Takens' Last Problem and Existence of Non-trivial Wandering Domains

S. Kiriki[*] T. Soma[†]

[*]Tokai Univ. (Japan)

[†]Tokyo Metro. Univ. (Japan)

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科研費 KAKENHI

GN:[*]25400112
[†]26400093

Contents

What is a wandering domain?

M: compact smooth manifold

Definition

D is a *wandering domain* for $f \in \mathrm{Diff}^r(M)$ if

- D : nonempty, connected, open set of M
- $f^i(D) \cap f^j(D) = \emptyset$ for $\forall i, j \in \mathbb{Z}$ with $i \neq j$

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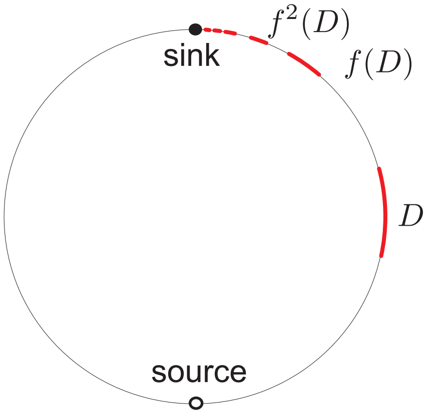
Trivial example



Non-trivial example

[Bohl(1916), Denjoy(1932)]

$\exists f \in \mathrm{Diff}^1(S^1)$, $\exists \{D_i\}_{i \in \mathbb{N}} \subset S^1$ with $f(D_i) = D_{i+1}$ s.t.

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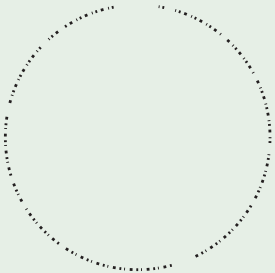
- $D_i \cap D_j = \emptyset$ for $\forall i, j \in \mathbb{N}$ with $i \neq j$;
- $\Lambda := S^1 \setminus \bigcup_{i\in\mathbb{N}} \mathrm{Int}(D_n)$ is a f-invariant Cantor set satisfying $\Lambda = \omega_D$.



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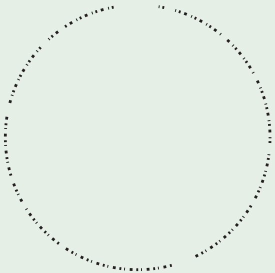

Note: $\forall f \in \mathrm{Diff}^2(S^2)$ has no wandering domain

In 1-dimension

Definition (de Melo & van Strien)

For a circle diffeomorphism f, an open interval in S^1 is a *non-trivial wandering domain* if

- $D, f(D), f^2(D), \ldots$ are pairwise disjoint;
- the ω-limit set of D is not equal to a single periodic orbit.

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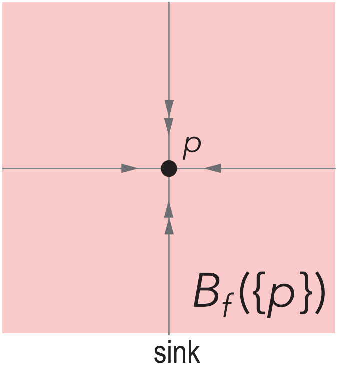
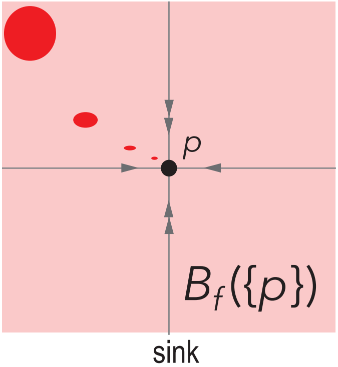


$$B_f(A) := \{x \in M; f^n(x) \to A \text{ as } n \to \infty\}$$



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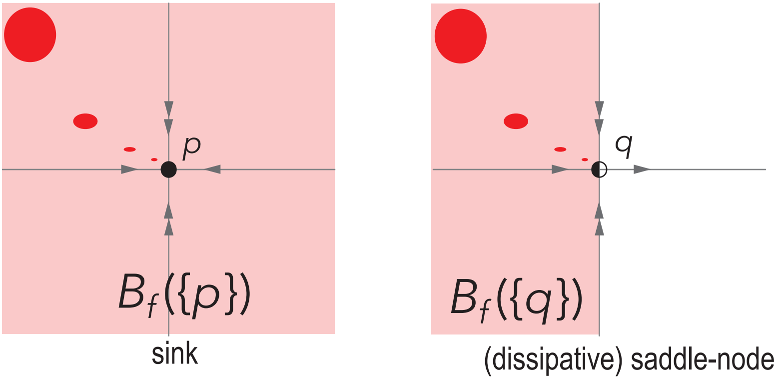

