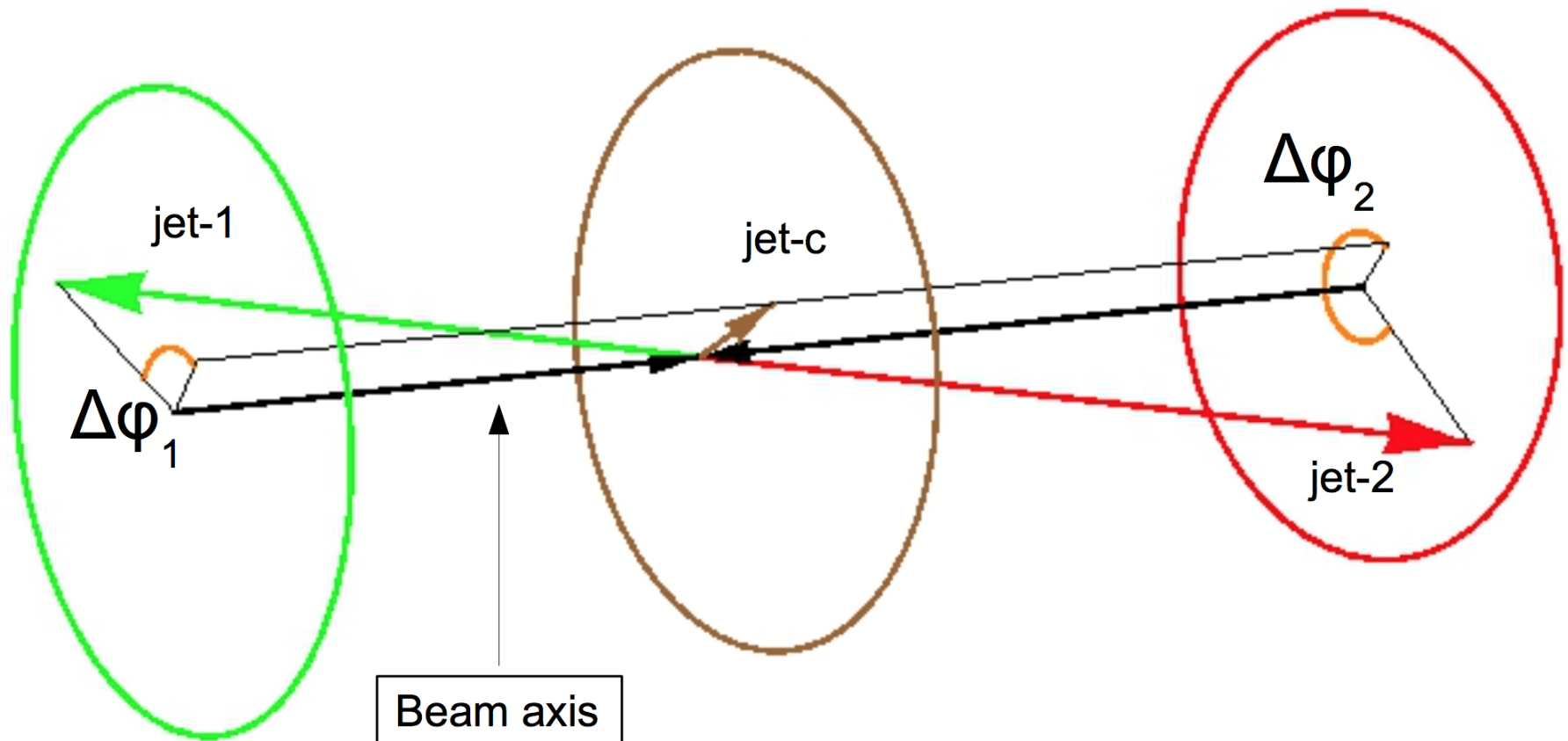
1. ...and then plot for different  $k_J$



# What to do next?

2. A second idea and by far more interesting is to integrate over all angles after using the projections on the two azimuthal angle differences between the central jet and  $k_A$  and  $k_B$  respectively

