

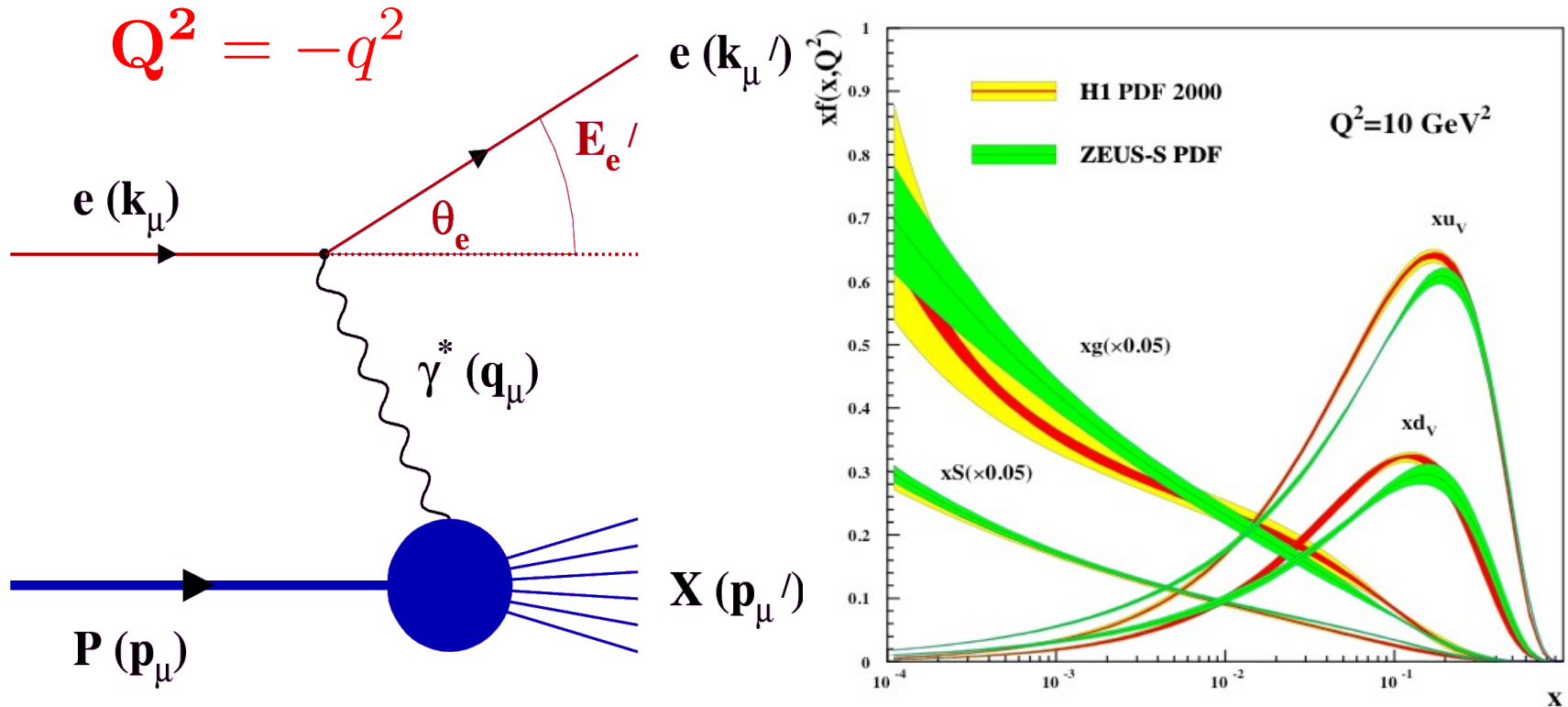
Multi(3)-particle production in DIS at small x

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New York, NY***

***MPI@LHC, 23-27 November 2015
ICTP, Trieste, Italy***

In collaboration with A. Ayala, M. Hentschinski and M.E. Tejeda-Yeomans

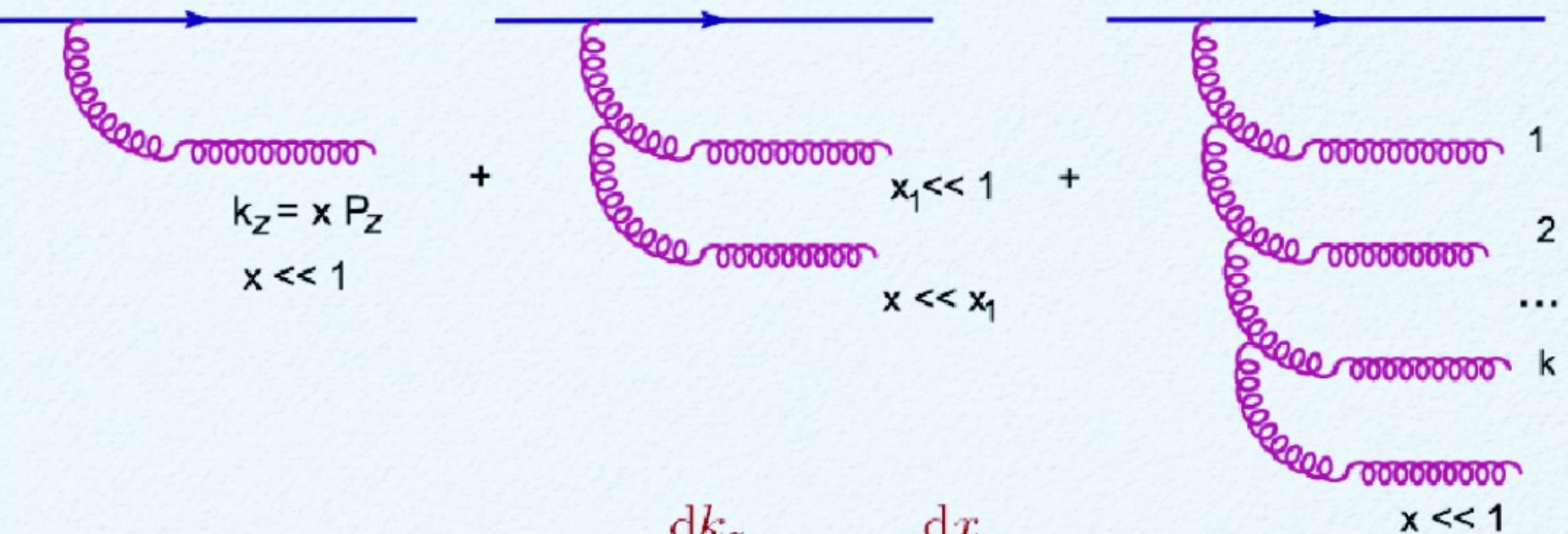
DIS at HERA: parton distributions



power-like growth of gluon and sea quark distributions with x
new QCD dynamics at small x ?

gluon radiation at small x : pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small- x) gluons



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

The 'price' of an additional gluon:

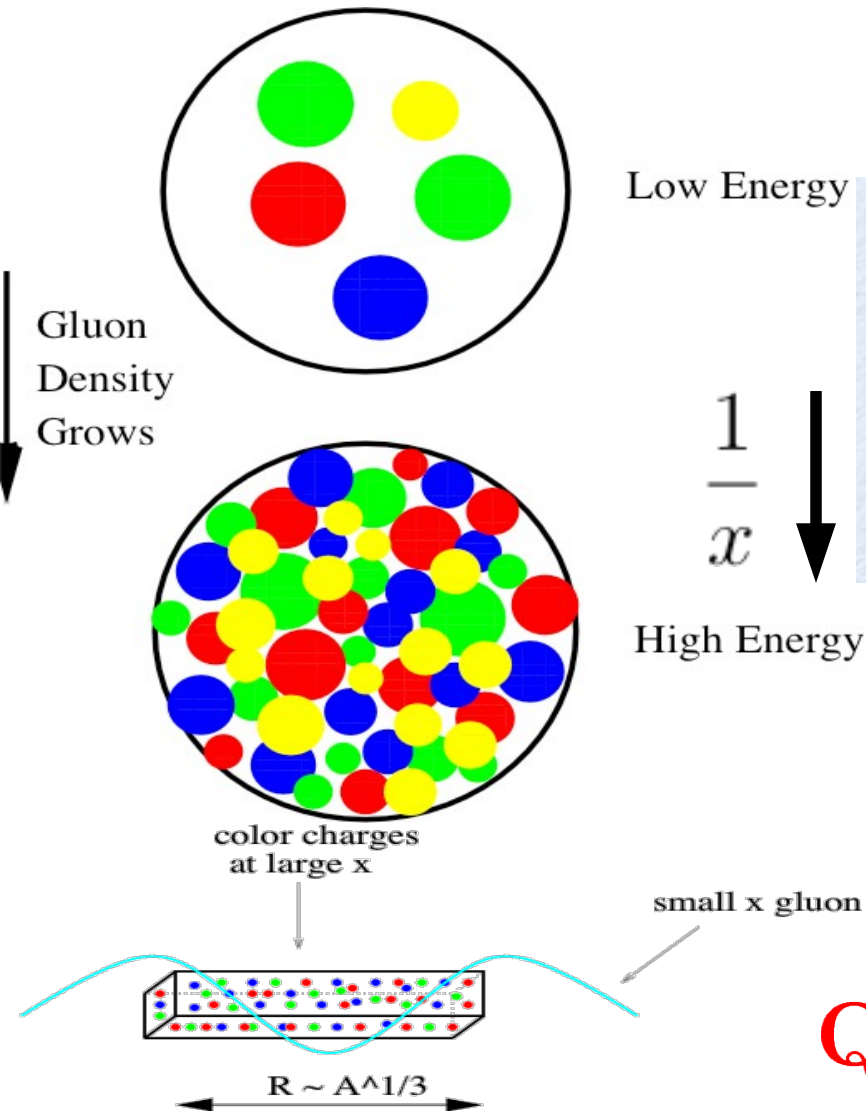
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

