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Generation and Dynamics of Ordered Sheared Zonal Flows from Drift Turbulence.

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Generation and Dynamics of Ordered Sheared Zonal Flows from Drift Turbulence

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This talk is made possible by the many discussions and contribution of materials and results from collaborators and colleagues:

Background and Motivation

• Turbulence in MFE Plasmas
• Why Is Turbulence of Interest in Fusion?
• What are Zonal Flows?
• Why Care About the Nonlinear Interactions Between Turbulence and Zonal Flows?
Images of Turbulence in Tokamaks

GYRO

DIII-D

J. Yu  C-III Emission (UCSD)
Imaging of Turbulent Density Fluctuations in the Core Region of DIII-D Tokamak

Ref: G. McKee, private communication
Result: Turbulent Transport in Confined Plasmas

Ref: Lackner, DEISY Talk 2005

\[ \frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_r f_\alpha + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \nabla_v f_\alpha = \left( \frac{\partial f_\alpha}{\partial t} \right)_{\text{coll.}} \]

distribution function Electric field Lorentz force Influence of collision

together with Maxwell’s equations

“small”-scale turbulence:
• strongly influenced by geometry
• single particle effects important which not captured by simple fluid picture
Turbulence Leads to Cross-field Transport

\[ \frac{\partial \bar{n}}{\partial t} + \nabla \cdot \bar{\Gamma} = 0 \]

\[ m\bar{n} \frac{\partial \bar{v}}{\partial t} + \nabla \cdot \bar{r}_R = + \frac{q}{m} \left( \bar{E} + \bar{v} \times \bar{B} \right) - \nabla \left( \bar{n} \bar{T} \right) \]

\[ \bar{r}_R \equiv \left\langle \begin{pmatrix} \frac{-\nabla \phi \times \bar{B}}{B^2} \\ \frac{-\nabla \phi \times \bar{B}}{B^2} \end{pmatrix} \right\rangle \]

\[ \frac{3}{2} \frac{\partial \bar{n} \bar{T}}{\partial t} + \frac{3}{2} \frac{\partial \langle \bar{n} \bar{T} \rangle}{\partial t} + \nabla \cdot \left[ \bar{Q}_{\text{cond}} + \bar{Q}_{\text{conv}} \right] = -\nabla \cdot \bar{q}_{\text{class}} + P_{\text{rad}} \]

\[ \bar{Q}_{\text{cond}} \equiv \frac{3}{2} \bar{n} \left( -\frac{\langle \bar{n} \nabla \phi \rangle \times \bar{B}}{B^2} \right) \]

\[ \bar{Q}_{\text{conv}} \equiv \frac{3}{2} \bar{T} \left( -\frac{\langle \bar{n} \nabla \phi \rangle \times \bar{B}}{B^2} \right) \]

Neglecting DC Convection, Magnetic Fluctuations, Parallel flow fluctuations, Viscosity, …

Assuming electrostatic ExB dynamics for velocity
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Zonal Flows are Common & Effect Transport in Many Systems
Zonal Flows are Common & Effect Transport in Many Systems

CASSINI Imaging Team, NASA
2D Dynamics in Magnetized Plasmas & Rotating Fluids

Navier-Stokes Eqn for Rotating Fluid:

\[
\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} + 2\Omega \times \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \Phi + \mathbf{v} \nabla^2 \mathbf{v}
\]

Momentum Eqn for Magnetized Plasma:

\[
\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} + \mathbf{B} \times \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \Phi + \mathbf{v} \nabla^2 \mathbf{v}
\]

I.e. momentum conservation same form for rotating fluid
And magnetized plasma ➔ DYNAMICS ARE SAME!
Result: Strong Similarity Between Planetary Flows & Magnetized Plasma Flows

- Turbulence driven
- No linear instability
- No direct radial transport

$ZFs$ are "mode", but:

1. Turbulence driven
2. No linear instability
3. No direct radial transport

Why ZFs are Important for Fusion?

Ref. Itoh APS 2005

1. Self-regulation
\[ \chi = R \chi_{gb} \]

2. Shift for onset of large transport

3. Identify transport control knob

4. Meso-scales and zonal field

\[ \tau_E = H \tau_E^L \]
"Intrinsic" H-factor (w/o barrier)

\[ H \propto R^{-0.6} \]

- Operation of ITER near marginality
- Flow damping?
- Intermittent flux, e.g., ELMs
- Impacts on RWM and NTM

Issues in fusion

- Realizability of ignition
  \[ L \propto R^{0.8} \]
  \[ L \propto H^{-1.3} \]

- \( P_{LH} \)- threshold
- Peak heat load
- \( \beta \)-limit
- Steady state

DW Turb. Coexists with Zonal Flows.
Outline

• Basic Physics of Drift Instabilities & Zonal Flows
  • Intro to Statistical Tools for Turbulence/Zonal Flow Studies
  • Laboratory Plasma Studies
  • Confinement Experiment Studies of Turbulence and Zonal Flows
• Conclusions & Open Issues
The Basic Picture of Turbulent Transport