# Back to the basic picture



## 2. Integrate over all angles after using projections

$$\begin{aligned} \frac{d^3 \sigma^{3\text{-jet}}}{d^2 \vec{k}_J dy_J} &= \frac{\bar{\alpha}_s}{\pi k_J^2} \int d^2 \vec{p}_A \int d^2 \vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B) \\ &\times \varphi(\vec{k}_A, \vec{p}_A, Y_A - y_J) \varphi(\vec{p}_B, \vec{k}_B, y_J - Y_B) \end{aligned}$$

$$\begin{aligned} &\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \\ &\quad \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3 \sigma^{3\text{-jet}}}{d^2 \vec{k}_J dy_J} \\ &= \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \\ &\quad \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta)}^N} \\ &\quad \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta, p_B^2, y_J - Y_B) \end{aligned}$$



## 2. Integrate over all angles after using projections

$$\begin{aligned}
 & \int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \\
 & \quad \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J} \\
 &= \bar{\alpha}_s \sum_{L=0}^N \binom{N}{L} (k_J^2)^{\frac{L-1}{2}} \int_0^\infty dp^2 (p^2)^{\frac{N-L}{2}} \\
 & \quad \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos(M\theta) \cos((N-L)\theta)}{\sqrt{(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta)}^N} \\
 & \quad \times \phi_M(p_A^2, p^2, Y_A - y_J) \phi_N(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos \theta, p_B^2, y_J - Y_B)
 \end{aligned}$$

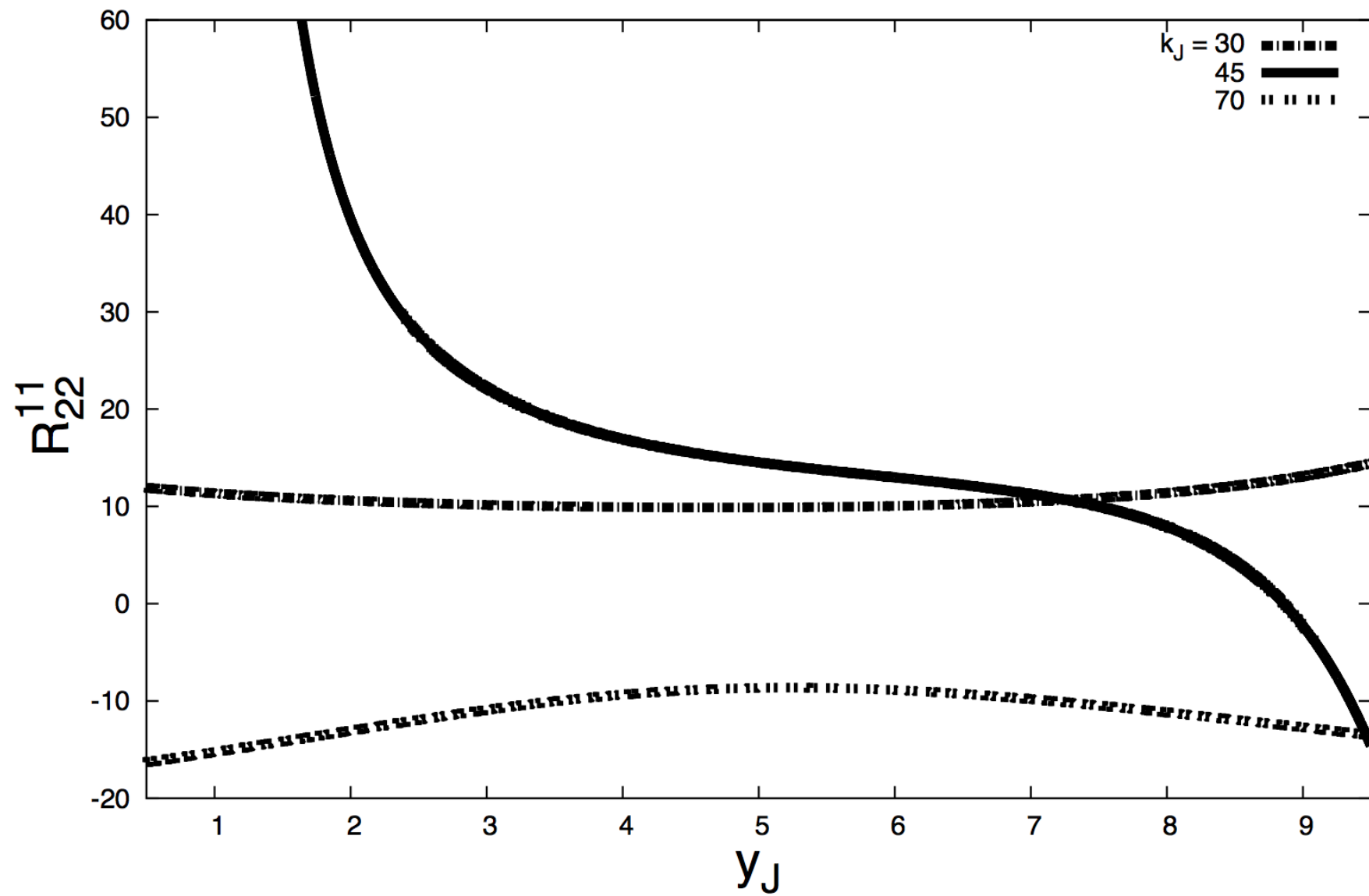
$$\begin{aligned}
 & \langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle \\
 &= \frac{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \frac{d^3\sigma^{3\text{-jet}}}{d^2\vec{k}_J dy_J}}
 \end{aligned}$$

