

# Fundamental aspects of the thermal neutron scattering



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Centro Atómico Bariloche Comisión Nacional de Energía Atómica

# ARGENTINA

Joint ICTP-IAEA School on Nuclear Data Measurements for Science and Applications October 27<sup>th</sup>, 2015



200441 (100002)

# **Research Reactors in Argentina**



RA-0



RA-1

Cordoba National University Academic use

First RR in Latin America (1958) Training professionals for NPPs



RA-3

10 MW 4% of world's Mo-99 production



RA-4

1 MW - Rosario National University Training, research, services for industry



1 MW - Bariloche Atomic Center & Instituto Balseiro Training of Nuclear Engineers Experimental facilities: PGAA, BNCT, difractometer, neutrography



RA-8

Critical facility. Test of CAREM reactor fuel design



RA-10

30 MW Multipurpose RR Under construction in Ezeiza. ARG

# **RA-10 Reactor**

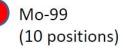
# Cold Neutron Source

OPAL Reactor (Australia)



**RA-10** 

Ir-192 MED/ Lu-177 (up to 4 positions)

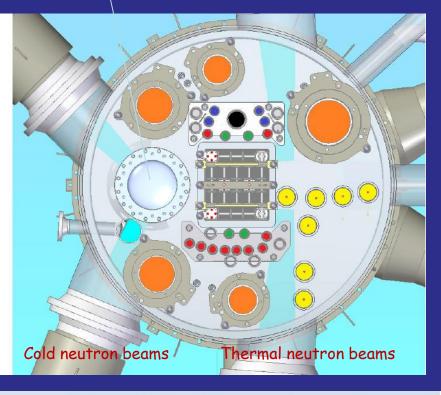


Ir-192 IND/ ORI (up to 4 positions)

LOOP

PNEUMATIC DEVICE (7 X 2 positions)

NTD (5 positions)



# **NPPs in Argentina**





#### Atucha - I 357 MWe PHWR First NPP in Latin America (1974)

# Embalse 648 MWe CANDU (1983)



Atucha - II 748 MWe PHWR (2014)



4<sup>th</sup> NPP

To be constructed Agreements with Chinese National Nuclear Corporation (CNNC)





# Argentinian RR's in the world



The Neutron Physics Department at Centro Atómico Bariloche was founded in 1969 by Hector Antunez , one of the alumni of the legendary neutron physics group at General Atomics in San Diego.

The group was created towards a small pulsed neutron source, a 25 MeV electron LINAC, similar to the accelerator at RPI.

Now we are 23 people (counting researchers, students and technical staff) working on neutron physics and applications to condensed matter research, materials science and nuclear engineering.

The main current group activity is the development of neutron scattering instruments for the forthcoming RA-10 reactor, which will be similar to the OPAL reactor that the Argentine company INVAP built in Australia. Thermal scattering nuclear data group (cross section libraries generation)



Rolando Granada Scattering theory and advanced neutron sources



Florencia Cantargi Cold moderator materials and neutron filters



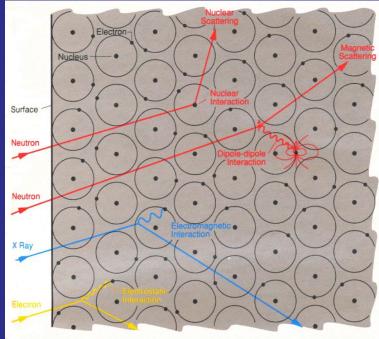
Ignacio Marquez Nuclear reactor applications and benchmarking

## Past members:

- Monica Sbaffoni (currently at IAEA),
- Victor Gillette (currently at University of Sharjah, U.A.E).

# Characteristics of neutrons

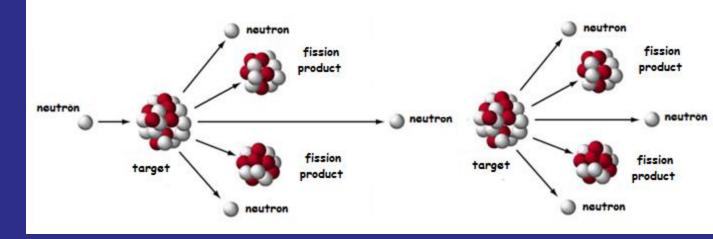
$\diamond$	Particle	Wave	- 
Charge	0	$\rightarrow \lambda \rightarrow$	
Mass	m <sub>n</sub> = 1.675 × 10 <sup>-27</sup> kg	and the state of the state of the	Sur
"Radius"	$r_0 = 6 \cdot 10^{-16} m$	Wave length $\lambda = \frac{h}{m \cdot v}$	Neu
Spin	1/2	Wave number k = $\frac{2\pi}{\lambda}$	Neu
Magn. Moment	μ = -1,9μ <sub>N</sub>		-
Momentum	$\vec{p} = \mathbf{m} \cdot \vec{\mathbf{v}}$	Momentum $\vec{p} = \frac{h \cdot \vec{k}}{2\pi} = h \cdot \vec{k}$	X R
Energy	$E = \frac{m}{2}v^2$	Energy $E = \frac{h^2}{2m\lambda^2} = \frac{\hbar^2 \cdot k^2}{2m}$	Ele
(v = velocity)		(h = Plank's constant)	<b>1</b>