**Drift waves:**
- ExB drift & density profile: $\vec{\phi}$ excites $\tilde{n}_e$
- $||$ electron and $\perp$ ion polarisation dynamics: $\tilde{\phi}$ tied to $\tilde{n}_e$
- Structure propagates in y-direction
- Resistivity: phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field

**Interchange forcing:**
- Compressibility $\perp$ drifts due to inhomogeneous B:
  - Energy path between $\tilde{n}_e$ and $\tilde{\phi}$
- Bigger phase shift between $\tilde{n}_e$ and $\tilde{\phi}$
- Transport across field

Ref: T. Ribiero IPP-Garching 2005 Summer School
The Basic Picture of Turbulent Transport

Particle transport caused by gradient driven turbulence:
phase shift of pressure ahead of the electrostatic potential

Ref: T. Ribiero IPP-Garching 2005 Summer School
TURBULENCE DRIVES CROSS-FIELD TRANSPORT OF PARTICLES, ENERGY AND MOMENTUM IN A MAGNETICALLY-CONFINED PLASMA

Complex, Highly Nonlinear, Self-Regulating System

Requires fluctuation diagnostics to characterize, understand and ultimately help control turbulence

\[ Q = n \langle \nabla_r \tilde{T} \rangle + T \langle \tilde{n} \nabla_r \tilde{T} \rangle \]

Energy
- Ohmic
- Neutral Beams
- Electron Cyclotron Heating
- Ion Cyclotron Heating

Profiles \((n, T_e, T_i, v_e)\)
\[ \nabla n, \nabla T \]
\[ \nabla E_r \]

Turbulence (ITG, TEM, ETG):
Drift wave?

Transport (>> neoclassical)
\[ \Gamma = \langle \tilde{n} \nabla_r \tilde{T} \rangle \]
\[ \tilde{n}, \tilde{T}, \tilde{\phi}, \tilde{B} \]
Origin of Zonal Flow Lies in 2D Dynamics

Navier-Stokes Eqn for Rotating Fluid:

\[
\left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v + 2\Omega \times v = -\frac{1}{\rho} \nabla p - \nabla \Phi + \nu \nabla^2 v
\]

For \( |\nabla \times v| \ll |\Omega| \) \( \nabla \cdot v = 0 \quad v = 0 \)

\[ \Omega \cdot \nabla v = 0 \]

I.e. no velocity gradient along direction of axis of rotation
Flow Generation from Turbulence: the Vortex Merging Picture

3-D

\[ \frac{\partial v_z}{\partial z} \neq 0 \Rightarrow \]

\[ \Rightarrow \text{Vortex Stretching} \]

\[ \ldots \text{Generate Flows on Smaller Scales} \]

2-D

\[ \frac{\partial v_z}{\partial z} = 0 \Rightarrow \text{Vortex Merging} \]
Flow Generation from Turbulence: Fourier Space

Usual Reynolds Stress Term in Simplified Momentum Eqn (ala Diamond et al. PRL 1994 and others)

\[
\frac{\partial \tilde{u}_\theta}{\partial t} + \left( \frac{\partial \langle \tilde{u}_r \tilde{u}_\theta \rangle}{\partial r} \right) = -v_{damp} \tilde{u}_\theta
\]

"Radial Transport of Angular Momentum"

Consider a Zonal Flow to Have:

\[
u = u^Z \hat{\theta} \quad k^Z_r = k^Z_r \hat{r}
\]

\[|k^Z_r| \ll |k_1|, |k_2|\]

\[
\tau_z \sim 1 / k^Z_r u^Z_\theta \gg t_{corr}
\]

F.T., Write as KE, and Average Energy Eqn over Z-flow scales:

\[
\frac{1}{2} \frac{\partial \langle u^2 (k^Z_Z) \rangle}{\partial t} - P^{turb}_{k^Z_Z} = -\mu \langle u^2 (k^Z_Z) \rangle
\]

where

\[
P^turb_{k^Z_Z} = \sum_{k_1, k_2, k^Z_Z = k_1 \pm k_2} Re \left\langle u^{\ast}_{\theta z} (k^Z_Z) (\tilde{u} (k_1) \cdot \nabla) \tilde{u}_\theta (k_2) \right\rangle
\]

NL Energy Transfer

Center for Energy Research

UCSD Jacobs | Mechanical and Aerospace Engineering

G.R. Tynan, Int'l Workshop on Frontiers in Plasma Physics, July 2008, ICTP Triest, Italy
Flow Generation from Turbulence: Fourier Space

- Free Energy Source Releases Energy On One Scale
- Nonlinear Energy Transfer Moves Energy to Dissipation Region
- Shear Flows Develop Via Transfer of Energy to LARGE SCALES (small k)
Schematic of NONLINEAR drift turbulence-zonal Flow interactions

