Generic Dynamics of Wind-Trees

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Aix Marseille université

August 6, 2015

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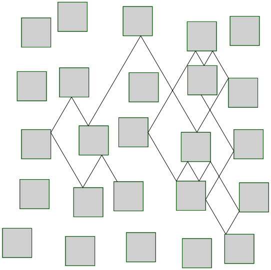
Overview

In a nutshell

The setting

A wind-tree: a billiard on the plane with an infinity of square scatterers.



Theorem

A generic wind-tree is ergodic and minimal.

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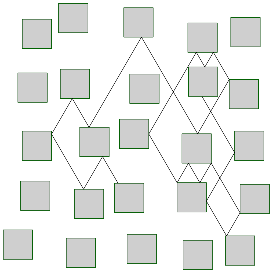

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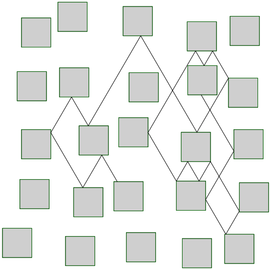

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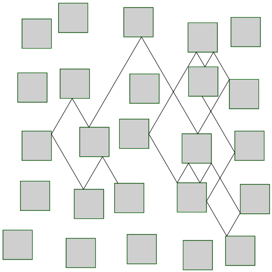

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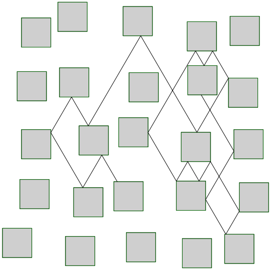

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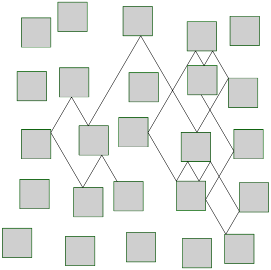

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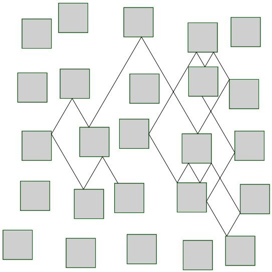

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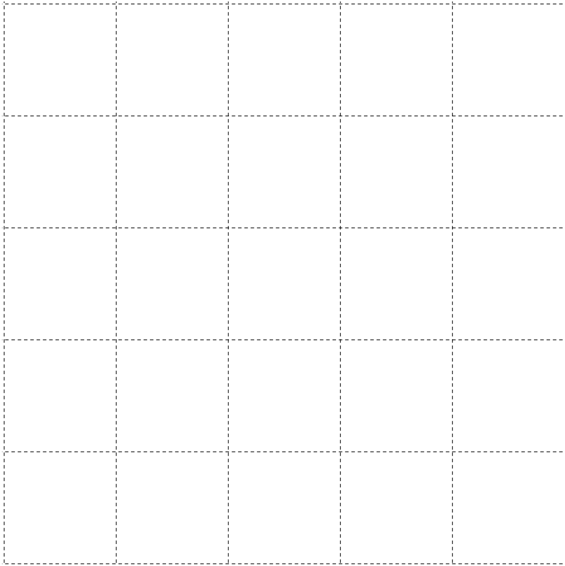
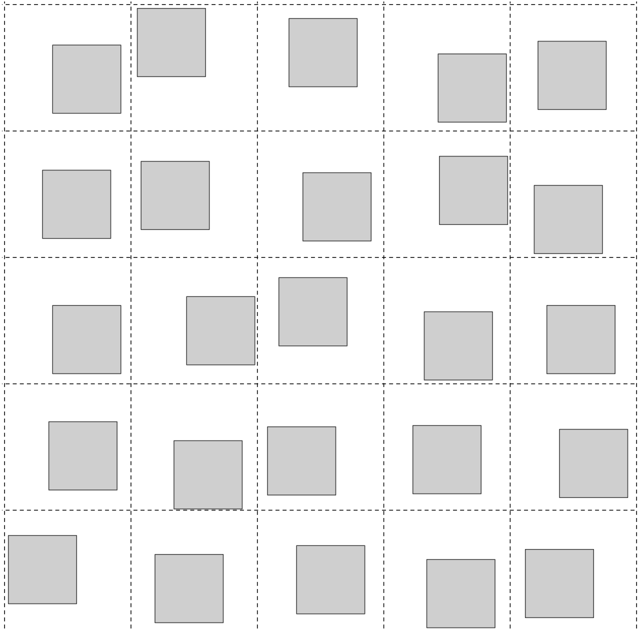
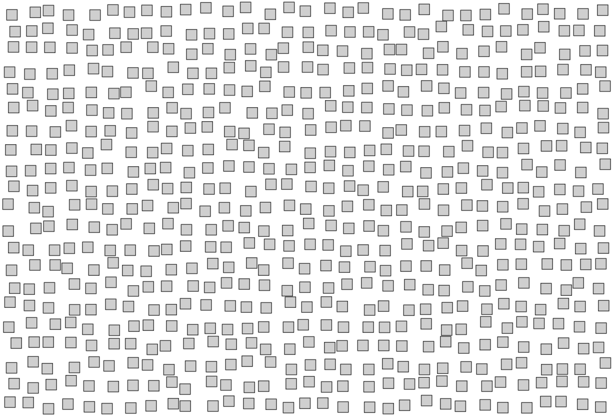
Tile the plane by square cells



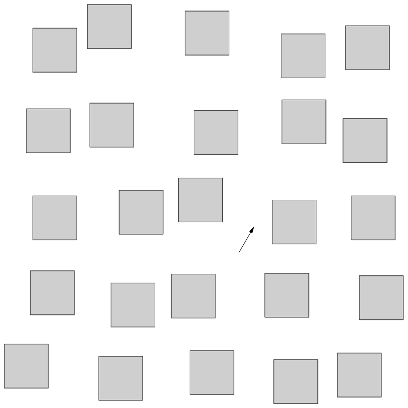
Put a tree ($2r$-side square) on each cell



You get a wind-tree table



Play billiard on it! You get the wind-tree flow



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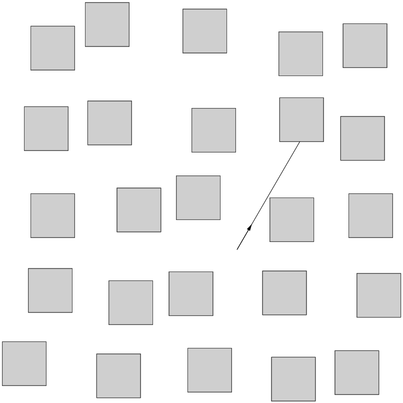

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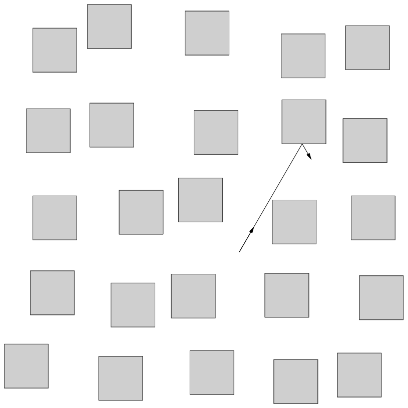