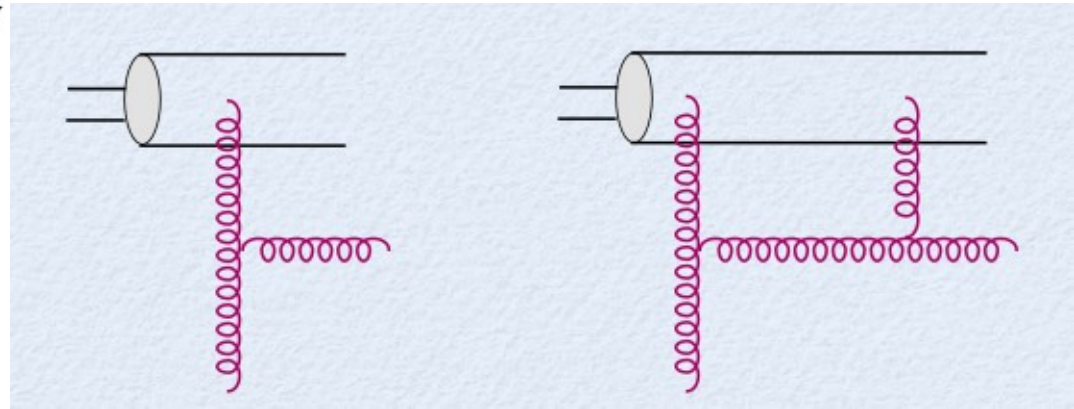
$$n \sim e^{\alpha_s \ln 1/x}$$

Gluon saturation

*Gribov-Levin-Ryskin
Mueller-Qiu*



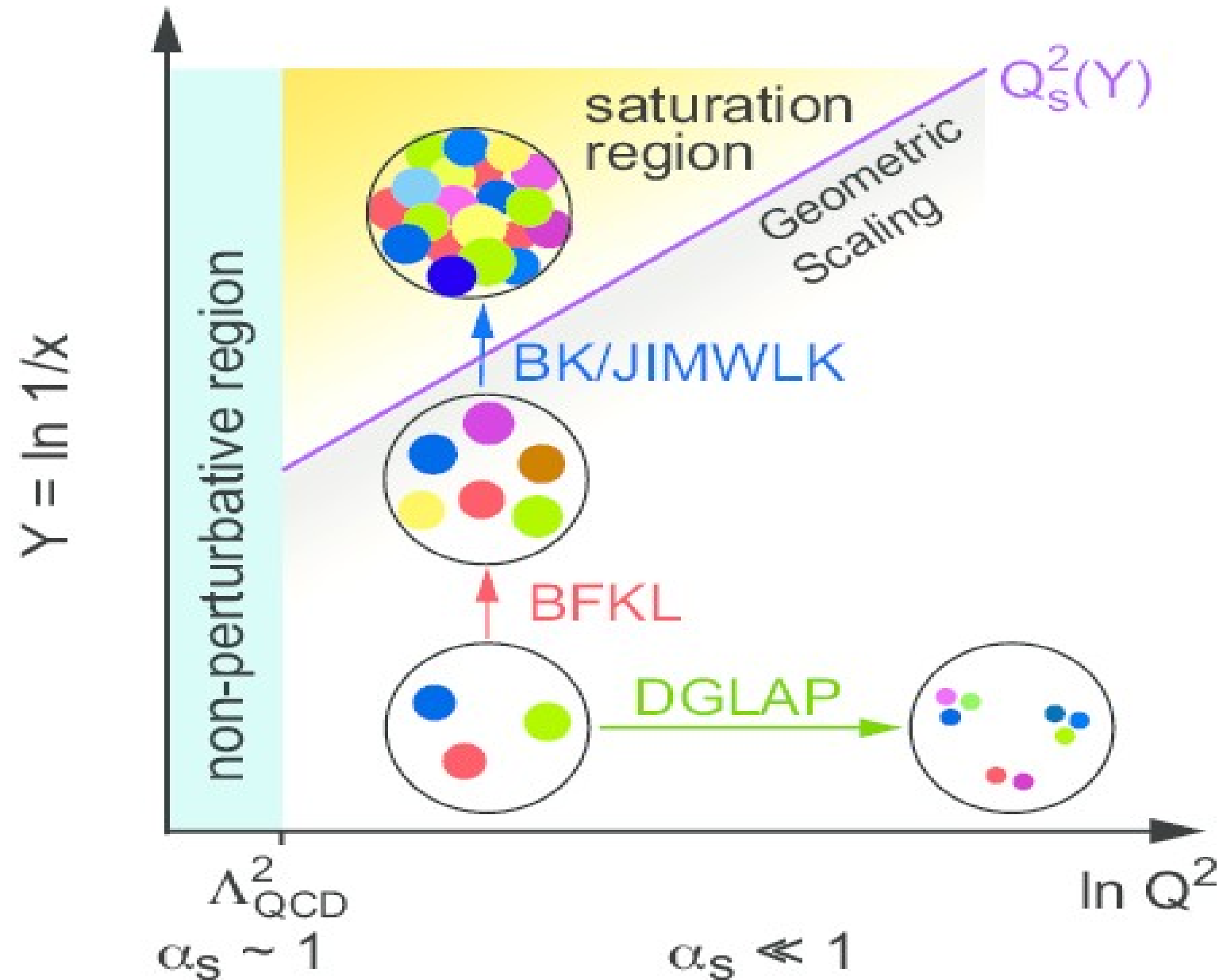
“attractive” bremsstrahlung vs.
“repulsive” recombination



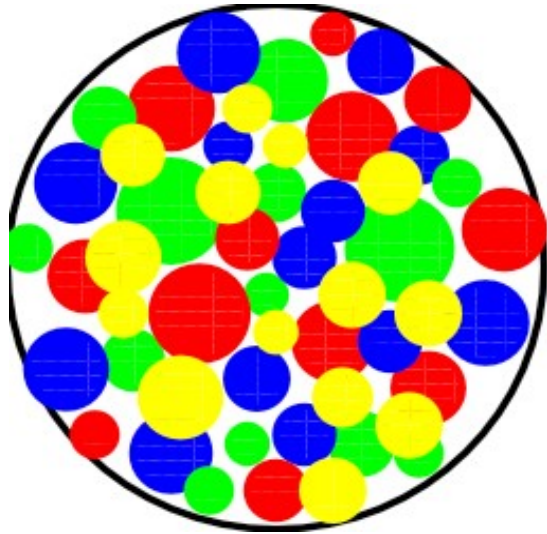
$$\frac{\alpha_s}{Q^2} \frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