2. ... so that you can define new observables:

$$\mathcal{R}_{P,Q}^{M,N} = \frac{\langle \cos (M (\theta_A - \theta_J - \pi)) \cos (N (\theta_J - \theta_B - \pi)) \rangle}{\langle \cos (P (\theta_A - \theta_J - \pi)) \cos (Q (\theta_J - \theta_B - \pi)) \rangle}$$

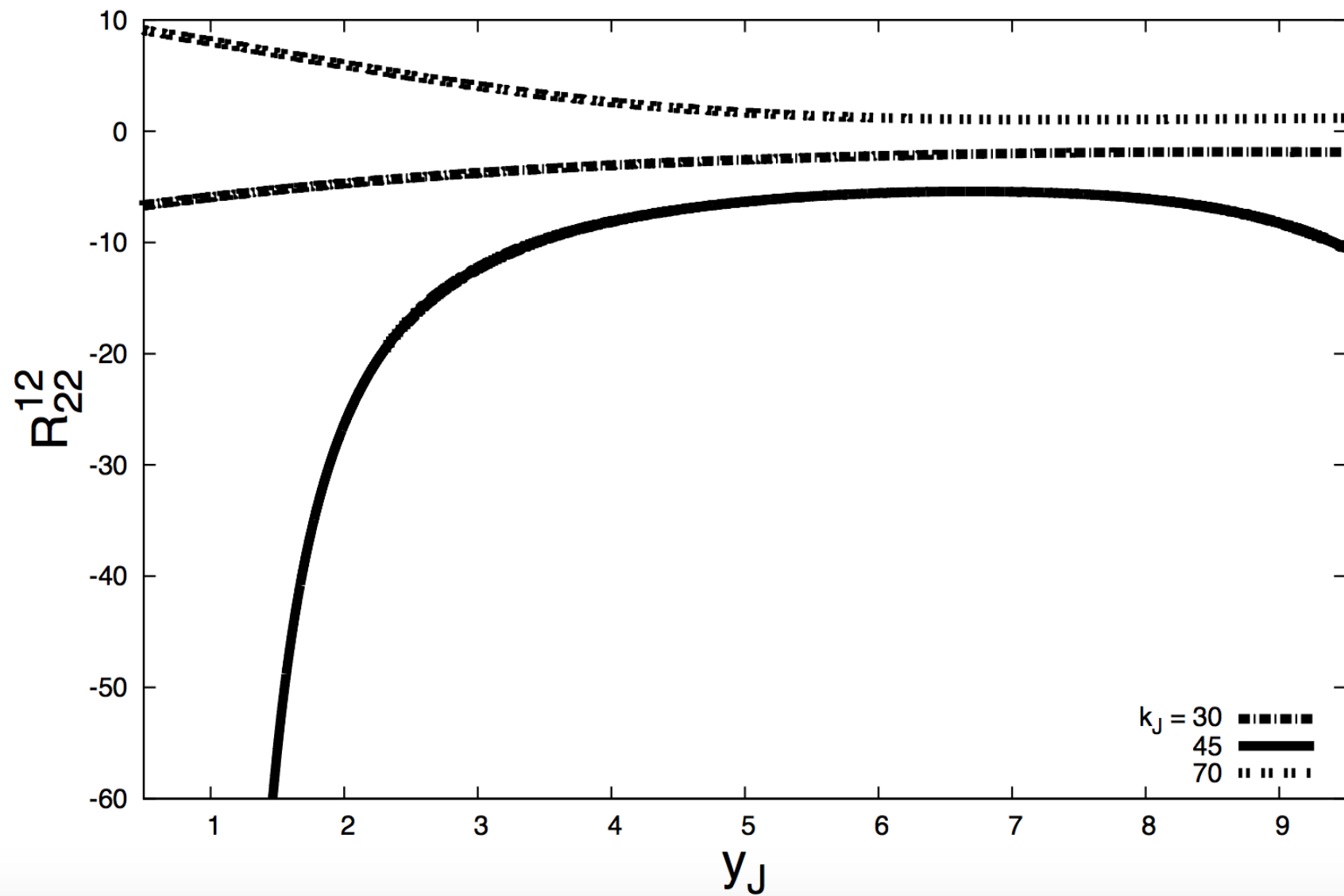
$$R_{22}^{11}$$

$$k_A = 40, k_B = 50, Y_A = 10, Y_B = 0$$



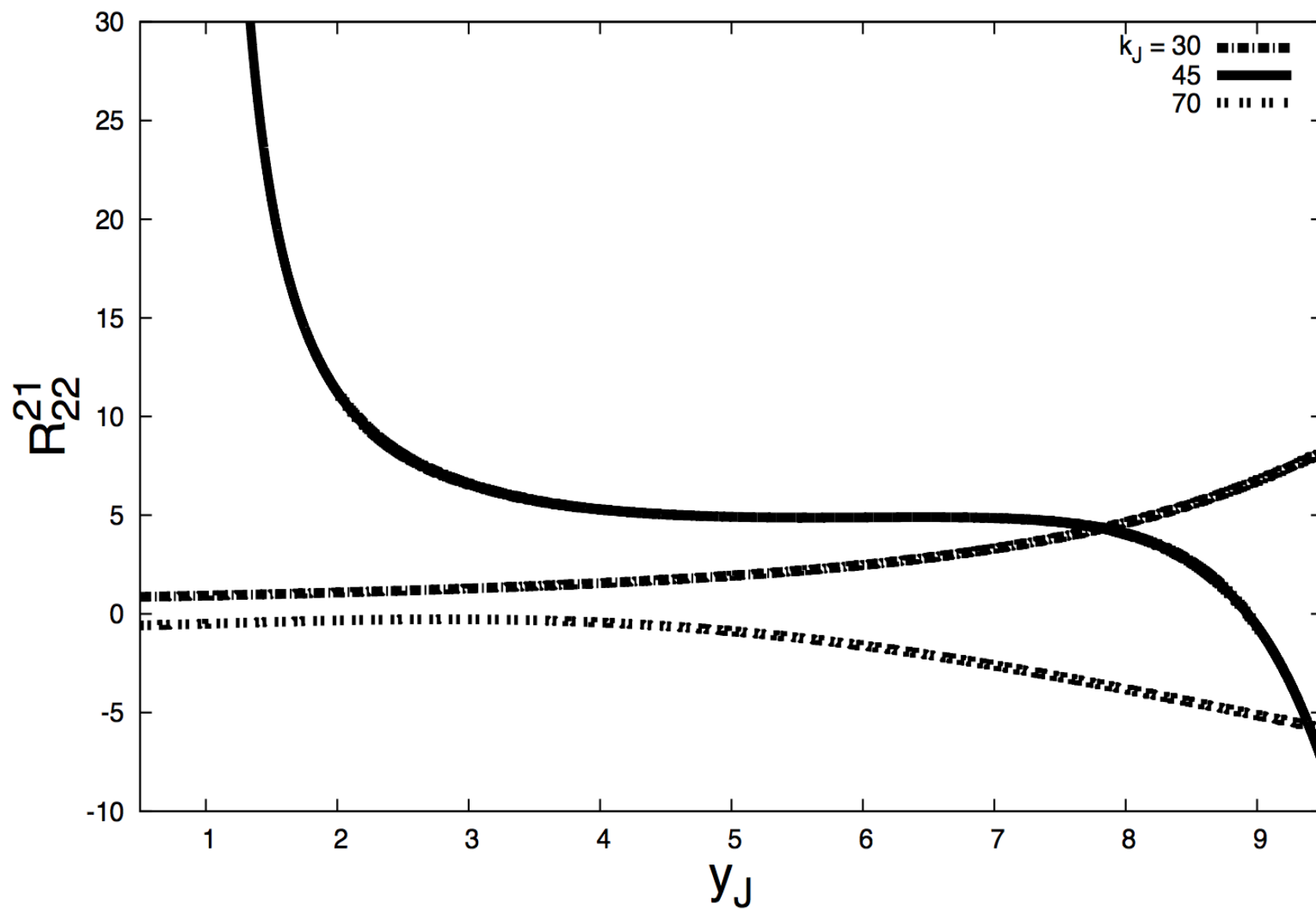
$$R_{22}^{12}$$

