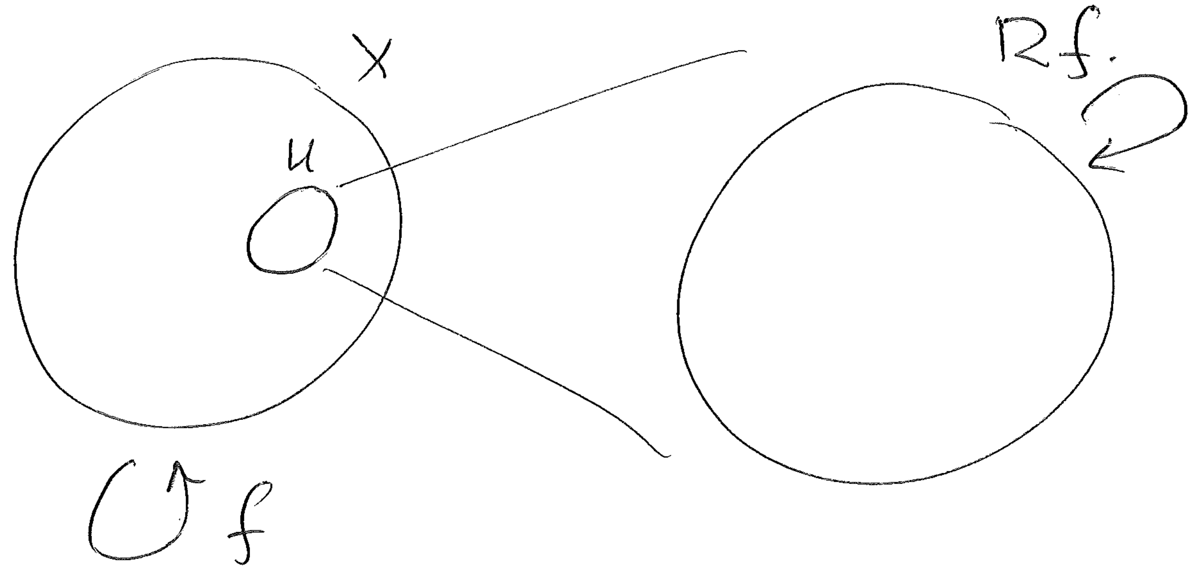
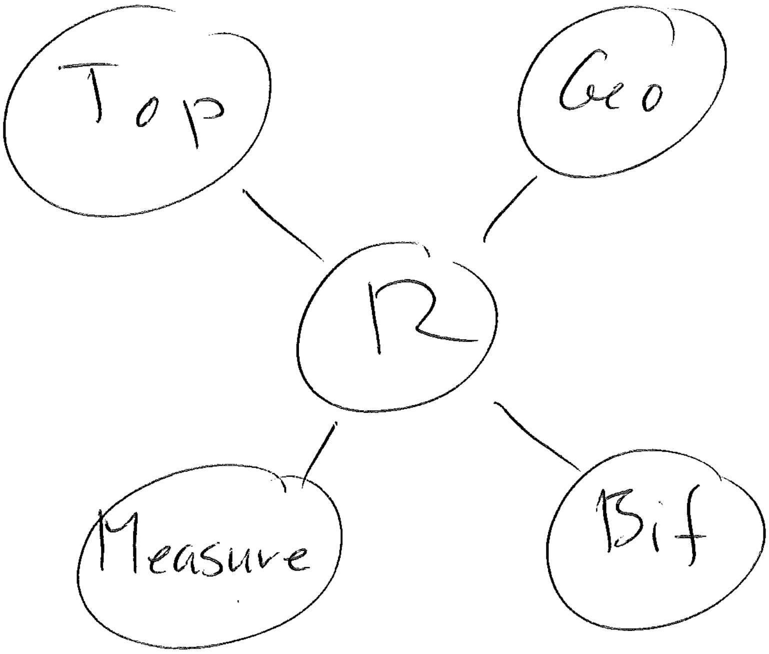
# Renormalization in low-dimensional Dynamics

## I] Renormalization




Rf rescaled version of the first return map to $U$.

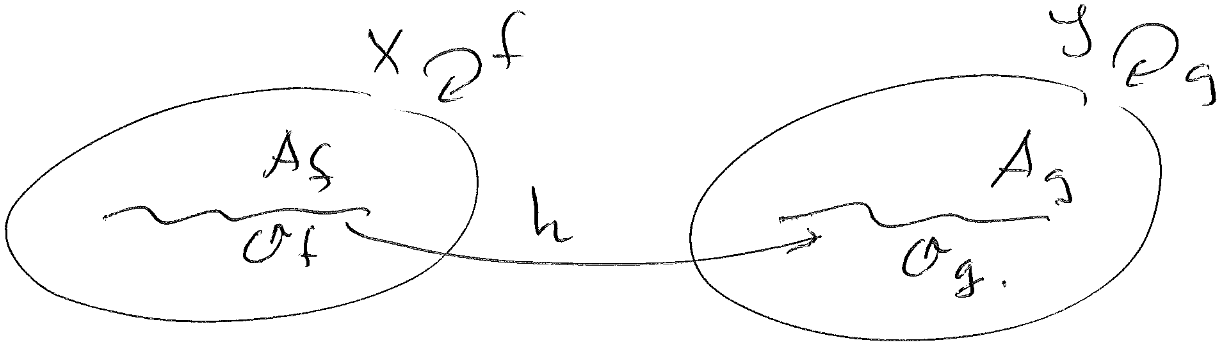
- Renormalization is like a microscope
- R unifies Topology, Geometry, Measure theory and Bifurcation.