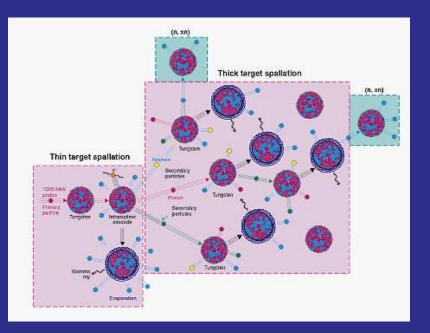


$$E = \frac{1}{2}m_{n}v^{2} = \frac{p^{2}}{2m_{n}} = KT_{m_{n}} \frac{h^{2}}{2m_{n}\Lambda^{2}}$$

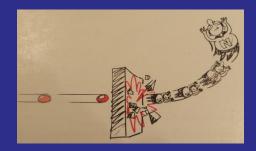
Mean life as a free particle: T=(888  $\pm$  3) sec

Neutrons for scattering experiments can be produced either by nuclear fission in a nuclear reactor





or by spallation when high-energy protons strike a heavy metal target (W, Ta, or U).



About 1.5 useful neutrons are produced by each fission event in a nuclear reactor whereas about 25 neutrons are produced by spallation for each 1-GeV proton incident on a tungsten target

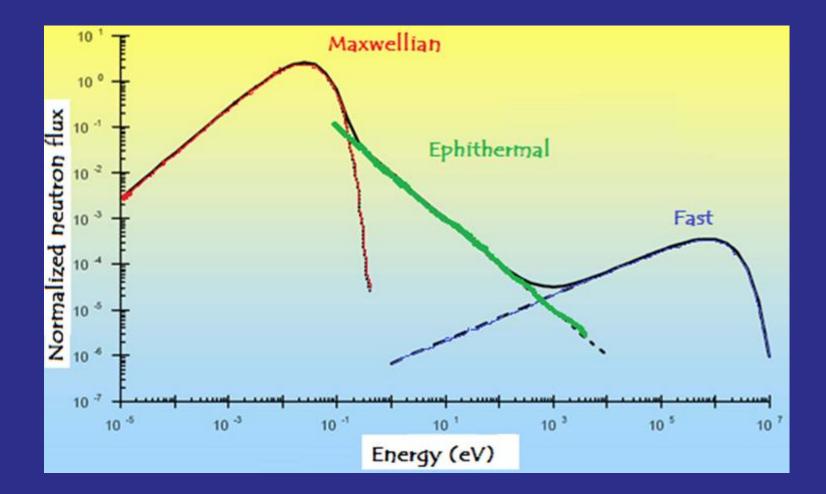
In general, reactors produce continuous neutron beams and spallation sources produce beams that are pulsed between 20 Hz and 60 Hz

Fission neutrons must be moderated to thermal energies to cause fission in other nuclei

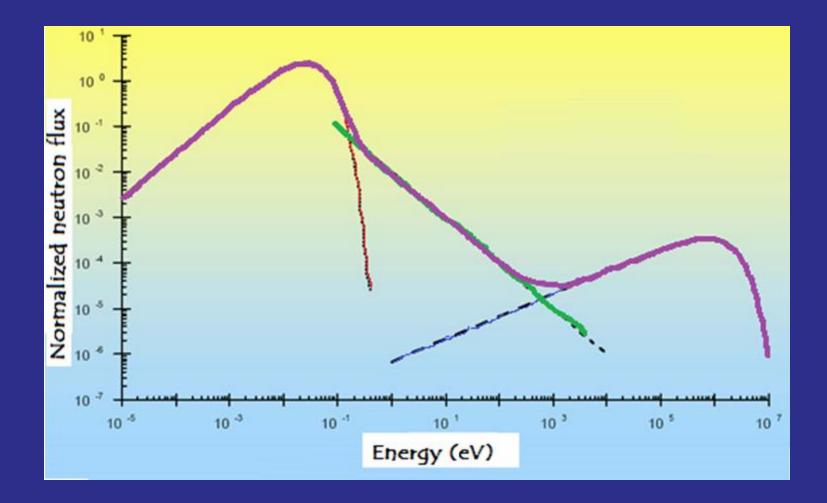
In contrast to fission, spallation cannot be selfsustaining