$$k_A = 40, k_B = 50, Y_A = 10, Y_B = 0$$

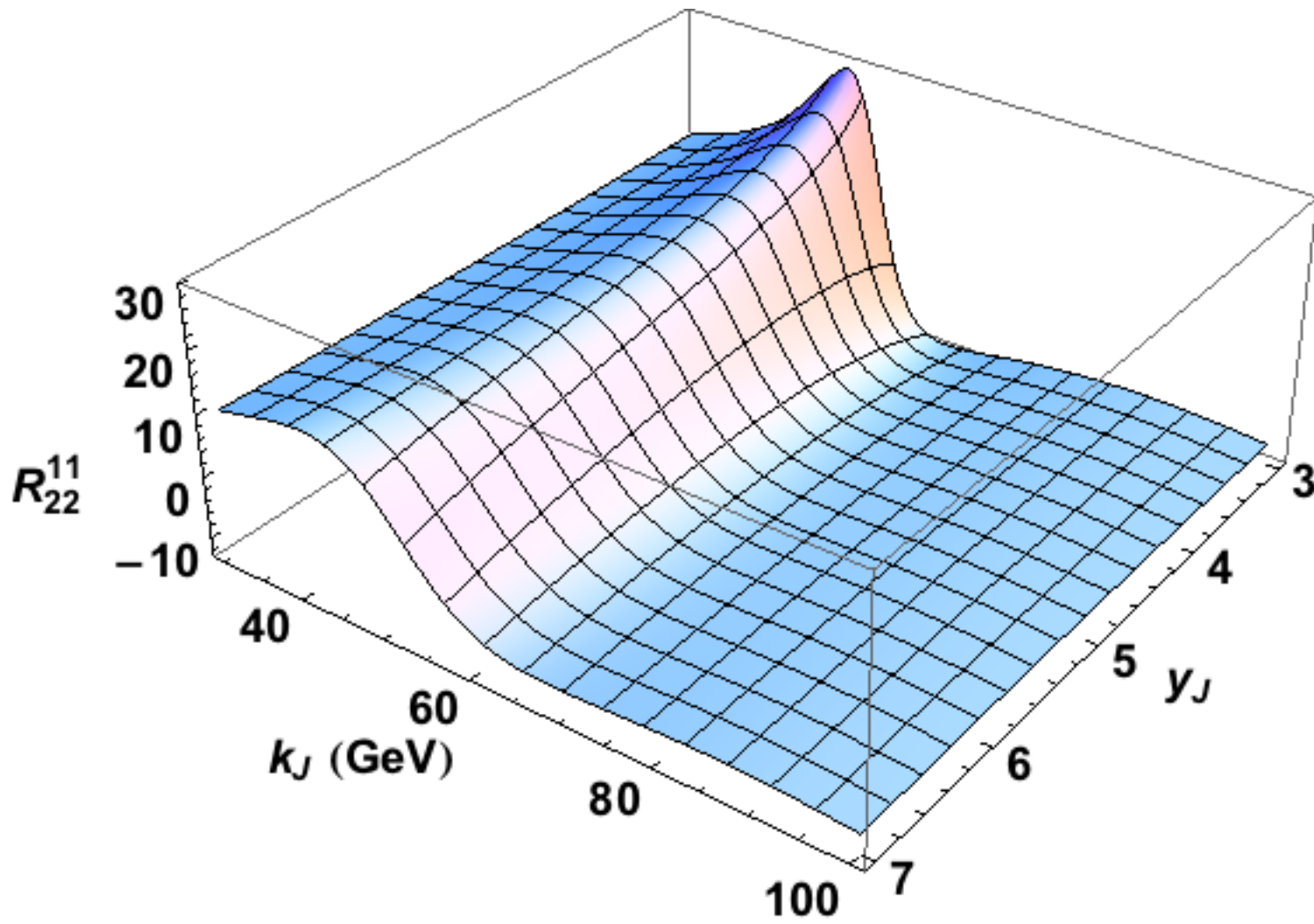


$$R_{22}^{21}$$

$$k_A = 40, k_B = 50, Y_A = 10, Y_B = 0$$

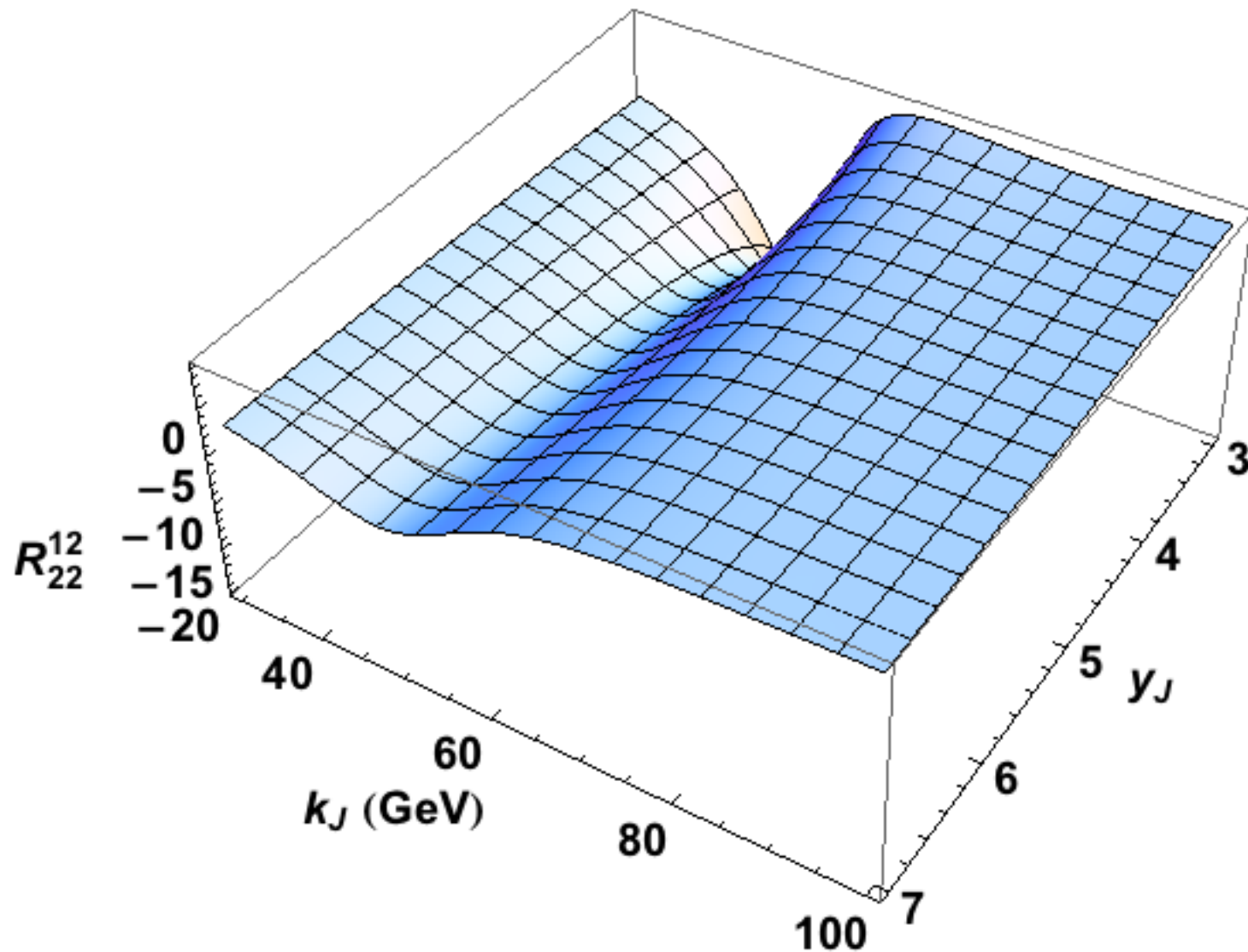


# 3D plot for $R_{22}^{11}$



$$k_A = 40 \text{ GeV}, k_B = 50 \text{ GeV}, Y_A = 10, Y_B = 0$$

# 3D plot for $R_{22}^{12}$



$$k_A = 40 \text{ GeV}, k_B = 50 \text{ GeV}, Y_A = 10, Y_B = 0$$

How about 4-jet events, is there something similar to be done there?