Ref: Itoh APS 2005

Suppression of DW by shearing

Damping by collisions

Generation By vortex tilting

Nonlinear flow damping

Collisional flow damping

Drift wave turbulence

Transport

Shearing

Energy return

VT, \n...
Large Scale Sheared Flows Can Develop

New feature: geodesic acoustic coupling (GAC)  B D Scott 2003
Impact of ZFs on Turbulence

Random stretching of DW eddies

\[ \langle \delta k^2 \rangle \sim D_k t \]

\[ k^2 \] of DW packet

DW energy \[ W_k = \omega_k N_k \]

Energy for ZFs excitation is extracted from DWs

Note: Conservation energy between ZF and DW

Ref: Itoh APS 2005
(1) Tilt of convection cell by a sheared flow

(2) Modulational Instability
External shear flow breaks streamers

**Potential**

- ![Potential plots](image)

**Pressure**

- ![Pressure plots](image)

**Flux**

- ![Flux plots](image)

Increasing velocity shear $V_{E'}$

RBM \[ [Beyer et al, 00] \]
Implication: Plasma Predicted to Sit At/Near Marginal Stability

Lackner DEISY Symposium 2005
Questions of Interest

• What Drives the Turbulence (I.e. Free Energy Source & Underlying Instability)?
• What are Spatio-temporal Scales of Turbulence?
• How Do Nonlinear Interactions Lead to Observed Spatio-temporal Scales?
• What does resulting flux v gradient curve look like?
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• **Intro to Statistical Tools for Turbulence/Zonal Flow Studies**
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Linear Signal Processing Gives **Average Spatio-temporal Scales** of **TIME STATIONARY Turbulence**

Fourier Transform:

\[ f(t) = \sum_{\omega} f(\omega) \exp(i\omega t) \Delta \omega \quad f(\omega) = \frac{1}{2\omega} \sum_{t} f(t) \exp(-i\omega t) \Delta t \]

Correlation Functions & Power Spectra:

\[ C_{\text{auto}}(\tau) = \frac{1}{N_{\text{ens}}} \sum_{n=1}^{N_{\text{ens}}} \frac{1}{T} \int_{-T/2}^{T/2} \tilde{f}_i(t) \tilde{f}_i(t + \tau) \quad P_{\text{auto}}(\omega) = \frac{1}{N_{\text{ens}}} \sum_{n=1}^{N_{\text{ens}}} \left| \tilde{f}_i(\omega) \right|^2 \]

\[ C_{\text{crs}}(\tau) = \frac{1}{N_{\text{ens}}} \sum_{n=1}^{N_{\text{ens}}} \frac{1}{T} \int_{-T/2}^{T/2} \tilde{f}_i(t) \tilde{g}_i(t + \tau) \quad P_{\text{crs}}(\omega) = \frac{1}{N_{\text{ens}}} \sum_{n=1}^{N_{\text{ens}}} \tilde{f}_i(\omega) \tilde{g}_i^*(\omega) \]

Inter-relation Between Two Signals: coherence and crossphase

\[ \gamma_{12}(\omega) = \frac{|P_{\text{crs}}(\omega)|}{\sqrt{P_1 P_2}}, \quad \tan \phi(\omega) = \frac{\text{Im}(P_{\text{crs}}(\omega))}{\text{Re}(P_{\text{crs}}(\omega))} \]
Nonlinear Energy Transfer Comes from 3rd Order Spectrum:

Example: Collisional Drift Turbulence Model (Hasegawa-Wakatani 1983)

\[
\frac{\partial W^n}{\partial t} = \frac{\Gamma_r}{L_n} + T^n + C_1 \left( W^n - \langle n^* \phi \rangle \right)
\]

\[
\frac{\partial W^\phi}{\partial t} = T^\phi - C_2 \left\langle \left| \nabla^2 \phi \right|^2 \right\rangle - C_3 W^\phi
\]

Internal and Kinetic Energies Defined As

\[
W^n = \sum_m |n_m(r,t)|^2
\]

\[
W^\phi = \sum_m |\nabla \phi_m(r,t)|^2
\]

Nonlinear Energy Transfer From Cross-Bispectrum:

\[
T^n_{\theta} (r, f, f') = - \text{Re} \left\langle \hat{n}^n(r, f) \hat{\nu}_{\theta}(r, f - f') \frac{1}{r} \frac{\partial \hat{n}}{\partial \theta} (r, f') \right\rangle
\]

\[
T^\phi_{ZF} (r, f, f') = - \text{Re} \left\langle \hat{V}_{ZF}^n(r, f) \hat{\nu}_{\theta}(r, f - f') \frac{1}{r} \frac{\partial \hat{\nu}_{r}}{\partial \theta} (r, f') \right\rangle
\]
Rate and Direction of Energy Transfer Determined by BiSpectrum

Bi-spectrum Defined As

\[ B(\omega_1, \omega_2) = \langle X(\omega_1) X(\omega_2) X^*(\omega_1 + \omega_2) \rangle \]