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Let M: closed surface and $f \in \mathrm{Diff}^r(M)$. An open set $D \subset M$ is a *non-trivial wandering domain* if

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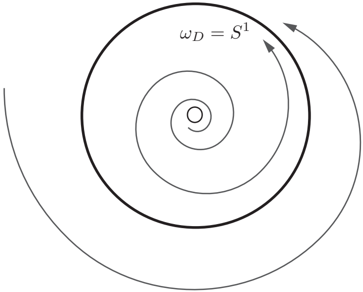
attracting irrational rotation



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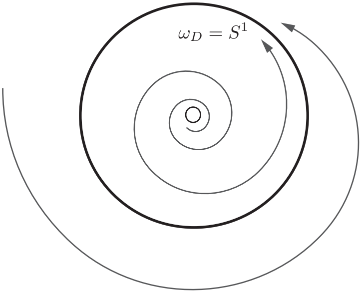

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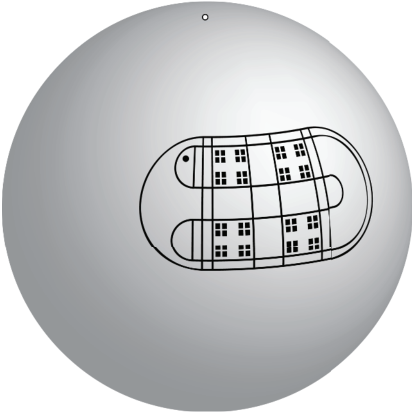
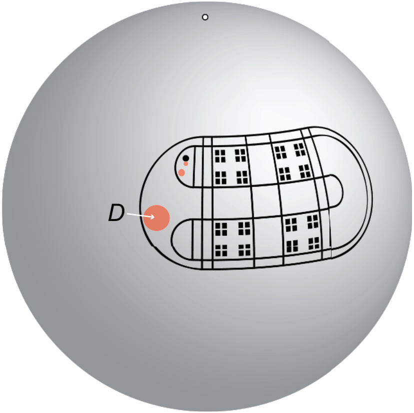


$f \in \mathrm{Diff}^r(S^2)$ with

- sink p_0;
- horseshoe Λ;
- source p_∞

In 2-dimension



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$$D \subset \text{basin of } p_0 \quad \Longrightarrow \quad \begin{array}{l} f^i(D) \cap f^j(D) = \emptyset \text{ if } i \neq j; \\ \omega_D = \{p_0\}; \\ \lim\limits_{n\to\infty} \mathrm{diam}(f^n(D)) \to 0 \end{array}$$

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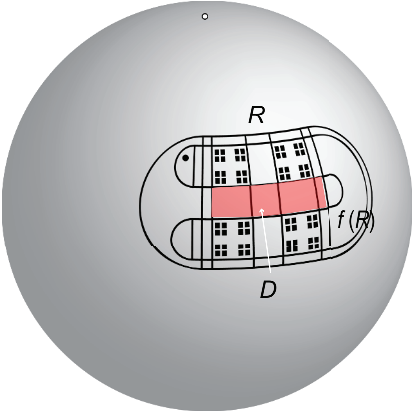

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$$D \subset R \setminus f(R) \quad \Longrightarrow \quad \begin{array}{l} f^i(D) \cap f^j(D) = \emptyset \text{ if } i \neq j,; \\ \omega_D = \{p_0\} \cup \Lambda; \\ \lim_{n\to\infty} \mathrm{diam}(f^n(D)) = c > 0 \end{array}$$

Non-trivial wandering domains in 2-dimension

M : closed surface

Definition

A nonempty connected open set D in M is a *non-trivial wandering domain* for $f \in \mathrm{Diff}^r(M)$ if

- $f^i(D) \cap f^j(D) = \emptyset$ for any $i, j \in \mathbb{Z}$ with $i \neq j$;
- there is a non-trivial basic set Λ such that, for any $x \in D$, the ω-limit set $\omega(x)$ contains Λ.

