Improved TMD factorization for forward di-jet production in pA collisions

Elena Petreska

École Polytechnique and Universidade de Santiago de Compostela

With Piotr Kotko, Krzysztof Kutak, Cyrille Marquet,

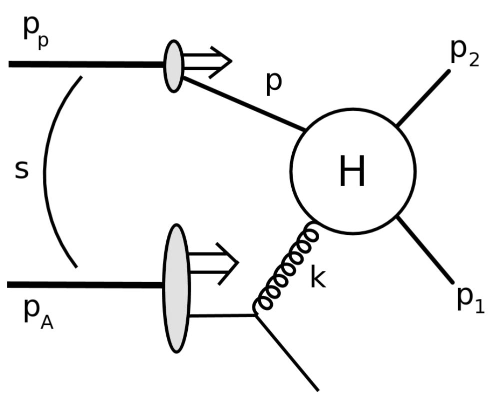
Sebastian Sapeta and Andreas van Hameren

JHEP 1509 (2015) 106 and preliminary work

MPI, Trieste

November 23-27, 2015

Forward di-jet production in dilute-dense collisions



The projectile is probed at large-x. The target is probed at small-x.

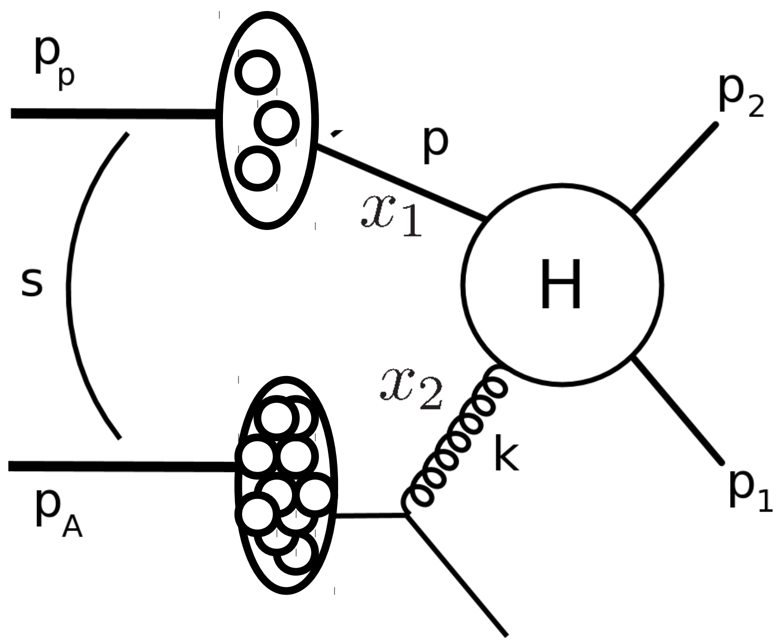
Optimal for studying small-x saturation effects.

***Motivation*: Unifying three theoretical approaches:**

Color Glass Condensate	High-Energy Factorization	Transverse Momentum Dependent factorization
	$Q_s \ll k_t \sim P_t$	$k_t \sim Q_s \ll P_t$
L. McLerran and R. Venugopalan, 1994 J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, 1997, 1999. E. Iancu, A. Leonidov, and L. D. McLerran,2001 A. H. Mueller, 2001 E. Ferreiro, E. Iancu, A. Leonidov, and L.McLerran, 2002 Marquet 2007	*S. Catani, M. Ciafaloni and F. Hautmann, 1991 M. Deak, F. Hautmann, H. Jung and K. Kutak, 2009 K. Kutak and S. Sapeta, 2012*	*C. J. Bomhof, P. J. Mulders and F. Pijlman 2006 F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan,2011*

Forward di-jet production in dilute-dense collisions

$$p(p_p) + A(p_A) \to j_1(p_1) + j_2(p_2) + X$$



$$x_1 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \sim 1$$

➢ Dilute

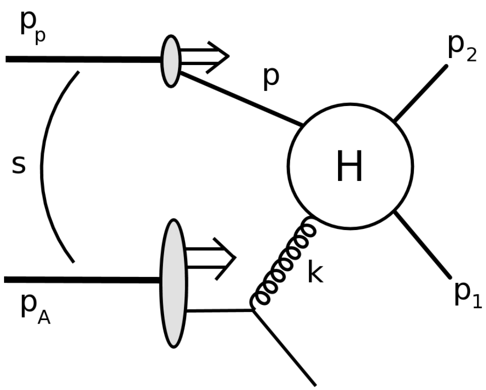
$$x_2 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \ll 1$$

➢ Dense

$$|k_t|^2 = |p_{1t} + p_{2t}|^2$$

➔ Momentum imbalance

Forward di-jet production in dilute-dense collisions



The projectile is probed at large-x. The target is probed at small-x.

Optimal for studying small-x saturation effects.

***Motivation*: Unifying three theoretical approaches:**

Color Glass Condensate	High-Energy Factorization	Transverse Momentum Dependent factorization
	$Q_s \ll k_t \sim P_t$	$k_t \sim Q_s \ll P_t$

L. McLerran and R. Venugopalan, 1994
J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, 1997, 1999.
E. Iancu, A. Leonidov, and L. D. McLerran, 2001
A. H. Mueller, 2001
E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, 2002

S. Catani, M. Ciafaloni and F. Hautmann, 1991
M. Deak, F. Hautmann, H. Jung and K. Kutak, 2009
K. Kutak and S. Sapeta, 2012

C. J. Bomhof, P. J. Mulders and F. Pijlman, 2006
F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan,2011

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate

  - Small-x limit;
    - Saturation and multigluon distributions;
      - No k_t factorization.

$$Q_s^2 \sim A^{\frac{1}{3}} \left(\frac{1}{x}\right)^{\lambda}$$ → Saturation scale;