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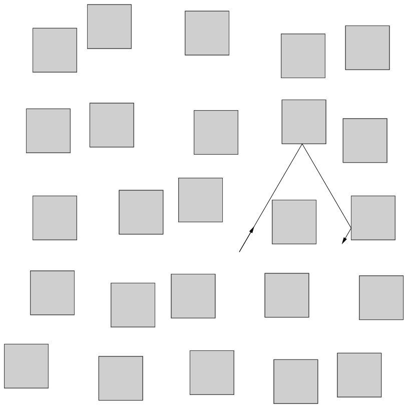

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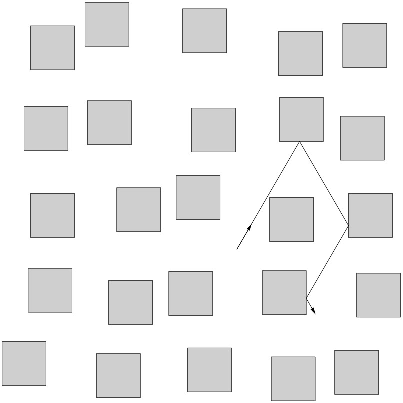

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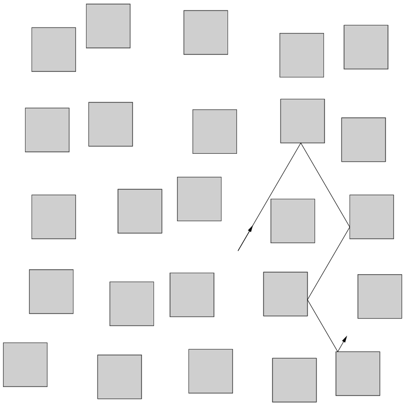

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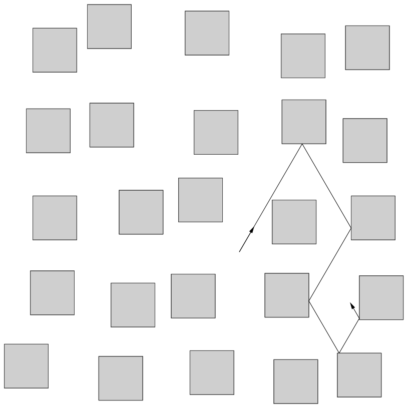

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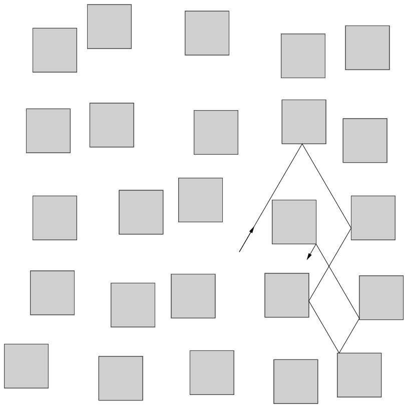

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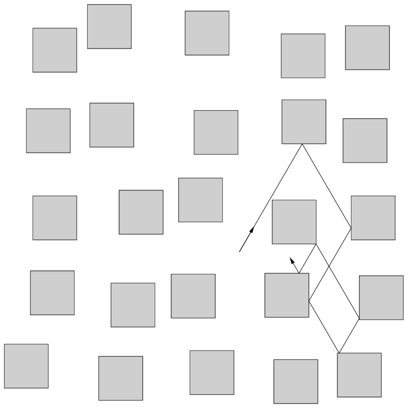

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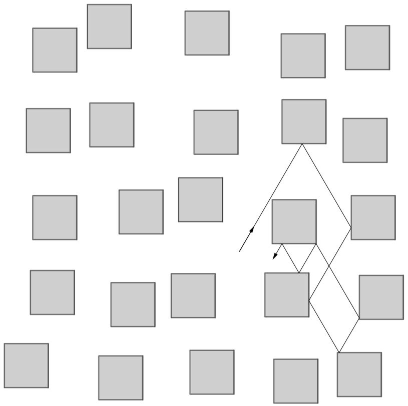

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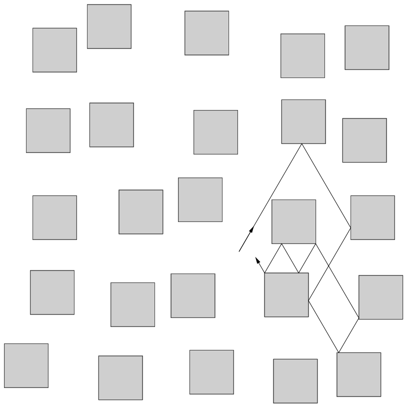

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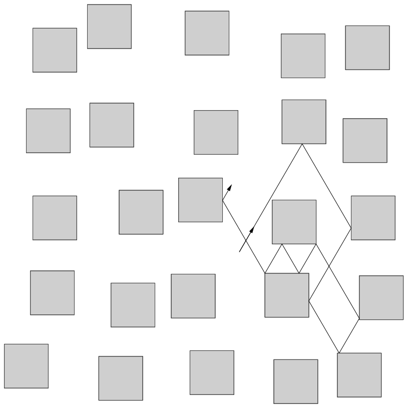

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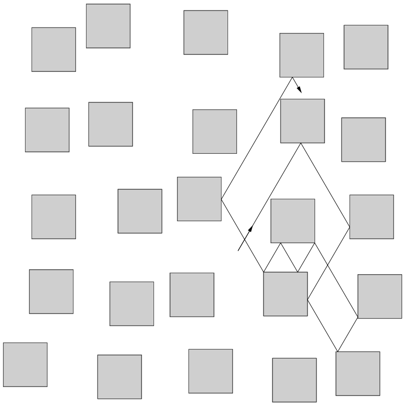

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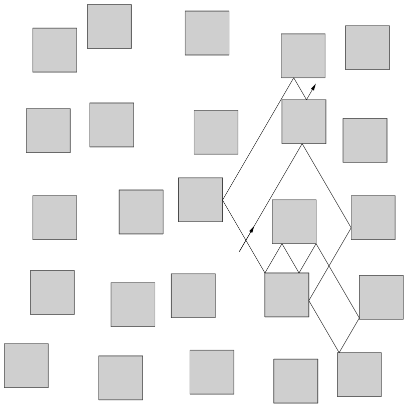

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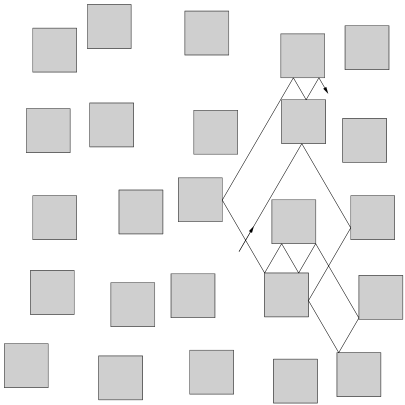

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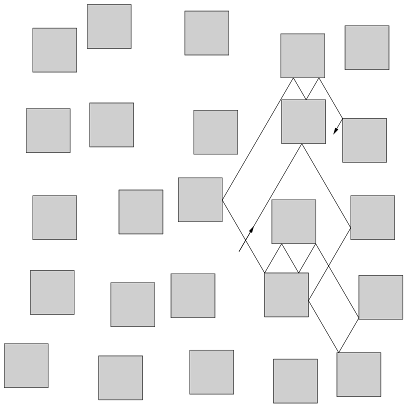