Definition

A non-trivial wandering domain D is called *contracting* if the diameter of $f^n(D)$ converges to zero as $n \to \infty$

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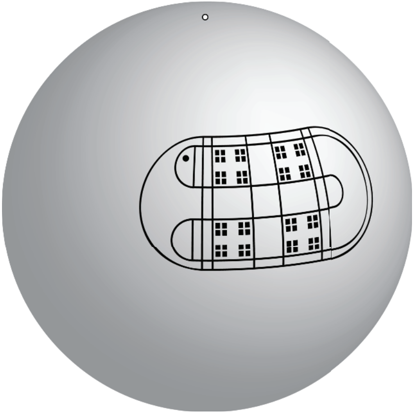
Smale horseshoe map



$f \in \mathrm{Diff}^r(S^2)$ with

- sink p_0;
- horseshoe Λ;
- source p_∞

There are trivial wandering domains but no non-trivial ones.

horseshoe & homoclinic tangency

Question

Does there exist a diffeomorphism having non-trivial wandering domains?

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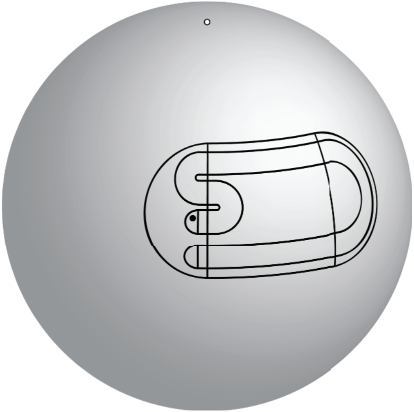
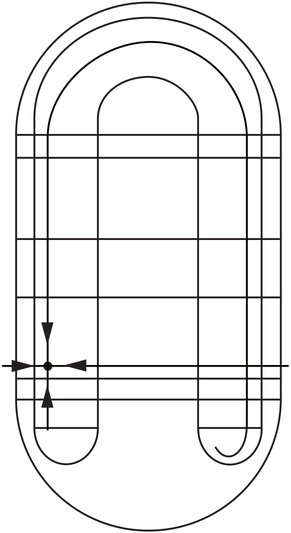


Colli-Vargas' example



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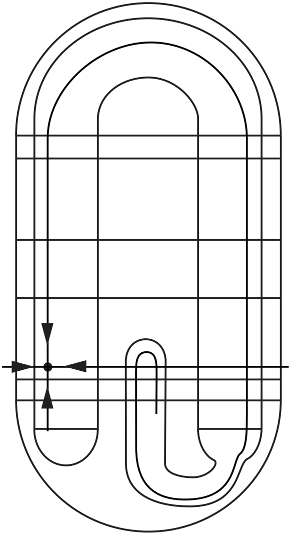

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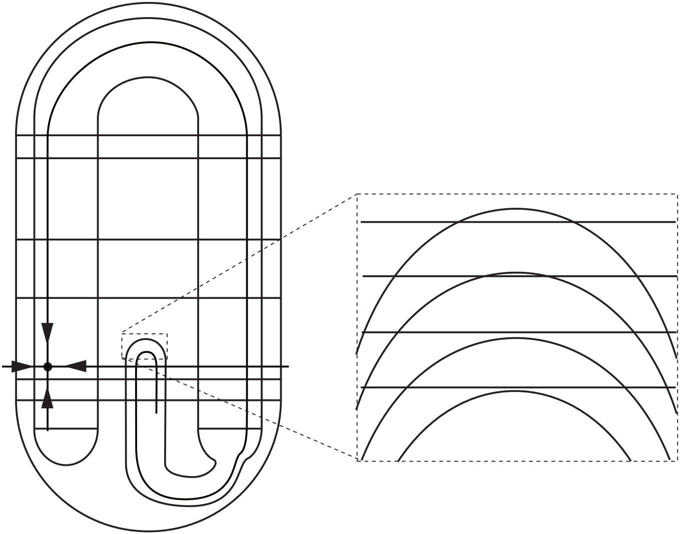