High occupation number,
classical fields solution of Yang-Mills equations
of motion:

$$A_\mu \sim \frac{1}{g}\,, \quad [D_\mu, F^{\mu\nu}] = J^\nu$$

Jamal's talk on Thursday

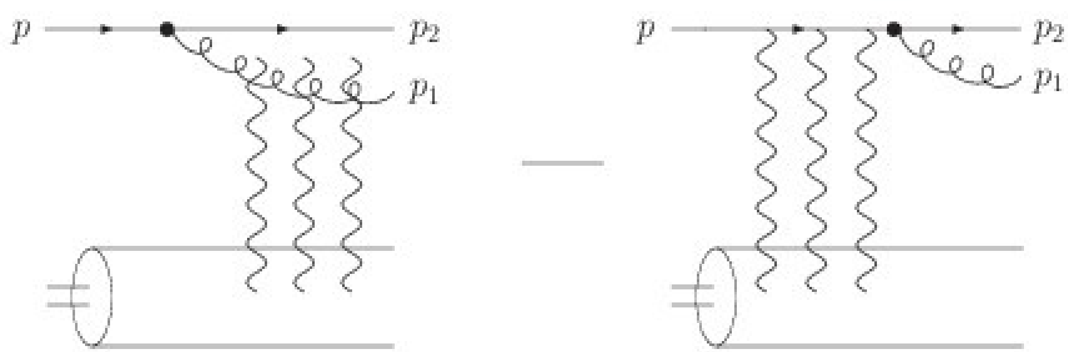
Prithwish's talk on Thursday

Quantum corrections: Non-linear renormalization group equation, JIMWLK.

J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, 1997, 1999.
E. Iancu, A. Leonidov, and L. D. McLerran, 2001

Forward di-jet production in dilute-dense collisions

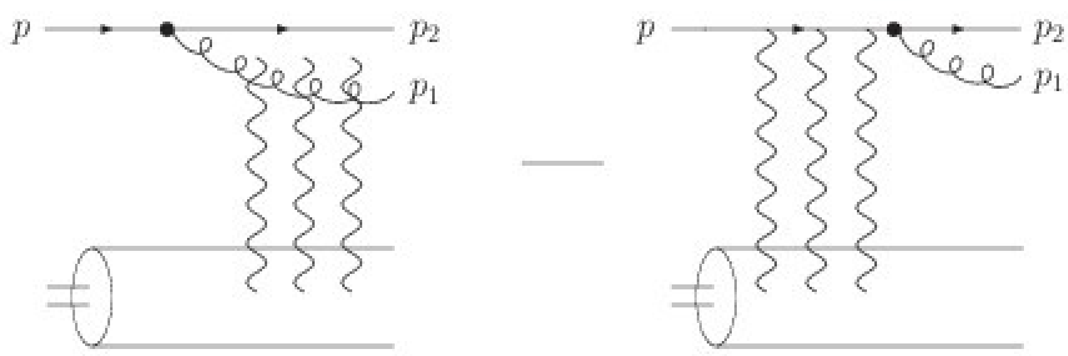
- Color Glass Condensate
  - Small-x limit;
    - Saturation and multigluon distributions;
      - No k_t factorization.



Forward di-jet production in dilute-dense collisions

• Color Glass Condensate

- Small-x limit;
  - Saturation and multigluon distributions;
    - No k_t factorization.

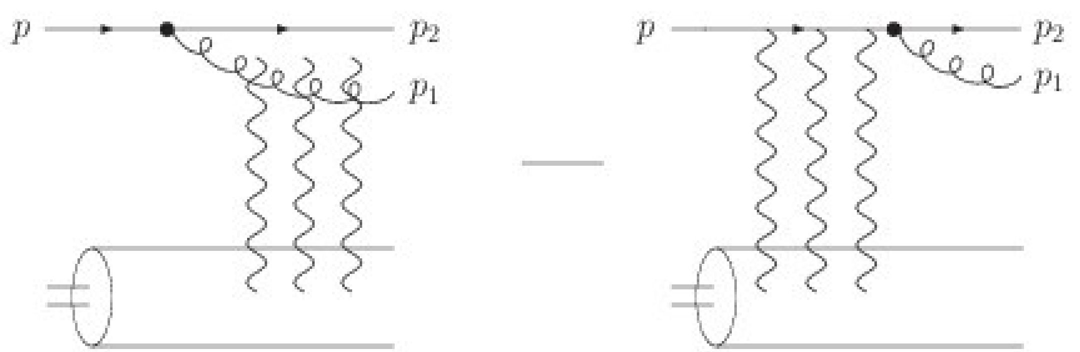


$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

$$V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})T^a\right]$$

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate
  - Small-x limit;
    - Saturation and multigluon distributions;
      - No k_t factorization.



$$U(\mathbf{x}) = \mathcal{P} \exp \left[ig \int dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

$$V(\mathbf{x}) = \mathcal{P} \exp \left[ig \int dx^+ A_a^-(x^+, \mathbf{x}) T^a \right]$$

$$|\mathcal{M}|^2 = \int \frac{d^2\mathbf{x}}{(2\pi)^2} \frac{d^2\mathbf{x}'}{(2\pi)^2} \frac{d^2\mathbf{b}}{(2\pi)^2} \frac{d^2\mathbf{b}'}{(2\pi)^2} e^{-ip_{1t}\cdot(\mathbf{x}-\mathbf{x}')} e^{-ip_{2t}\cdot(\mathbf{b}-\mathbf{b}')} \sum_{\lambda\alpha\beta} \phi^{\lambda^*}_{\alpha\beta}(p, p_1^+, \mathbf{x}'-\mathbf{b}') \phi^{\lambda}_{\alpha\beta}(p, p_1^+, \mathbf{x}-\mathbf{b})$$