A proton at high energy: **saturation**

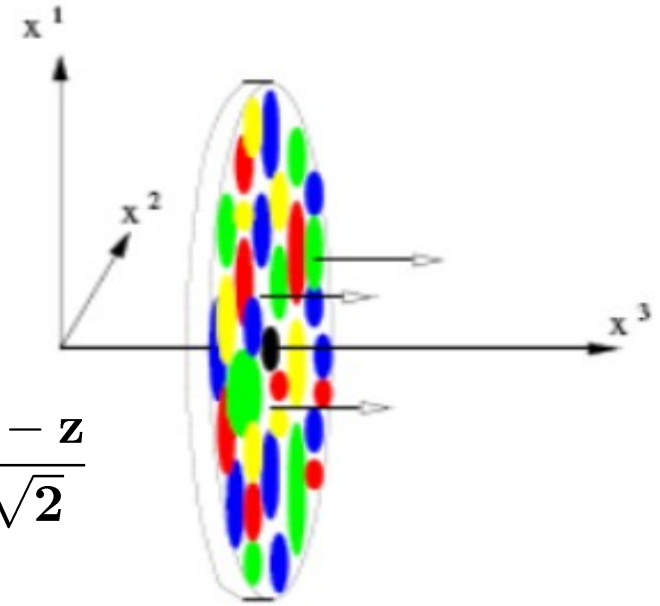


Large A is cheaper than high energy



boost

$$x^+ \equiv \frac{t + z}{\sqrt{2}} \quad x^- \equiv \frac{t - z}{\sqrt{2}}$$



sheet of color charge moving along x^+ and sitting at $x^- = 0$

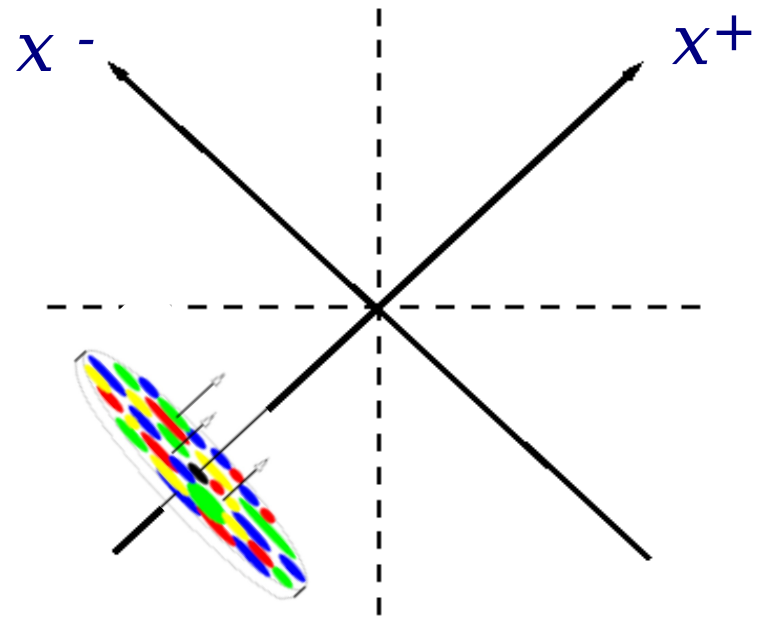
$$\mathbf{J}_a^\mu(\mathbf{x}) \equiv \delta^{\mu+} \delta(\mathbf{x}^-) \rho_a(\mathbf{x}_t)$$

*color
current*

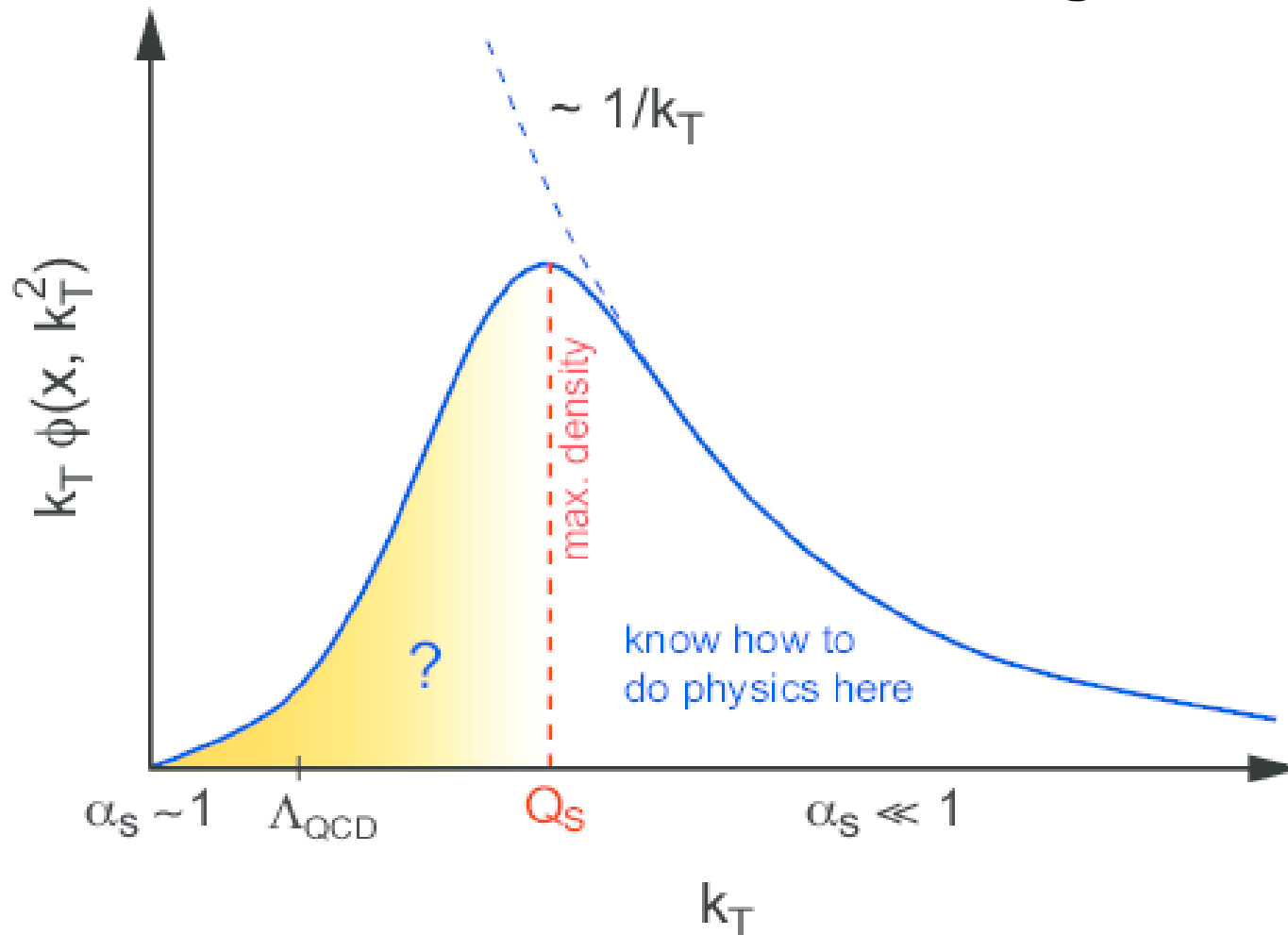
*color
charge*

$$\mathbf{A}_a^+(z^-, z_t) = \delta(z^-) \alpha_a(z_t)$$

with $\partial_t^2 \alpha_a(z_t) = g \rho_a(z_t)$



typical momentum of small x gluons: Q_s^2



one can compute parton multiplicities

low x QCD in a background field: CGC

(a high gluon density environment)

two main effects:

*“multiple scatterings” encoded in classical field (**p_t broadening**)*

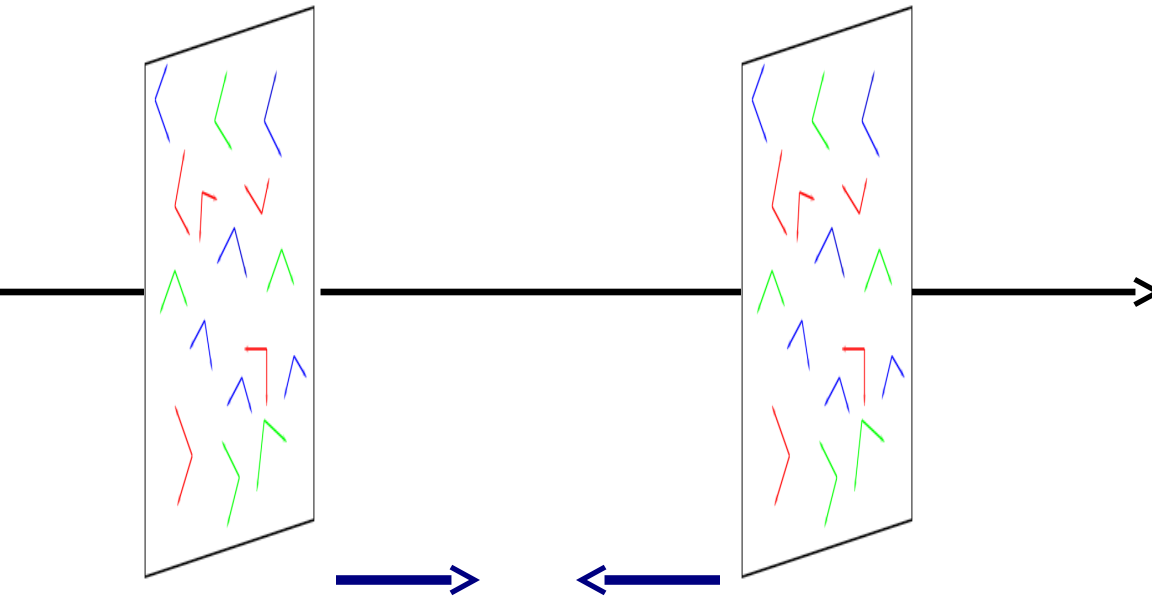
*evolution with $\ln(1/x)$ a la BK/JIMWLK equation (**suppression**)*