$$\tau^s \tau^u > 1$$

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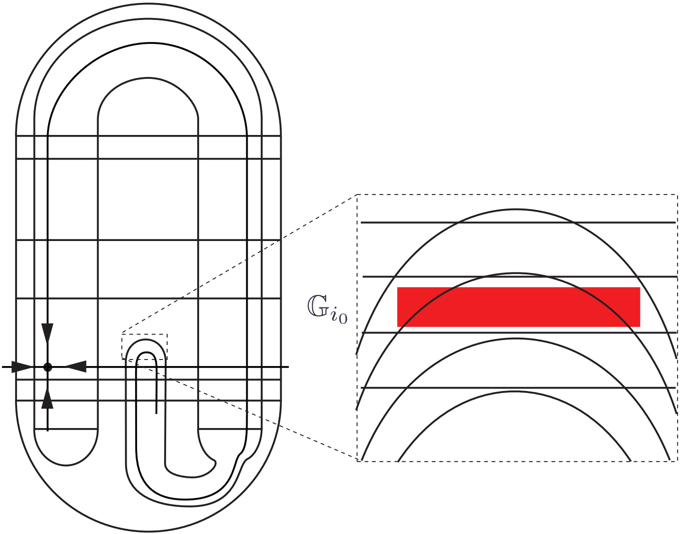

$D_0 \subset \mathbb{G}_{i_0}$

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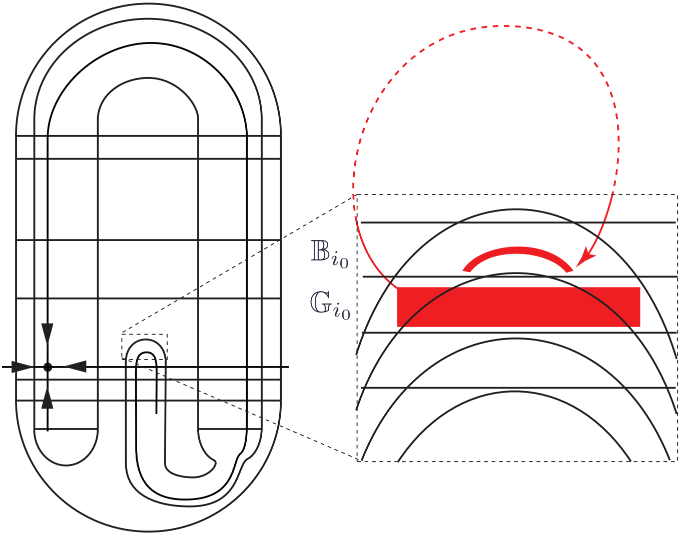
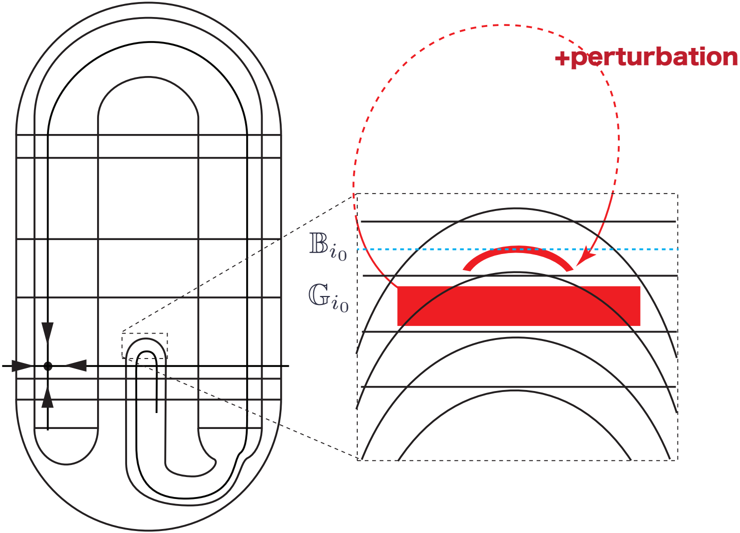