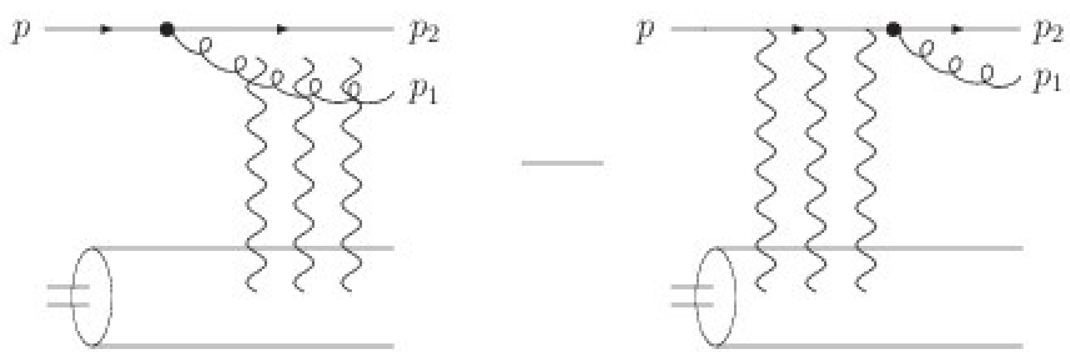
$$\times \left\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b}, \mathbf{x}, \mathbf{b}', \mathbf{x}'; x_2] - S^{(3)}_{qg\bar{q}}[\mathbf{b}, \mathbf{x}, \mathbf{b}' + z(\mathbf{x}'-\mathbf{b}'); x_2] \right.$$

$$\left. - S^{(3)}_{qg\bar{q}}[\mathbf{b} + z(\mathbf{x}-\mathbf{b}), \mathbf{x}', \mathbf{b}'; x_2] + S^{(2)}_{q\bar{q}}[\mathbf{b} + z(\mathbf{x}-\mathbf{b}), \mathbf{b}' + z(\mathbf{x}'-\mathbf{b}'); x_2] \right\}$$

C. Marquet (2007)

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate
  - Small-x limit;
    - Saturation and multigluon distributions;
      - No k_t factorization.



$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

$$V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int dx^+ A_a^-(x^+,\mathbf{x})T^a\right]$$

$$S_{q\bar{q}}^{(2)}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

$$S_{qg\bar{q}}^{(3)}(\mathbf{b},\mathbf{x},\mathbf{z}') = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U^\dagger(\mathbf{z}')t^c U(\mathbf{b})t^d\right) V^{cd}(\mathbf{x})\right\rangle$$

$$S_{qg\bar{q}g}^{(4)}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{b})U^\dagger(\mathbf{b}')t^d t^c\right)\left[V(\mathbf{x})V^\dagger(\mathbf{x}')\right]^{cd}\right\rangle$$

Forward di-jet production in dilute-dense collisions

- Color Glass Condensate

  - No k_t factorization.
    - Three- and four-point correlators of Wilson lines

- High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

K. Kutak and S. Sapeta (2012)

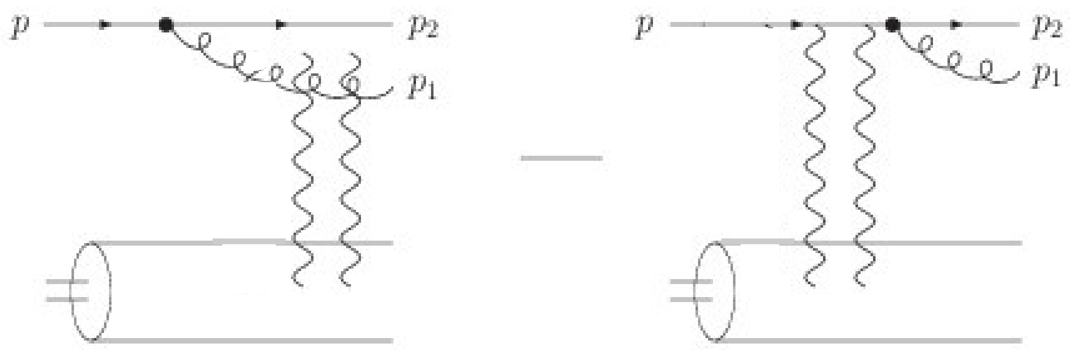
- Parton distributions of collinear factorization for the large-x projectile;
  - One k_t dependent unintegrated gluon distribution for the small-x target;

$$\mathcal{F}_{g/A} \simeq k_t^2 \int d^2 r_t e^{-ik_t\cdot r_t} S^{(2)}_{q\bar{q}}(0,r_t)$$

- Off-shell matrix elements.

Forward di-jet production in dilute-dense collisions

CGC in the dilute target limit $Q_s \ll k_t \sim P_t$



$$U(\mathbf{x}) \approx 1 + ig\int dx^+ A^-(x^+,\mathbf{x}) - \frac{g^2}{2}\int dx^+ dy^+ \mathcal{P}\left\{A^-(x^+,\mathbf{x})A^-(y^+,\mathbf{x})\right\} + \mathcal{O}(A^3)$$

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}') = S^{(2)}(\mathbf{b}-\mathbf{b}') - \frac{C_A}{C_F}S^{(2)}(\mathbf{x}-\mathbf{x}') - \frac{C_A}{C_F} - \frac{C_A}{2C_F}\left[S^{(2)}(\mathbf{x}'-\mathbf{b}) + S^{(2)}(\mathbf{x}-\mathbf{b}') - S^{(2)}(\mathbf{x}-\mathbf{b}) - S^{(2)}(\mathbf{x}'-\mathbf{b}')\right]$$

$$S^{(3)}_{qg\bar{q}}(\mathbf{b},\mathbf{x},\mathbf{v}') = \frac{C_A}{2C_F}\left[S^{(2)}(\mathbf{b}-\mathbf{x}) + S^{(2)}(\mathbf{x}-\mathbf{v}')\right] - \frac{1}{2C_AC_F}S^{(2)}(\mathbf{b}-\mathbf{v}') - \frac{C_A}{2C_F}$$

$$S^{(2)}_{q\bar{q}}(\mathbf{z},\mathbf{z}') = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(U(\mathbf{z})U^\dagger(\mathbf{z}')\right)\right\rangle$$