LT pQCD with collinear factorization:

single scattering

evolution with $\ln Q^2$

High energy collisions: colliding Sheets of Color Glass Condensates



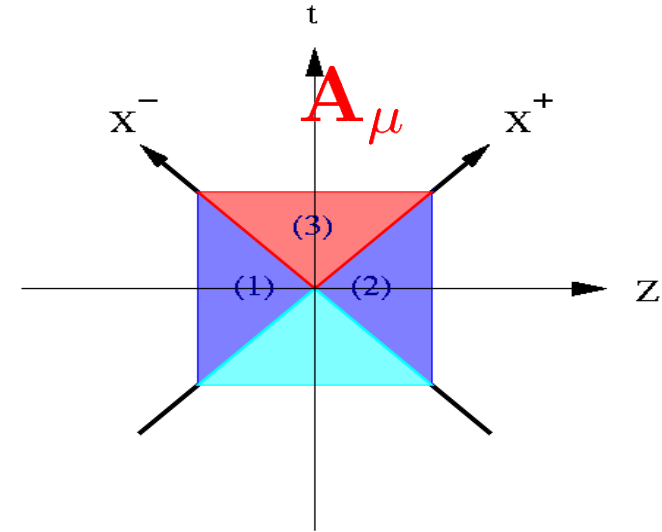
before the collision:

$$A^+ = A^- = 0$$

$$A^i = A_1^i + A_2^i$$

$$A_1^i = \theta(x^-)\theta(-x^+)\alpha_1^i$$

$$A_2^i = \theta(-x^-)\theta(x^+)\alpha_2^i$$



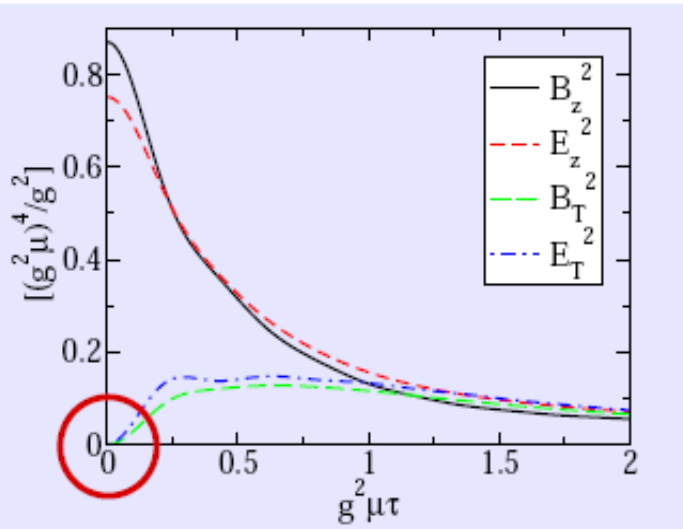
after the collision:

solve for A_μ

in the forward LC

GLASMA:

gluon fields produced in collision of two sheets of color glass



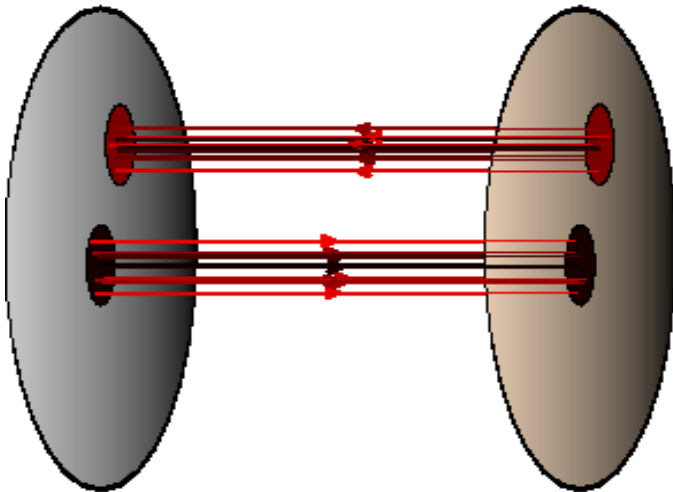
early on fields (E,B) are
longitudinal

classical solutions are
boost invariant

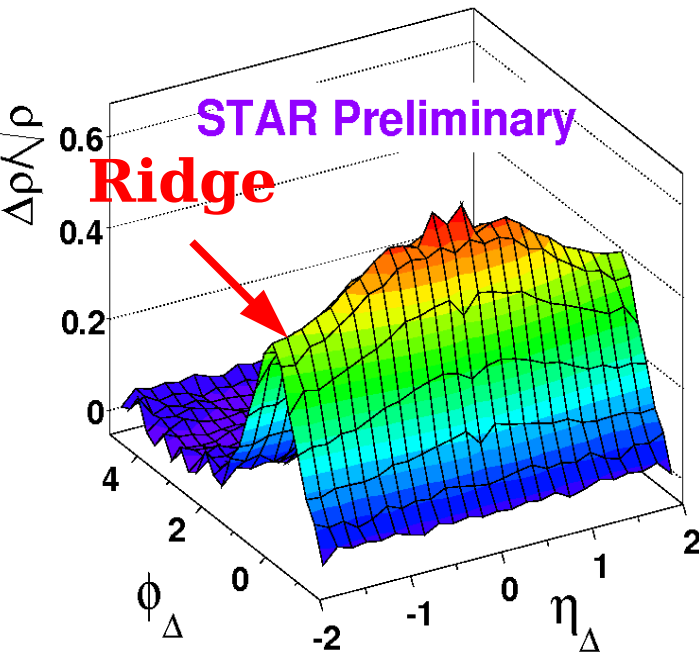
transverse size of
these flux tubes is $\sim \frac{1}{Q_s}$

$$\frac{1}{A_{\perp}} \frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

$$\frac{1}{A_{\perp}} \frac{dE_{\perp}}{d\eta} = \frac{0.25}{g^2} Q_s^3$$

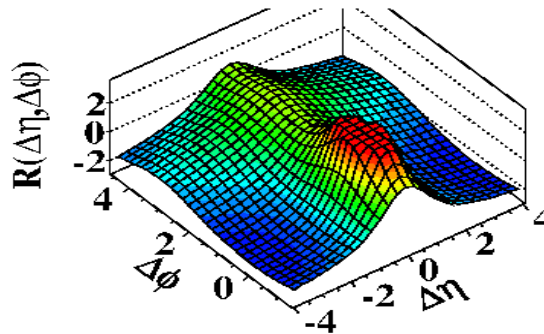


long-range rapidity correlations: the ridge

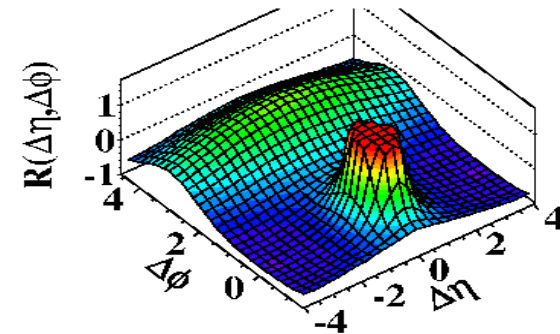


AA at RHIC

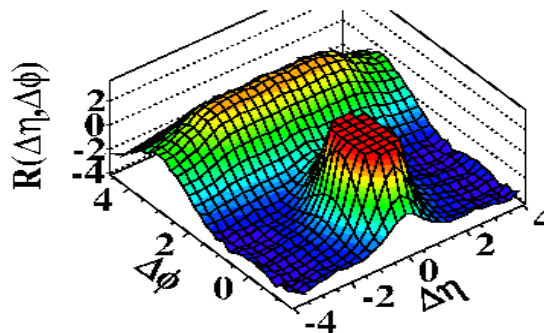
(a) CMS MinBias, $p_T > 0.1 \text{ GeV}/c$