	Ultracold	Cold	Thermal	Epithermal	Fast
Energy	0.25 μeV	1 meV	25 meV	1 eV	2 MeV
Temperatura	3 mK	12 K	290 K	12000 K	2.32 10 <sup>10</sup> K
Longitud de onda	570 Å	9 Å	1.8 Å	0.29 Å	2.03 10 <sup>-4</sup> Å
Velocidad	6.9 m/seg	440 m/seg	2200 m/seg	14000 m/seg	10 <sup>7</sup> m/seg

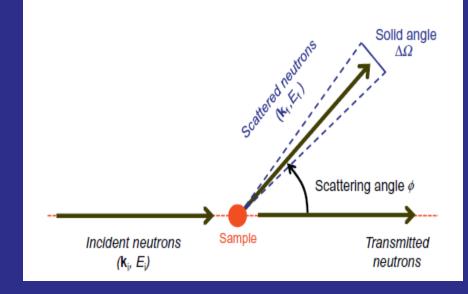
# Neutron spectra in a nuclear reactor



# Neutron spectra in a nuclear reactor



# Neutron Scattering Theory

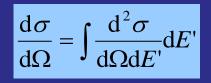


*Incident flux* ( $\Phi$ ) : neutrons crossing per unit area and per unit time

Double differential cross section

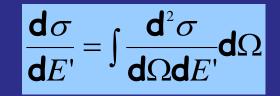


number of neutrons scattered per second into the solid angle d $\Omega$  in the ( $\theta$ ,  $\varphi$ ) direction and with final energies between E' and E'+dE'/  $\Phi d\Omega dE$ 



Differential cross section is the number of scattered neutrons per second into the solid angle  $d\Omega$  in the  $(\theta, \phi)$ direction  $/ \Phi dE$ 

The energy kernel is the number of scattered neutrons per second and energy unit and with final energies between E' and E'+dE'



*Total scattering cross section* is the total number of scattered neutrons per second in any direction and with any energy .

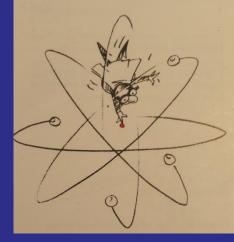
$$\sigma_{\text{tot}} = \int \frac{\mathbf{d}\sigma}{\mathbf{d}\Omega} \mathbf{d}\Omega = \int \frac{\mathbf{d}\sigma}{\mathbf{d}E'} \mathbf{d}E'$$



The effective area presented by a nucleus to an incident neutron is the total cross secction.

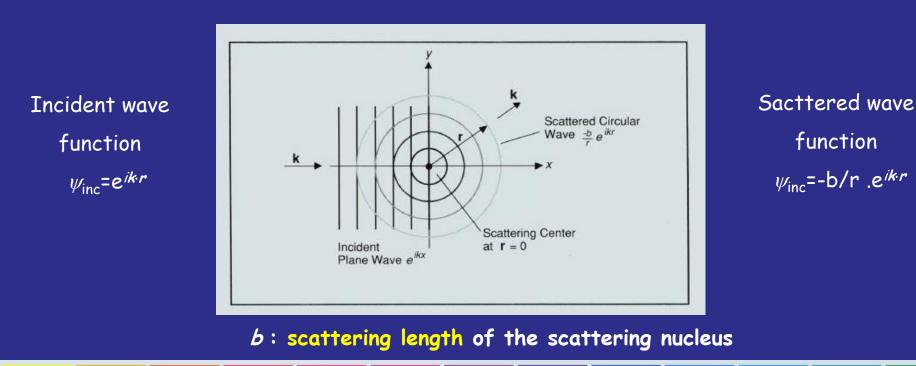
# Scattering by a Single (fixed) Nucleus

Range of nuclear force (~ 1fm) << thermal neutron wavelength (~2  $10^{-10}$  m) scattering is "point-like"



function

Energy of neutron is too small to change energy of nucleus & neutron cannot transfer kinetic energy to a fixed nucleus  $\rightarrow$  scattering is elastic