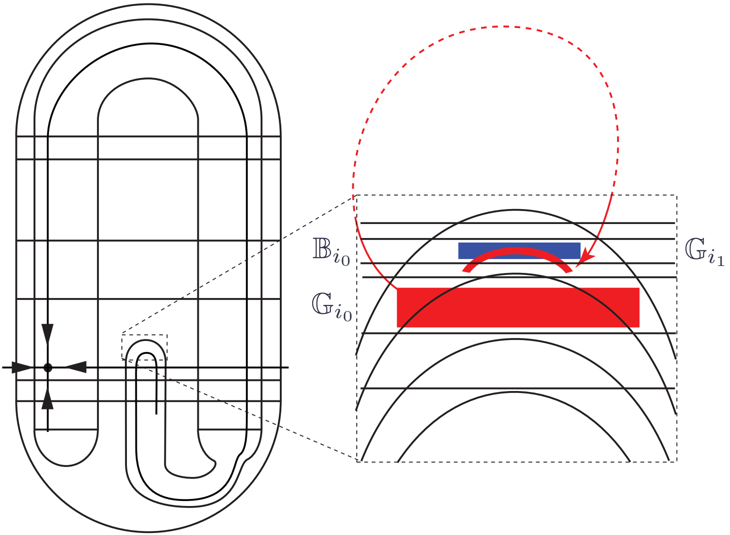
$\exists k_0 \in \mathbb{N}$ s.t. $f^{k_0}(D_0) \cap \mathbb{B}_{i_0} \neq \varnothing$

Colli-Vargas' example



center of $f_{\delta_1}^{k_1}(D_0)$ = center of $\mathbb{B}_{i_0}$

Colli-Vargas' example



$D_1 \subset \mathbb{G}_{i_1}, \; D_1 \cap f_{\delta_0}^{k_0}(D_0) \neq \varnothing$

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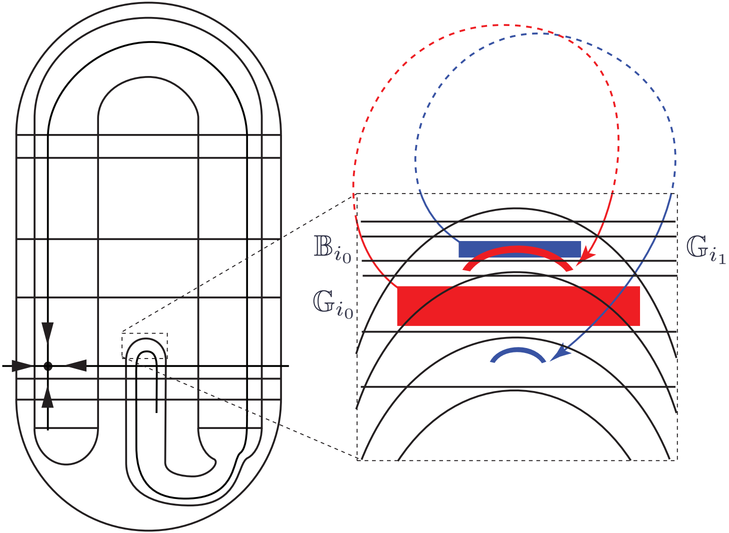

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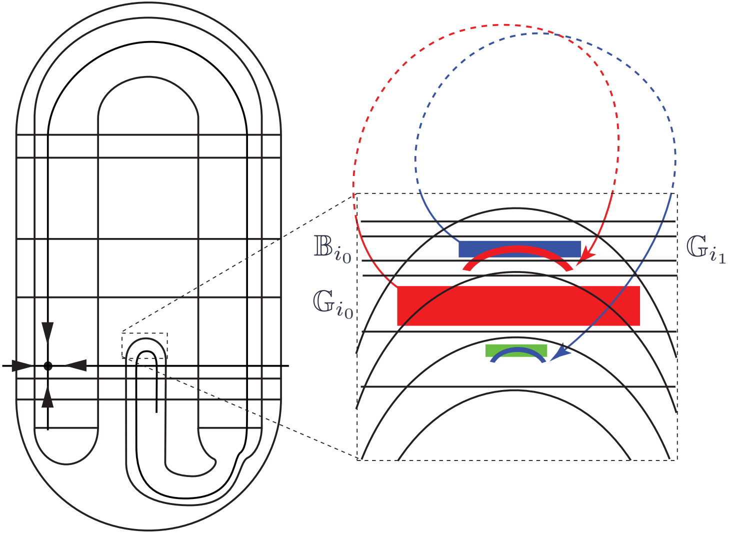

$$D_0 \xrightarrow{f_{\delta_0}^{k_0}} D_1 \xrightarrow{f_{\delta_1}^{k_1}} D_2$$