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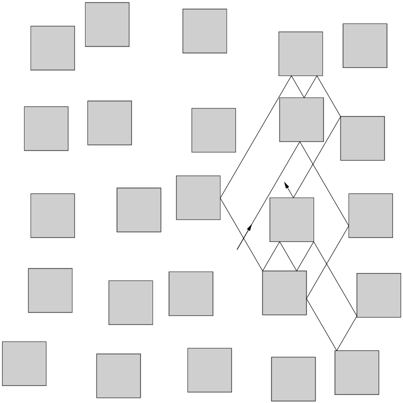

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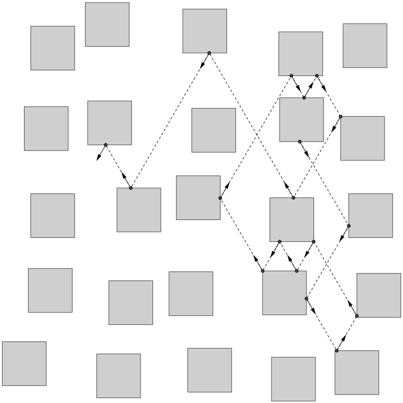

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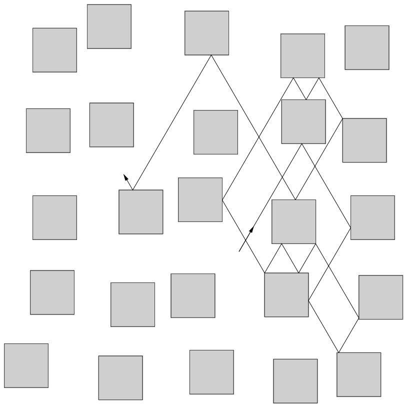

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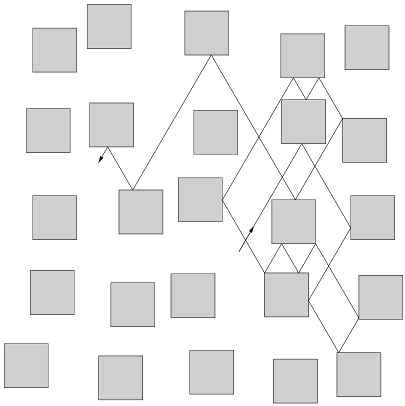

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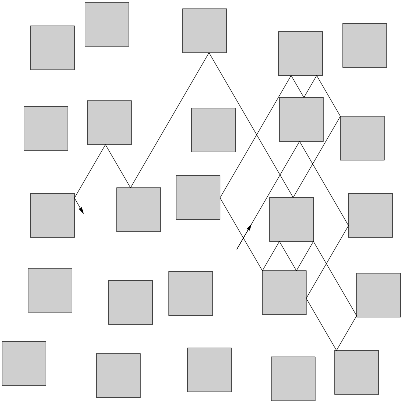

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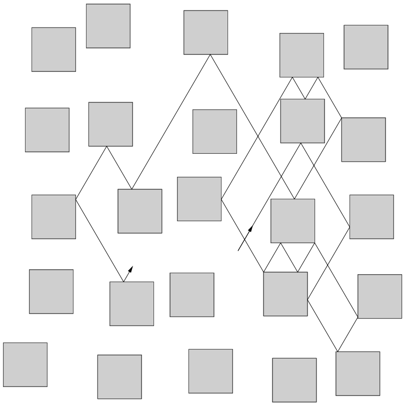

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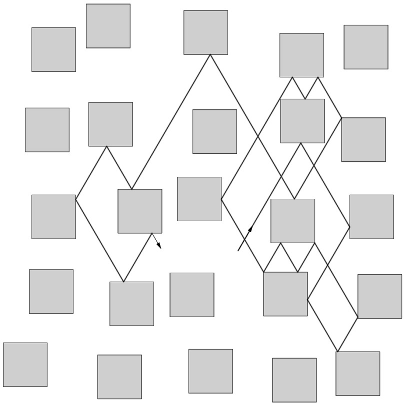

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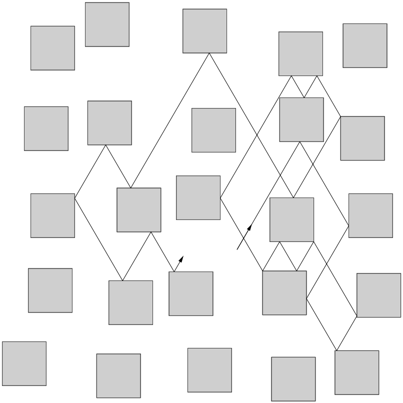
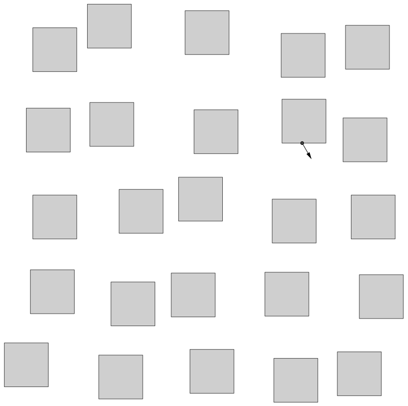


This was a picture of the billiard flow. Let consider the first return map to the border of the table: the billiard map.

Wind-tree map sends a point in the border and a direction to the point and the direction corresponding to the next bouncing.



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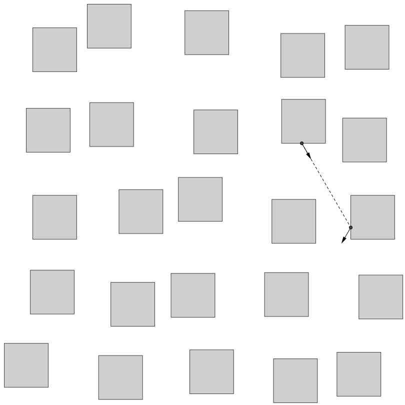

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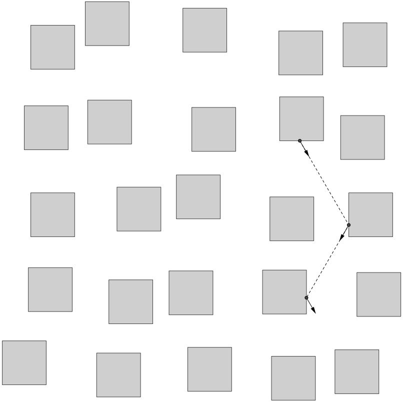

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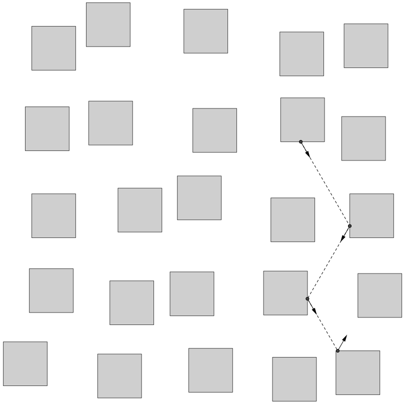