Degree of Phase Coherence Determined by BiCoherence

\[ \hat{b}^2 (\omega_1, \omega_2) = \frac{B(\omega_1 + \omega_2)}{\left| X(\omega_1) X(\omega_2) \right|^2 \left| X^*(\omega_1 + \omega_2) \right|^2} ; \ 0 < \hat{b} < 1 \]

BiPhase Determines Phase Delay Between Interacting Waves:

\[ \Theta(\omega_1, \omega_2) \equiv Tan^{-1} \left\{ \frac{Im[B(\omega_1, \omega_2)]}{Re[B(\omega_1, \omega_2)]} \right\} ; \ -\pi < \Theta < \pi \]

Energy Transfer Direction and Rate:

\[ Re\left[ B(\omega_1, \omega_2) \right] = |B(\omega_1, \omega_2)| \cos[\Theta(\omega_1, \omega_2)] \]
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**Controlled Shear De-Correlation Experiment (CSDX)**

<table>
<thead>
<tr>
<th>CSDX parameter</th>
<th>Typical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas pressure</td>
<td>0.5 - few mTorr</td>
</tr>
<tr>
<td>$T_e$</td>
<td>$\sim$ 3 eV</td>
</tr>
<tr>
<td>$T_i$</td>
<td>$\sim$ 0.7 eV</td>
</tr>
<tr>
<td>$n_e$</td>
<td>1-10 x 10^{12} cm^{-3}</td>
</tr>
<tr>
<td>Source (Helicon 13.56 MHz)</td>
<td>1500W (typically)</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>Up to $\sim$ 1000 G</td>
</tr>
</tbody>
</table>

m=0 Helicon Plasma Source

![Diagram of CSDX experiment](image)

- Exit Pump
- RF source
- Multi-tip Langmuir probe was inserted in this port
- Ar gas injection

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Dimensionless Scales in CSDX

Length Scales

\[ \frac{\rho_i}{L_n} \sim 0.1 \]
\[ \frac{\rho_s}{L_n} \sim 0.5 \]
\[ \frac{\lambda_{mfp}^e}{L_{||}} \sim 0.05 - 0.1 \]

Collision Rates

\[ \frac{\nu_e}{\Omega_{C_e}} \sim 0.005 \]
\[ \frac{\nu_{ii}}{\Omega_{C_i}} \sim 1 \]
\[ \frac{\nu_{i0}}{\Omega_{C_i}} \sim 0.01 \]

Parallel Dissipation Rate

\[ \frac{k_{||}^2 C_s^2}{\nu_e \Omega_{C_i}} \sim 1 \]

Non-ambipolar G.C. Drifts

Polarization Drift

\[ \frac{\omega}{\Omega_{C_i}} \sim 0.1 - 0.3 \]

Ion-Ion Collisional Drift

\[ \frac{\mu_{ii}}{L_{\perp}^2 \Omega_{C_i}} \approx 0.01 - 0.1 \]
DEVELOPMENT OF DRIFT TURBULENCE FROM LINEAR DRIFT WAVES
Equilibrium Profiles Evolve as B Field Increases in CSDX Helicon Plasma

Density

Electron Temperature

Source Width

Source Width

Burin et al, May 2005 PoP
Wave at Onset Consistent w/ Collisional Drift Wave

\[ V_{De} = \nabla \rho \times \vec{B} \]

\( \vec{B} \times \nabla \rho \)
direction of wave propagation

end on view from vacuum pump side

probe 1

probe 2

1.0mT 630 gauss
signal 2 (right)
signal 1 (left)

Time

probe 1

probe 2

probe 1

probe 2

probe 1

probe 2

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Linear Eigenmodes of DW w/ Flow Shear Match Observations Near Onset

Burin et al, May 2005 PoP
Drift Wave Dispersion Agrees with Linear Theory

Burin Phys. Plasmas 2005
EVIDENCE FOR INVERSE ENERGY TRANSFER DURING DEVELOPMENT OF TURBULENCE
Development of Wave-Wave Interactions

Keep Convective Derivative (Nonlinear(!)) in Momentum Equation, e.g. $\theta$ component:

$$\frac{\partial \tilde{u}_\theta}{\partial t} + \frac{\partial \tilde{u}_r \tilde{u}_\theta}{\partial r} = -\nu_{\text{damp}} \tilde{u}_\theta$$

F.T., Write as KE, and Average Energy Eqn for wavenumber $k$:

$$\frac{1}{2} \frac{\partial \left\langle u_{\theta}^2(k) \right\rangle}{\partial t} - P_{k_\text{turb}} = -\mu \left\langle u_{\theta}^2(k) \right\rangle$$

where $P_{k_\text{turb}} = \sum_{k_1, k_2, k = k_1 + k_2} \text{Re}\left\langle u^{*}_\theta(k) (\tilde{u}(k_1) \cdot \nabla) \tilde{u}_\theta(k_2) \right\rangle$

*Represents NL Energy Transfer due to Wave-Wave Interaction*
Local k-spectra Are Constructed from 2-Point Measurements


