Gauge-Invariant Gluon TMD from large- to small-x in the coordinate space

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Outline:

- **Operator definition** of **TMD**: gauge invariance, path dependence, universality, factorisation
- **Path-dependence** as the central issue: can one make any use of it?
- **Stokes-Mandelstam** gTMD: fully gauge-invariant, maximally path-dependent
- Evolution in the coordinate space: **equations of motion** in the loop space
- **Abelian case**: exponentiation, area derivative
- **Outlook**

Definitions of TMD/uPDF

- TMD **factorisation** $\rightarrow$ **operator definition**
- uPDF via **evolution/resummation**: DGLAP, BFKL, CCFM
- **High-energy/small-x** regime: Balitsky, Kovchegov + extentions
- Looking for the an alternative approach

Operator structure of TMD

'Standard' approach:

factorisation in a convenient gauge (small- or large-x regime) $\to$ gauge-dependent pdf $\to$ gluon resummation $\to$ gauge-invariant pdf with Wilson lines, path-dependence as prescribed by the factorisation

Alternative approach:

operator structure related to a given pdf $\to$ generic gauge-invariant path-dependent object $\to$ evolution in the coordinate space to fit the factorisation scheme (small- or large-x regime) $\to$ gauge-invariant pdf with Wilson lines, path-dependence as prescribed by the factorisation

Gluon TMD: from Small-x to Large-x

@ [Mulders, Rodrigues (2001); Collins (2011); Dominguez, Marquet, Xiao, Yuan (2011); Balitsky, Tarasov (2014, 2015)]

Small-x

$$\mathbf{G}^{ij}_{\rm small-x}(x, \mathbf{k}_\perp; P, S) = \int dz^- \int d^2 z_\perp \mathrm{e}^{ik_\perp z_\perp}$$

$$\langle h| \, D^i \mathcal{W}_{\rm LC}(z^-, \mathbf{z}_\perp) \, {\mathcal{W}^\dagger}_{\rm LC}(z^-, \mathbf{z}_\perp) D^j \mathcal{W}_{\rm LC}(0^-, \mathbf{0}_\perp) \, {\mathcal{W}^\dagger}_{\rm LC}(0^-, \mathbf{0}_\perp) \, |h\rangle$$

$$= \int dz^- \int d^2 z_\perp \mathrm{e}^{ik_\perp z_\perp}$$

$$\langle h| \, \mathcal{F}^{il}(z^-, \mathbf{z}_\perp) {\mathcal{W}^\dagger}_{\rm LC}(z) \; \mathcal{W}_{\rm LC}(0) \; \mathcal{F}^{lj}(0^-, \mathbf{0}_\perp) \; |h\rangle$$

$$= \int dz^- \int d^2 z_\perp \mathrm{e}^{ik_\perp z_\perp} \langle h| \, \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z, 0) \; \mathcal{F}^{lj}(0) \; |h\rangle$$

Rapidity cutoff: $\ln x$; single-logs $\alpha_s \ln x$; non-linear dynamics, BK Eq.

Gluon TMD: from Small-x to Large-x

@ [Mulders, Rodrigues (2001); Collins (2011); Dominguez, Marquet, Xiao, Yuan (2011); Balitsky, Tarasov (2014, 2015)]

Large/Moderate-x

$$\mathbf{G}^{ij}_{\rm small-x}(x, \mathbf{k}_\perp; P, S) = \int dz^- \int d^2 z_\perp \mathrm{e}^{-ixp^+ z^- + ik_\perp z_\perp}$$

$$\langle h| \mathcal{F}^{il}(z^-, \mathbf{z}_\perp) \mathcal{W}^\dagger{}_{\rm LC}(z)\ \mathcal{W}_{\rm LC}(0)\ \mathcal{F}^{lj}(0^-, \mathbf{0}_\perp)\ |h\rangle$$

$$= \int dz^- \int d^2 z_\perp \mathrm{e}^{-ixp^+ z^- + ik_\perp z_\perp} \langle h| \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z, 0)\ \mathcal{F}^{lj}(0)\ |h\rangle$$

Rapidity cutoff: $\eta \neq \ln x$; double-logs are possible $\alpha_s \eta \ln x$; linear evolution.

Gluon TMD: from Small-x to Large-x

Two definitions in two regimes—look so similar, but in fact very different:

$$\text{small} = \int dz^- \int d^2 z_\perp \mathrm{e}^{ik_\perp z_\perp} \langle h| \, \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z,0) \, \mathcal{F}^{lj}(0) \, |h\rangle$$

VS

$$\text{large} = \int dz^- \int d^2 z_\perp \mathrm{e}^{-ixp^+ z^- + ik_\perp z_\perp} \langle h| \, \mathcal{F}^{il}(z) \mathcal{W}_{\gamma_{\rm LC}}(z,0) \, \mathcal{F}^{lj}(0) \, |h\rangle$$

Factorization schemes are different, evolution is different: how to relate?
Very complicated connection @ [Balitsky, Tarasov (2014, 2015)]
However: the operator structure is the same. Let us start with it and forget (for a while) about the factorization issues.