Forward di-jet production in dilute-dense collisions

CGC in the limit $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma(pA \to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{2\pi} x_1 f_{q/p}(x_1,\mu^2) z(1-z) \hat{P}_{gq}(z) \left[1 + \frac{(1-z)^2 p_{1t}^2}{P_t^2} - \frac{1}{N_c^2}\frac{z^2 p_{2t}^2}{P_t^2}\right] \frac{\mathcal{F}_{g/A}(x_2,k_t)}{p_{1t}^2\ p_{2t}^2}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

Forward di-jet production in dilute-dense collisions

CGC in the limit $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma(pA \to qgX)}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{2\pi} x_1 f_{q/p}(x_1,\mu^2) z(1-z) \hat{P}_{gq}(z) \left[1 + \frac{(1-z)^2 p_{1t}^2}{P_t^2} - \frac{1}{N_c^2} \frac{z^2 p_{2t}^2}{P_t^2}\right] \frac{\mathcal{F}_{g/A}(x_2,k_t)}{p_{1t}^2\, p_{2t}^2}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

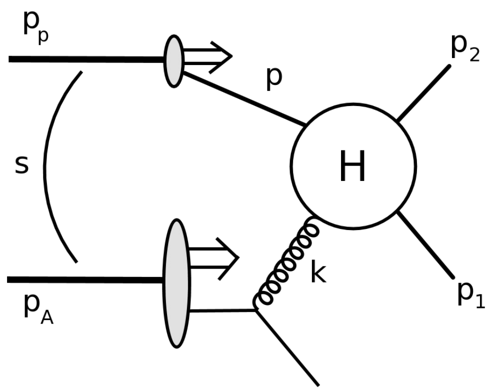
Forward di-jet production in dilute-dense collisions

High-Energy Factorization is equivalent to the CGC theory in the limit

$$Q_s \ll k_t \sim P_t$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

Forward di-jet production in dilute-dense collisions



The projectile is probed at large-x. The target is probed at small-x.

Optimal for studying small-x saturation effects.

***Motivation*: Unifying three theoretical approaches:**

Color Glass Condensate	High-Energy Factorization	Transverse Momentum Dependent factorization
	$Q_s \ll k_t \sim P_t$	$k_t \sim Q_s \ll P_t$

L. McLerran and R. Venugopalan, 1994
J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, 1997, 1999.
E. Iancu, A. Leonidov, and L. D. McLerran, 2001
A. H. Mueller, 2001
E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, 2002

S. Catani, M. Ciafaloni and F. Hautmann, 1991
M. Deak, F. Hautmann, H. Jung and K. Kutak, 2009
K. Kutak and S. Sapeta, 2012

C. J. Bomhof, P. J. Mulders and F. Pijlman, 2006
F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan,2011

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization $k_t \sim Q_s \ll P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

F. Dominguez, C. Marquet, B. Xiao and F. Yuan (2011)

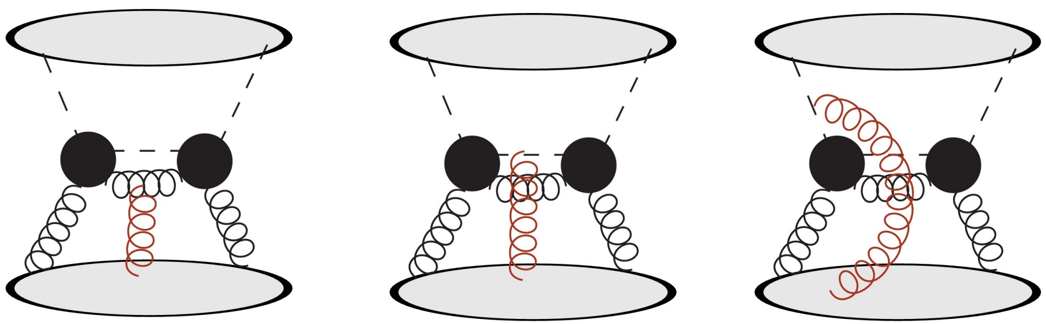
- Valid in the large-N_c limit;
  - Parton distributions of collinear factorization for the large-x projectile;
    - Five k_t dependent unintegrated gluon distributions for the small-x target;
      - On-shell hard factors.
        - Equivalent to CGC at large N_c and in the collinear limit.

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2) \otimes H_{ag\to cd} \otimes \mathcal{F}_{ag}(x_2,k_t)$$

$$\mathcal{F}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \langle A|\text{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}\right) F^{i-}(0)\right]|A\rangle$$



Igor's talk on Wednesday

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;\mathbf{0})U(\pm\infty,\xi^+;\boldsymbol{\xi})$$

$$\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

$$U(a,b;\mathbf{x}) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,\mathbf{x})t^a\right]$$