# Scattering length characteristics

It is a constant that depends on the nucleid and on the spin of the neutron-nucleus system

## It is a constant, it does not depend on $\theta$ or $\phi$

It is a complex number. The imaginary part is related to resonance phenomena between certain nuclei and the neutron (a few cases)

When b is positive the potential V(r) is repulsive

In some cases b is negative: this allows for contrast (e.g. Ti-Zr alloys)

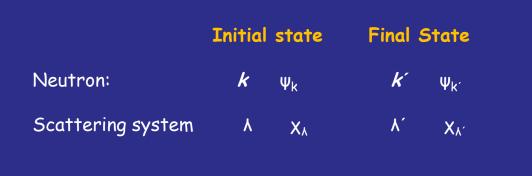
There is not a theory of nuclear forces that allows predicting the value of b It is taken as a phenomenological parameter and determined experimentally for each nucleus (tabulated).

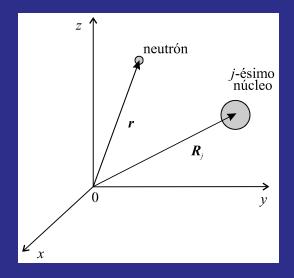
b varies erratically as a function of Z. <u>BIG advantage over X-rays</u>

b is the analog of the "form factor" in X-rays which is not isotropic.

$$\sigma_{scatt} = 4\pi b^2$$

# NUCLEAR SCATTERING





The diferential cross section represents the sum of all processes in which the state of the scattering system changes from  $\lambda$  to  $\lambda'$ , and the state of the neutron changes from k to k'

$$\left(\frac{\mathbf{d}\sigma}{\mathbf{d}\Omega}\right)_{\boldsymbol{\lambda}\to\boldsymbol{\lambda}'} = \frac{1}{\Phi} \frac{1}{\mathbf{d}\Omega} \sum_{\mathbf{k}'} W_{\mathbf{k},\boldsymbol{\lambda}\to\mathbf{k}',\boldsymbol{\lambda}'}$$
  
en d $\Omega$ 

where  $W_{\mathbf{k},\lambda\to\mathbf{k}',\lambda'}$  is the number of transitions per second from the state  $\{\mathbf{k},\lambda\}$  to the state  $\{\mathbf{k}',\lambda\}$ , and  $\Phi$  is the incident neutron flux.

# By means of Fermi Golden Rule:

$$\sum_{\mathbf{k}'} W_{\mathbf{k},\lambda \to \mathbf{k}',\lambda'} = \frac{2\pi}{\hbar} \rho_{\mathbf{k}'} |\langle \mathbf{k}' \lambda' | V | \mathbf{k} \lambda \rangle|^2$$
  
en dΩ

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\lambda\to\lambda'} = \frac{k'}{k} \left(\frac{m}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k'}\lambda' | V | \mathbf{k}\lambda \right\rangle\right|^2$$

$$E_{\scriptscriptstyle \lambda} + E = E_{\scriptscriptstyle \lambda'} + E'$$

$$\left(\frac{\mathbf{d}^{2}\sigma}{\mathbf{d}\Omega\mathbf{d}E'}\right)_{\lambda\to\lambda'} = \frac{k'}{k} \left(\frac{m}{2\pi\hbar}\right)^{2} \left|\left\langle \mathbf{k}'\lambda'\right|V\right|\mathbf{k}\lambda\left\langle \right|^{2}\delta\left(E_{\lambda}-E_{\lambda'}+\hbar\omega\right)$$

where  $\hbar \omega = E - E'$ 

The interaction potential between the neutron and the j-th nucleus is proportional to  $V_j(r-R_j)$ , in such a way that for the whole system

$$V(\mathbf{r}) = \sum_{j} V_{j}(\mathbf{r} - \mathbf{R}_{j})$$
*Fermi Pseudopotencial*

$$V_{j}(\mathbf{r} - \mathbf{R}_{j}) = \frac{2\pi\hbar^{2}}{m} b_{j} \delta(\mathbf{r} - \mathbf{R}_{j})$$
*Neutron position Scattering length of the j*-th nucleus
$$V(\mathbf{Q}) = \frac{2\pi\hbar^{2}}{m} \sum_{j} \mathbf{b}_{j}$$

where 
$$\hbar Q = \hbar (k - k')$$

$$\left( \frac{\mathrm{d}^2 \sigma}{\mathrm{d}\Omega \,\mathrm{d}E'} \right)_{\lambda \to \lambda'} = \frac{k'}{k} \sum_{j,j'} b_{j'}^* b_j \langle \lambda' | \exp(-i\boldsymbol{Q} \cdot \boldsymbol{R}_{j'}) | \lambda \rangle \langle \lambda' | \exp(i\boldsymbol{Q} \cdot \boldsymbol{R}_{j}) | \lambda \rangle \times \frac{1}{2\pi \,\hbar} \int_{-\infty}^{\infty} \exp\left( i \frac{E_{\lambda'} - E_{\lambda}}{\hbar} t \right) \exp(-i\omega t) \,\mathrm{d}t$$