How about 4-jet events, is there something similar to be done there?

More generally, what about:

- Including PDF's?
- Jet clustering algorithms?

**Other observables beyond azimuthal correlations?**

# BFKLex Monte Carlo

This is an implementation of the iterative solution of the BFKL equation as a Monte Carlo code.

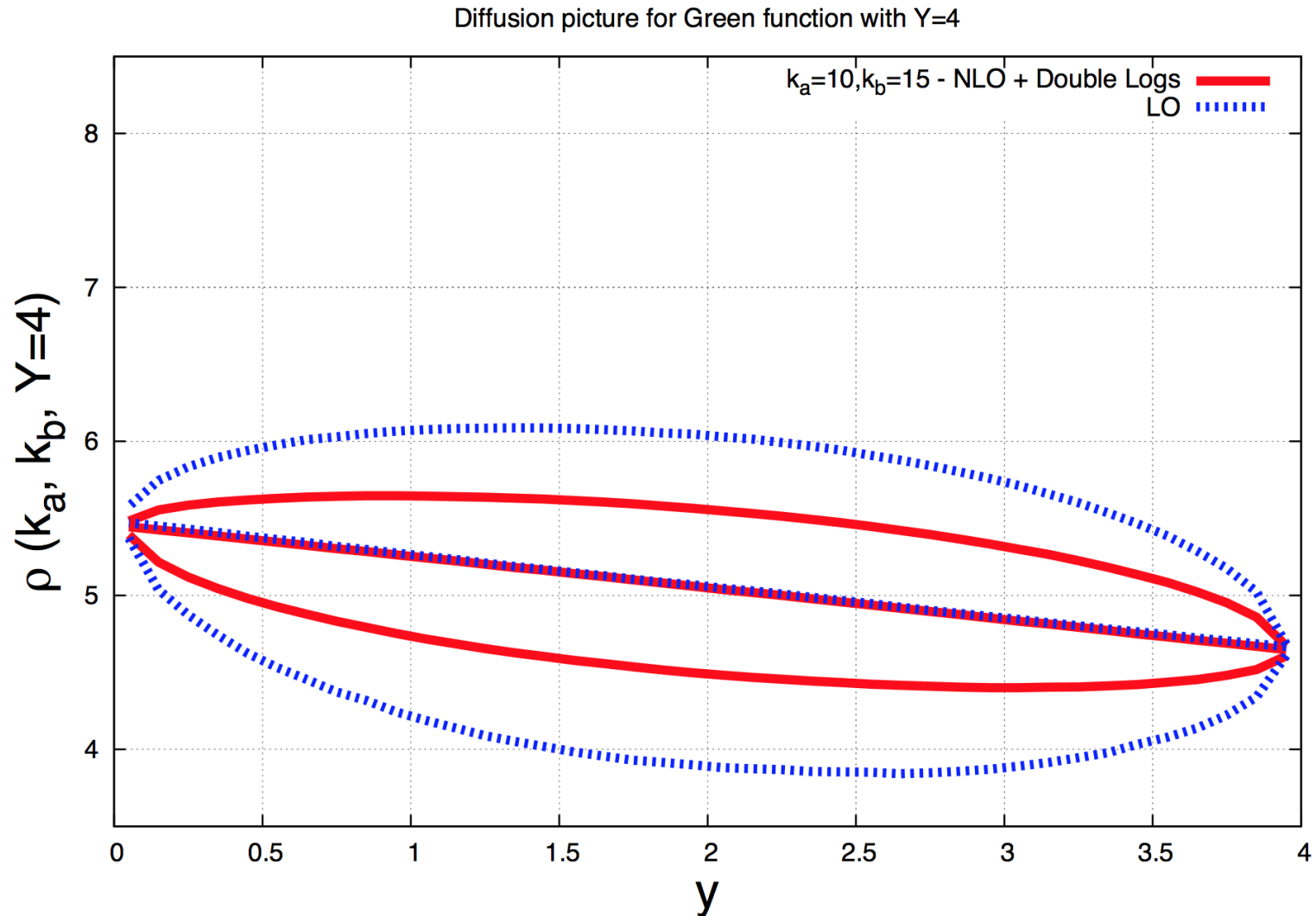
GC, A. Sabio Vera, [arXiv:1508:07711](#)