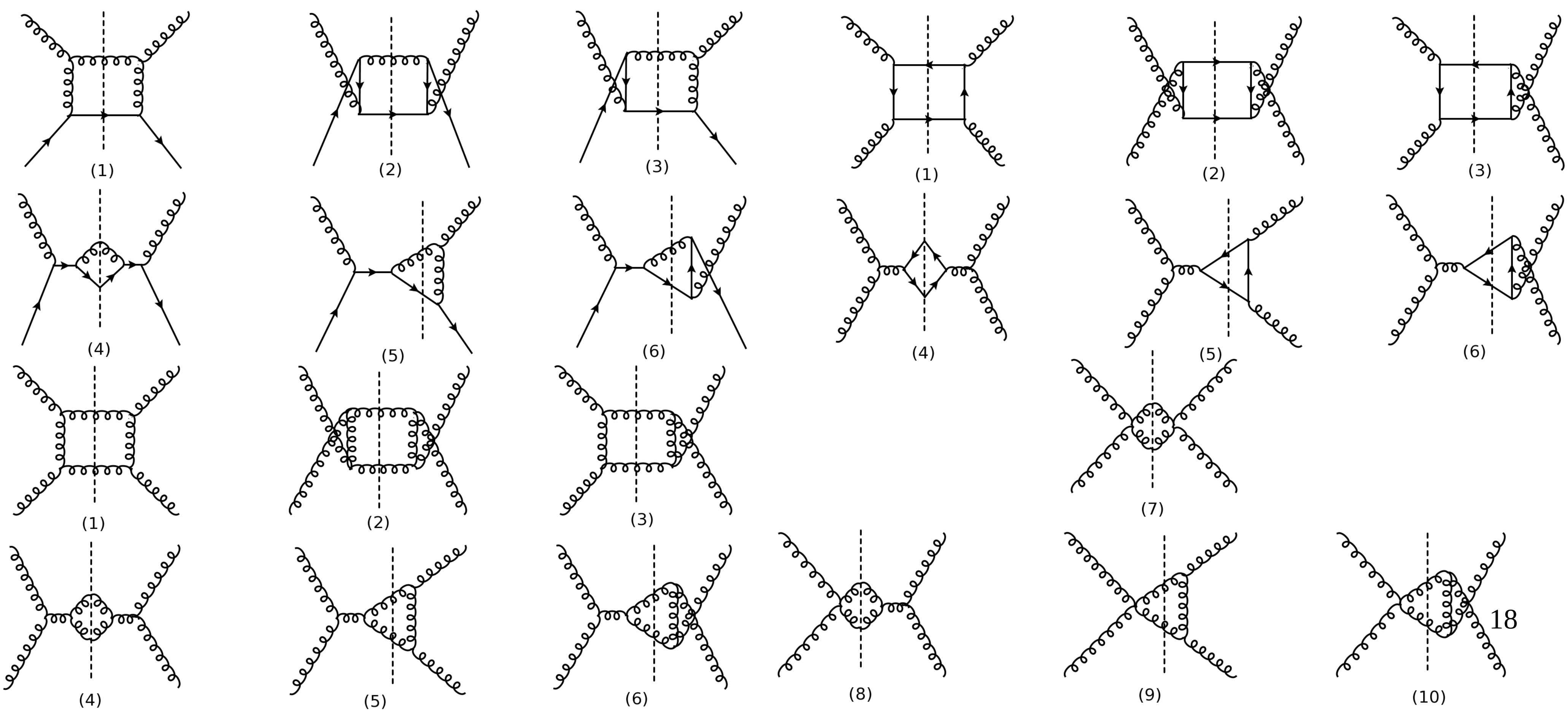
- TMD gluon distributions are gauge invariant, but process dependent.

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto x_1 f_{a/p}(x_1,\mu^2)\otimes H_{ag\to cd}\otimes \mathcal{F}_{ag}(x_2,k_t)$$

C. J. Bomhof, P. J. Mulders and F. Pijlman (2006)



Forward di-jet production in dilute-dense collisions

- Improved Transverse Momentum Dependent factorization

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

- Includes all finite N_c corrections;
- Three new TMDs, eight in total;
- Reduces the number of independent distributions to two per channel;

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd} \frac{1}{1+\delta_{cd}}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

$$\mathcal{F}_{qg}^{(1)} = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}} \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)} \propto \frac{1}{N_c}\left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[F(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[-]\dagger} F(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)} \propto \left\langle \mathrm{Tr}\left[F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right]\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\right\rangle$$

Forward di-jet production in dilute-dense collisions

- Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\ \frac{1}{1+\delta_{cd}}$$

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

	$K^{(1)}_{ag\to cd}$	$K^{(2)}_{ag\to cd}$
$qg\to qg$	$-\frac{\hat{s}^2+\hat{u}^2}{2\hat{t}^2\hat{s}\hat{u}}\left[\hat{u}^2+\frac{\hat{s}^2-\hat{t}^2}{N_c^2}\right]$	$-\frac{C_F}{N_c}\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{\hat{t}^2\hat{u}}$
$gg\to q\bar{q}$	$\frac{1}{2N_c}\frac{(\hat{t}^2+\hat{u}^2)^2}{\hat{s}^2\hat{t}\hat{u}}$	$-\frac{1}{2C_F N_c^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$gg\to gg$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2(\hat{t}^2+\hat{u}^2)}{\hat{t}^2\hat{u}^2\hat{s}^2}$	$\frac{2N_c}{C_F}\frac{(\hat{s}^2-\hat{t}\hat{u})^2}{\hat{t}\hat{u}\hat{s}^2}$

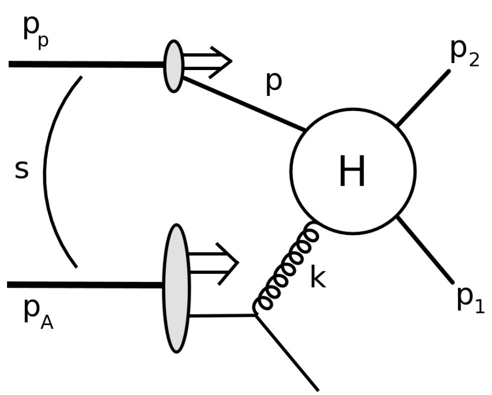
Forward di-jet production in dilute-dense collisions

- Improved Transverse Momentum Dependent factorization

P. Kotko, K. Kutak, C. Marquet, EP, S. Sapeta, A. van Hameren (2015)

- Includes all finite N_c corrections;
  - Eight unintegrated gluon distributions, two independent per channel;
  - Outlook: Show equivalence between CGC and HEF at finite N_c

Forward di-jet production in dilute-dense collisions



The projectile is probed at large-x. The target is probed at small-x.

Optimal for studying small-x saturation effects.