In a real case, we have to sum all the final states keeping the initial state fixed and then average over all initial states of the target.

$$\frac{\mathrm{d}^2 \sigma}{\mathrm{d}\Omega \,\mathrm{d}E'} = \frac{k'}{k} \frac{1}{2\pi \,\hbar} \sum_{j,j'} b_{j'} \,b_j \int_{-\infty}^{\infty} \left\langle \exp\left(-i\boldsymbol{Q}\cdot\boldsymbol{R}_{j'}(0)\right) \exp\left(i\boldsymbol{Q}\cdot\boldsymbol{R}_{j}(t)\right) \right\rangle \exp(-i\omega t) \,\mathrm{d}t$$

This is the master formula and it is the basis for the interpretation of all neutronscattering experiments. Each scatterer has it own b which varies between one nucleus to another (different isotopes, nuclear spin, different species)

$$\frac{\mathrm{d}^2 \sigma}{\mathrm{d}\Omega \,\mathrm{d}E'} = \frac{k'}{k} \frac{1}{2\pi \,\hbar} \sum_{j,j'} \overline{b_{j'} \,b_j} \int_{-\infty}^{\infty} \exp(-i\omega t) \,\chi_{jj'}(t) \,\mathrm{d}t$$

$$\chi_{jj'}(t) = \left\langle \exp\left(-i\mathbf{Q}\cdot\mathbf{R}_{j'}(0)\right) \exp\left(i\mathbf{Q}\cdot\mathbf{R}_{j}(t)\right) \right\rangle$$

The system will have an average  $\overline{b}$  and averaged  $\overline{b}^2$ 

The measured cross-section will be an average. Taking that there is NO correlation among the b of different nuclei then

$$\overline{b_j b_{j'}} = \overline{b}^2 \qquad j' \neq j$$
$$\overline{b_j b_{j'}} = \overline{b}^2 \qquad j' = j$$

# Coherent scattering: depends of Q direction

1



$$\left(\frac{\mathrm{d}^{2}\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right)_{\mathrm{coh}} = \frac{\sigma_{\mathrm{coh}}}{4\pi}\frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j,j'}\int_{-\infty}^{\infty}\exp(-i\omega t)\,\chi_{jj'}(t)\,\mathrm{d}t$$
$$\sigma_{\mathrm{coh}} = 4\pi\overline{b}^{2}$$

Interference effects

# Incoherent scattering: independent of Q

$$\left(\frac{\mathbf{d}^{2}\sigma}{\mathbf{d}\Omega\mathbf{d}E'}\right)_{inc} = \frac{\sigma_{inc}}{4\pi}\frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j}\int_{-\infty}^{\infty}\exp(-i\omega t)\chi_{jj}(t)\mathbf{d}t$$



# $\sigma_{inc} = 4\pi \left\{ \overline{b^2} - \overline{b}^2 \right\}$

### No interference effects

We can also write

$$\frac{\mathrm{d}^{2}\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{1}{4\pi}\frac{k'}{k}N\left(\sigma_{\mathrm{coh}}\,S_{\mathrm{coh}}(\boldsymbol{Q},\omega) + \sigma_{\mathrm{inc}}\,S_{\mathrm{inc}}(\boldsymbol{Q},\omega)\right)$$

where  $S_{coh}(Qw)$  and  $S_{mc}(Qw)$  are the coherent and incoherent scattering laws

$$S_{\rm inc}(\boldsymbol{Q},\omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \exp(-i\omega t) \,\chi_{\rm inc}(\boldsymbol{Q},t) \,\mathrm{d}t \qquad S_{\rm coh}(\boldsymbol{Q},\omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \exp(-i\omega t) \,\chi_{\rm coh}(\boldsymbol{Q},t) \,\mathrm{d}t$$