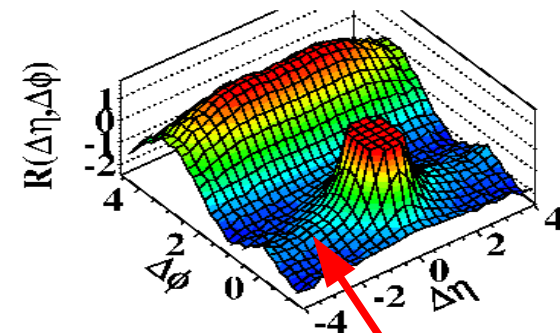
(b) CMS MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



(c) CMS $N \geq 110$, $p_T > 0.1 \text{ GeV}/c$



(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



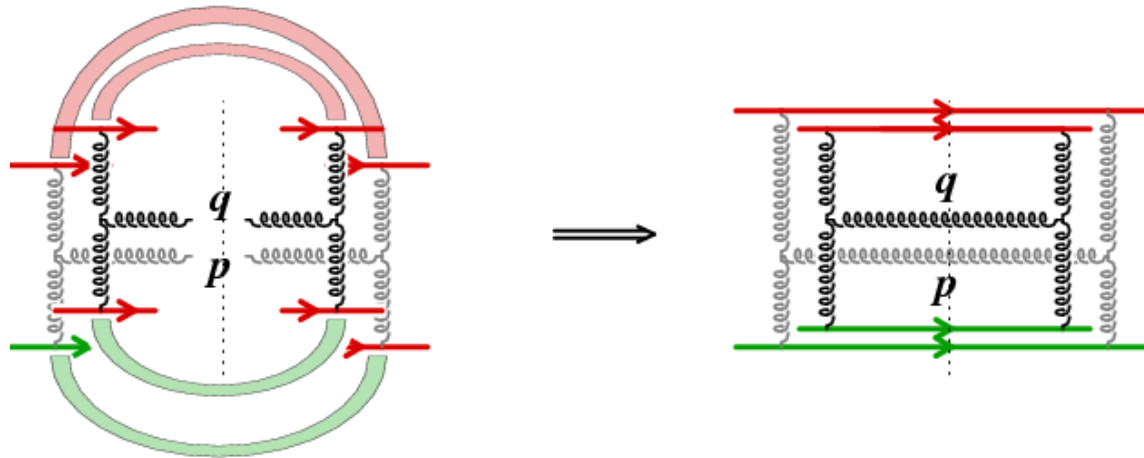
PP at LHC

Ridge

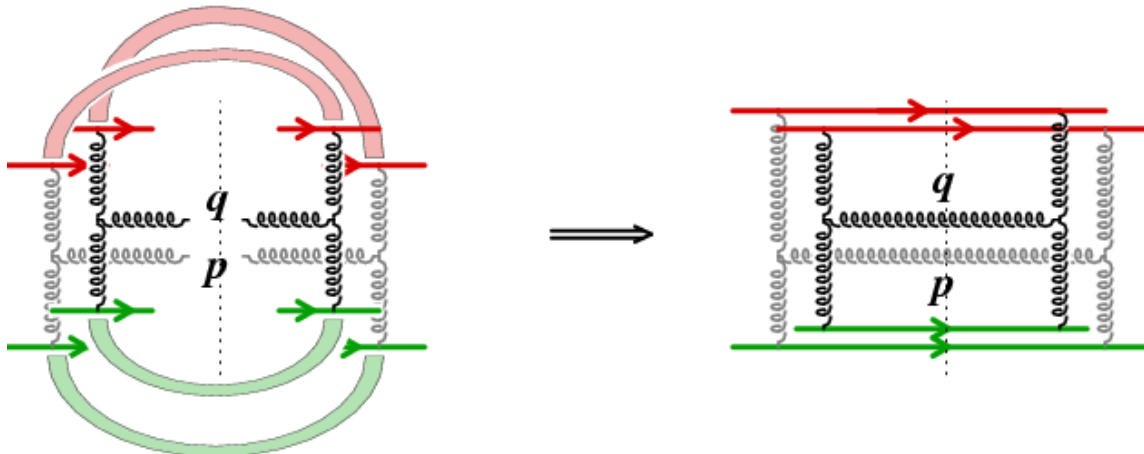
multi-gluon correlations:
need to go beyond single parton distributions

two-gluon production

Independent production of two gluons:

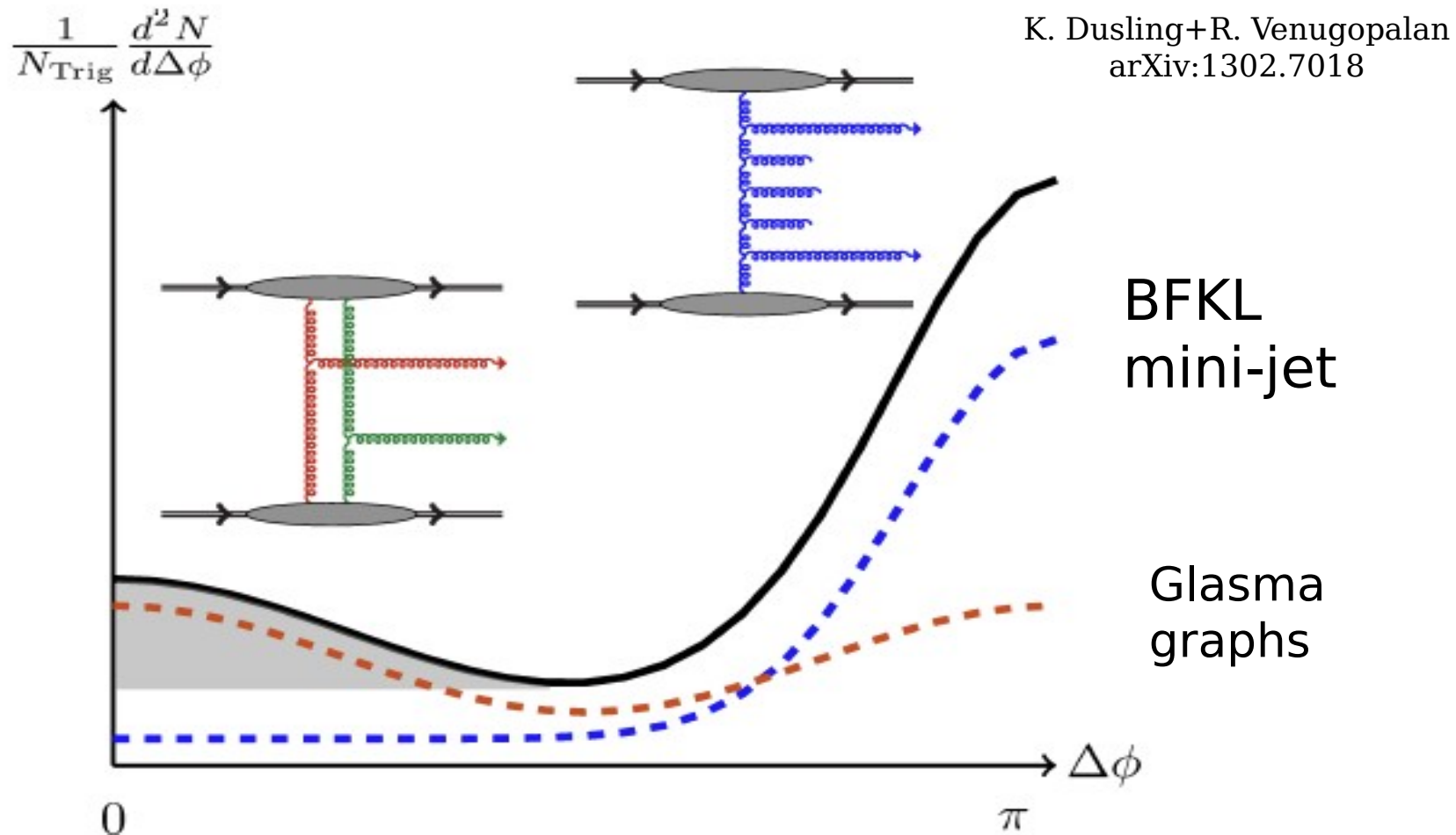


correlated two-gluon production:



DGMV:
NPA810 (2008) 91

Anatomy of long range di-hadron collimation



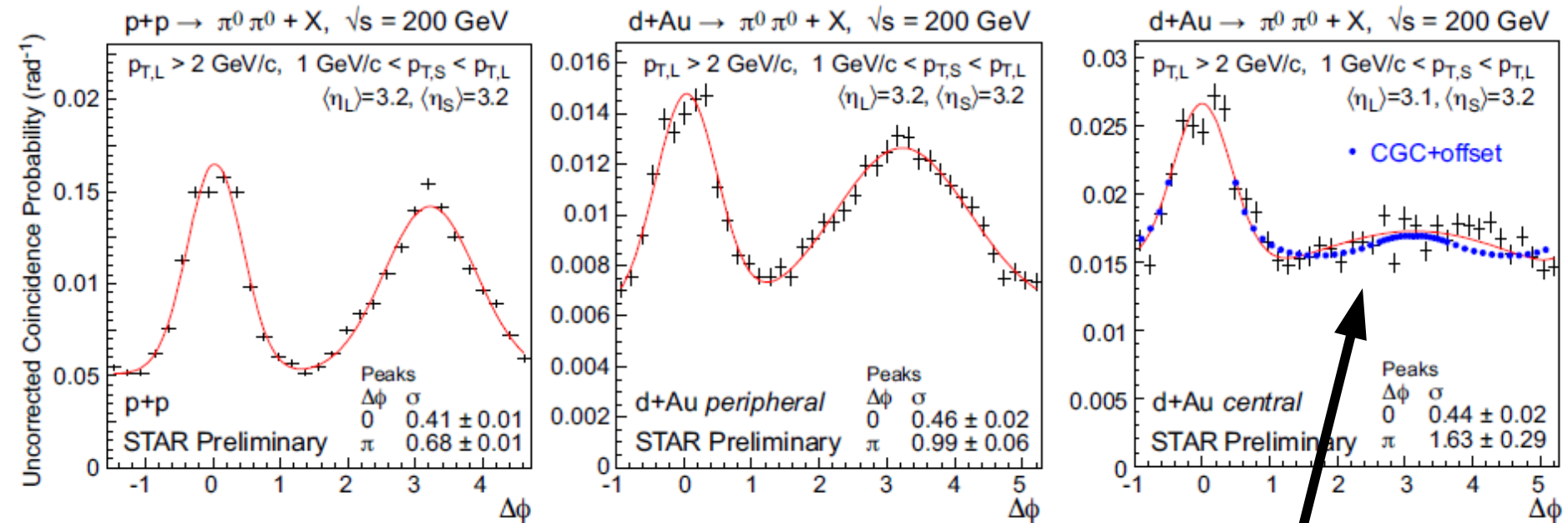
Talk by P. Tribedy

*late time/medium
effects?*

A “simpler” system: di-hadron correlations in pA

Talk by E. Petreska

Recent STAR measurement (arXiv:1008.3989v1):



Marquet, NPA (2007), Albacete + Marquet, PRL (2010)
 Tuchin, NPA846 (2010)
 A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)
 T. Lappi + H. Mantysaari, NPA908 (2013)

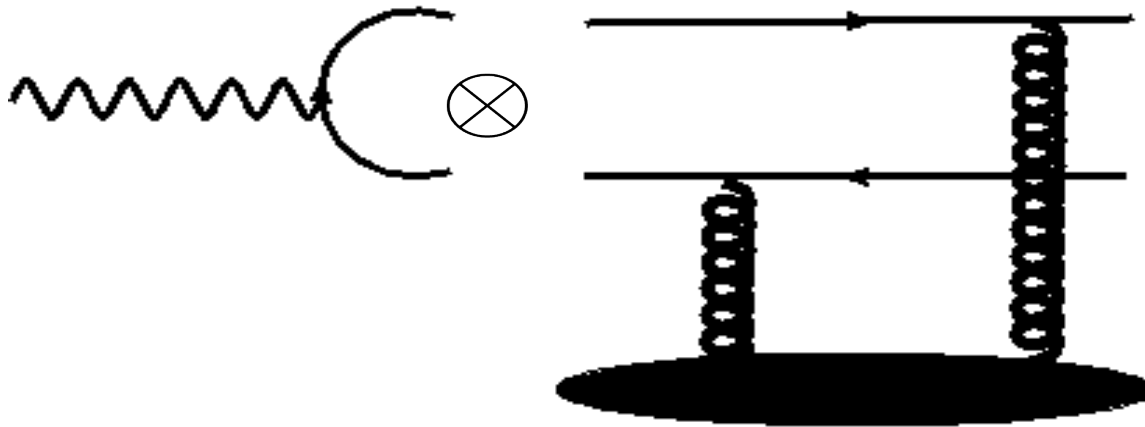
***saturation effects
de-correlate
the hadrons***

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

DIS total cross section

$$\sigma_{\text{DIS}}^{\text{total}} = 2 \int_0^1 dz \int d^2 \mathbf{x}_t d^2 \mathbf{y}_t \left| \Psi(\mathbf{k}^\pm, \mathbf{k}_t | z, \mathbf{x}_t, \mathbf{y}_t) \right|^2 \mathbf{T}(\mathbf{x}_t, \mathbf{y}_t)$$

$$\mathbf{T}(\mathbf{x}_t, \mathbf{y}_t) \equiv \frac{1}{N_c} \text{Tr} \langle 1 - \mathbf{V}(\mathbf{x}_t) \mathbf{V}^\dagger(\mathbf{y}_t) \rangle$$



$$\mathbf{V} \equiv \text{Wilson line} \equiv \text{multiple scatterings} \sim 1 + \mathcal{O}(g A) + \mathcal{O}(g^2 A^2)$$

Wilson line encodes multiple scatterings from the color field of the target

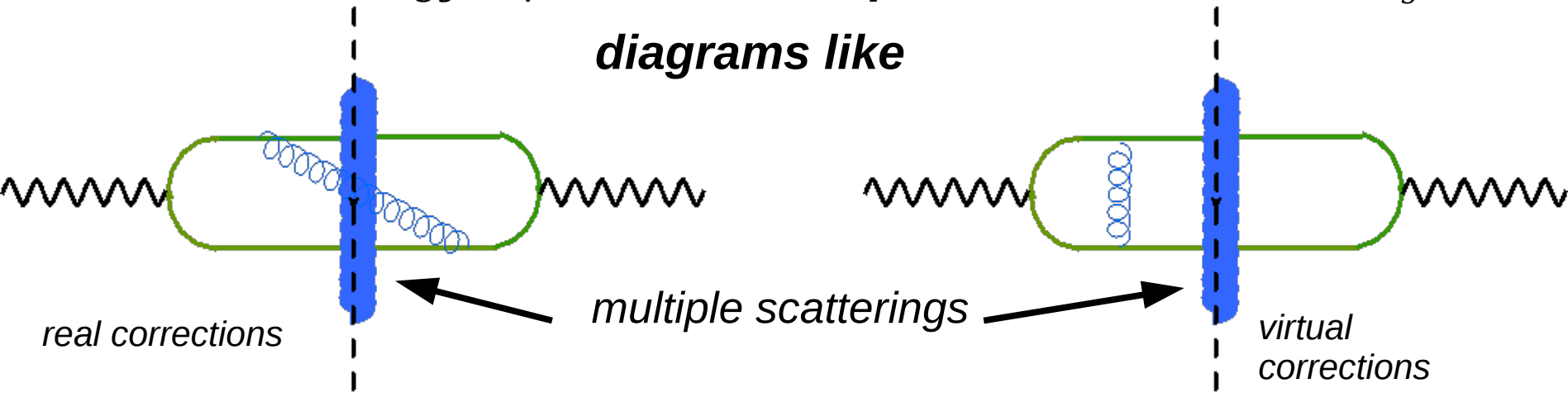
DIS total cross section: energy (x) dependence

recall the parton model was scale invariant, scaling violation (dependence on Q^2) came after quantum corrections - $O(\alpha_s)$

what we have done so far is to include high gluon density effects but no energy dependence yet