***Motivation*: Unifying three theoretical approaches:**

Color Glass Condensate	High-Energy Factorization	Transverse Momentum Dependent factorization
	$Q_s \ll k_t \sim P_t$	$k_t \sim Q_s \ll P_t$

L. McLerran and R. Venugopalan, 1994
J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, 1997, 1999.
E. Iancu, A. Leonidov, and L. D. McLerran, 2001
A. H. Mueller, 2001
E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, 2002

S. Catani, M. Ciafaloni and F. Hautmann, 1991
M. Deak, F. Hautmann, H. Jung and K. Kutak, 2009
K. Kutak and S. Sapeta, 2012

C. J. Bomhof, P. J. Mulders and F. Pijlman, 2006
F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan,2011

Forward di-jet production in dilute-dense collisions

- High-Energy Factorization $Q_s \ll k_t \sim P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}_{g/A}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

- Parton distributions of collinear factorization for the large-x projectile;
  - One k_t dependent unintegrated gluon distribution for the small-x target;
    - Off-shell hard factors.

- Transverse Momentum Dependent factorization $k_t \sim Q_s \ll P_t$

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd}(k_t)\, \frac{1}{1+\delta_{cd}}$$

- Parton distributions of collinear factorization for the large-x projectile;
  - Six k_t dependent unintegrated gluon distributions for the small-x target;
    - On-shell hard factors.

Forward di-jet production in dilute-dense collisions

• Transverse Momentum Dependent factorization at finite N_c

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

Restore the transverse momentum dependence in the matrix elements

Off-shell matrix elements calculated with two methods:

• Feynman rules + gauge vector defined by the target four-momentum
+ longitudinal polarization vector for the off-shell gluon

S. Catani, M. Ciafaloni and F. Hautmann, 1991

• Color ordered amplitudes

Gauge invariance on the level of amplitudes;
Redundancy in hard factors removed from the start.

Andreas talk on Thursday

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

Restore the transverse momentum dependence in the matrix elements

i	1	2
$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\frac{(\bar{s}^4+\bar{t}^4+\bar{u}^4)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\frac{(\bar{t}^2+\bar{u}^2)(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}(\bar{s}^2+\bar{u}^2)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right)$	$-\frac{C_F}{N_c}\frac{\bar{s}(\bar{s}^2+\bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

$$\bar{s}=(x_2p_A+p)^2 \qquad \bar{t}=(x_2p_A-p_1)^2 \qquad \bar{u}=(x_2p_A-p_2)^2$$

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\ \frac{1}{1+\delta_{cd}}$$

Unifies the regions of validity of HEF and TMD; It can be used to study forward di-jet production for any value of the momentum imbalance between the saturation scale and the moment of the jets.

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t) \Phi^{(i)}_{ag\to cd}(k_t) \frac{1}{1+\delta_{cd}}$$

Phenomenological study

- Start with the large N_c case;
- First gluon input: Analytical model expressions for the gluon distributions;
- Improve: Numerical inputs with small-x evolution.

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

- Gluon distributions in the Golec-Biernat-Wusthoff model

$$S^{(2)}(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2Q_s^2}{4}\right]$$

K. Golec-Biernat and M. Wusthoff (1998)

$$\mathcal{F}^{(1)}_{qg}(x_2,k_t) = 2\gamma\frac{S_\perp}{Q_s^2(x_2)}k_t^2\exp\left[-\frac{k_t^2}{Q_s^2(x_2)}\right]$$

$$\mathcal{F}^{(2)}_{qg}(x_2,k_t) = \gamma\left[\mathrm{Ei}\left(-\frac{k_t^2}{Q_s^2(x)}\right) - \mathrm{Ei}\left(-\frac{k_t^2}{3Q_s^2(x)}\right)\right]$$

$$\mathcal{F}^{(1)}_{gg}(x_2,k_t) = \frac{\gamma}{4}e^{-\frac{k_t^2}{2Q_s^2(x_2)}}\left(2+\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}^{(2)}_{gg}(x_2,k_t) = \frac{\gamma}{4}e^{-\frac{k_t^2}{2Q_s^2(x_2)}}\left(2-\frac{k_t^2}{Q_s^2(x_2)}\right)$$

$$\mathcal{F}^{(6)}_{gg}(x_2,k_t) = \gamma\left[\mathrm{Ei}\left(-\frac{k_t^2}{2Q_s^2(x)}\right) - \mathrm{Ei}\left(-\frac{k_t^2}{4Q_s^2(x)}\right)\right]$$

$$\gamma = N_cS_\perp/4\pi^3\alpha_s$$

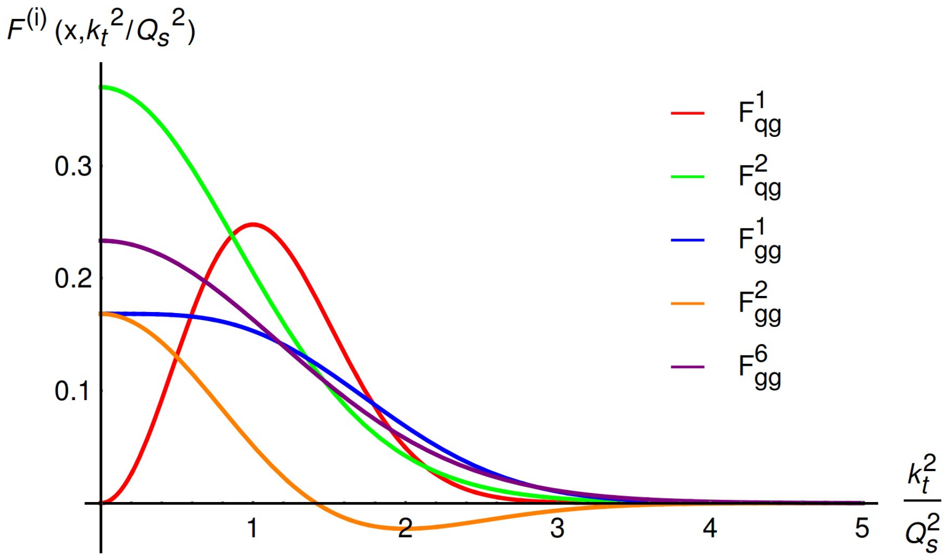
Soon to be published

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\, \frac{1}{1+\delta_{cd}}$$