The Fourier transform are called *Intermediate Scattering Functions* 

$$\chi_{\rm coh}(\boldsymbol{Q},t) = \frac{1}{N} \sum_{j,j'} \left\langle \exp\left(-i\boldsymbol{Q}\cdot\boldsymbol{R}_{j'}(0)\right) \exp\left(i\boldsymbol{Q}\cdot\boldsymbol{R}_{j}(t)\right) \right\rangle \qquad \chi_{\rm inc}(\boldsymbol{Q},t) = \frac{1}{N} \sum_{j} \left\langle \exp\left(-i\boldsymbol{Q}\cdot\boldsymbol{R}_{j}(0)\right) \exp\left(i\boldsymbol{Q}\cdot\boldsymbol{R}_{j}(t)\right) \right\rangle$$

These correlation functions, scattering laws and intermediate scattering functions contain all the information on the structure and dynamics of the scattering system. This information is obtained in a direct way in the measurement of the double differential scattering cross section. We have a Bravais lattice with only one atom in the unit cell, where each position is determined by a vector  $I_i$ 

Allowing the thermal moving of the j-th nucleus, its position will be Rj=  $I_j + u_j$ , where  $u_j$  is the displacement from the equilibrium position and  $I_j$  is a constant.

# Coherent scattering

U and V are operators related to displacements of the nuclei from equilibrium at time 0 and t

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{coh} = \frac{\sigma_{coh}}{4\pi} \frac{k'}{k} \frac{N}{2\pi\hbar} \exp\left(\frac{U^2}{2}\right) \sum_{-\infty} \exp(i\mathbf{\kappa} \cdot \mathbf{I}) \int_{-\infty}^{\infty} \exp(-i\omega t) dt$$

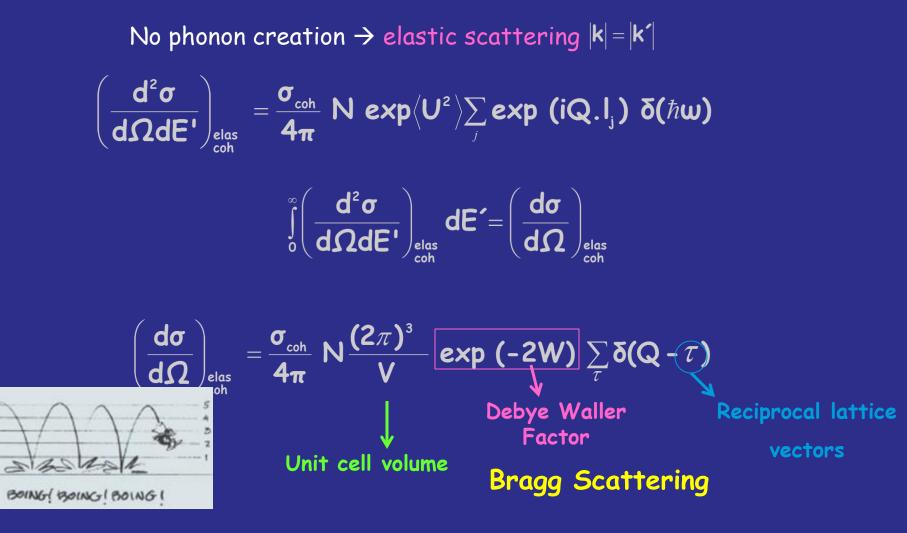
Debye Waller Factor: related to the meansquare displacement of each atom around its equilibribum position → depends on the temperature

# $exp\langle U^{2}\rangle = exp(-2W) = exp(-\langle (Q. u_{o}(0))^{2}\rangle)$

Related to the creation of phonons



# Coherent elastic scattering

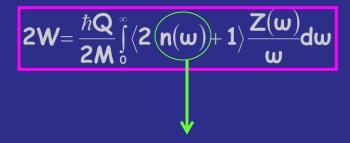


(for non- Bravais crystals a form factor appears in this expresion)

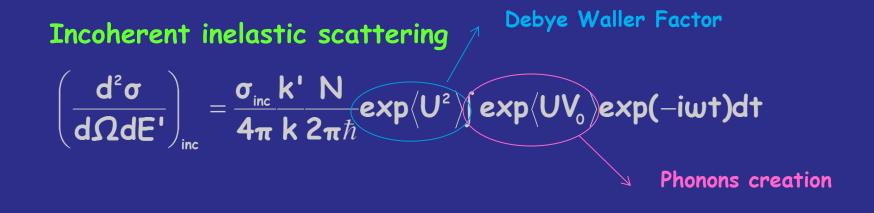
Mean-square displacements are associated to the oscillation modes. Since the number of degrees of freedom is close to  $10^{23}$ , oscillation modes are expressed as an integral over all the frequencies by means of the density of states  $Z(\omega)$  (or frequency spectra)

 $Z(\omega)$  d $\omega$  is the fraction of normal modes between w and w+dw

The Debye Waller factor is written as:



Bose occupation number



# Incoherent elastic scattering

 $\left( \frac{d\sigma}{d\Omega} \right)_{elas} = \frac{\sigma_{incoh}}{4\pi} N exp (-2W)$ 

No phonon creation  $\rightarrow$  elastic scattering  $|\mathbf{k}| = |\mathbf{k}'|$ 

The only dependence in the scattering direction apears in the Debye Waller factor.