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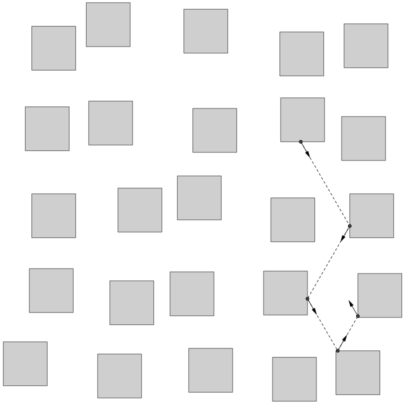

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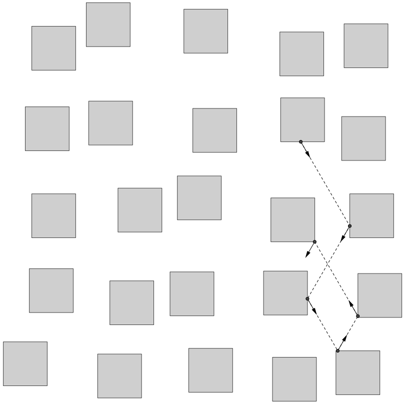

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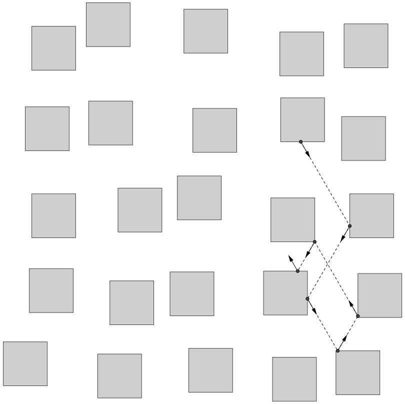

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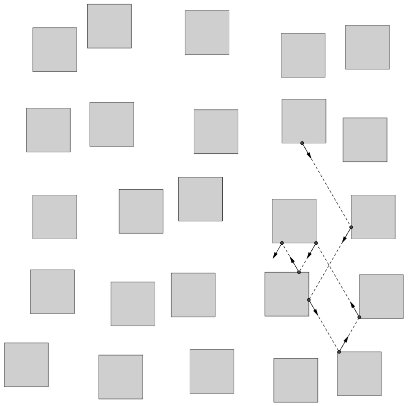

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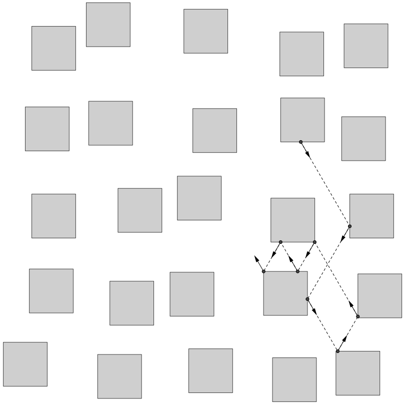

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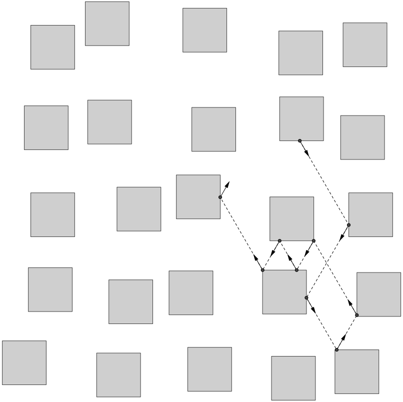

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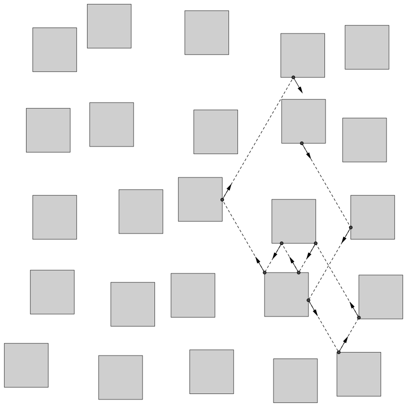

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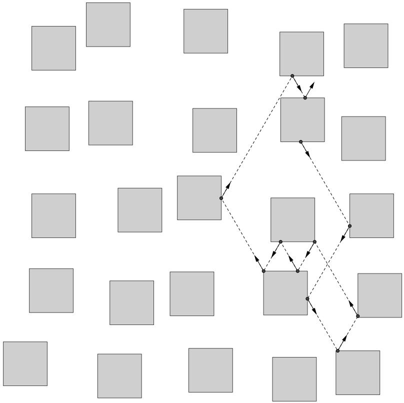

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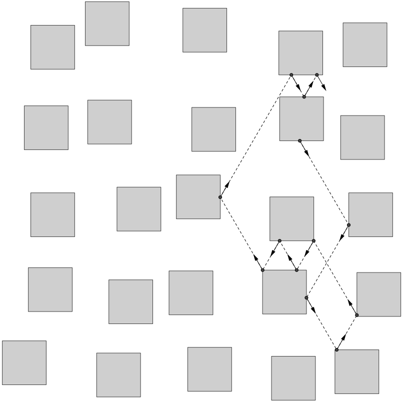

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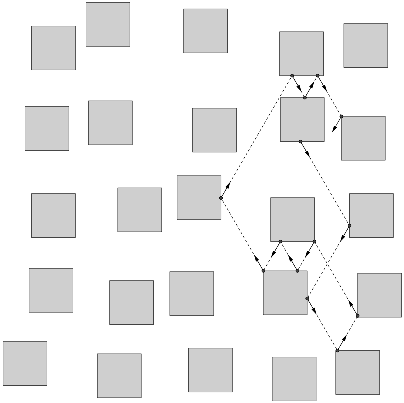

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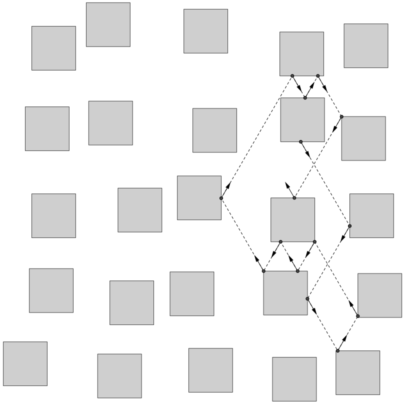

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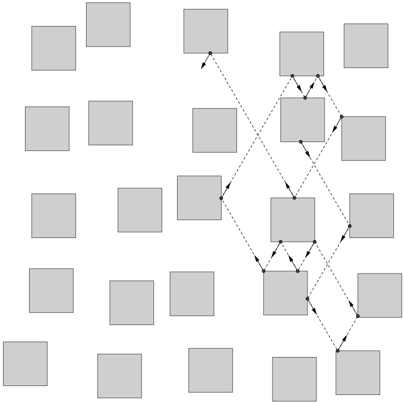

