PROBLEM LIST 3. INVARIANT MANIFOLDS.

SYLVAIN CROVISIER AND RAFAEL POTRIE
SCHOOL ON DYNAMICAL SYSTEMS, ICTP, JULY 2015

(1) Let $f \in \mathrm{Diff}^1(M)$ be a globally partially hyperbolic diffeomorphism with a splitting $TM = E^s \oplus E^c$ such that $\dim E^s = 1$.
   (a) Use Peano's existence theorem for differential equations to show that every point $x \in M$ is contained in a curve η_x everywhere tangent to E^s.
   (b) Deduce that there exists a foliation tangent to E^s which is invariant by f.

(2) Consider a surface diffeomorphism which preserves a C^1-circle C. Assume $T_C M = E \oplus E^s$ is dominated.
   (a) Prove that E is tangent to C.
   (b) Prove that for any diffeomorphism g that is C^1-close to f, there exists an invariant C^1-circle C_g that is C^1-close to C.

(3) Let $f \in \mathrm{Diff}^1(M)$ be a globally partially hyperbolic diffeomorphism with a splitting $TM = E^s \oplus E^c \oplus E^u$ such that $\dim E^c = 1$ and γ be a closed arc tangent to E^c.
   Show that the saturation of γ by local strong stable manifolds

$$D = \bigcup_{x \in \gamma} W^{ss}_{loc}(x)$$

   is an embedded C^1-submanifold tangent to $E^s \oplus E^c$.