- Gluon distributions in the GBW model



- High-kt behavior in the McLerran-Venugopalan model

L. D. McLerran and R. Venugopalan, (1994)

The scattering is dominated by one hard exchange

$$S^{(2)}(\mathbf{r}) = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\log\frac{1}{\Lambda r}\right]$$

$$\mathcal{F}^{(i)} \simeq \gamma \frac{Q_s^2(x_2)}{k_t^2} + \mathcal{O}\left(\frac{Q_s^4(x_2)}{k_t^4}\log\frac{k_t^2}{\Lambda^2}\right)$$

$$\mathcal{F}^{(2)}_{gg} \simeq 0$$

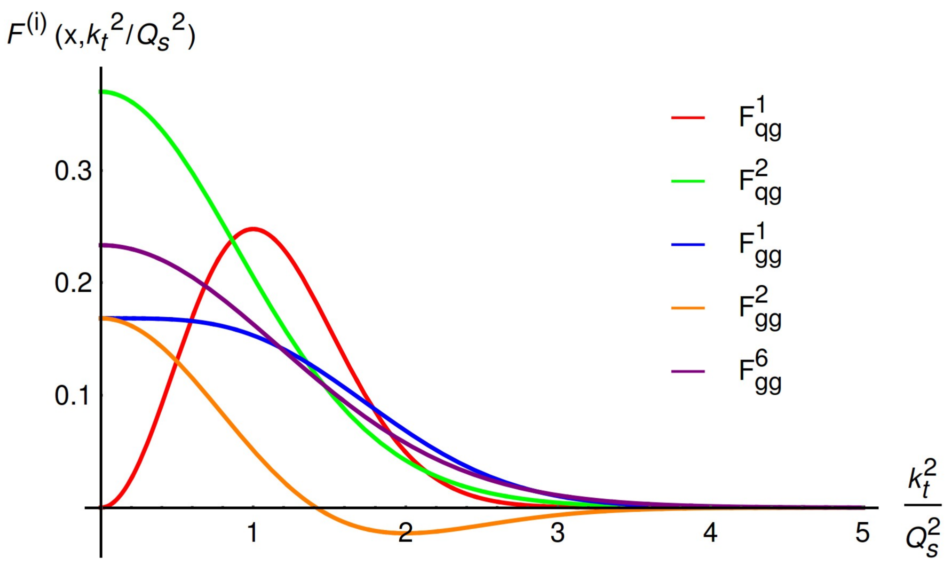
Soon to be published

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

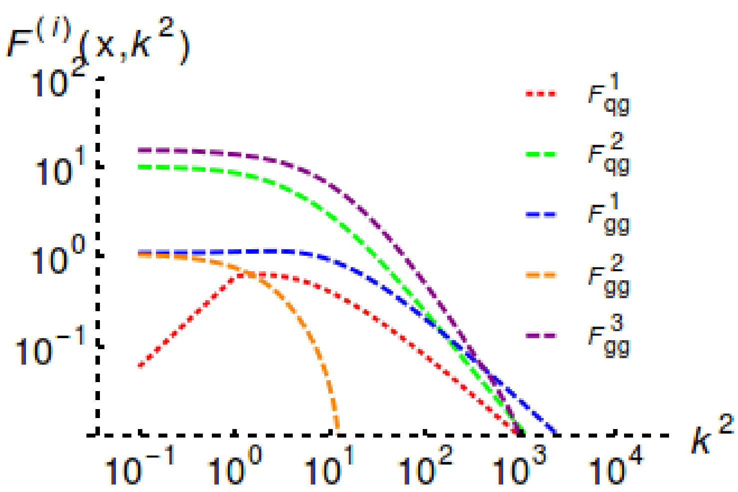
$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\,\frac{1}{1+\delta_{cd}}$$

- Gluon distributions in the GBW model



Kutak-Sapeta gluons with non-linear evolution:

K. Kutak and S. Sapeta, (2012)



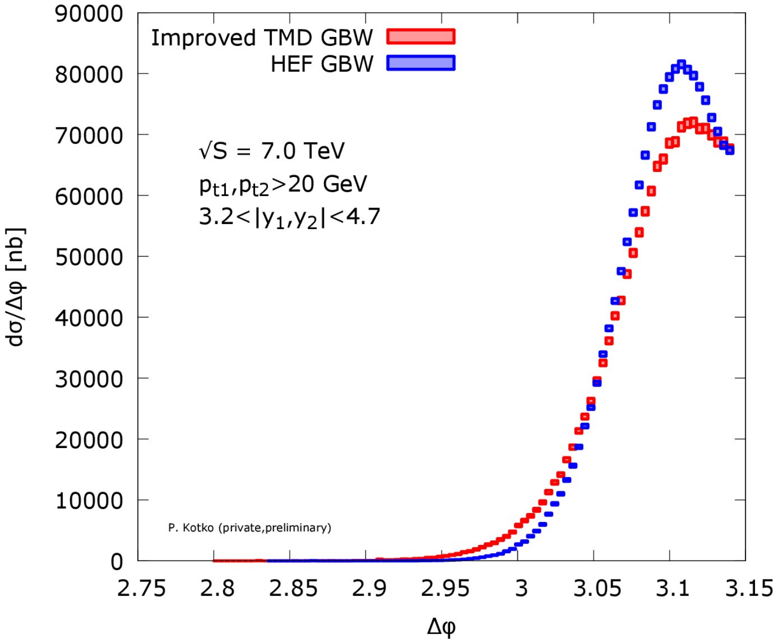
Soon to be published

Forward di-jet production in dilute-dense collisions

Unifying factorization formula

$$\frac{d\sigma^{pA\to\text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t) \frac{1}{1+\delta_{cd}}$$

Azimuthal correlations in forward di-jet production.