At low temperatures exp(-2W) is close to  $1 \rightarrow$ scattering es isotropic scattering

# Phonons

Through a Taylor expansion of exp<UV> y exp<UVo> in the coherent and incoherent scattering expresions respectively

$$exp\langle UV \rangle = 1 + \langle UV \rangle + \frac{1}{2} \langle UV \rangle^{2} + \dots + \frac{1}{p!} \langle UV \rangle^{p} \rightarrow p \text{ phonons}$$
  
elastic 1 phonon 2 phonons

# Coherent elastic 1 phonon scattering

$$\frac{d^2\sigma}{d\Omega dE'}\Big|_{coh\pm 1f} = \frac{\sigma_{coh}k'}{4\pi}\frac{(2\pi)^3}{k}\frac{\hbar}{v_o}\frac{e^{-2W}}{2M}\sum_s\sum_r\frac{(\mathbf{Q}\cdot\hat{\mathbf{e}_s})^2}{\omega_s}\left\{\begin{array}{l} < n_s + 1 > \\ < n_s > \end{array}\right\}\,\delta(\hbar\omega\pm\hbar\omega_s)\delta(\mathbf{Q}\pm\mathbf{q}-\tau)$$

# Incoherent inelastic 1 phonon scattering

$$\frac{d^2\sigma}{d\Omega dE'}\Big|_{inc\,\pm 1f} = \frac{\sigma_{inc}\,k'}{4\pi}\frac{N}{k}\frac{Q^2}{4M}Q^2e^{-2W}\frac{Z(\omega)}{\omega}\left[\coth\left(\frac{\beta\,\hbar\,\omega}{2}\right)\pm 1\right]$$

Density of states  $\rightarrow$ 

# Information of the dynamics of the scattering system





#### coherent scattering

Scattering in which an incident neutron wave interacts with all the nuclei in a sample in a coordinated fashion; that is, the scattered waves from all the nuclei have definite relative phases and can thus interfere with each other.



#### incoherent scattering

Scattering in which an incident neutron wave interacts independently with each nucleus in the sample; that is, the scattered waves from different nuclei have random, or indeterminate, relative phases and thus cannot interfere with each other.

# Summary



BOING BOING ! BOING !

#### elastic scattering

Scattering with no change in the energy of the incident neutron; or, in terms of the wave vector of the neutron, scattering in which the direction of the vector changes but not its magnitude.



#### inelastic scattering

Scattering in which exchange of energy and momentum between the incident neutron and the sample causes both the direction and the magnitude of the neutron's wave vector to change.



The NJOY Nuclear Data Processing System is a commercial code that generates scattering laws, kernels and cross sections for different materials at different temperatures

It is a modular computer code designed to read evaluated data in ENDF format, transform the data in various ways, and output the results as libraries designed to be used in various applications.

Each module performs a well defined processing task. The modules are essentially independent programs, and they communicate with each other using input and output files,

plus a very few common variables.

$$\alpha = \frac{E' + E - 2\sqrt{E'E}\mu}{AK_BT} \qquad \beta = \frac{E' - E}{K_BT}$$

$$S(\alpha,\beta) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\beta t} e^{-\alpha \Gamma(t)} dt$$

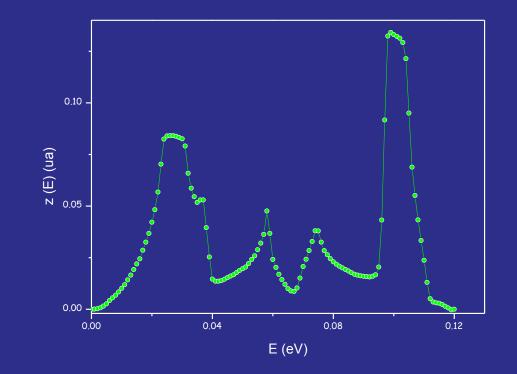
The dynamics of the scattering system is in the frequency spectra

$$Z(E) = \mathcal{O}_{cont} Z_{cont}(E) + \sum_{vibr_i=1}^{N} \mathcal{O}_{vibr_i} Z_{vibr_i}(E)$$