Measure Fluctuations at 2 Points:

Find Phase Delay due to Propagation

\[ \delta \theta = \tan^{-1} \frac{\text{Im}(X_1(f)X_2^*(f))}{\text{Re}(X_1(f)X_2^*(f))} \]

Local Wavenumber \( k_{\theta_{\text{local}}} = \frac{\delta \theta}{\delta x} \)

Build-up Local k-spectra from Multiple Realizations:

Burin et al, May 2005 PoP
Evolution of $(\phi_f / kT_e)^2$ Power Spectrum with Increasing Magnetic Field

- Coherent Drift Waves Appear at $\sim 400$G
- Harmonics Develop As B Increases
- Coherent Modes at Intermediate B
- Broadband Spectra at 1kG

Burin et al, May 2005 PoP
Intensity of 3-wave Interactions Increases as Turbulence Develops

Tynan, PoP 2004

Burin, PoP 2005
Evolution of Energy Spectrum w/ Magnetic Field

\[ E_{k_\theta} = \left( \frac{n_{k_\theta}}{n} \right)^2 + k_\theta^2 \left( \frac{e\phi_{k_\theta}}{k_B T_e} \right)^2 \]

Low-k Region Fills In

Spectral Index Region Develops

Burin et al, May 2005 PoP
Real and Imaginary Frequencies of Linear Eigenmodes
Free Energy Sources Are Known ➔ Can Find Linearly Unstable Region

- Include \( \text{grad-P, } \) Vshear Free Energy Sources
- Include Neutral Flow Drag (Effective at high \( k \)), FLR Damping
- Find Stable & Unstable Regions

Implies Energy MUST Be Transferred Into Low-k Region Via Nonlinear Processes

Tynan et al Nov 2004 PoP
Conclusions from Laboratory Plasma Experiment

- Drift Waves Develop if Grad-P is High Enough and Damping Small Enough
- Coherent Waves Obey Linear Dispersion Relation
- Weakly Dispersive Waves Allow NONLINEAR 3-Wave Coupling to Begin
- Nonlinear Energy Transfer Re-arranges Spectrum
- Broad Spectrum Emerges & Shows Indicates of INVERSE ENERGY TRANSFER…
- PART II… EMERGENCE OF ORDERED FLOWS OUT OF TURBULENCE, CONFINEMENT DEVICE RESULTS, BIFURCATIONS,…
EVIDENCE FOR EXISTENCE OF SHEAR LAYER SUSTAINED BY TURBULENT REYNOLDS STRESS
Mach Probe Measures $V_\theta, V_z$ Profiles

Observe Fluctuation Propagation Speed w/ Multipoint Probe Array or Fast Imaging

\[ R(y) \]

\[ \tau (\mu s) \]

- $\Delta y = 0.37 \text{ cm}$
- $\Delta y = 0.74 \text{ cm}$
- $\Delta y = 1.10 \text{ cm}$
- $\Delta y = 1.46 \text{ cm}$
- $\Delta y = 1.83 \text{ cm}$

\[ y = 0.37 \text{ cm} \]
\[ y = 0.74 \text{ cm} \]
\[ y = 1.10 \text{ cm} \]
\[ y = 1.46 \text{ cm} \]
\[ y = 1.83 \text{ cm} \]
Direct Imaging of Turbulence Density Fluctuations and Shear Flow
We Can Also Infer Flowfield from Motion of Density Perturbations
Independent Measurements of Shear Layer

Sound speed

\[ c_s = \left( \frac{T_e}{M_i} \right)^{1/2} \]

\[ = 2.8 \times 10^5 \text{ cm/s} \]
Use Measured Reynolds Stress in Azimuthal Momentum Balance & Solve for V Profile

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \langle \tilde{V}_r \tilde{V}_\theta \rangle \right) = -v_{io} \bar{V}_{i\theta} + \mu_{ii} \nabla^2 \bar{V}_{i\theta} \]

Tynan et al April 2006 PPCF  Holland et al, in press, PRL
Spectral properties of Reynolds stress

(a) Absolute magnitude of the power spectrum of the turbulent Reynolds stress. Color scale is logarithmic (base 10)

(b) Cross-phase $\alpha_{\delta v_r, \delta v_\theta}$ between radial and azimuthal turbulent velocity fields

(c) Squared cross-coherence $\gamma^2_{\delta v_r, \delta v_\theta}$ between radial and azimuthal turbulent velocity fields.
Estimate Dissipation from Measurements

Measure:

\[
\int_{-a}^{a} T_i dl = 0.7 \text{eV}
\]

\[
\int_{-a}^{a} T_{gas} dl = 0.4 \text{eV}
\]

Assume:

\[T_i(0) > T_i(a)\]

\[T_{gas}(a) = T_{wall}\]

\[
\mu_{ii} = \frac{3}{10} \rho_i v_{ii}
\]

\[P_{gas} = n_{gas} T_{gas} = \text{const}\]

\[\mu_{ii} \propto n_i T_i^{1/2}\]

\[\bar{\mu}_{ii} \approx 4 \times 10^4 \text{cm}^2/\text{sec}\]

\[\mu_{ii}(0) > \mu_{ii}(a)\]

\[v_{i0} \sim 6 \times 10^3 \text{ sec}^{-1}\]