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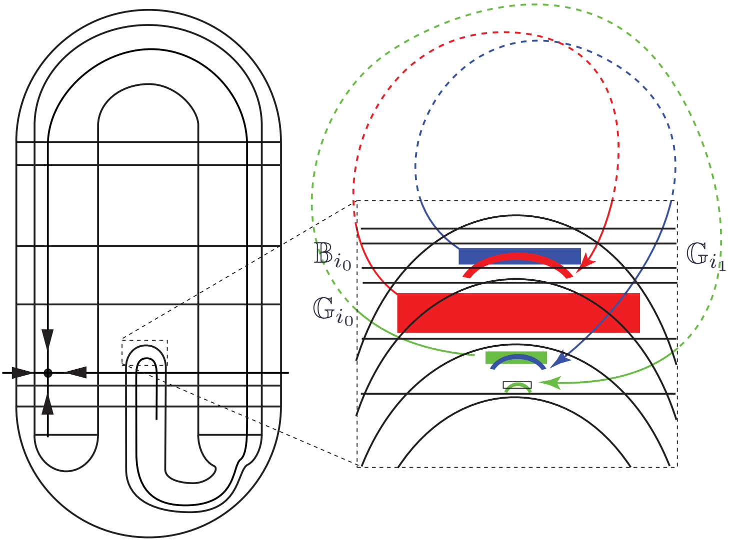
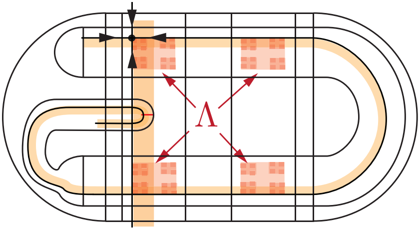
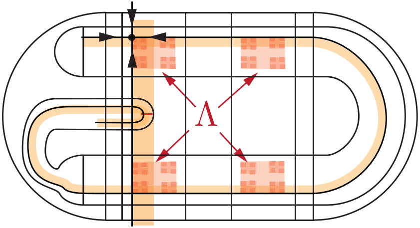


$$D_0 \xrightarrow{f_{\delta_0}^{\kappa_0}} D_1 \xrightarrow{f_{\delta_1}^{\kappa_1}} D_2 \xrightarrow{f_{\delta_2}^{\kappa_2}} D_3 \longrightarrow \cdots\cdots$$



[Colli-Vargas '01]

There exists a 2-dimensional C^r, $r \geq 2$, diffeomorphism having a contracting non-trivial wandering domain whose ω-limit set is contained in the horseshoe Λ with its homoclinic tangencies.



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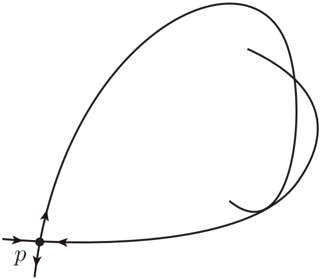
Main results



M: closed surface
$f \in \mathrm{Diff}^r(M)$, $r \geq 2$, with

- saddle fixed point p;
- homoclinic tangency for p

$\exists$ open set $\mathcal{N}_f \subset \mathrm{Diff}^r(M)$ s.t. $f \in \mathrm{Cl}(\mathcal{N}_f)$ and $\mathcal{N}_f$ has persistent tangenvies.

$\mathcal{N} \overset{\text{def}}{=} \bigcup_f \mathcal{N}_f$: Newhouse open set

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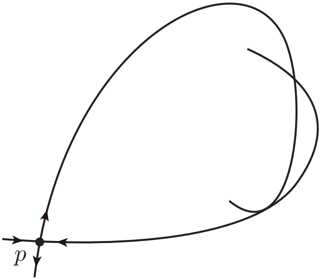

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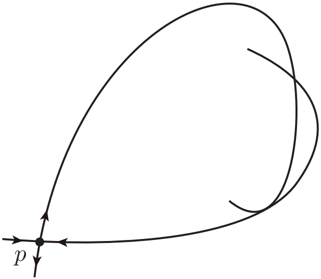

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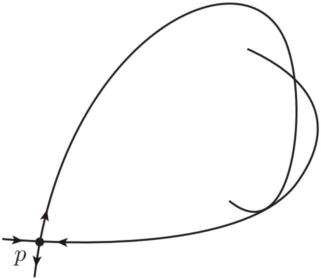