Quark and Gluon TMD: Generic Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

Highly **gauge-dependent quark correlator** for a hadron h with a momentum P and spin S

$$\mathbf{Q}_{\rm g-d}(x, \mathbf{k}_\perp; P, S) = \int d^4z \ {\rm e}^{-ikz} \ \langle h| \ \bar{\psi}(z)\psi(0) \ |h\rangle$$

Highly **gauge-dependent gluon correlator** for a hadron h with a momentum P and spin S

$$\mathbf{G}^{\mu\nu}_{\rm g-d}(x, \mathbf{k}_\perp; P, S) = \int d^4z \ {\rm e}^{-ikz} \ \langle h| \ \mathcal{A}^\mu(z)\mathcal{A}^\nu(0) \ |h\rangle$$

Gluon TMD: Gauge-Invariant Operator Definition

@ [Mulders, Rodrigues (2001); Collins (2011)]

$$\mathbf{G}^{\mu\nu|\rho\sigma}(x, \mathbf{k}_\perp; P, S) = \int d^4z \, \mathrm{e}^{-ikz} \, \langle h| \, \mathcal{F}^{\mu\nu}(z) \, \mathcal{W}_\gamma \, \mathcal{F}^{\rho\sigma}(0) \, |h\rangle$$

Wilson line (system of lines) $\mathcal{W}_\gamma$ in the adjoint representation

$$\mathcal{F}_{\mu\nu} = F^a_{\mu\nu} T^a$$

Respects the desirable operator structure

Knows nothing about any factorization scheme: maximally path-dependent, γ is entirely arbitrary

Still difficult to evaluate

Gluon TMD: Several Operator Definitions

@ [Mulders, Rodrigues (2001); Collins (2011)]

Gluon TMD from the generic **gauge-invariant correlator**

$$\mathbf{G}^{\mu\nu|\rho\sigma}(k; P, S) =$$

$$\int d^4 z \ \mathrm{e}^{-ikz} \ \langle h| \ \mathcal{F}^{\mu\nu}(z) \ \mathcal{W}_\gamma \ \mathcal{F}^{\rho\sigma}(0) \ |h\rangle$$

$$\rightarrow$$

$$\mathbf{G}^{ij}(x, k_\perp; P, S) \sim \int dk^- \ \mathbf{G}^{+i|+j}(k; P, S) =$$

$$\int dz^- d^2 z_\perp \ \mathrm{e}^{-ikz} \ \langle h| \ \mathcal{F}^{+i}(z) \ \mathcal{W}_\gamma \ \mathcal{F}^{+j}(0) \ |h\rangle$$

Equations of Motion in the Loop Space

@ [Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt et al. (1981, 1982)]

Wilson loops as the (fundamental) gauge-invariant degrees of freedom:

$$\langle \mathcal{W}_\gamma \rangle = \langle \mathcal{P}_\gamma \exp \oint_\gamma d\zeta_\mu \mathcal{A}^\mu(\zeta) \rangle$$

or

$$\langle \mathcal{W}^n_{\gamma_1, \dots \gamma_n} \rangle = \langle \mathcal{T} \mathcal{W}_{\gamma_1} \cdots \mathcal{W}_{\gamma_n} \rangle$$

The Wilson functionals obey the **Makeenko-Migdal** loop equations:

$$\partial^\nu \frac{\delta}{\delta \sigma_{\mu\nu}(x)} \langle \mathcal{W}^1_\gamma \rangle = N_c g^2 \oint_\gamma dz^\mu \, \delta^{(4)}(x - z) \langle \mathcal{W}^2_{\gamma_{xz}\gamma_{zx}} \rangle$$

Loop space and differential operators

Area derivative:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\langle\mathcal{W}_\gamma\rangle = \lim_{|\delta\sigma_{\mu\nu}(z)|\to 0}\frac{\langle\mathcal{W}_{\gamma\delta\gamma_x}\rangle - \langle\mathcal{W}_\gamma\rangle}{|\delta\sigma_{\mu\nu}(z)|}$$

Path derivative:

$$\partial_\mu\langle\mathcal{W}_\gamma\rangle = \lim_{|\delta z_\mu|\to 0}\frac{\langle\mathcal{W}_{\delta z_\mu^{-1}\gamma\delta z_\mu}\rangle - \langle\mathcal{W}_\gamma\rangle}{|\delta z_\mu|}$$

Differential operators in the loop space $\to$ evolution of the Wilson loops in the coordinate representation $=$ equations of motion in the loop space

Stokes-Mandelstam Gluon TMD

Non-Abelian Stokes' theorem

@ [Arefeva (1980) etc.]

$$\mathcal{P}_\gamma \exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right] = \mathcal{P}_\gamma \mathcal{P}_\sigma \exp\left[\int_\sigma d\sigma_{\rho\rho'}(\zeta)\mathcal{F}^{\rho\rho'}(\zeta)\right]$$