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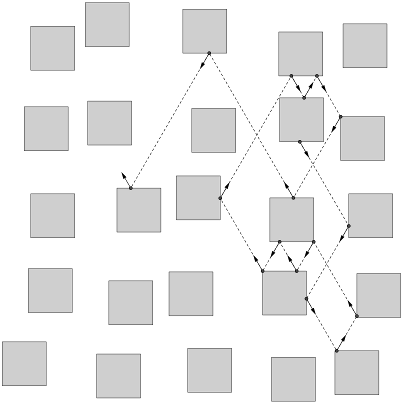

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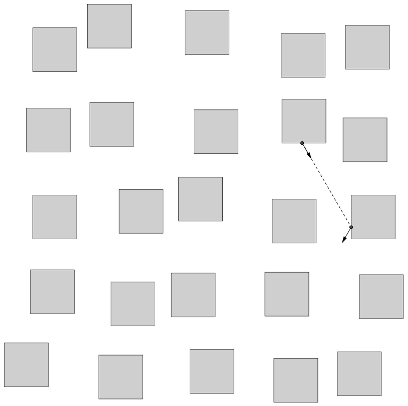

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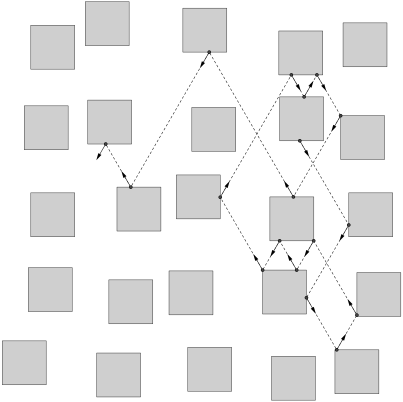

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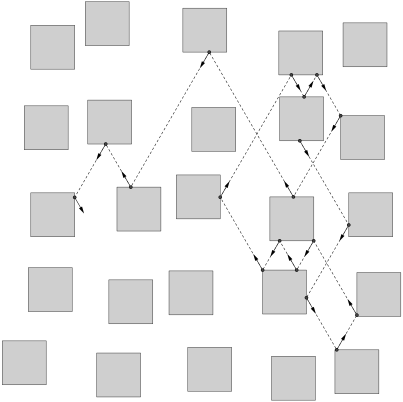

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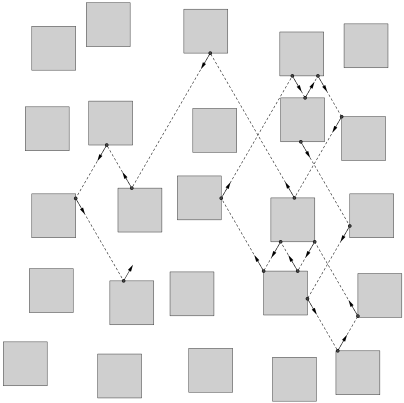

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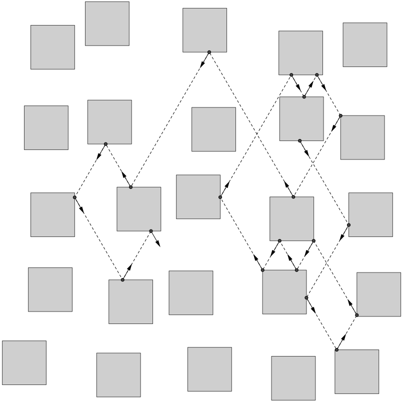

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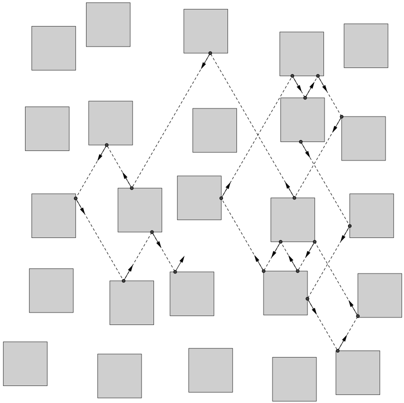

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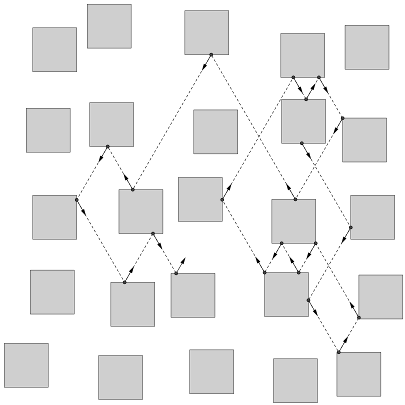
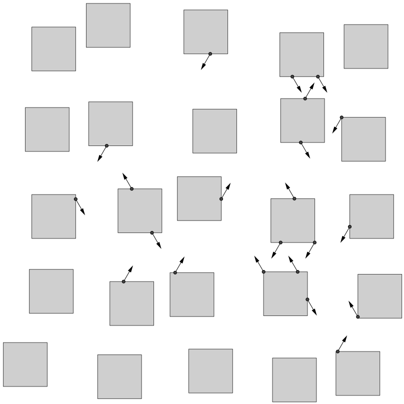
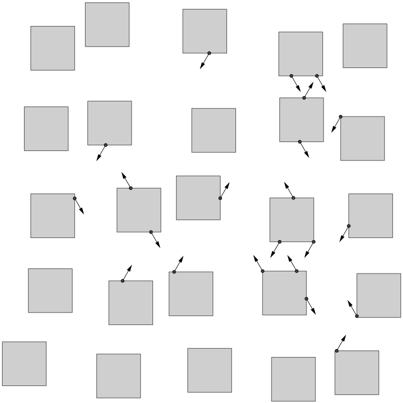


Note that the map is not everywhere defined.



Note that for any starting point and direction, only four directions may be reached.



Parameter space

The set of $2r$ by $2r$ squares, with vertical and horizontal sides, centered at (a, b) contained in the unit cell $[0, 1]^2$, is naturally parametrized by

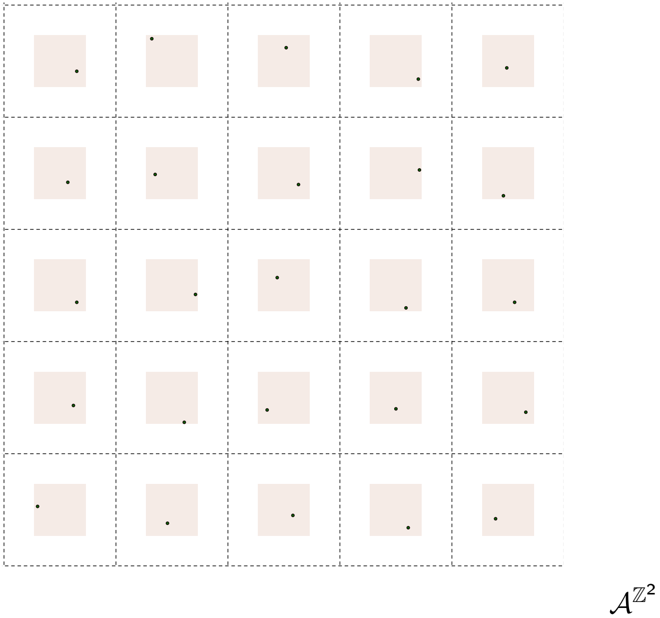
$$\mathcal{A} := \{t = (a, b) : r \leq a \leq 1 - r,\ r \leq b \leq 1 - r\}$$

Our parameter space is $\mathcal{A}^{\mathbb{Z}^2}$ with the product topology.