to include the energy dependence, need quantum corrections - $O(\alpha_s)$

diagrams like



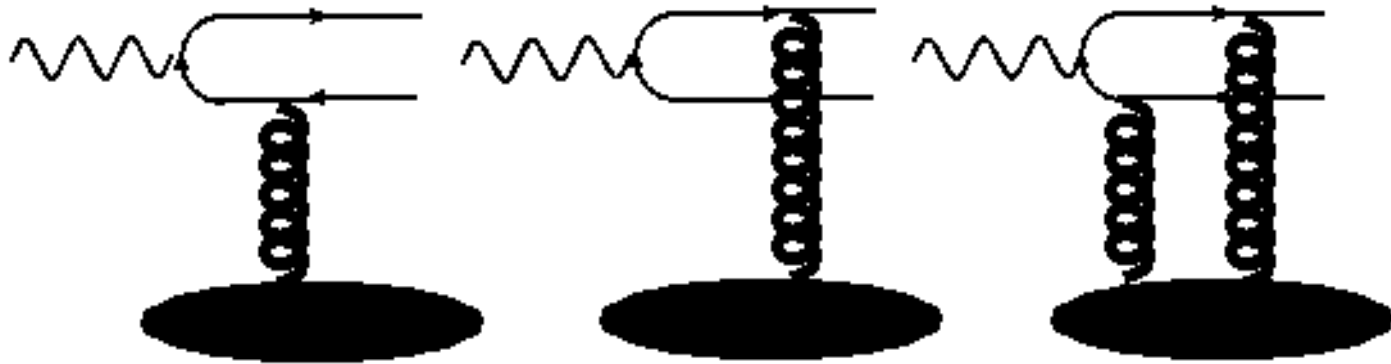
**x dependence of dipole cross section:
BK/JIMWLK evolution equation**

***NLO corrections
recently computed***

Extensive phenomenology at HERA

something with more discriminating power:
di-hadron correlations in DIS

LO: $\gamma^* T \rightarrow q \bar{q} X$

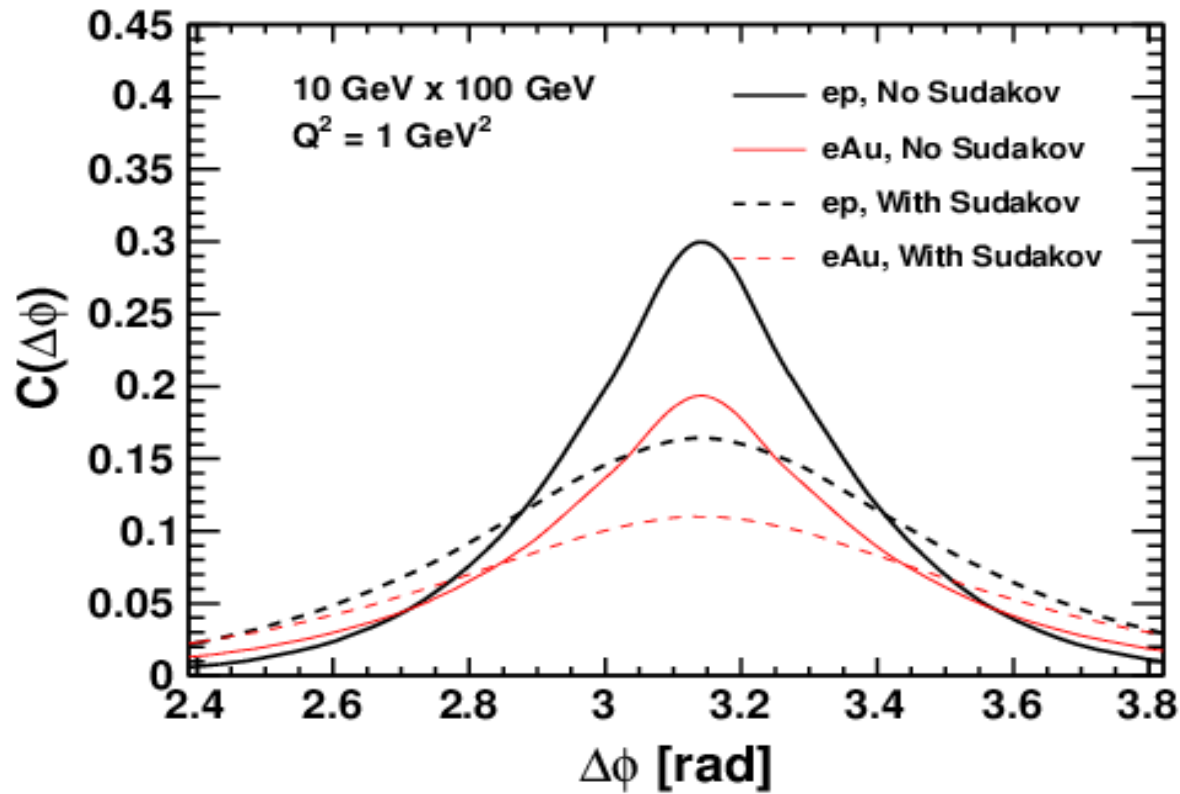


propagator in the background color field

$$S_F(q, p) \equiv (2\pi)^4 \delta^4(p - q) S_F^0(p) + S_F^0(q) \tau_f(q, p) S_F^0(p)$$

$$\begin{aligned} \tau_f(q, p) \equiv & (2\pi) \delta(p^- - q^-) \gamma^- \int d^2 x_t e^{i(q_t - p_t) \cdot x_t} \\ & \{ \theta(p^-) [V(x_t) - 1] - \theta(-p^-) [V^\dagger(x_t) - 1] \} \end{aligned}$$

Azimuthal correlations in DIS



Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

Toward precision CGC: *NLO corrections*

DIS total cross section:

photon impact factor
evolution equations

pA collisions:

Single inclusive particle production

NLO di-jet production in DIS

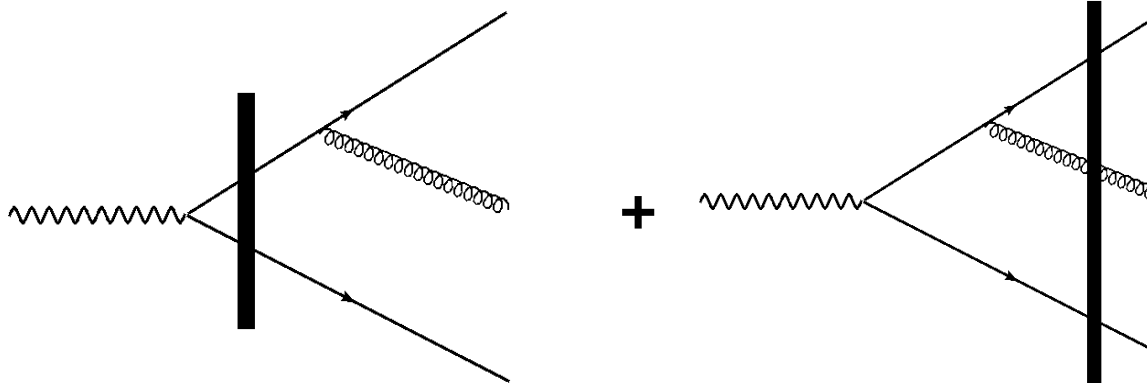
LO 3-jet production

Azimuthal correlations in DIS

*di-jet production in DIS: **NLO***

real contributions: $\gamma^* \mathbf{T} \rightarrow q \bar{q} g \mathbf{X}$

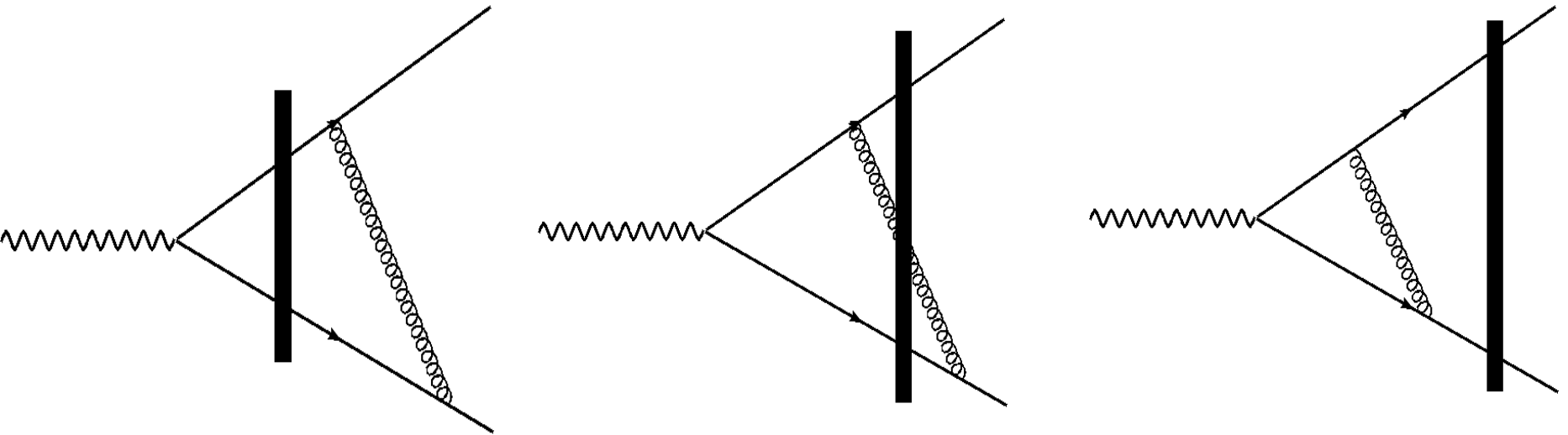
integrate out one of the produced partons