Tynan et al, April 2006 PPCF,
Holland et al, in press, PRL
Measured Velocity Profile Consistent with Turbulent Momentum Balance

Tynan et al, April 2006 PPCF, , Holland et al, PRL 2006
No Turbulent Particle Transport Across Shear Layer

Tynan et al, April 2006 PPCF, Holland et al, In press, PRL

G.R. Tynan, Int'l Workshop on Frontiers in Plasma Physics, July 2008, ICTP Triest, Italy
Fast-frame imaging showing the dynamics of the shear layer
Net Nonlinear Energy Transfer into/out-of Frequency $f$

\[ T_n \equiv \left\langle -\mathrm{Re} \sum_{\omega_1, \omega_2} (\hat{z} \times \nabla_{\perp} \phi_{\omega_1}^*) \cdot \left( (\hat{z} \times \nabla_{\perp} \phi_{\omega_1} \cdot \nabla_{\perp}) (\hat{z} \times \nabla_{\perp} \phi_{\omega_2}^*) \right) \right\rangle \]

\[ T_n \equiv \left\langle -\mathrm{Re} \left[ \sum_{\omega_1, \omega_2} n_{\omega_1}^* (\hat{z} \times \nabla_{\perp} \phi_{\omega_1} \cdot \nabla_{\perp}) n_{\omega_2} \right] \right\rangle \]
Shear Layer Formation in Collisional Drift Turbulence Simulations
(2D) Hasegawa-Wakatani model in cylindrical geometry.
- Includes ion-neutral flow damping effect $\nu$, **neglects** nonlocal (finite $\rho_s / L_n$) terms, **fixed parallel wavenumber**.

$$\left( \frac{\partial}{\partial t} + \vec{V} \times \vec{B} \cdot \vec{\nabla} \right) n + \frac{V^*}{r} \frac{\partial n}{\partial \theta} + \frac{k_{\|}^2 \nu_{\text{th}}^2}{\omega \nu_e} (n - \phi) = D_n \nabla_\perp^2 n$$

$$\left( \frac{\partial}{\partial t} + \vec{V} \times \vec{B} \cdot \vec{\nabla} \right) \nabla_\perp^2 \phi + \frac{k_{\|}^2 \nu_{\text{th}}^2}{\omega \nu_e} (n - \phi) = \nu \nabla_\perp^2 \phi + \mu \nabla_\perp^4 \phi$$

- Parameters used reflect best estimates for average CSDX values:
  - $\rho_s = 1 \text{ cm}$, $L_n = 2 \text{ cm}$, $\omega_{||} = 1$, $\nu = 0.03 C_s / L_n$,
  - $D_n = 0.01 \rho_s^2 C_s / L_n$, $\mu = 0.4 \rho_s^2 C_s / L_n$
- Advances eqns by combination of 2nd order RK and implicit treatment of diffusive terms (conserves energy to within 1%).
Zonal Flow Forms from Vortex Merging
Simulations Show Zonal Flow Formation
Vortex Merging

Iso-Potential Contours:

- Simulation uses 64 x 64 pts, results insensitive to changes in $D_n$, $v$, $\mu$
- Changing $L_n$ to 10 cm does not qualitatively affect results
Measured Velocity Profile Consistent with Turbulent Momentum Balance

Tynan et al April 2006 PPCF
Evidence for Modulational Instability Behavior in Lab Plasma DW Turbulence-ZF Interactions
Turbulence Measurements in Tokamak Core Region
Visualizations of Core Plasma Turbulence Obtained with High-Sensitivity BES System Across Much of Minor Radius

- 1 MHz sampling
- L-mode discharges (1.2 MA, 2.0 T, 5 MW NBI)
- Frequency-filtered per spectrum
- 2D spline
- $10^5$ time dilation ($5\mu s = 1\ sec$)

\[\frac{\tilde{n}}{n}\]

<table>
<thead>
<tr>
<th>Minor Radius (r/a)</th>
<th>0.4</th>
<th>0.46</th>
<th>0.64</th>
<th>0.7</th>
<th>0.82</th>
<th>0.91</th>
<th>0.88</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{n}/n$</td>
<td>0.5%</td>
<td>0.75%</td>
<td>0.9%</td>
<td>2-8%</td>
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Fluctuation Spectra and amplitude vary strongly with radius

- Density fluctuation amplitude in L-mode discharges shows wide dynamic range across plasma radius
- Spectra strongly Doppler-shifted to higher frequency towards core
2D Correlation and $S(k_r, k_\theta)$ Spectra Confirm Spatial Asymmetry

- Exhibits radially decaying, poloidally wavelike structure, $L_{c,\theta} > L_{c,r}$
- Wavenumber spectra can be compared with turbulence simulations

$\rho(R, \theta, \tau=0)$
$\rho$ (Correlation, $\tau=0$)

$S(k_r, k_\theta)$

Wavenumber spectrum obtained from DCT

Peak:
$k_r=0$ cm$^{-1}$
$k_\theta=1.25$ cm$^{-1}$
Existence of Zonal Flows & GAMs in Confined Plasmas
Zonal flows really do exist!