Phase space

Once launched in the direction θ, the billiard direction can only achieve four directions $\{\pm\theta, \pm(\theta - \pi)\}$; thus the phase space Ω^g_θ of the billiard map T^g_θ is a subset of the cartesian product of the boundary with these four directions. It contains precisely the pairs (s, ϕ) such that at s the direction ϕ points to the interior of the table.

Parameter space



Phase space

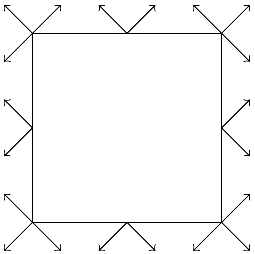


Figure: The contribution of one tree.

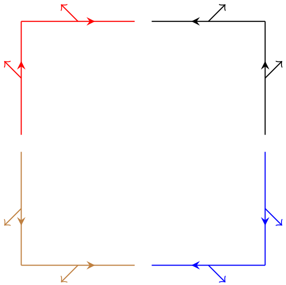


Figure: The phase space decomposes into four oriented "intervals".

Theorem (Generic minimality)

There is a dense G_δ set of parameters $\mathcal{G}$ such that for each $g \in \mathcal{G}$:

- *for a dense-G_δ set of full measure of θ the wind-tree map T_θ^g is minimal and has forward and backward escape orbits,*
- *the map T^g has a dense set of periodic points,*
- *if r is rational, then the map T^g has a locally dense set of periodic points,*
- *no two trees intersect.*

Definition

- A dense G_δ set is a countable intersection of open dense sets.

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Definition

- A map or a flow is minimal if and only if all its orbits are dense.

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Definition

- An orbit or half-orbit is an escape orbit if every compact set is visited only finitely many times.

Theorem (Generic minimality)

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- *if r is rational, then the map T^g has a locally dense set of periodic points,*
- *no two trees intersect.*

Definition

- The set of periodic points is called locally dense if there exists a G_δ-subset of the boundary which is of full measure, such that for every s in this set, there is a dense set of inner-pointing directions $\theta \in \S 1$ for which (s, θ) is periodic.

Theorem (Generic minimality)

There is a dense G_δ set of parameters $\mathcal{G}$ such that for each $g \in \mathcal{G}$:

- *for a dense-G_δ set of full measure of θ the wind-tree map T_θ^g is minimal and has forward and backward escape orbits,*
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Theorem (Generic ergodicity)

There is a dense G_δ subset $\mathcal{G}$ of parameters $\mathcal{A}^{\mathbb{Z}^2}$ such that for each $g \in \mathcal{G}$ there is a dense G_δ subset of directions $\mathcal{H} \subset \S 1$ of full measure such that the billiard flow on T_g in the direction θ is ergodic for every $\theta \in \mathcal{H}$.

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Definition

- A map or a flow is ergodic if and only if every measurable invariant set is of zero measure or has a complementary of zero measure.

Theorem (Generic ergodicity)

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The key idea

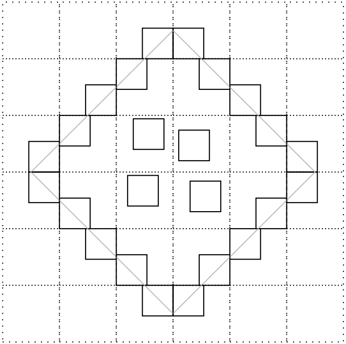


Figure: A 2-ringed configuration.

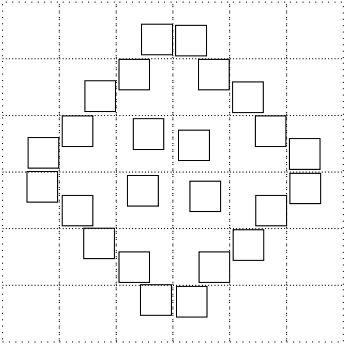
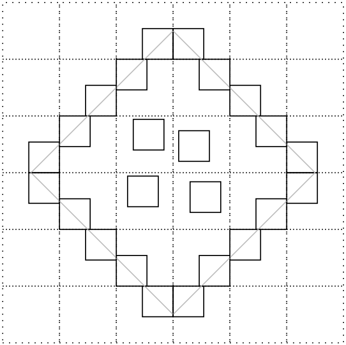


Figure: A small perturbation.

The key idea: use known results on compact invariant subsets



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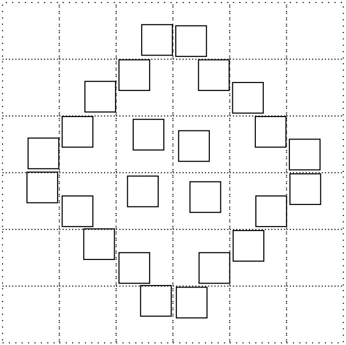
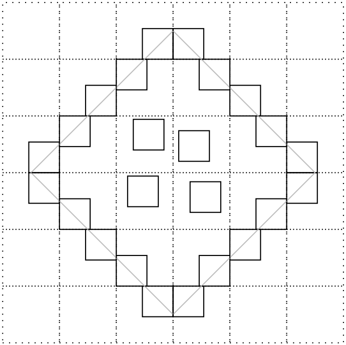


Figure: A small perturbation.

For minimality:

In a polygonal billiard, any direction without saddle connections gives rise to a minimal billiard map. (Keane)

The key idea: use known results on compact invariant subsets



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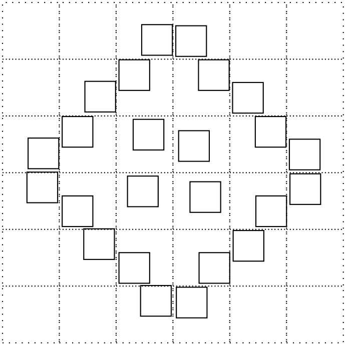
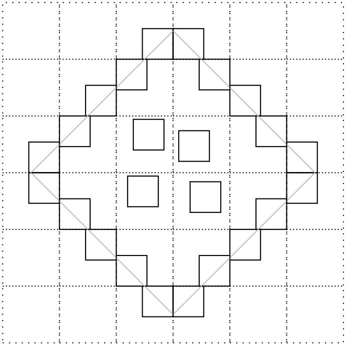


Figure: A small perturbation.

(A saddle connection is loosely speaking a T_{θ}^{g}-orbit going from a corner of a tree to some corner (maybe the same one).)

The key idea: use known results on compact invariant subsets



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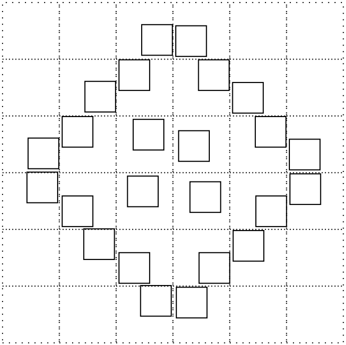

Figure: A small perturbation.

For ergodicity:

KMS: In every rational polygonal billiard, almost every direction gives rise to ergodic billiard map/flow.

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aufgebauten Folgerungen bleiben der folgenden Entwicklungsphase vorbehalten[48]).