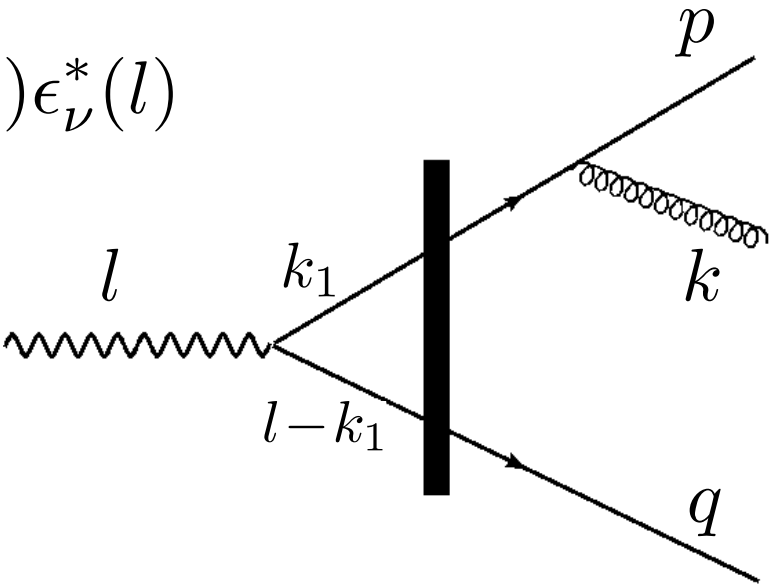
work in progress: Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans

di-jet azimuthal correlations in DIS

virtual contributions: $\gamma^* T \rightarrow q \bar{q} X$



+ “*self-energy*” diagrams

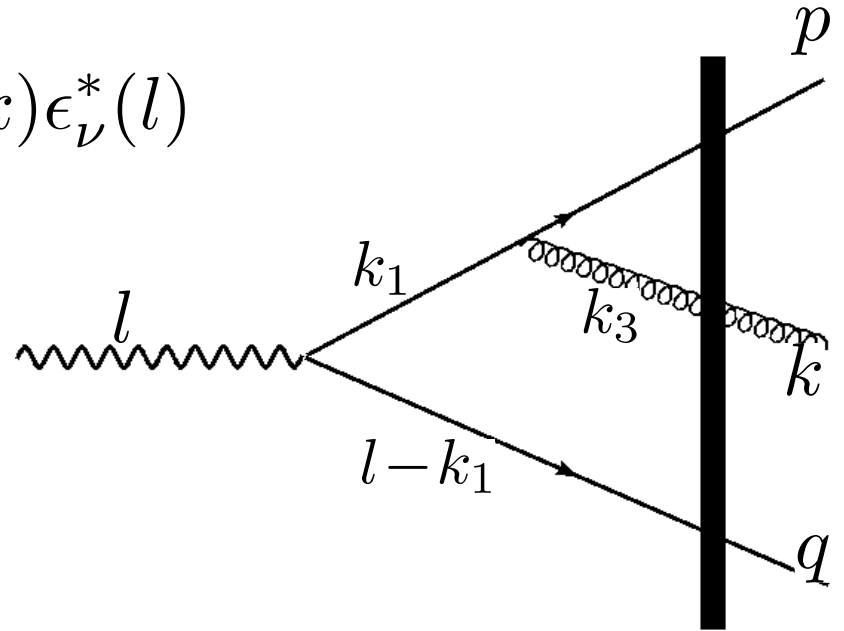
$$\mathcal{A} \equiv -eg \bar{u}(p) [A]^{\mu\nu} v(q) \epsilon_\mu(k) \epsilon_\nu^*(l)$$


$$A_1^{\mu\nu} = \gamma^\mu t^a S_F^0(p+k) \tau_F(p+k, k_1) S_F^0(k_1) \gamma^\nu S_F^0(l-k_1) \tau_F(l-k_1, q) \frac{d^4 k_1}{(2\pi)^4}$$

with interactions given by

$$\tau_F(p, q) \equiv (2\pi) \delta(p^- - q^-) \gamma^- \int d^2 z_t e^{-i(p_t - q_t) z_t} [\theta(p^-) V(z_t) - \theta(-p^-) V^\dagger(z_t)]$$

$$\mathcal{A} \equiv -eg \bar{u}(p) [A]^{\mu\nu} v(q) \epsilon_\mu(k) \epsilon_\nu^*(l)$$



$$A_2^{\mu\nu} = \tau_F(p, k_1 - k_3) S_F^0(k_1 - k_3) \gamma^\mu t^a S_F^0(k_1) \gamma^\nu S_F^0(l - k_1) \tau_F(l - k_1, q) \\ G_\mu^{0\lambda}(k_3) \tau_g^{ac}(k_3, k) \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4}$$

Two different ways of evaluating these:

1: Use the 1-d delta function, do k^+ integration using contour integration, reduced to 2-d transverse integration over k_t

2: promote all to 4-d, use “standard” momentum space techniques

A 1-loop example

$$I(p_1, p_2) = \int \frac{d^d k_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l - k_1)^2]} e^{iz_\epsilon(\cdot k_{1,\epsilon} - p_{1,\epsilon})} e^{-iy_\epsilon(\cdot k_{1,\epsilon} + p_{2,\epsilon})} (2\pi)^2 \delta(p_1^- - k_1^-) \delta(l^- - k_1^- - p_2^-)$$

complete it to 4-d

$$I(p_1, p_2) = 2\pi \delta(l^- - p_1^- - p_2^-) e^{-iy_\epsilon(\cdot p_{1,\epsilon} + p_{2,\epsilon})} \int dr^+ \int dr^- \delta(r^+) \int \frac{d^d k_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l - k_1)^2]} e^{ir \cdot k_1}$$

use Schwinger parameters

$$\left(\frac{i}{k^2 - m^2 + i0} \right)^\lambda = \frac{1}{\Gamma(\lambda)} \int_0^\infty d\alpha \alpha^{\lambda-1} e^{i\alpha(k^2 - m^2 + i0)}$$

complete the square, Gaussian integration,

$$I(p_1, p_2) = 8\pi^2 \delta(l^- - p_1^- - p_2^-) \frac{e^{-iz_\epsilon \cdot p_{1,\epsilon}}}{l^-} e^{-iy_\epsilon \cdot p_{2,\epsilon}} K_0 \left(\sqrt{\alpha(1-\alpha)} Q^2 (\mathbf{x} - \mathbf{y})^2 \right), \quad \alpha = p_1^- / l^-$$

we are developing a Mathematica package to do this

di-jet azimuthal correlations in DIS

$$\text{NLO: } \gamma^* \text{T} \rightarrow \text{h h X}$$

integrate out one of the produced partons - there are divergences:

rapidity divergences: JIMWLK evolution of n-point functions

colinear divergences: DGLAP evolution of fragmentation functions

infrared divergences cancel

the finite pieces are written as a factorized cross section

work in progress: Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans

related work by:

Boussarie, Grabovsky, Szymanowski, Wallon, JHEP1409, 026 (2014)

Balitsky, Chirilli, PRD83 (2011) 031502, PRD88 (2013) 111501

Beuf, PRD85, (2012) 034039

SUMMARY

CGC is a systematic approach to high energy collisions

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

Leading Log CGC works (too) well for a qualitative/semi-quantitative description of data, NLO is needed

Need to eliminate/minimize late time/hadronization effects

Di-jet angular correlations offer a unique probe of CGC

3-hadron/jet correlations should be even more discriminatory

an EIC is needed for precision CGC studies