Soon to be published

Result I

High-Energy Factorization can be derived from the Color Glass Condensate in the dilute target limit.

Result II

Simplified Transverse Momentum Dependent factorization

Extension to finite N_c ; Three new TMD gluon distributions;
+ Reduction to 2 independent TMD gluons per channel.

Result III

Unifying Transverse Momentum Dependent factorization formula:

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd}(k_t)\Phi^{(i)}_{ag\to cd}(k_t)\ \frac{1}{1+\delta_{cd}}$$

The new formula:

- Is valid for for an arbitrary value of the momentum imbalance of the jets;
- It encompasses the regimes of validity of both HEF and TMD.

Forward di-jet production in dilute-dense collisions

Outlook

- Show equivalence between CGC and TMD at finite Nc.

- Find scattering processes that will probe each of the TMD distributions individually and that will separate them experimentally.

- Derive analytical expressions for the TMD gluons present at finite Nc.

- Phenomenology of the new formula with evolved gluons and comparison to data.

Thank you

Forward di-jet production in dilute-dense collisions

Work in progress

- Gluon distributions in the Golec-Biernat-Wusthoff model

$$S(\mathbf{r}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle = \exp\left[-\frac{\mathbf{r}^2 Q_s^2}{4}\right]$$

- At large N_c there are two fundamental gluon distributions:
  - Dipole distribution $\mathcal{F}_{qg}^{(1)} = \frac{N_c\ k_t^2\ S_\perp}{2\pi^2\alpha_s}\int \frac{d^2r}{(2\pi)^2} e^{-i\mathbf{k}\cdot\mathbf{r}}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{r})U^\dagger(\mathbf{0})\right]\right\rangle$

- Weizsäcker-Williams distribution $x_2 G_1(x_2,k_t) = \frac{C_F}{2\alpha_s\pi^4}\int d^2b\int\frac{d^2r}{r^2}\ e^{-i\mathbf{k}\cdot\mathbf{r}}\ N_A(x,r,\mathbf{b})$

$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = \int d^2q_t\, x_2G_1(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \int d^2q_t\, x_2G_2(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\int d^2q_t\frac{(k_t-q_t)\cdot q_t}{q_t^2}x_2G_2(x_2,q_t)F(x_2,k_t-q_t)$$

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = \int d^2q_t d^2q_t'\, x_2G_1(x_2,q_t)F(x_2,q_t')F(x_2,k_t-q_t-q_t')$$

TMD factorizaion in the large *Nc* limit

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_i^n H^{(i)}_{ag\to cd}\mathcal{F}^{(i)}_{ag}(x_2,k_t)\frac{1}{1+\delta_{cd}}$$

Dominguez, Marquet, Xiao and Yuan (2011)

$$qg \to qg: \qquad n=2$$

$$gg \to q\bar{q}: \qquad n=2$$

$$gg \to gg: \qquad n=3$$

$$\mathcal{F}^{(1)}_{qg} = xG^{(2)}(x,q_\perp),\quad \mathcal{F}^{(2)}_{qg} = \int xG^{(1)}(q_1)\otimes F(q_2)\ ,$$

$$\mathcal{F}^{(1)}_{gg} = \int xG^{(2)}(q_1)\otimes F(q_2),\quad \mathcal{F}^{(2)}_{gg} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)}(q_1)\otimes F(q_2)\ ,$$

$$\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1)\otimes F(q_2)\otimes F(q_3)\ ,$$

TMD factorizaion at finite *Nc*

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_i^n H^{(i)}_{ag\to cd} \mathcal{F}^{(i)}_{ag}(x_2,k_t) \frac{1}{1+\delta_{cd}}$$

$qg \to qg:\quad n=2$

$gg \to q\bar{q}:\quad n=3$

$gg \to gg:\quad n=6$

Kotko, Kutak, Marquet, EP,
Sapeta, van Hameren (2015)

Not all the hard factors are independent

$\Leftrightarrow$

Only two independent gluon distributions

TMD factorizaion at finite *Nc* and arbitrary k_t

$$\frac{d\sigma^{pA\to \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2}\sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\sum_{i=1}^{2} K^{(i)}_{ag\to cd}\Phi^{(i)}_{ag\to cd}\,\frac{1}{1+\delta_{cd}}$$

colour-ordered amplitude squared	gluon TMD
$\lvert\mathcal{M}_{gg^*\to gg}(1^*,2,3,4)\rvert^2$ $\lvert\mathcal{M}_{gg^*\to gg}(1^*,3,2,4)\rvert^2$	$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\big(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg}$ $+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\big)$
$\mathcal{M}_{gg^*\to gg}(1^*,2,3,4)\,\mathcal{M}^*_{gg^*\to gg}(1^*,3,2,4)$ $\mathcal{M}^*_{gg^*\to gg}(1^*,2,3,4)\,\mathcal{M}_{gg^*\to gg}(1^*,3,2,4)$	$\Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\big(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg}$ $+\mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\big)$

TMD's and hard factors from color ordered amplitudes:

$gg \to gg$

$$\mathcal{M}^{a_1a_2a_3a_4}_{gg^*\to gg}\left(n_1,\varepsilon_2^{\lambda_2},\varepsilon_3^{\lambda_3},\varepsilon_4^{\lambda_4}\right) = f_{a_1a_2c}f_{ca_3a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}\right)$$
$$+f_{a_1a_3c}f_{ca_2a_4}\,\mathcal{M}_{gg^*\to gg}\left(1^*,3^{\lambda_3},2^{\lambda_2},4^{\lambda_4}\right)$$