Zwischenstück: Mit Rücksicht auf einige spätere Erörterungen empfiehlt es sich an einem aufs Äusserste vereinfachten Modell zu erläutern, welche Stellung der Stosszablansatz in den zuletzt erwähnten *Maxwell-Boltzmann*schen Untersuchungen einnimmt.

In der unbegrenzten Zeichenebene bewege sich eine sehr grosse, aber *endliche* Zahl (N) von materiellen Punkten: die „P-Moleküle". Sie seien für einander vollständig durchdringlich. Sie bewegen sich kräftefrei, ausser dass sie *elastische* Zusammenstösse mit den nun einzuführenden Q-Molekülen erfahren. — Die „Q-Moleküle" sind Quadrate von der Seitenlänge a, in *unendlicher* Zahl regellos über die unendliche Zeichenebene verteilt, und zwar *unbeweglich* (starr befestigt), auf jedes grössere Gebiet sollen nahe gleichviel entfallen, die mittlere Distanz A der nächstbenachbarten soll gross gegen a sein, und *die Diagonalen jedes Q-Moleküles seien exakt parallel der x- resp. y-Axe.*

Zur Zeit t_0 mögen alle P-Moleküle dieselbe Absolutgeschwindigkeit c und nur die folgenden vier Bewegungsrichtungen besitzen:

$$(1) \longrightarrow \quad (2) \uparrow \quad (3) \longleftarrow \quad (4) \downarrow$$

Wegen der Unbeweglichkeit der Q-Moleküle und der *exakten* Orientierung ihrer Diagonalen wird diese Verfügung sich dauernd aufrecht erhalten. — *Hingegen ändern sich durch die Stösse*, welche die P-Moleküle an den Q-Molekülen erfahren, die Zahlen

$$f_1, f_2, f_3, f_4,$$

die angeben, wie viele Moleküle in einem bestimmten Zeitpunkt die angeführten vier Bewegungsrichtungen besitzen: es ändert sich die „Geschwindigkeitsverteilung".

Das Analogon zur *Maxwell*schen-Verteilung bildet hier die Verteilung:

$$f_1{}^0 = f_2{}^0 = f_3{}^0 = f_4{}^0 = \frac{N}{4}. \tag{5}$$

Es handelt sich also um den Nachweis, dass unter der Wirkung der Zusammenstösse ein sukzessiver Ausgleich der Zahlen f_i stattfindet, und dass die Verteilung (5) sich aufrecht erhält, sobald sie einmal eingetreten ist.

$N_{12}\,\Delta t$ bezeichne die Zahl der P-Moleküle, die im Zeitelement Δt durch einen Zusammenstoss aus der Bewegungsrichtung (1) in die Richtung (2) geworfen werden. Es sind das offenbar alle und nur diejenigen Moleküle, welche zu Beginn des Zeitelementes Δt zugleich folgende beiden Bedingungen erfüllen:

A. Sie besitzen die Bewegungsrichtung 1.

B. Sie liegen in irgendeinem der Streifen S (Fig. 1). [An jedes der unendlich vielen Q-Moleküle ist ein solcher Streifen angelegt zu denken.]

Die Angabe der Zahlen f_1, f_2, f_3, f_4, genügt offenbar noch nicht, um zu bestimmen, wie viele P-Moleküle ausser der Bedingung A) *auch noch die Be-*

48) Der hypothetische Charakter des Stosszahlansatzes wurde lange Zeit durchaus nicht empfunden. Zum Beleg vgl. in *Boltzmann* [4] (1871) die Schlussbemerkung: „. . . so habe ich in jener Abhandlung" — gemeint ist die auf dem Stosszahlansatz basierte Abhandlung [3] — „den weitläufigeren, aber von jeder Hypothese freien Weg eingeschlagen."

dingung B) *erfüllen.* — Das Analogon zu dem mehrfach genannten *Stosszahlansatz* besteht nun in folgender *Behauptung:*

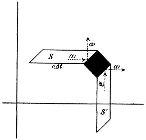

Fig. 1.

Von den P-Molekülen jeder einzelnen Bewegungsrichtung entfällt auf die Streifen S ein solcher Bruchteil, als dem Verhältnis der Gesamtfläche aller S zur totalen freien Fläche entspricht. Dieses Verhältnis sei bezeichnet mit

$$k \cdot \Delta t. \tag{6}$$

Danach würden im Zeitelement Δt

$$N_{12}\,\Delta t = f_1 \cdot k\,\Delta t \tag{7}$$

Moleküle von (1) nach (2) geworfen; analog

$$N_{21}\,\Delta t = f_2 \cdot k\,\Delta t \tag{8}$$

im selben Zeitelement Δt umgekehrt von (2) nach (1). (Hier sind die Streifen S durch die *flächengleichen* Streifen S' — Fig. 1 — zu ersetzen.)

Die Gegenüberstellung der Gleichung (7) und (8) zeigt unmittelbar, dass bei den Stössen vom obigen Typus das grössere f an das kleinere f während Δt in Summe

$$|f_1 - f_2| \cdot k\,\Delta t \tag{9}$$

Moleküle verliert. — Analog für jedes andere Paar von Stosstypen.

Wenn bei der Berechnung der Zahlen N_{12}, N_{21}, N_{23}, N_{32} etc. für jedes Zeitelement Δt immer wieder der Stosszahlansatz (7) *zugrunde gelegt wird, so erhält man eine monotone Abnahme für die Unterschiede der Zahlen f_1, f_2, f_3, f_4. (Einseitige Annäherung an Verteilung 5.)*

6. Das Boltzmannsche H-Theorem: Die kinetische Deutung einseitig verlaufender Prozesse[49]). Die Kritik des Stosszahlansatzes und seiner Folgerungen setzte ein, sobald man es als Paradoxon empfand, dass die durchaus *reversiblen* Gasmodelle der kinetischen Theorie imstande sein sollen, wesentlich einseitig verlaufende, *irreversible* Prozesse zu deuten. Nun wurde gerade durch das *Boltzmannsche H-Theorem* das Studium der nichtstationären[50]), irreversiblen Prozesse in den Vordergrund gestellt: um zu zeigen, wie jede Nicht-*Maxwell*sche Verteilung sich einseitig der *Maxwell*schen nähert, fasst dieses Theorem alle dabei stattfindenden Einzelprozesse (darunter Wärmeleitung und innere Reibung) zu einem einzigen, *einseitig verlaufenden* Totalprozess zusammen und gipfelt in der *kinetischen Deutung des Postulats der*

49) *Boltzmann* [6, 7, 16]; *Lorentz* [1] (1887).

50) Allerdings hatten schon viel früher *Clausius* und *Maxwell* Reibung, Wärmeleitung und Diffusion kinetisch gedeutet. Da sie sich aber auf *stationäre* Fälle beschränkten, so trat dabei das im Text angedeutete Paradoxon noch nicht zum Bewusstsein.

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the Maxwell-Boltzmann distribution is still stationary with respect to molecular collisions.

Boltzmann (1872, as one of the corollaries of the H-theorem)[47]—The Maxwell-Boltzmann distribution is the only distribution which can maintain its invariance,[48] and any other distribution under the influence of collisions finally goes over into the Maxwellian one.