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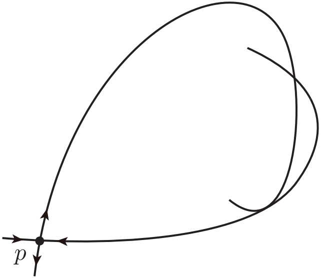

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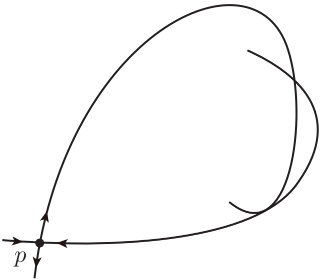
Theorem A (Colli-Vargas' conjecture '01)

The Newhouse open set $\mathcal{N}$ is contained in the *closure* of a subset of $\mathrm{Diff}^r(M)$, $2 \leq r < \infty$, whose any diffeomorphism has a contracting non-trivial wandering domains.

Main results



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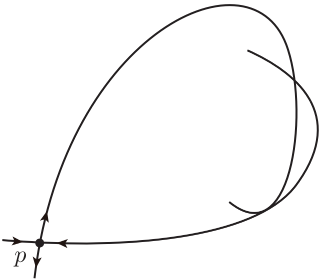

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Takens' Last Problem

X: compact state space
$\varphi : X \to X$: continuous map

Definition

An orbit $\{x, \varphi(x), \varphi^2(x), \dots\}$ has *historic behavior* if the measure

$$\mu_{x,n} := \frac{1}{n+1} \sum_{i=0}^{n} \delta_{\varphi^i(x)},$$

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Takens' Last Problem

X: compact state space
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Taknes' last problem [Taknes 2008]

Whether are there *persistent classes* of smooth dynamical systems such that the set of initial states which give rise to orbits with historic behavior has *positive Lebesgue measure*?

Theorem B (Answer to Takens' last problem)

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An application to Hénon maps

Hénon family $f_{a,b} : \mathbb{R}^2 \to \mathbb{R}^2$

$$f_{a,b}(x, y) = (1 - ax^2 + y, bx)$$

where a, b: real parameters

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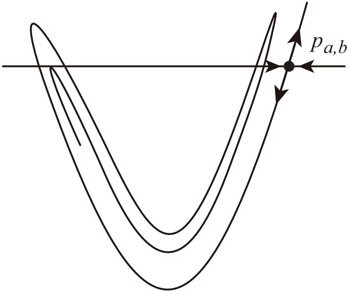

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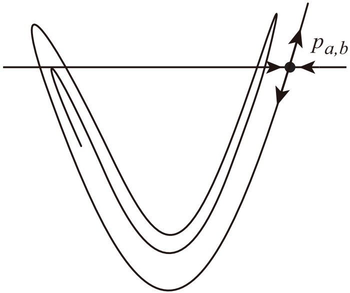

Open problem [van Strien '10], [Lyubich-Martens '11]

Does the Hénon family have a non-trivial wandering domain?

Corollary C

There is an open set $\mathcal{O}$ of the parameter space of Hénon family with $\mathrm{Cl}(\mathcal{O}) \ni (2, 0)$ such that for every $(a, b) \in \mathcal{O}$, $f_{a,b}$ is approximated by C^r diffeomorphisms, $2 \leq r < \infty$, which have historic & non-trivial wandering domains.

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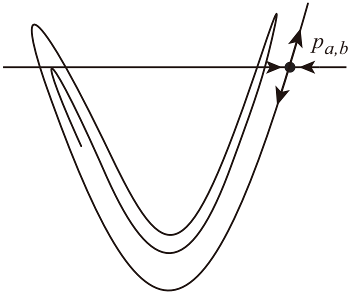

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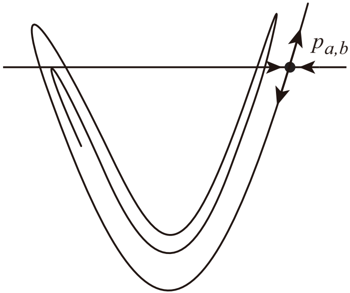

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Thank you for your kind attention!

The paper can be downloaded from:

arXiv:1503.06258

or

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