Present status:

- NLO BFKL, collinearly improved
- Interfaced with PDF's and FastJet

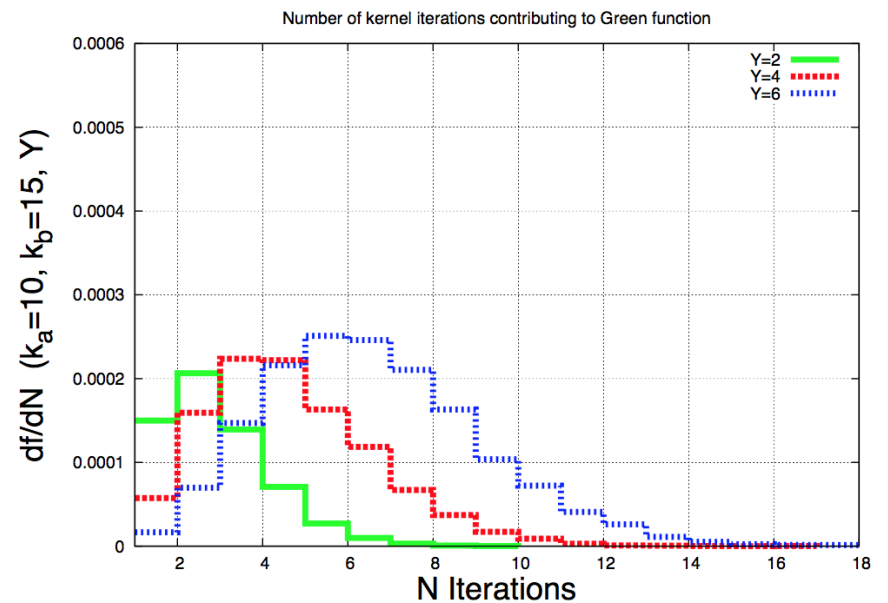
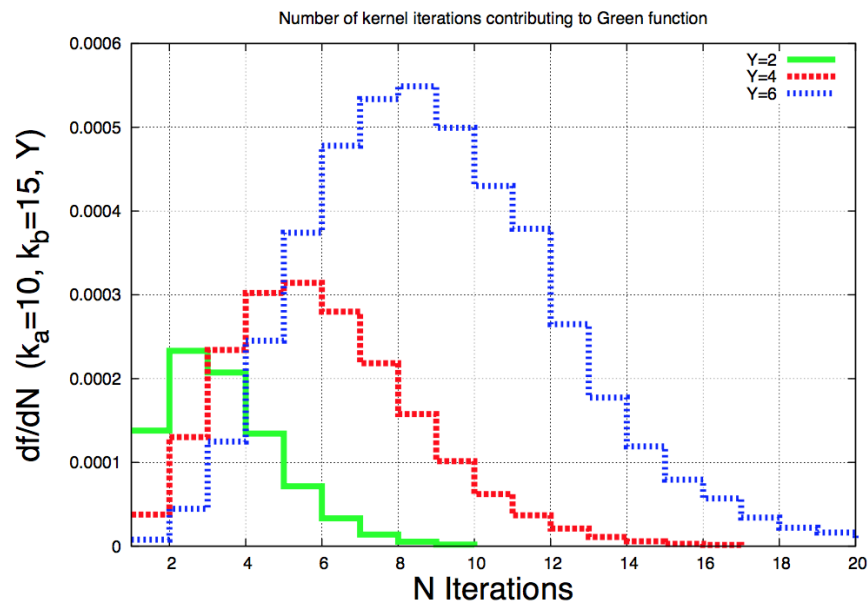
# BFKLex Monte Carlo

## interesting facts: Diffusion



# BFKLex Monte Carlo

interesting facts: “multiplicity”



# Conclusions & Outlook

- We use three tagged jets to propose new observables with a distinct signal of BFKL dynamics, 4-jet case is also ready
- We use ratios of correlation functions to minimize the influence of higher order corrections
- For a realistic comparison against experimental data we need to integrate over a range of  $k_A$  and  $k_B$  and introduce PDFs and a jet algorithm
- Comparison with **BFKLex** results and other Monte Carlo codes is underway