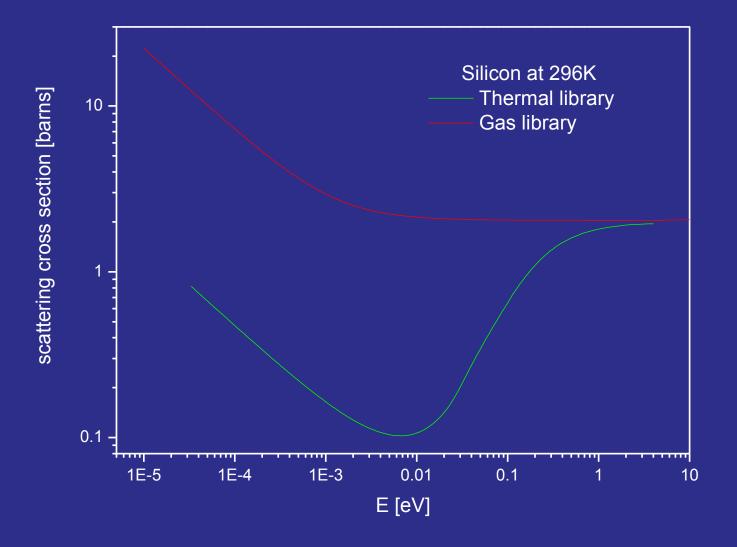
Each spectrum is composed of a continuous part, associated to translational and rotational motions of the molecular system

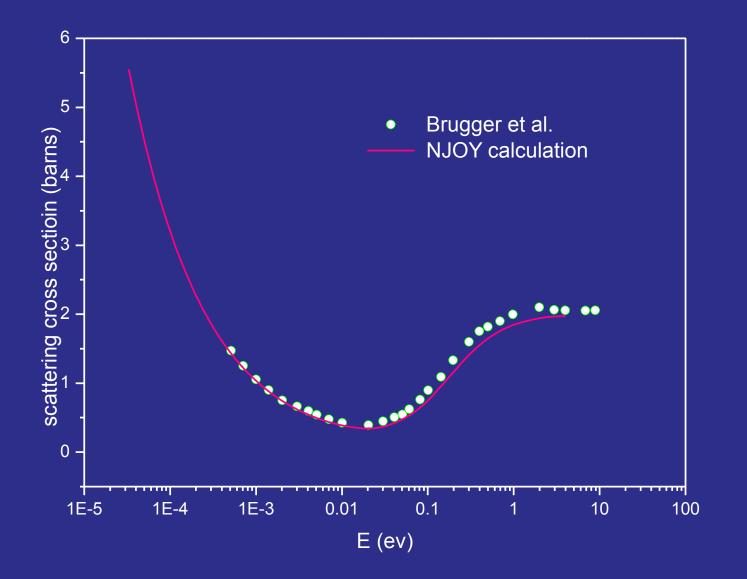
and a set of discrete oscillators related to the molecule's internal degrees of freedom

# Silicon single-crystal



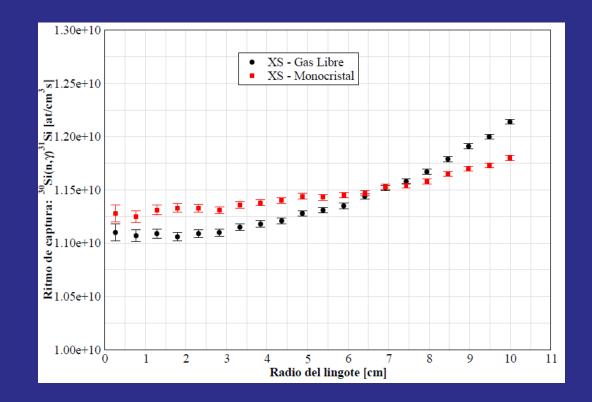
R. Meyer y D. Comtesse, Phys. Rev. B 83 (2011) 014301





# Capture reaction rate for ${}^{30}$ Si (n, $\gamma$ ) ${}^{31}$ Si with both libraries in the RA10 MCNP model

The reaction rate indicates the amount of <sup>31</sup>P generated in the silicon lingote.





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vou on an educational tour of the subject of nuclear data. What are nuclear data? Where Who does nuclear

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