We want to emphasize the following features with regard to the foundations of these investigations:

1. The calculation uses, at least partially, the mechanical properties of the gas model. In particular it uses the laws for the collisions of two molecules in order to calculate, when the type of collision is specified, the domains of state ($\Delta\tau$) into which two molecules are thrown from given domains of state.

2. The calculation employs some of the postulates about equal frequencies which are discussed in Section 3. In particular, when the number of collisions of various kinds in time element Δt is sought, it uses an assumption which is essentially identical with the *Stosszahlansatz* discussed in Section 3.

As to the frequency postulates, which lie at the basis of the kinetic theory of gases, the above investigations give the following results: With a partial application of the mechanical properties of the gas model, they prove statements about the relative frequencies of non-*gleichberechtigt* occurrences (the Maxwell-Boltzmann distribution law). In doing so they take for granted in the calculations certain assumptions about equal frequencies (especially the *Stosszahlansatz*).

Thus the *Stosszahlansatz* assumes a central position. Criticism of this postulate and revision of its corollaries will be treated later.[49]

Appendix to Section 5

Because of certain later discussions it seems advisable to explain on a much-simplified model what the position

of the *Stosszahlansatz* is in the Maxwell-Boltzmann investigations.

Let us consider a large but finite number (N) of material points moving in the infinite plane. We will call these points the "P-molecules." We will assume that they can completely penetrate each other. They move in the absence of forces, except for elastic collisions that they undergo with the "Q-molecules." The "Q-molecules" are defined as squares with sides of length a; there are an infinite number of them, distributed irregularly over the infinite plane, and they are fixed. Every portion of the plane contains about the same number of them (i.e., the distribution is uniform over the plane), and the average distance A of the neighboring squares is large compared to a. The diagonal of each Q-molecule is exactly parallel to the x and y axes respectively.

We assume that at time t_0 all P-molecules have the same speed c and that they can move in the following four directions:

$$(1) \rightarrow \quad (2) \uparrow \quad (3) \leftarrow \quad (4) \downarrow .$$

Because the Q-molecules are fixed and because their diagonals are oriented exactly, this assumption will hold true at any time. On the other hand, however, the numbers

$$f_1, f_2, f_3, f_4, \tag{4}$$

which are the numbers of molecules moving in the four directions at any given time, will be changed by the collisions of the P-molecules with the Q-molecules. In other words, the "velocity distribution" will change.

The distribution analogous in this example to the Maxwell distribution is

$$f_1^0 = f_2^0 = f_3^0 = f_4^0 = \frac{N}{4} . \tag{5}$$

Therefore in our case we have to show that a gradual

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equalization of the four f_i's takes place under the influence of the collisions and that the distribution (Eq. 5) maintains itself once it has been achieved.

Let us denote by $N_{12}\Delta t$ the number of P-molecules whose motion is changed by the collision from direction (1) to direction (2). These are all those and only those molecules which at the beginning of time interval Δt satisfy simultaneously the following two requirements:

A. They move in direction (1).

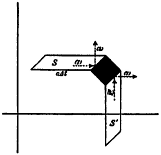

Figure 1

B. They lie in one of the strips S. (See Figure 1; one should imagine that each of the infinitely many Q-molecules has such a strip attached to it.)

It is clear that knowledge of the numbers f_1, f_2, f_3, f_4 is not enough to determine how many P-molecules satisfying condition A will also satisfy condition B.

The analogy to the *Stosszahlansatz* can now be expressed by the following statement:

The fraction of P-molecules of each single direction of motion which lie in the strips S is the same as the ratio of the total area of the strips to the total free area in the plane. Let us denote this ratio by

(6) $$k \cdot \Delta t.$$

Then in time interval Δt

(7) $$N_{12}\Delta t = f_1 \cdot k\Delta t$$

molecules are thrown from (1) to (2); similarly

(8) $$N_{21}\Delta t = f_2 \cdot k\Delta t$$

molecules are thrown in the same time interval from (2) to (1). (Here the strips S are to be replaced by the strips S' which have the same area; see Figure 1.)

A comparison of Eqs. (7) and (8) shows immediately that, because of collisions of the type discussed above, in time interval Δt the larger f loses

(9) $$|f_1 - f_2| \cdot k\Delta t$$

molecules to the smaller f. An analogous statement can be made about every other pair of collision types.

If we use the *Stosszahlansatz* given in Eq. (7) for the calculation of the numbers N_{12}, N_{21}, N_{23}, N_{32}, etc., in each time interval Δt, we get a monotonic decrease in the differences between the numbers f_1, f_2, f_3, f_4. Distribution (Eq. 5) is therefore reached monotonically in time.

6. The Boltzmann H-theorem. The kinetic interpretation of irreversible processes[50]

Criticism of the *Stosszahlansatz* and its corollaries arose as soon as it was recognized as paradoxical that the completely reversible gas model of the kinetic theory was apparently able to explain irreversible processes, i.e., phenomena whose development shows a definite direction in time. These nonstationary,[51] irreversible processes were brought into the center of interest by the H-theorem of Boltzmann. In order to show that every non-Maxwellian distribution always approaches the Maxwell distribution in time, this theorem synthesizes all the special irreversible processes (like heat conduction and

The Ehrenfest placed their obstacles "irregularly"

elastische Zusammenstösse mit den nun einzuführenden Q-Molekülen erfahren. — Die „Q-Moleküle" sind Quadrate von der Seitenlänge a, in *unendlicher* Zahl regellos über die unendliche Zeichenebene verteilt, und zwar *unbeweglich* (starr befestigt), auf jedes grössere Gebiet sollen nahe gleichviel entfallen, die mittlere Distanz A der nächstbenachbarten soll gross gegen a sein, und *die Diagonalen jedes Q-Moleküles seien exakt parallel der x- resp. y-Axe.*

they undergo with the "Q-molecules." **The "Q-molecules" are defined as squares with sides of length a; there are an infinite number of them, distributed irregularly over the infinite plane, and they are fixed.** Every portion of the plane contains about the same number of them (i.e., the distribution is uniform over the plane), and the average distance A of the neighboring squares is large compared to a. The diagonal of each Q-molecule is exactly parallel to the x and y axes respectively.

The Ehrenfest placed their obstacles "irregularly" , but...

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!!!

Their used a kind of mixing

true at any time. On the other hand, however, the numbers

(4) $$f_1, f_2, f_3, f_4,$$

which are the numbers of molecules moving in the four directions at any given time, will be changed by the collisions of the P-molecules with the Q-molecules. In other words, the "velocity distribution" will change.

The distribution analogous in this example to the Maxwell distribution is

(5) $$f_1^0 = f_2^0 = f_3^0 = f_4^0 = \frac{N}{4}.$$

Therefore in our case we have to show that a gradual equalization of the four f_i's takes place under the influence of the collisions and that the distribution (Eq. 5) maintains itself once it has been achieved.

in order to show that directions become "decorrelated".

Their used a kind of ~~mixing~~

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Their used *k*-fold ergodicity

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Work in Progress

Ergodicity of any positive integer power of the wind-tree flow is generic.

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Thank you for your attention.

arXiv:1506.00814