CHS Dual HIBP System

90 degree apart

E_r(r,t)

High correlation on magnetic surface,
Slowly evolving in time,
Rapidly changing in radius.

Fujisawa, PRL 2004
Suppression of transport by ZF

\[ \Gamma(\omega) = \frac{1}{B} \left\langle \hat{E}_\theta, \omega \hat{n}_\omega \right\rangle \]

ZF

HIBP on CHS

Fujisawa, PPCF 2006

Regulation of transport by GAMs

GAMs

\[ \Gamma(t) \]

DW

HIBP on JFT-2M

Modulation of envelope and transport by GAMs were confirmed.

Ido, submitted to NF
GAM-ZF Observed in Edge Region

- BES measures localized, long-wavelength \((k_r \rho_i < 1)\) density fluctuations
  - Can be radially scanned shot to shot to measure turbulence profiles
  - Recent upgrades allow for BES to measure core fluctuations

- Time-delay estimation (TDE) technique uses cross-correlations between two poloidally separated measurements to infer velocity

McKee PoP 2001
Measured $V_\theta$ Spectra Exhibit Signatures of Both ZMF Zonal Flows and GAMs

- Spectra indicate broad, low-frequency structure with zero measurable poloidal phase shift
  - Consistent with low-m ($m=0$?)
  - Peaks at/near zero frequency
- GAM also clearly observed near 15 kHz
- ZMF zonal flow has radial correlation length comparable to underlying density fluctuations
  - Necessary for effective shearing of turbulence

Gupta et al., PRL 97 125002 (2006)
GAMs Observed to Peak in Plasma Edge

- GAM velocity oscillation amplitude peaks near $r/a \sim 0.9 - 0.95$
  - Decays near separatrix
  - Decays inboard, still detectable to $r/a \sim 0.75$
  - Consistent with HIBP measurements on JFT-2M (Ido et al., PPCF 2006)

- ZMF zonal flows not observed for $r/a > \sim 0.9$, but do increase towards core
  - Harder to quantify radial dependence because of broad spectral characteristics

McKee et al., PPCF 2006
Observe Transition from ZMF-Dominated Core to GAM-Dominated Edge

- Velocity spectra show broad ZF spectrum for $r/a < 0.8 \rightarrow$ ZMF flow
- Superposition of broad spectrum and GAM peak near $r/a = 0.85$
- GAM dominates for $r/a > 0.9$
- Consistent with theory/simulation expectations that GAM strength increases with $q$
  - Increase in GAM strength with $q_{95}$ also observed (McKee et al., PPCF 2006)
- GAM is highly coherent, with correlation time $\tau_{GAM} > 1$ ms, two orders of magnitude larger than turbulence decorrelation $\tau_{turb} \sim 10 \mu s$
  - Indicates GAM is “slow” relative to edge turbulence timescales, and so can effectively interact with turbulence (Hahm et al., PoP ’99)

McKee IAEA 2006
Measuring Nonlinear Effect of Zonal Flows on Turbulence in a Tokamak
BES System Configured to Provide Zonal Flow Measurements Over Large Fraction of Plasma

- BES measures localized, long-wavelength \((k_r \rho_i < 1)\) density fluctuations
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\[
\langle \tilde{n}(f) \rangle^2 \\
\langle |V_\theta(f)|^2 \rangle \\
\tilde{n}(r, \theta, t) \\
\tilde{V}_\theta(r, \theta, t) \\
\text{Time Delay Estimation} \\
\text{GAM}
\]
Measuring Nonlinear Energy Transfer in Tokamaks

\[
T_n^Y (f', f) = -\text{Re} \left( n^*(f)V_y(f - f') \frac{\partial n}{\partial y}(f') \right)
\]

- \( T_n^Y(f', f) \) measures the transfer of energy between density fluctuations at \( f \) and poloidal density gradient fluctuations at \( f' \) at a specific spatial location due to poloidal convection
  - A **positive value** of \( T_n^Y \) indicates that \( n(f) \) is **gaining energy** from \( \frac{\partial n}{\partial y}(f') \)
  - A **negative value** indicates that \( n(f) \) is **losing energy** to \( \frac{\partial n}{\partial y}(f') \)
GAMs Drive Clear Forward Transfer of Internal Energy in Frequency Space