Mandelstam formula

@ [Mandelstam (1968)]

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\mathcal{P}_\gamma \exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right] = \mathcal{P}_\gamma \mathcal{F}^{\mu\nu}(x) \exp\left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta)\right]$$

Stokes-Mandelstam Gluon TMD

$$\tilde{\mathbf{G}}^{\mu\nu|\mu'\nu'}(z;P,S) = \frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu'\nu'}(0)}\langle h|\mathcal{W}_{\gamma^{[z,0]}}\ |h\rangle =$$

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\frac{\delta}{\delta\sigma_{\mu'\nu'}(0)}\sum_X \langle h|\mathcal{W'}_{\gamma^{[z]}}|X\rangle\langle X|\mathcal{W'}_{\gamma^{[0]}}\ |h\rangle$$

Non-Abelian exponentiation

$$\langle W_{\gamma^{[z,0]}}\rangle = \exp\left[\sum a_n W^{(n)}\right]\ ,\ W^{(n)} = \text{hadronic correlators}$$

Gauge invariance, Path dependence, Universality

Evolution in the Coordinate Space: Abelian Case

Abelian exponentiation

$$\langle \mathcal{W}_\gamma \rangle = \langle h | \mathcal{P}_\gamma \exp \left[\oint_\gamma d\zeta_\rho \mathcal{A}^\rho(\zeta) \right] | h \rangle =$$

$$\exp \left[-\frac{g^2}{2} \oint_\gamma d\zeta_\mu \oint_\gamma d\zeta'_\nu \, D_{\mu\nu}(\zeta - \zeta') \right]$$

Basic hadronic correlator

$$D_{\mu\nu}(\zeta - \zeta') = \langle A_\mu(\zeta) A_\nu(\zeta') \rangle$$

Evolution in the Coordinate Space: Abelian Case

Parameterization

$$D_{\rho\rho'}(z) =$$
$$g_{\rho\rho'}\ D_1(z,P) + \partial_\rho\partial_{\rho'}\ D_2(z,P) + \{P_\rho\partial_{\rho'}\}\ D_3(z,P) + P_\rho P_{\rho'}\ D_4(z,P)$$

In general, the hadronic correlator contains all necessary information

$$D_{\rho\rho'}(\zeta-\zeta') = \langle P,S|A_\rho(\zeta)A_{\rho'}(\zeta')|P,S\rangle$$

Evolution in the Coordinate Space: Abelian Case

Area derivative

$$\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\langle h|\mathcal{W}_\gamma|h\rangle =$$
$$-\frac{g^2}{2}\left[\frac{\delta}{\delta\sigma_{\mu\nu}(z)}\oint_\gamma d\zeta_\rho \oint_\gamma d\zeta'_{\rho'}\, D_{\rho\rho'}(\zeta-\zeta')\right]\langle h|\mathcal{W}_\gamma|h\rangle$$

Non-vanishing terms after taking the path-derivative ∂_ν

– standard Makeenko-Migdal term

$$\sim \oint_\gamma d\zeta^\nu\ \partial^2\ D_1(z^2, P^2)$$

– hadron momentum-dependent term

$$\sim \oint_\gamma d\zeta^\nu\ (P\partial)^2\ D_4(z^2, P^2)$$

Evolution in the Coordinate Space: Abelian Case

Shape evolution equation

$$\partial^z_\mu \frac{\delta}{\delta\sigma_{\mu\nu}(z)} \langle h|\mathcal{W}_\gamma|h\rangle =$$

$$-\frac{g^2}{2}\left[\oint_\gamma \; d\zeta^\nu \; \left(\partial^2 \; D_1(z,P) + (P\partial)^2 \; D_4(z,P)\right)\right] \langle h|\mathcal{W}_\gamma|h\rangle$$

Consistency check: Wilson loops in vacuum

$$\partial^2 D_1(z) = -\delta^{(4)}(z), \;\; D_4 = 0$$

$$\partial^z_\mu \frac{\delta}{\delta\sigma_{\mu\nu}(z)} \langle 0|\mathcal{W}_\gamma|0\rangle = g^2 \oint_\gamma d\zeta^\nu \;\; \delta^{(4)}(z-\zeta)$$

= Makeenko-Migdal Eq. in the LO.

Outlook:

Gluon TMD distribution function can be formulated within fully gauge-invariant, generically path-dependent framework based on the loop space formalism in the coordinate representation. It is not associated with any factorization framework and respects the operator structure.

This approach goes the other way round wrt to the standard one: one starts with a maximally general object and then extracts a gluon TMD which is adjustable to any specific factorization scheme by means of the geometrical evolution in the coordinate space.

The main ingredients of this approach are the hadronic matrix elements of Wilson loops $\langle h| \mathcal{W}_\gamma |h\rangle$.

Non-Abelian exponentiation enables separation of the non-local path-dependence and local UV-divergent contributions and appropriate parameterisation of various gTMD functions.

The work in progress: arXiv:1511.00517 [hep-ph]