## Topology

A renormalization step has a "type" (Combinatorial Information)

Two Systems which are $\infty$-finitely renormalizable of the Same type have topologically conjugated attractors



## Geometry:

Often the renormalizations of two systems of the same type converge

$$d_{C^1}(R^n f, R^n g) \to 0$$

Universal geometry at the limit of zooming in

Often $h: A_f \longrightarrow A_g$ is differentiable

(Rigidity).

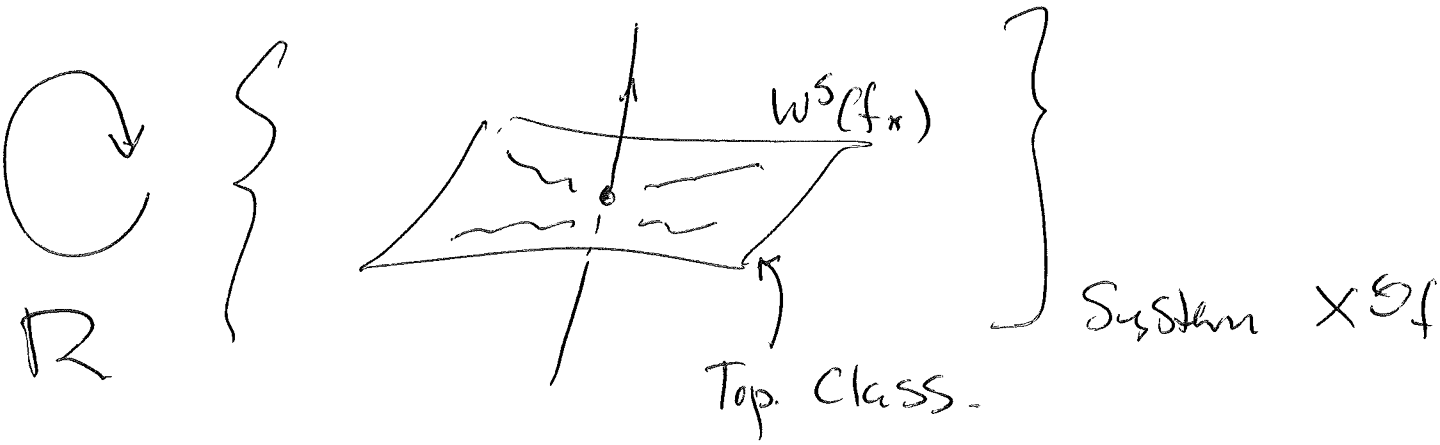
## Measure Theory:

The renormalization proces allows to construct relevant invariant measures.

$$\frac{1}{n}\sum_{i=0}^{n-1} \varphi(f^i(x)) \longrightarrow \int \varphi \, d\mu. \quad \text{a.e. } x.$$

## Bifurcation

Often topological classes coincide with Stable manifolds of the renormalization operator.




Renormalization allows to deal with these main issues in dynamics.

The underlying R-phenomena (as Universality and Rigidity) makes this possible.

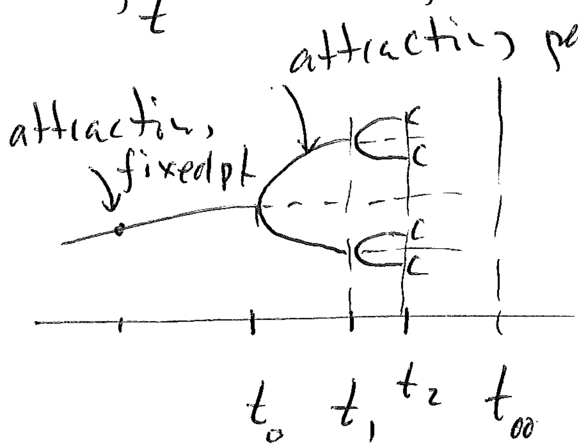
These phenomena are fascinating by themselves

# Example

## Period Doubling Cascade

- Period Doubling bifurcation

$f_t : X \circlearrowleft$ family




- Period Doubling Cascade: Route to chaos are observed in many real world systems. Each time (if there is friction).

$$t_n \xrightarrow{\exp} t_\infty$$

with rate $\dfrac{1}{4.669\ldots}$

$\boxed{\text{parameter Universality}}$

(Elec. circuit, Fluid Dyn, ..)