- \( T_n^Y(f', f) \) clearly shows that fluctuations at \( f = f' + f_{GAM} \) gain energy, while those at \( f = f' - f_{GAM} \) lose energy
- Density fluctuations gain energy from lower frequency gradient fluctuations, and lose energy to higher frequency gradient fluctuations
- Simple picture: energy moves between \( n, \partial n/\partial y \) to high \( f \) in “steps” of \( f_{GAM} \)
  **Net transfer of energy to high \( f \)**
- Demonstrates that the convection of density fluctuations by the GAM leads to a cascade of internal energy to high \( f \)

\[
T_n^Y(f', f) = -\text{Re} \left( \tilde{n}^*(f)V_y(f-f') \frac{\partial \tilde{n}}{\partial y}(f') \right)
\]

Holland PoP 2007
GAMs Nonlinearly Transfer Density Fluctuation Power from f<75kHz region to f>75kHz region
Implications for Transport
External shear flow breaks streamers

RBM  [Beyer et al, 00]

Increasing velocity shear $V_E'$
Implication: Plasma Predicted to Sit At/Near Marginal Stability

Lackner DEISY Symposium 2005
Experiments Confirm Essential Elements of Nonlinear Turbulence-Zonal Flow Interactions

- Inverse Energy Transfer Exists in Drift Turbulence
- Zonal Flow Can Be Self-consistent w/ Turbulent R/S
- Particle Transport Across Shear Layer Inhibited
- Fluid-based Turbulence Simulations are Consistent w/ CSDX Experiments
- Kyushu LMD Experiments Show Inverse Kinetic Energy Transfer
- Zonal Flows (ZMF and GAMs) Exist in Confinement Devices
- They Regulate Spatial Scale of Turbulence
- They Regulate Cross Field Particle Flux
Open Questions

• Can We Measure the NL Kinetic Energy Transfer?
• Can We Show Self-consistent Picture of Linear Instability, NL Energy Transfer, and Saturated Spectra?
• Is the ZF Really Driven by Turbulence in Hot Fusion-grade Plasmas?
• Do Reynolds-stress Driven Flows Trigger Transport Barrier Formation?
• Is the NL Turbulence-ZF Interaction Consistent with Observed Critical Gradient Behavior?
References: Drift Turbulence-Zonal Flows

• **Theory:**

• **Experiments:**
  - Gupta DK, Fonck RJ, McKee GR, et al., Phys. Rev. Lett. 97 (12); Art. No. 125002
  - All the papers in the April 2006 special issue of Plasma Physics and Controlled Fusion.
Homework: Analyzing Phase coherence nonlinear interactions Numerically using Digital Signal Processing:

1) Generate Test Signal:

\[ x(t) = \sin(\omega_1 t + \theta_1) + \sin(\omega_2 t + \theta_2) + \frac{3}{4} \sin(\omega_3 t + \theta_3) + \frac{1}{4} \sin(\omega_4 t + \theta_4) \]

\[ \omega_3 = \omega_1 + \omega_2 ; \omega_4 = \omega_1 - \omega_2 \quad \theta_1 : [0,2\pi] \text{ random; } \theta_2 : [0,2\pi] \text{ random} \]

And consider three different cases:

a) \[ \theta_3 \in [0,2\pi] \text{ random } \theta_4 = \theta_1 \pm \theta_2 \]

b) \[ \theta_3 = \theta_1 \pm \theta_2 \quad \theta_4 \in [0,2\pi] \text{ random} \]

c) \[ \theta_3, \theta_4 \in [0,2\pi] \text{ random} \]
Show that the bicoherence measures the degree of phase coherency between interacting triplets:

BiCoherence is defined as

\[
\hat{b}^2(\omega_1, \omega_2) = \frac{B(\omega_1 + \omega_2)}{\langle |X(\omega_1)X(\omega_2)|^2 \rangle \langle |X^*(\omega_1 + \omega_2)|^2 \rangle} ; \quad 0 < \hat{b} < 1
\]

Where the Bi-spectrum Defined As

\[
B(\omega_1, \omega_2) = \langle X(\omega_1)X(\omega_2)X^*(\omega_1 + \omega_2) \rangle
\]

\(X(\omega)\) denotes the fourier transform of \(x(t)\), and \(<>\) denotes An ensemble average
Homework Question

Suppose the mean pressure gradient driving turbulence is constant, and the turbulence is coupled to a zonal flow such as discussed in this talk. How does the turbulence amplitude respond to an increase or decrease in the damping rate of the zonal flow?
Hint for HW Problem: Self-regulation - Predator-prey model, Malkov 2001

\[
\begin{align*}
\text{DW} & \quad \frac{\partial}{\partial t} \mathcal{E}_d = \gamma_L \mathcal{E}_d - \gamma_2 \mathcal{E}_d^2 - \alpha \mathcal{V}_{ZF}^2 \mathcal{E}_d \\
\text{ZF} & \quad \frac{\partial}{\partial t} \mathcal{V}_{ZF}^2 = -\gamma_{\text{damp}} \mathcal{V}_{ZF}^2 + \alpha \mathcal{V}_{ZF}^2 \mathcal{E}_d
\end{align*}
\]

DW amplitude is influenced by ZF damping.